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Super-horizon primordial black holes: How do they grow?

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Abstract. Primordial black holes have important observational implications through Hawking evaporation and gravitational radiation as well as being a candidate for cold dark matter. Those black holes may have formed in the early universe typically with the mass scale contained within the Hubble horizon at the formation epoch and subsequently accreted the mass surrounding them. Numerical relativity simulation shows that primordial black holes of different masses do not accrete much, which contrasts with a simplistic Newtonian argument. Primordial black holes larger than the cosmological horizon have non-standard global structure, suggesting that they may have formed in inflationary cosmology.

1. Introduction

Primordial black holes (PBHs) may have formed in the early universe [1]. Those black holes may contribute to current gamma-ray and cosmic ray backgrounds through Hawking evaporation. They may also contribute to the cosmic density and behave as cold dark matter. They could be a promising target for ground-based interferometric gravitational wave detectors [2, 3]. Thus we can obtain information of the early universe. In particular, we can constrain the probability of PBH formation in the early universe [4]. See [5] for a recent review of theoretical and observational background and development in the studies of PBHs.

PBHs are usually assumed to have formed with the mass scale $M_{\rm h,f} \simeq G^{-1}c^3 t_{\rm f}$ which was contained within the Hubble horizon at the formation epoch, where G, c and $t_{\rm f}$ are the gravitational constant, the light speed and the formation time from big bang, respectively. This is based on the argument of the Jeans scale, gravitational radius and separate universe condition [4, 6]. This picture was subsequently modified after the discovery of critical behaviour in the formation of PBHs [7]. Nevertheless, the typical mass scale of the formed PBHs is still the horizon mass scale at the formation epoch, i.e.,

$$M_{\rm PBH,f} \simeq M_{\rm h,f} \simeq \frac{c^3 t_{\rm f}}{G} \simeq 1 M_\odot \left(\frac{T_{\rm f}}{100 \text{ MeV}}\right)^{-2}. \tag{1}$$

2. Newtonian argument of PBH growth

The accretion onto a PBH could change the mass scale of PBHs in principle. If we assume spherically symmetric and quasi-stationary flow onto a PBH and also neglects cosmological expansion, we can estimate the mass accretion rate of the black hole as

$$\frac{dM}{dt} = 4\pi\alpha r_{\rm A}^2 v_{\rm s} \rho_\infty, \tag{2}$$

where $r_{\mathrm{A}} = GM/v_{\mathrm{s}}^2$ is the accretion radius, v_{s} is the sound speed, ρ_∞ is the density at infinity and α is a constant of order unity. To apply the above equation for the growth of PBHs in the early universe, we assume that ρ_∞ is given by the density of the background Friedmann universe and v_{s} is the order of the speed of light. Then the above equation is integrated to give [6, 8]

$$M = \frac{At}{1 + \frac{t}{t_{\mathrm{f}}}\left(\frac{At_{\mathrm{f}}}{M_{\mathrm{f}}} - 1\right)}, \tag{3}$$

where $A \simeq c^3/G$ is a constant and M_{f} is the PBH mass at the time t_{f} of formation. Figure 1 shows three categories of solutions expressed by Eq. (3). In this argument, the effects of cosmological expansion are neglected. Since such effects will be important only for the cosmological horizon scale, the above analysis for PBHs much smaller than the cosmological horizon scale is expected to be valid. On the other hand, for PBHs as large as or larger than the cosmological horizon, which are naturally realised just after the formation, the above analysis is suspect.

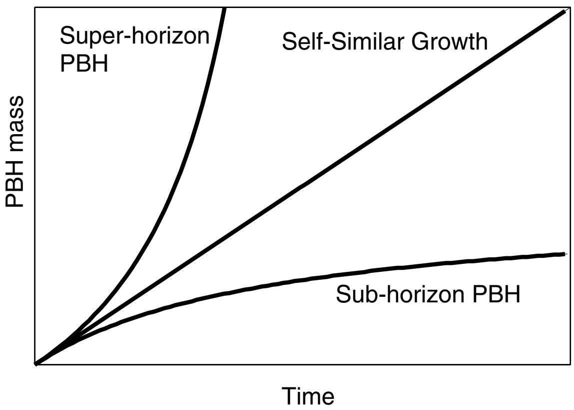

Figure 1. PBH mass growth based on the Newtonian argument. Three categories of solutions, sub-horizon, self-similar and super-horizon, are shown.

