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$K \to \pi\ell^+\ell^-$: Status and update

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Abstract. $K \to \pi\ell^+\ell^-$ decays, $\ell = e, \mu$, will be one of the goals of future kaon facilities, like NA62 (CERN) and K^0TO (J-PARC). We review briefly the theoretical status of those processes.

1. Introduction

Suppressed (rare) kaon decays in the Standard Model (SM) constitute an interesting benchmark where to look for New Physics. Common sense and logic tell us that, in any observable, in order to look for physics beyond the SM one needs to have a reliable SM prediction and this is not obvious in many kaon decays. Indeed low-energy strong interacting effects, of difficult implementation, may be involved in those determinations [1].

$K \to \pi\nu\overline{\nu}$ are atypical, poorly measured, decays that are dominated by short-distance interactions [2, 3] and, accordingly, have a very accurate prediction within the SM. This is the reason why they are the main goal of present dedicated facilities: $K^+ \to \pi^+\nu\overline{\nu}$ at NA62 (CERN) and $K_L \to \pi^0\nu\overline{\nu}$ at K^0TO (J-PARC) [4]. These devices should be able, in addition, to get samples of other related processes like $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e, \mu$) at NA62 or $K_L \to \pi^0\gamma\gamma$ at K^0TO (as we will comment the latter is relevant to determine the CP-conserving contribution to $K_L \to \pi^0\ell^+\ell^-$).

In this note we summarize the theoretical status of the $K \to \pi\ell^+\ell^-$ decays. These are $\Delta S = 1$ weak neutral current transitions in the SM and, hence, very suppressed . Contrarily to the case above, these decays are fairly dominated by strongly interacting low-energy dynamics which is always harder to determine. As in many other processes of analogous features, Chiral Perturbation Theory (ChPT) is the appropriate tool to handle these decays [5, 6]. They were first studied within this framework by Ecker, Pich and De Rafael [7, 8] when they computed the dominant octet ($\Delta I = 1/2$) contributions at the leading $\mathcal{O}(p^4)$ in ChPT. This set up a systematic approach that would pave the way for later improvements, as we will see. The subdominant contributions of $SU(3)$ 27-representation operators, at $\mathcal{O}(p^4)$, have been carried out in Ref. [9].

$K^+, K_S \to \pi\ell^+\ell^-$ decays are dominated by single virtual-photon exchange, $K \to \pi\gamma^*$. However this contribution is CP violating for $K_L \to \pi^0\ell^+\ell^-$ and, consequently, the latter is driven by substantially different dynamics. The theoretical status of the first processes will be set forth in Section 1. Meanwhile Section 2 will be dedicated to the description of the K_L decay.

A more detailed account of $K \to \pi\ell^+\ell^-$ decays and their corresponding bibliography can be looked up in Ref. [1].

2. $K^+, K_S \to \pi\ell^+\ell^-$

CP allowed $K(k) \to \pi(p)\ell^+\ell^-$, $\ell = e, \mu$, decays are dominated by single virtual photon exchange $K \to \pi\gamma^*$. This is the case for K^+ and K_S decays. Hence the amplitude is determined by one electromagnetic transition form factor in the presence of nonleptonic weak interactions [7, 10]:

$$i \int d^4x \, e^{iqx} \langle \pi(p)| \, T \left\{ J_{em}^\mu(x) \, \mathcal{L}_{\Delta S=1}(0) \right\} |K_j(k)\rangle = \frac{G_F M_K^2}{(4\pi)^2} \, V_j(z) \left[z(k+p)^\mu - (1 - r_\pi^2)q^\mu \right], \quad (1)$$

with $j = +, S$, to distinguish both decays, and $q = k - p$, $z = q^2/M_K^2$, $r_P = M_P/M_K$. Here J_{em}^μ is the electromagnetic current and $\mathcal{L}_{\Delta S=1}$ the strangeness changing nonleptonic weak Lagrangian. The spectrum in the dilepton invariant mass is then given by:

$$\frac{d\Gamma_j}{dz} = \frac{G_F^2 \alpha^2 M_K^5}{12\pi(4\pi)^4} \, \lambda^{3/2}(1, z, r_\pi^2) \sqrt{1 - 4\frac{r_\ell^2}{z}} \left(1 + 2\frac{r_\ell^2}{z} \right) |V_j(z)|^2, \quad (2)$$

with $\lambda(a, b, c) = (a + b - c)^2 - 4ab$ and $4r_\ell^2 \le z \le (1 - r_\pi)^2$.

