(EXP)

SLAC-PUB-788
UCRL-20606
February 1971

THE REACTION $\pi^- p \to \pi^+ \Delta^-$ AT CMS ENERGIES 1640-1760 MeV*

A. D. Brody,** R. J. Cashmore, A. Kernan,*** D. W. G. S. Leith,
B. G. Levi, and B. C. Shen***

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

D. J. Herndon, L. R. Price,† A. H. Rosenfeld, P. Soding††

Lawrence Radiation Laboratory
University of California, Berkeley, California 94720

ABSTRACT

A partial wave analysis of the quasi-two body reaction $\pi^- p \to \Delta^- \pi^+$ has led to two solutions. In both solutions, A and B, the total $\pi\pi N$ decay of the $N^*(D_{15})$ is accounted for by the $\Delta\pi$ channel whereas only in solution B does the $\Delta\pi$ channel correspond to the total $\pi\pi N$ decay of the $N^*(F_{15})$. The relative sign of the couplings of the resonances to $\Delta\pi$ is also different between solutions, being negative in solution A and positive in solution B. The results are used to predict the decay rates of other members of the octets.

(Submitted to Physics Letters B)

---

*Work supported by the U. S. Atomic Energy Commission.

**Present address is CERN, Geneva, Switzerland.

***Present address is University of California, Riverside, California.

†Present address is University of California, Irvine, California.

††Present address is DESY, Hamburg, West Germany.

An energy dependent partial wave analysis of the quasi-two body reaction

$$\pi^- p \rightarrow \pi^+ \Delta^- \quad (1)$$

has been performed at 11 c. m. energies in the range 1645 MeV to 1766 MeV in an attempt to determine the coupling of the F_{15} (1688) and D_{15} (1670) resonances to the $\Delta\pi$ system. (Preliminary results, superceded here, were presented at the 1970 Kiev Conference.) The data are obtained from 20, 248 events of the reaction

$$\pi^- p \rightarrow \pi^+ \pi^- n \quad (2)$$

observed in exposures of the Berkeley 72 in and argonne 30 in liquid hydrogen bubble chambers to π^- beams. Details of the data reduction will be given elsewhere.[1] These data are a subsample of the total exposure which spans the c. m. energy range 1400 MeV to 2000 MeV. Examples of reaction (1) are obtained from (2) by selecting those events for which[2]

$$1140 < M(\pi^- n) < 1320 \quad (3)$$

No subtraction of background events, included by this selection, has been made. This point is discussed in the section 'Results.'

The partial wave analysis has been performed using the method described by A. D. Brody and A. Kernan,[3] where the following partial waves in the $\Delta\pi$ system are considered (the notation employed is L2J, where L is the $\Delta\pi$ orbital angular momentum, indicated by S, P, D, and F and J is the total angular momentum):

(i) P5 — containing the F_{15} resonance

(ii) D5 — containing the D_{15} resonance

(ii) S3, P1, D1, F7 — containing background amplitudes.

The following parameterizations of the T matrix were used for the resonances and backgrounds

(i) resonances — a nonrelativistic Breit-Wigner amplitude with mass dependent width

$$T = \frac{1}{2} \frac{\sqrt{\Gamma_{el}(E)\,\Gamma_{\Delta\pi}(E)}}{(M_R - E) - i\,\frac{\Gamma_{tot}(E)}{2}} \tag{4}$$

where M_R = resonance mass

$\Gamma_{el}(E)$ = elastic decay width

$$= X_{el}\,\Gamma^R_{tot}\,\frac{B_L(q_{el})}{B_L\left(q^R_{el}\right)}\,\frac{q_{el}}{q^R_{el}}\,\frac{M_R}{E} \tag{5}$$

$\Gamma_{\Delta\pi}(E)$ = $\Delta\pi$ decay width

$$= x_{\Delta\pi}\,\Gamma^R_{tot}\,\frac{B_{L_1}(q_{inel})}{B_{L_1}\,q^R_{inel}}\,\frac{q_{inel}}{q^R_{inel}}\,\frac{M_R}{E} \tag{6}$$

$\Gamma_{tot}(E)$ = total decay width, approximated by[4]

$$\approx \Gamma_{el}(E) + \Gamma_{\Delta\pi}(E)\left(\frac{1 - x_{el}}{x_{\Delta\pi}}\right) \tag{7}$$

and q_{el}, q_{inel} are the momenta in the elastic and $\Delta\pi$ channels

q^R_{el}, q^R_{unel} are the momenta in the elastic and $\Delta\pi$ channels at the resonance mass

L, L_1 are the orbital angular momenta in the elastic and $\Delta\pi$ channels

$B_L(q)$ is a barrier penetration factor.[5] A value of 1 fermi was used for the interaction radius.

x_{el} is the elasticity parameter $\Gamma^R_{el}/\Gamma^R_{tot}$.