3. Numerical relativity of PBH growth in a scalar field universe

Since the observation of cosmological acceleration at the present epoch, matter fields with huge negative pressure have attracted much attention. One of the typical models is a scalar field slowly rolling down on the slope of the potential, which is called quintessence. Recently, Bean and Magueijo [9] applied a quasi-Newtonian argument for the accretion of a quintessence field onto a PBH, which leads to solutions given by Eq. (3), and claimed that PBHs of inflation origin could be the seeds for supermassive black holes.

To get insight into the growth of horizon-scale PBHs in the context of the quintessence/scalar field cosmology, we implement fully general relativistic numerical simulation of the growth of PBHs in a universe containing a massless scalar field. We solve the Einstein equation and the equation for the scalar field. Since the system is inhomogeneous and dynamical, we solve these equations numerically. The line element in spherically symmetric spacetimes is given by

$$ds^2 = -a^2(u,v)dudv + r^2(u,v)(d\theta^2 + \sin^2\theta d\phi^2), \tag{4}$$

in the double-null coordinates and henceforth we use the geometrised units $G = c = 1$. The equations in this coordinate system are given in [10] explicitly. For the present problem, this scheme is advantageous because it has no apparent coordinate singularity and it also fits the characteristics of the propagation of the scalar field. The details of the numerical scheme and implementation are described in [10, 11].

In the double null-formulation, it is most natural to provide initial data on the null hypersurfaces $u = u_0$ and $v = v_0$ and solve a diamond region $u_0 < u < u_1$ and $v_0 < v < v_1$. As for initial data we adopt the simplest model, in which the Schwarzschild region is surrounded by the flat Friedmann background. To avoid the discontinuity at the matching surface, we set the smoothing region to retain the numerical accuracy.

Since we have null infinity in the flat Friedmann spacetime, we can define an event horizon. On the other hand, the notion of trapping horizons [12], which is very similar to apparent horizons, is also useful. This is defined as a hypersurface foliated by marginal surfaces and the definition is local. Although the event horizon and the trapping horizon coincide for the Schwarzschild black hole, they are different from each other for general dynamical cases.

4. Numerical results

Here we present the results for three models. See [11, 13] for the details of the chosen parameter values. We define the mass of the black hole using the Misner-Sharp mass [14] on the event horizon. Figure 2 shows the time evolution of the PBH mass and the mass contained within the cosmological apparent horizon denoted as 'POTH' as the abbreviation of the past outer trapping horizon. We can see that sub-horizon PBHs do not accrete much, the accretion onto horizon-scale PBHs is suppressed and super-horizon PBHs decrease their mass.