The $V_j(z)$ form factor can be determined within the ChPT framework, i.e. in a model-independent way. Because gauge invariance, $V_j(z) = 0$ at $\mathcal{O}(p^2)$ in the chiral expansion. Hence the leading contribution is $\mathcal{O}(p^4)$ that was computed in Ref. [7]. Although a complete study to next-to-leading $\mathcal{O}(p^6)$ has not been performed yet, the dominant unitarity corrections from $K \to \pi\pi\pi$, shown in Fig. 1, were studied in Ref. [10], that constitute the present *state-of-the-art* (there is also a tiny contribution from a kaon running in the loop, that obviously has no absorptive part). The full result can be decomposed in a polynomial contribution (linear in z
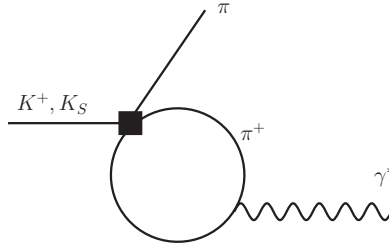

Figure 1. $K \to \pi\pi\pi$ contribution to $K^+, K_S \to \pi\gamma^*$.

up to this order) plus a unitarity loop correction :

$$V_j(z) = a_j + b_j z + V_j^{\pi\pi}(z), \qquad (j = +, S). \quad (3)$$

The polynomial part is, unfortunately, not fully given by chiral symmetry. It translates, mainly, into the Low Energy Couplings (LECs) of the ChPT framework, and it encodes contributions from higher energy spectra that have been integrated out in the process to reach the low-energy effective theory. a_j gets contributions starting at $\mathcal{O}(p^4)$ while b_j, but for a tiny $\mathcal{O}(p^4)$ kaon loop input in Fig. 1, starts at $\mathcal{O}(p^6)$. As we will comment we have, up to now, only model-dependent theoretical predictions for these parameters.

The unitarity corrections in $V_j(z)$ are given by :

$$V_j^{\pi\pi}(z) = \overbrace{\frac{\alpha_j + \beta_j(z - z_0)/r_\pi^2}{G_F M_K^2 r_\pi^2}}^{K \to \pi\pi\pi} \overbrace{\left[\frac{4}{9} - \frac{4}{3z} + \frac{4}{3z}\left(1 - \frac{z}{4}\right) G(z) \right]}^{\text{1-loop}} \overbrace{\left[1 + \frac{z}{r_\rho^2} \right]}^{F_V(z)}, \quad (4)$$

where $z_0 = r_\pi^2 + 1/3$. We now explain the structure of the unitarity corrections:

- The interaction of the vertex $K \to \pi\pi\pi$ in Fig. 1 is driven by the α_j and β_j constants that are expressed in terms of the parameterizations of Ref. [11]:

$$
\begin{aligned}
\alpha_+ &= \beta_1 - \frac{1}{2}\beta_3 + \sqrt{3}\gamma_3\,, & \beta_+ &= 2(\xi_1 + \xi_3 - \xi_3') \\
\alpha_S &= -\frac{4}{\sqrt{3}}\gamma_3\,, & \beta_S &= \frac{8}{3}\xi_3'.
\end{aligned}
\tag{5}
$$

  It can be noticed that α_+, β_+ are given by both $\Delta I = 1/2, 3/2$ transitions, while α_S, β_S come only from $\Delta I = 3/2$ transitions, because these are the only ones that drive the CP conserving contribution to $K_S \to \pi^0\pi^+\pi^-$. As a consequence $V_S^{\pi\pi}(z)$ is fairly suppressed in comparison with $V_+^{\pi\pi}(z)$.