$x_{\Delta\pi}$ is the $\Delta\pi$ 'elasticity' parameter $\Gamma^R_{\Delta\pi}/\Gamma^R_{tot}$.

(ii) backgrounds:

$$T = (a + b\,q_{inel}) + i(c + d\,q_{inel}) \tag{8}$$

The parameters $\sqrt{x_{el}x_{\Delta\pi}}$, M_R, Γ^R_{tot}, of the resonances, and a, b, c, d of the backgrounds, were then determined by a fit to the experimental distributions by minimizing the function, F, where

$$F = \sum_{\text{energies}} \sum_{\substack{\text{production}\\ \text{angular}\\ \text{bins}}} \left[\frac{\left(\frac{dN}{d\Omega}\right)_{obs} - \left(\frac{dN}{d\Omega}\right)_{pred}}{\delta\left(\frac{dN}{d\Omega}\right)_{obs}}\right]^2 + \sum_{\text{energies}} \left[\frac{\sigma^{\Delta\pi}_{obs} - \sigma^{\Delta\pi}_{pred}}{\delta\,\sigma^{\Delta\pi}_{obs}}\right]^2 \qquad (9)$$

using the minimization program MINFUN.[6]

Results:

We fit the production angular distribution of the Δ^- and the cross sections $\sigma(\Delta^-\pi^+)$. Starting from a number of sets of random parameter values, two distinct solutions, A and B, have been obtained and are summarized in Table 1. The major differences between these two solutions are in the size of the P5 amplitude and its relative sign to the D5 amplitude. Although solution A has a lower value of the function F there is no compelling reason to prefer it to solution B, and in the remaining discussion we retain both solutions. In Fig. 1 we display the variations of the complex amplitudes from these two solutions. In Fig. 2 examples of the fits to the production angular distributions are given while in Fig. 3 the contributions of the various partial waves to the $\Delta^-\pi^+$ cross section are indicated (the Δ^- is still defined by Eq. (3)). To determine the errors on the resonance parameters, we varied each parameter in turn, reminimized the function and studied the shape of F as we did so.

We have also made fits including the decay angular distributions of the Δ^- and the two solutions, A and B, remain distinct with the resonance parameters unaltered within errors. However, we feel that the results derived from fitting only the production distributions are more reliable than those from the fit including the decay angular distributions, since the decay distributions will be more sensitive to the presence of $\pi\pi N$ background and interference with the Δ^+.

In performing the analysis we have ignored the fact that some of the events included by the mass selection (3) are not produced through the intermediate state $\pi^+\Delta^-$. These events[7] ($\sim 25\%$ of the sample) are accounted for by the background amplitudes (ii). Since they are not described by the resonance partial waves (i) the determination of the F_{15} and D_{15} parameters in the $\Delta\pi$ system will be unaffected.

We have attempted to determine the sign of the $D_{13}(1520)$ coupling to the $\Delta\pi$ channel, relative to the D_{15} and F_{15} resonances, by introducing lower energy data. This was inconclusive due to the large correlation between the s-wave decay of the $D_{13}(1520)$ and our s-wave background.