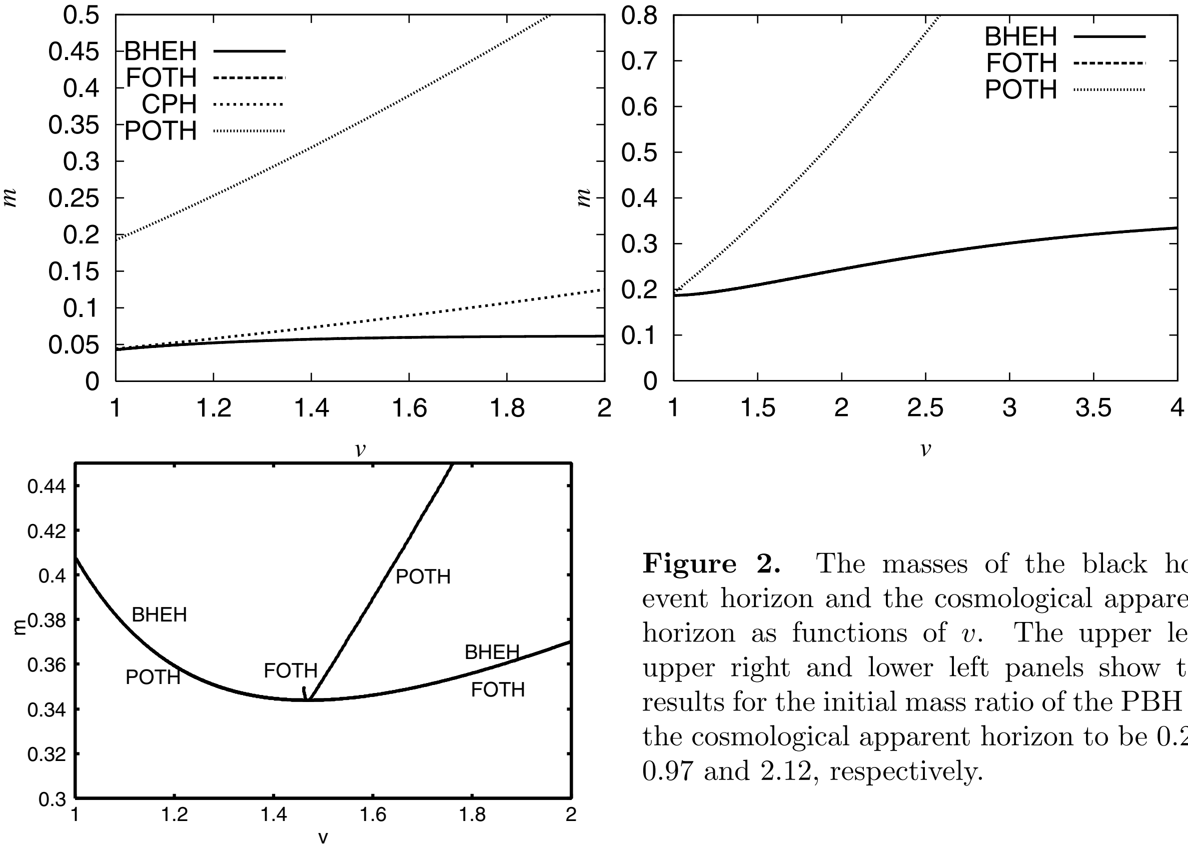

Figure 2. The masses of the black hole event horizon and the cosmological apparent horizon as functions of v. The upper left, upper right and lower left panels show the results for the initial mass ratio of the PBH to the cosmological apparent horizon to be 0.22, 0.97 and 2.12, respectively.

The evolution of PBHs of different masses is understood in terms of the mass accretion equation [11, 13]:

$$m_{\mathrm{BHEH},v} = -\frac{8\pi r^2 r_{,u}(\Psi_{,v})^2}{a^2}, \tag{5}$$

where the right-hand side should be estimated on the event horizon. The sign of the mass growth rate is governed by $r_{,u}$ on the event horizon. This corresponds to the expansion of ingoing null geodesic congruence on the event horizon. This is negative inside the cosmological horizon, but zero on and positive outside it. Figure 3 schematically summarises the results of general relativistic numerical simulations in contrast to Fig. 1.

For the models where the initial event horizon is past trapped, we can show that both first outgoing null ray $u = u_0$ and ingoing null ray $v = v_0$ reach infinity. This means we have two distinct null infinities and this results cannot be embedded into the standard diagram of the PBH. This implies the inflationary origin of super-horizon PBHs.

5. Summary

The cosmic expansion is crucial for the growth of horizon-scale PBHs. The numerical relativity of scalar field PBHs shows that the accretion onto a PBH is significantly suppressed when the PBH is as large as the cosmological apparent horizon. The mass of super-horizon PBHs decreases although it always swallows the scalar field. In any case, PBHs do not accrete very much even during a scalar-field-dominated era. A complementary work is in preparation on the non-existence of PBHs growing self-similarly in a universe containing a scalar field whether massless or with a potential.

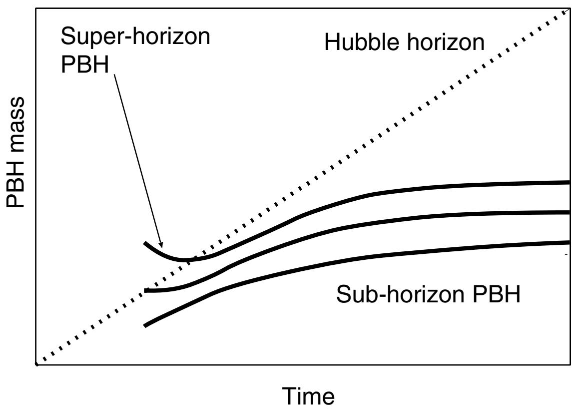

Figure 3. Schematic figure showing the mass growth of PBHs in a universe containing a massless scalar field, based on general relativistic numerical simulations.

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