- The 1-loop function $G(z)$ is given in Ref. [1].

- The $\pi^+\pi^-\gamma^*$ vertex in the one-loop diagram is dressed through the vector form factor in the Resonance Chiral Theory framework [12]:

$$
F_V(z) = \frac{M_\rho^2}{M_\rho^2 - M_K^2 z} \simeq 1 + \frac{z}{r_\rho^2}\,,
\tag{6}
$$

If, as expected, $a_j, b_j \sim \mathcal{O}(1)$, the polynomial contribution fairly dominates over the unitarity-cut loop corrections given by $V_j^{\pi\pi}(z)$. Hence these decays are very sensitive to the chiral LECs. Several models have been considered for their computation. One of them comes to mind immediately from the comparison of the electromagnetic form factor in Eq. (6) and the weak polynomial term in $V_j(z)$ (3). One may consider that this later term could also be dominated by a resonance transition (VMD). This would predict:

$$
\left\{\; a_j\, b_j > 0, \qquad \frac{b_j}{a_j} = \frac{M_K^2}{M_\rho^2} \simeq 0.4 \;\right\}^{\text{VMD model}}.
\tag{7}
$$

Other models have also been employed: weak deformation model [13], minimal hadronic approximation [14], Padé type [15] or large-N_C considerations [16]. Recently RCB and UKQCD collaborations [17, 18] have studied these decays within lattice QCD and have produced:

$$
a_+ = 1.4 \pm 0.7, \qquad b_+ = 0.7 \pm 0.8\,,
\tag{8}
$$

that are in good agreement with the VMD model prediction (7). It has to be noted that this is a first try in the lattice. They are working with unphysically heavy pion and kaon masses and, accordingly, an extrapolation to the physical point is needed. We should expect an improvement from the lattice in the near future.

The values of a_j, b_j, can be extracted from the experimental data by using our model-independent description of $V_j(z)$ in Eq. (3) as they are the only unknowns. The present situation, given by NA48/1 and NA48/2 is depicted in Table 1. While in K^+ decay both, branching and spectrum, have been determined, in the K_S decay we only have the branching ratio. As a result only the a_S parameter has been resolved in the neutral decay, while we have results for both a_+ and b_+. By comparing the prediction of the VMD model in Eq. (7) with the phenomenological results for a_+ and b_+ in Table 1 we notice that the only agreement is in the common sign of both parameters but not in their ratio. Hence one could consider either that the model is not correct or that higher order chiral corrections are not negligible. This later hypothesis would be rather unexpected.

The spectra of $K^+ \to \pi^+e^+e^-$ decay is shown in Figure 2 while the one of $K^+ \to \pi^+\mu^+\mu^-$ is shown in Figure 3. In those figures we also show the theoretical prediction as given by the

Process	BR $\times 10^9$	a_j	b_j	b_j/a_j
$K^+ \to \pi^+ e^+ e^-$ [19]	314 ± 10	-0.578 ± 0.016	-0.779 ± 0.066	~ 1.35
$K^+ \to \pi^+ \mu^+ \mu^-$ [20]	96.2 ± 2.5	-0.575 ± 0.039	-0.813 ± 0.145	~ 1.41
$K_S \to \pi^0 e^+ e^-$ [21]	$5.8^{+2.9}_{-2.4}$	$|1.06|^{+0.26}_{-0.21}$	-	-
$K_S \to \pi^0 \mu^+ \mu^-$ [22]	$2.0^{+1.5}_{-1.2}$	$|1.54|^{+0.40}_{-0.32}$	-	-

Table 1. Experimental determinations of $K \to \pi \ell^+ \ell^-$ by NA48/1 (K_S) and NA48/2 (K^+).