In order to give $\Delta\pi$ branching fractions for the D_{15} and F_{15} resonances from our measured values of $\sqrt{x_{el}\, x_{\Delta\pi}}$ we have to assume values for x_{el}. We have used average values for the πN branching fractions determined in the elastic phase shift analyses.[8] (D_{15}: 0.42 ± 0.04, F_{15}: 0.62 ± 0.06) Table 2 includes corrections for those events lying outside the selection band (3). From solution A we obtain $\Delta\pi$ branching fractions of

$$0.63 \pm 0.07 \text{ and } 0.13^{+0.03}_{-0.04}$$

for the D_{15} and F_{15} resonances respectively, while from solution B the corresponding numbers are

$$0.63^{+0.07}_{-0.11} \text{ and } 0.39^{+0.06}_{-0.10}$$

The values of the $\Delta\pi$ branching fractions derived from solution B imply that the total $\pi\pi N$ widths of both the F_{15} and D_{15} are accounted for by the $\pi\Delta$ decay mode,[8] whereas in solution A only the D_{15} is completely accounted for and the F_{15} still has an appreciable $\pi\pi N$ decay. Finally we wish to point out that any background present in the P5 and D5 partial waves, not included in the amplitudes (i) for these waves, would change the $\Delta\pi$ branching ratios of the F_{15} and D_{15} resonances.

SU(3) Discussion:

These determinations of the couplings of the $D_{15}(1680)$ and $F_{15}(1690)N^*$ resonances to the $\Delta\pi$ system allow us to determine the SU(3) coupling constants for the decay of the $\frac{5^-}{2}$ and $\frac{5^+}{2}$ octects into the $\frac{3^+}{2}$ baryon decuplet and 0^- meson octet. We have followed the prescription of R. Levi-Setti and others[9] in writing the partial width as

$$\Gamma = (cg)^2 M_p \frac{\langle B_L(q_R)q_R \rangle}{M_R} \tag{10}$$

where c is the SU(3) Clebsch-Gordon coefficient for $\{8\} \longrightarrow \{10\} \times \{8\}$

g is the SU(3) coupling constant

q_R is the momentum of the decay particles[10]

$B_L(q_R)$ is the barrier penetration factor.[5] The radius of interaction was assumed to be 1 fermi.[10]

M_p is the proton mass

M_R is the resonance mass

In Table 2 we give the values of g^2 for each octet, corresponding to the results of the two solutions A and B, together with the predicted partial widths of the other octet members. Unfortunately, few decays of the other members of the octets into decuplet and octet have been measured. However, where a comparison can be made the results derived from solution A show better agreement (cf. $\Lambda(1815)$ branching fractions). Measurably large widths are predicted for the decays

$$\Lambda(1830) \longrightarrow \Sigma(1385)\pi \longrightarrow \Lambda\pi\pi$$

$$\Sigma(1915) \longrightarrow \Sigma(1385)\pi \longrightarrow \Lambda\pi\pi$$

$$\Sigma(1915) \longrightarrow \Delta(1238)\overline{K} \longrightarrow N\pi\overline{K}$$

and thus a quasi-two body analysis of the reaction

$$K^-p \longrightarrow \Sigma(1385)\pi \longrightarrow \Lambda\pi\pi$$

in the c.m. energy range 1700 MeV - 2000 MeV should allow identification of the $\Lambda(1830)$ and $\Sigma(1915)$ resonances and further measurements of the $\Sigma(1765)$ and $\Lambda(1815)$ branching fractions. The determination of the relative sign of the $\Lambda(1830)$ and $\Lambda(1915)$

resonance couplings to $\Sigma(1385)\pi$ channel should distinguish the correct solution in our results. Owing to the lack of information on the Ξ branching fractions quantitative statements are difficult to make, although no violent discrepancies are apparent.

REFERENCES

1. A. D. Brody et al., "$\pi^- p$ elastic scattering in the cms energy range 1400-2000 MeV," submitted to Phys. Rev. (1971); A. D. Brody et al., "Experimental results on the reactions $\pi^- p \rightarrow \pi\pi N$ in the c.m. energy range 1400-2000 MeV," to be submitted to Phys. Rev.

2. In performing this mass selection not all of the Δ^- is included. In order to estimate the correction factor we chose to express the Δ amplitude T_Δ in terms of the empirical πN phase shifts, and then determined the fraction of the Δ falling within our mass selection band for the appropriate values of the orbital angular momentum in the $\Delta\pi$ final state. We used the form

$$|T_\Delta|^2 \propto Q^{2L} \frac{\sin^2 \delta_{33}}{\Gamma(M)} \frac{M}{q}; \ \tan \delta_{33} = \frac{M\Gamma(M)}{M_\Delta^2 - M^2} ,$$

where $M_\Delta = 1.236$ GeV/c^2, q is the momentum of the particles in the decay of the Δ, Q is the c.m. momentum of the Δ and L its orbital angular momentum in the c.m. Those corrections were not applied until after the partial wave fit; the $\Delta\pi$ branching ratios for the p-wave decay of the F_{15} and d-wave decay of the D_{15} were then raised by 1.22 and 1.20 respectively.