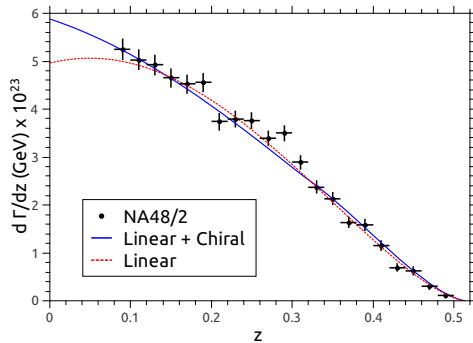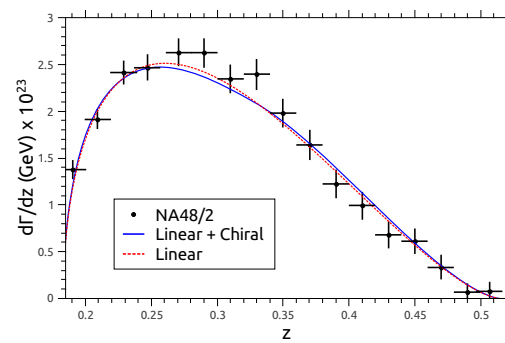**Figure 2.** Spectra for $K^+ \to \pi^+ e^+ e^-$ in the dilepton invariant mass. NA48/2 data from [19].

Figure 3. Spectra for $K^+ \to \pi^+ \mu^+ \mu^-$ in the dilepton invariant mass. NA48/2 data from [20].

full $V_+(z)$ in Eq. (3) (Linear + Chiral) and keeping the polynomial part only (Linear) for the a_+ and b_+ parameters in Table 1. As can be noticed this latter contribution dominates and the role of the unitarity loop is rather small.

In addition to the dominant $K^\pm \to \pi^\pm \gamma^*$ amplitude, the form factor $V_+(z)$ receives a short-distance contribution from the operator Q_{7V} from the effective Lagrangian [1]. Although negligible in the branching ratios and spectra, its interference with the long-distance amplitude leads to a CP-violating charge asymmetry in $K^\pm \to \pi^\pm \ell^+ \ell^-$ [8], defined by:

$$\Delta\Gamma \equiv \frac{\Gamma(K^+ \to \pi^+ e^+ e^-) - \Gamma(K^- \to \pi^- e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-) + \Gamma(K^- \to \pi^- e^+ e^-)} \sim 0.07 \, \mathrm{Im}\, \lambda_t, \tag{9}$$

where $\lambda_q = V_{qd} V_{qs}^*$. For the numerical determination we have used that $\mathrm{Im}\, b_+/\mathrm{Im}\, a_+ \simeq M_K^2/M_\rho^2$ (VMD model). With $\mathrm{Im}\, \lambda_t \sim \eta \lambda^5 A^2 \sim 10^{-4}$, this asymmetry is tiny within the SM [10, 23]. Experimentally the present bound is given by $\Delta\Gamma < 0.021$ at 90% C.L. [19].

An interesting observable could be the unintegrated charge asymmetry [10] defined by:

$$\delta\Gamma(z) \equiv \frac{\frac{d\Gamma}{dz}(K^+ \to \pi^+ e^+ e^-) - \frac{d\Gamma}{dz}(K^- \to \pi^- e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-) + \Gamma(K^- \to \pi^- e^+ e^-)}. \tag{10}$$

In order to skip the background produced by the Dalitz decay of a π^0, namely $K^\pm \to \pi^\pm \pi_D^0$, $\pi_D^0 \to e^+ e^- \gamma$, a cut in the experimental data excludes events with $z < 0.08$. The unintegrated asymmetry, as seen in Fig. 4, stays a higher values.

3. $K_L \to \pi^0 \ell^+ \ell^-$

The decay $K_L \to \pi^0 \gamma^*$ is CP violating. As a consequence the dynamics of the $K_L \to \pi^0 \ell^+ \ell^-$ decay is more involved than the processes above. Indeed both long-distance and short-distance
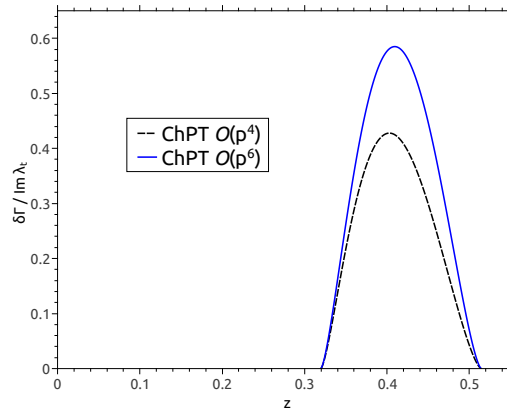
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Figure 4. ChPT predictions for the unintegrated charge asymmetry for the process $K^+ \to \pi^+ e^+ e^-$ in the z variable.