3. A. D. Brody and A. Kernan, Phys. Rev. 182, 1785 (1969).

4. The total decay width is written as

$$\Gamma_{tot}(E) = \Gamma_{el}(E) + \Gamma_{\Delta\pi}(E) + \Gamma_r(E)$$

where $\Gamma_r(E)$ is the decay width for all remaining channels and represents only a few percent of the total decay. We assume that $\Gamma_r(E)$ has the same energy

dependence as $\Gamma_{\Delta\pi}(E)$ so that we can write Eq. (7),

$$\Gamma_{\Delta\pi}(E) + \Gamma_r(E) = \left(1 + \frac{x_r}{x_{\Delta\pi}}\right)\Gamma_{\Delta\pi}(E) = \left(\frac{1 - x_{el}}{x_{\Delta\pi}}\right)\Gamma_{\Delta\pi}(E)$$

5. J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, Inc., New York, 1956).
6. G. C. Sheppey, MINFUN, CERN 6600 Computer Program Library Write Up.
7. Maximum likelihood fits to the Dalitz Plot populations in the final state $\pi^-\pi^+n$ indicate that there are $\sim 60\%\ \Delta^-$, $\sim 10\%\ \Delta^+$, $\sim 30\%$ phase space, leading to an estimate $\sim 25\%$ background within our mass selection.
8. D. J. Herndon, A. Barbaro-Galtieri, A. H. Rosenfeld, UCRL-20030 πN (1970).
9. R. Levi-Setti, Rapporteur Talk on Strange Baryon Resonances, Proceedings of the Lund International Conference on Elementary Particles, Lund (1969);
R. D. Tripp, Rapporteur Talk on Strange Baryon Resonances, Proceedings of the 14th International Conference on High Energy Physics, Vienna (1968);
R. D. Tripp et al., Nucl. Phys. B3, 10 (1967).
10. An integration was performed over the width of the decuplet member of determine $\langle q\, B_L\, (q_R) \rangle$ in Eq. (10).
11. Particle Data Group, Rev. Mod. Phys. 42, 87 (1970).

FIGURE CAPTIONS

1. (a) Variation of partial wave amplitudes from solution A; (b) Variation of partial wave amplitudes from solution B. Tick marks correspond to the c. m. energies 1647 MeV, 1685 MeV and 1740 MeV. P5 is derived from the F_{15} resonance and D_5 from the D_{15} resonance.
2. Examples of fits at three energies to the production angular distributions from solutions A and B.
3. The experimental $\pi^+\pi^-n$ cross section (1140 MeV/c^2 < $M(\pi^-n)$ < 1320 MeV/c^2) together with the contributions of the various partial waves in solutions A and B.

TABLE 1

FIT PARAMETERS

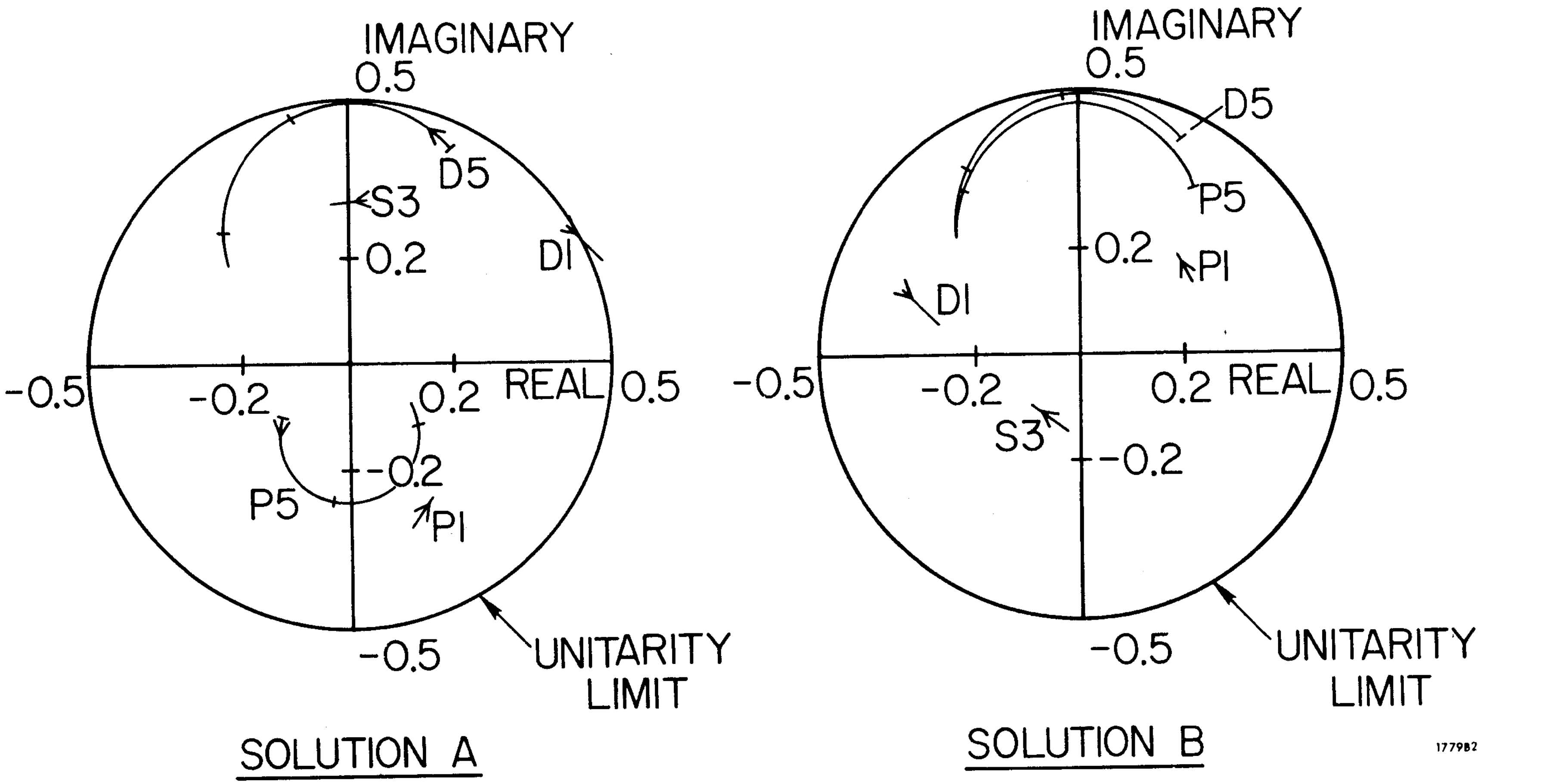
Solution	F ($\approx \chi^2$)	No. of Data Points	F_{15} Mass	F_{15} Width	F_{15} $\sqrt{x_{el} x_{\Delta\pi}}$	D_{15} Mass	D_{15} Width	D_{15} $\sqrt{x_{el} x_{\Delta\pi}}$
A	219.12	231.	1.690 ± 0.005	0.077 ± 0.022	− 0.252 {+ 0.039, − 0.024}	1.671 ± 0.004	0.112 ± 0.017	+ 0.468 {+ 0.012, − 0.018}
B	228.04	231	1.686 ± 0.009	0.130 {+ 0.035, − 0.053}	+ 0.447 {+ 0.025, − 0.053}	1.680 ± 0.009	0.158 {+ 0.090, − 0.020}	+ 0.468 {+ 0.014, − 0.035}
	S3				P1			
	a	b	c	d	a	b	c	d
A	0.284	− 0.733	0.337	− 0.072	− 0.009	0.393	− 0.527	0.638
B	0.219	− 0.726	− 0.333	0.553	0.314	− 0.290	− 0.078	0.615
	D1				F7			
	a	b	c	d	a	b	c	d
A	0.191	0.712	0.512	− 0.734	− 0.138	−	0.086	−
B	− 0.581	0.718	0.373	− 0.726	− 0.071	−	0.072	−