contributions drive their observables. The structure of the amplitude includes, in addition to the vector form factor $V_L(z)$, other form factors, namely scalar, pseudoscalar and axial-vector, may contribute to this decay [24].

There are three dominating contributions:

- An indirect CP-violating transition due to the K^0-$\overline{K}^0$ mixing, proportional to $V_S(z)$ in Eq. (3):

$$V_L^{\text{ind}}(z) = \pm \varepsilon \, V_S(z) \,. \tag{11}$$

- A direct CP-violating contribution arising from the four-fermion effective operators Q_{7V} and Q_{7A} given in [1]. The contributions to the form factors are proportional to Im λ_t and the semileptonic form factors of $\langle \pi(p_\pi)|\bar{s}\gamma_\mu u|K(p_K)\rangle$ [24].

- A CP-conserving contribution from $K_L \to \pi^0\gamma\gamma \to \pi^0\ell^+\ell^-$ [25], as given by the Feynman diagram in Figure 5.
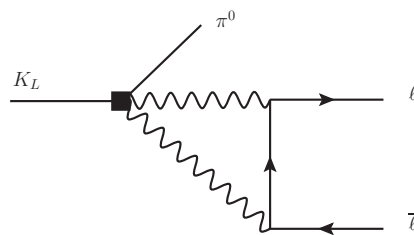

Figure 5. CP-conserving contribution to $K_L \to \pi^0\ell^+\ell^-$.

The addition of both CP-violating contributions gives (using the VMD model to eliminate b_S):

$$10^{12} \times \text{BR}(K_L \to \pi^0 e^+ e^-)\Big|_{\text{CPV}} = 15.7 \, |a_S|^2 \pm 6.2 \, |a_S| \left(\frac{\text{Im}\,\lambda_t}{10^{-4}}\right) + 2.4 \left(\frac{\text{Im}\,\lambda_t}{10^{-4}}\right)^2,$$

$$10^{12} \times \text{BR}(K_L \to \pi^0 \mu^+ \mu^-)\Big|_{\text{CPV}} = 3.7 \, |a_S|^2 \pm 1.6 \, |a_S| \left(\frac{\text{Im}\,\lambda_t}{10^{-4}}\right) + 1.0 \left(\frac{\text{Im}\,\lambda_t}{10^{-4}}\right)^2. \tag{12}$$

Process	Theory CP-violating	Theory CP-conserving	Experiment (90% C. L.)
$K_L \to \pi^0 e^+ e^-$	3.1 ± 0.9	~ 0	< 28 [26]
$K_L \to \pi^0 \mu^+ \mu^-$	1.4 ± 0.5	0.52 ± 0.16	< 38 [27]

Table 2. Comparison between theoretical and experimental results for $\mathrm{BR}(K_L \to \pi^0 \ell^+ \ell^-)$. The numbers correspond to $\mathrm{BR} \times 10^{11}$. We assume positive interference between both CP-violating contributions (theoretically preferred [28]).

It can be seen that for $|a_S| \sim 1$ (as the phenomenological results of Table 1 suggest) the indirect CP-violating contribution should be dominating in the electronic case while for the muon both are alike.

In Table 2 we compare the present experimental bounds with both CP-violating and CP-conserving pieces. The dominating contribution to the latter is helicity suppressed ($\propto M_\ell$) and, therefore, it is negligible in the electron case. However, although smaller than the CP-violating, the CP-conserving contribution is not tiny in $K_L \to \pi^0 \mu^+ \mu^-$. It is interesting to note how close to the present experimental bounds are the SM predictions. We are almost there.

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