TABLE 2

COUPLING CONSTANTS AND PARTIAL WIDTHS FOR DECAY OF $5/2^-$ AND $5/2^+$ OCTETS INTO $3/2^+$ DECUPLET AND 0^- OCTET

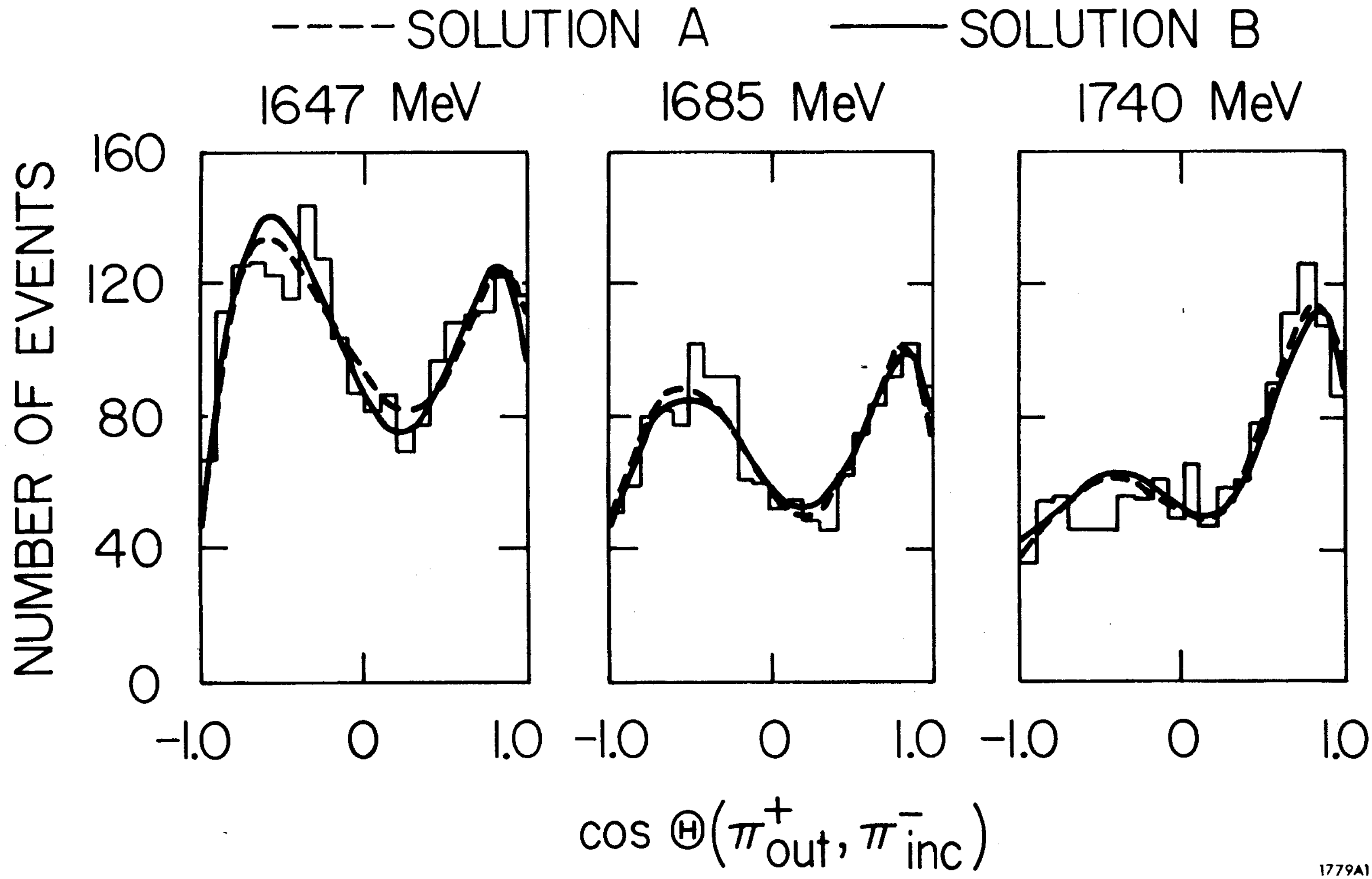
Octet	Particle	Mass and width[c]	Decay Mode	c^2	$\frac{\langle B_L(q_R)q_R\rangle M_p}{M_R}$	SOLUTION A: Observed branching fraction	SOLUTION A: Predicted branching fraction	SOLUTION A: g^2	SOLUTION A: Predicted width	SOLUTION B: Observed branching fraction	SOLUTION B: Predicted branching fraction	SOLUTION B: g^2	SOLUTION B: Predicted width
$5/2^-$	N(1680)	1.672 0.142	$\Delta(1238)\pi$	4/5	0.074	0.63 $\pm$0.07 (a)	Input	1.506 $\pm$.167	Input	0.63 +0.07 −0.11 (a)	Input	1.506 +.167 −.263	Input
	$\Sigma(1765)$	1.765 0.100	$\Sigma(1385)\pi$	2/15	0.047	0.13 $\pm$0.02 (c)	.09 $\pm$.01	-	0.009 $\pm$.001	0.13 $\pm$0.02 (c)	.09 +.01 −.02	-	.009 +.001 −.002
			$\Delta(1238)\bar{K}$	8/15	0.013		.11 $\pm$.01		0.011 $\pm$.001		.11 +.01 −.02		.011 +.001 −.002
	$\Lambda(1830)$	1.835 0.075	$\Sigma(1385)\pi$	3/5	0.076	-	.92 $\pm$.11	-	0.069 $\pm$.008	-	.92 +.11 −.16	-	.069 +.008 −.012
	$\Xi(1930)$	1.930 0.110	$\Xi(1530)\pi$	1/5	0.052	-	.16 $\pm$.02	-	0.016 $\pm$.002	-	0.16 +.02 −.03	-	.016 +.002 −.003
			$\Sigma(1385)\bar{K}$	1/5	0.008		.02 $\pm$.01		0.002 $\pm$.001		0.02 $\pm$.01		.002 $\pm$.001
$5/2^+$	N(1690)	1.688 0.125	$\Delta(1238)\pi$	4/5	0.160	0.13 +0.03 −0.04 (b)	Input	0.127 +.029 −.039	Input	0.39 +0.06 −0.10 (b)	Input	0.380 +.058 −.097	Input
	$\Sigma(1915)$	1.910 0.050	$\Sigma(1385)\pi$	2/15	0.178	-	.06 $\pm$.02	-	0.003 $\pm$.001	-	.18 +.02 −.06	-	0.009 +.001 −.003
			$\Delta(1238)\bar{K}$	8/15	0.140		.18 +.04 −.06		0.009 +.002 −.003		.56 +.08 −.14		0.028 +.004 −.007
	$\Lambda(1815)$	1.815 0.075	$\Sigma(1385)\pi$	3/5	0.142	0.17 $\pm$0.03 (c)	.15 $\pm$.04	-	0.011 $\pm$.003	0.17 $\pm$0.03 (c)	.43 +.07 −.11	-	0.032 +.005 −.008
	$\Xi(2030)$	2.030 0.050	$\Xi(1530)\pi$	1/5	0.159	-	.08 $\pm$.02	-	0.004 $\pm$.001	-	.24 +.04 −.06	-	0.012 +.002 −.003
			$\Sigma(1385)\bar{K}$	1/5	0.119		.06 $\pm$.02		0.003 $\pm$.001		.18 +.02 −.04		0.009 +.001 −.002

(a) assumes $x_{el} = 0.42 \pm 0.04$ (b) assumes $x_{el} = 0.62 \pm 0.06$ (c) See Ref. 11



IMAGINARY
0.5
D5
S3
0.2
DI
-0.5
-0.2
0.2
REAL
0.5
-0.2
P5
PI
-0.5
UNITARITY
LIMIT
SOLUTION A
IMAGINARY
0.5
D5
P5
0.2
PI
DI
-0.5
-0.2
0.2
REAL
0.5
S3
-0.2
-0.5
UNITARITY
LIMIT
SOLUTION B
177982

Fig. 1



----SOLUTION A
——SOLUTION B
1647 MeV
1685 MeV
1740 MeV
NUMBER OF EVENTS
160
120
80
40
0
-1.0
0
1.0
cos Θ(π+out, π-inc)
1779A1

Fig. 2

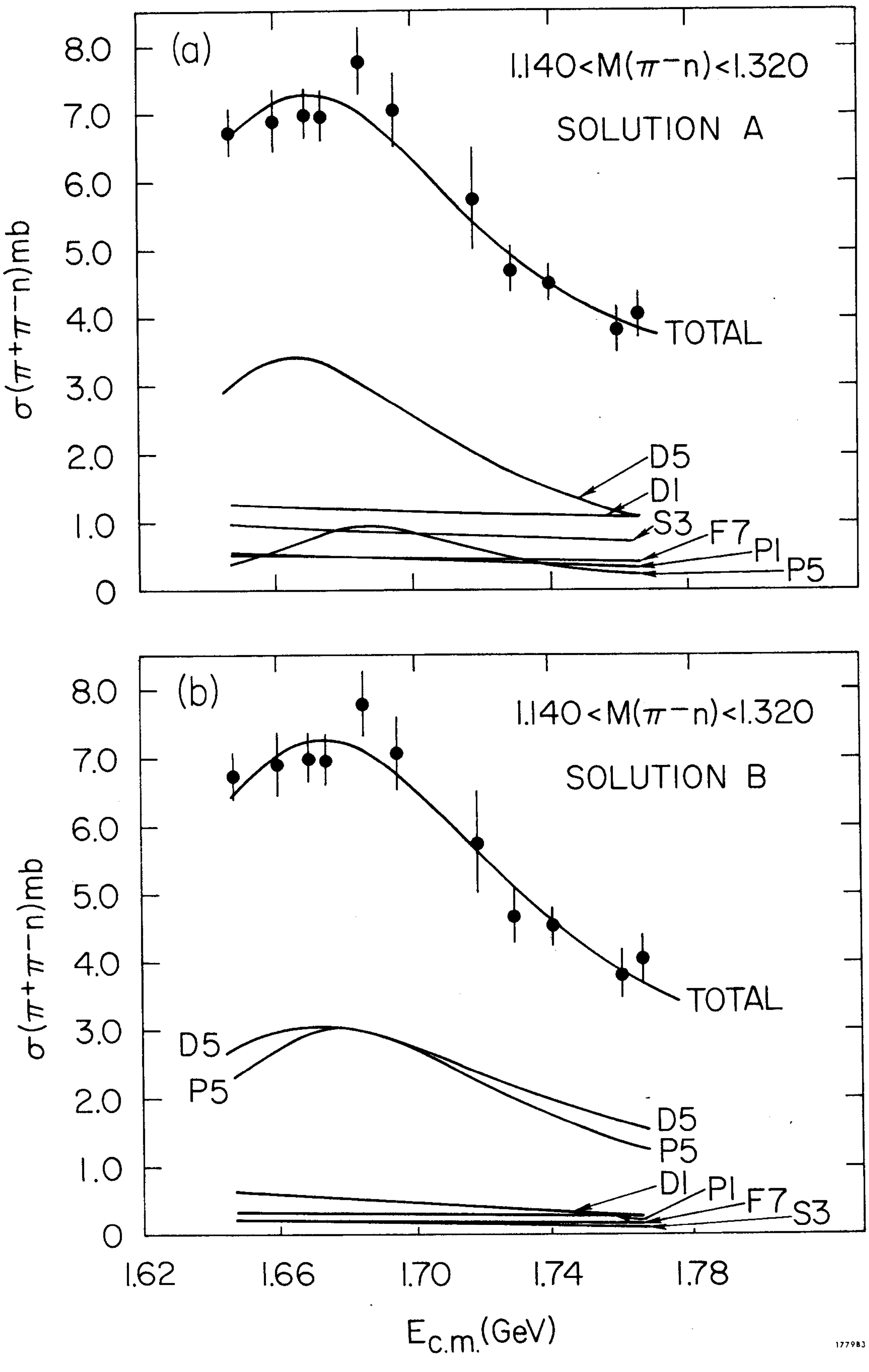

(a)
1.140<M(π⁻n)<1.320
SOLUTION A
TOTAL
D5
D1
S3
F7
P1
P5
σ(π⁺π⁻n)mb
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
(b)
1.140<M(π⁻n)<1.320
SOLUTION B
TOTAL
D5
P5
D5
P5
D1
P1
F7
S3
σ(π⁺π⁻n)mb
1.62
1.66
1.70
1.74
1.78
Ec.m.(GeV)
177983

Fig. 3