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SLED ENERGY, SPECTRUM WIDTH, PEAK AND AVERAGE CURRENT

Introduction

P. Wilson[1] has calculated the energy V_0, the spectrum width $\Delta V'$, the peak current i_p and the average current i_a, the beam power P, as a function of beam pulse width T_b, and the values of two currents, one during the first half and the other during the second half of a .33μsec beam pulse necessary for gain compensation of a SLEDed 5.3 μsec RF pulse.

In this note, the above parameters will be calculated for RF turnoff times $t_2 = 2.7$, 3.5, and 5.0 μsec. Unless otherwise noted, all times are measured from the instant of RF turn-on and the phase flipping time, t_1, is $t_2 - .83$ μsec. The steady state emitted field α is 1.67, the coupling coefficient β is 5, and the unloaded quality factor Q_0 is 10^5. These are the measured values for the SLED cavities constructed at SLAC. It is assumed that the non-SLED energy is 26 GeV.

In addition the following will be discussed: Compensation with both beam loading and stagger of either turn-on or phase flipping times; partial SLED operation; and practical operation.

Compensation with Beam Loading

The zero current energy, V_s, of the accelerator when it is partially or completely SLEDed is:

$$V_s = G(t)nNE + \ell NE = V_0 + \Delta V_s(t)$$

The energy gain at t_a is $V_0 = (nG_a + \ell)NE$ and the change in energy after t_a is

$$\Delta V_s(t-t_a) = [G(t-t_a) - G_a]nNE$$

G(t) = normalized SLED gain
E = energy per klystron
N = total active klystron population
nN = number of SLEDed klystrons
ℓ = number of normal klystrons
G_a = SLED gain at t_a

All energies are in GeV.

Fig. 1 is a plot of G(t) in the range 2.4 to 2.8 μsec for t_2 equal to 2.7 μsec. G(t) just before it reaches its maximum at t_2 is a monotonically rising function of time.

The beam-induced energy is zero when the beam is injected and rises monotonically with time. Therefore by choosing the proper current amplitude it is possible to obtain zero net energy change from t_a, the time the current is injected, to t_2, the time when the gain is maximum. The current amplitude, i_p, is obtained as follows. The change in beam-induced energy during $T_b = t_2 - t_a$ is $V_b = .0843(1 - T_b/1.66)T_b$.[1] The change in energy due to SLED gain variation is $\Delta V_s = (M-G_a)nNE$. M is the gain at t_a. Equating V_b to ΔV_s we obtain

$$i_p = (M-G_a)nNE/.0843(1-T_b/1.66)T_b$$

Define $m_a = (M-G_a)/T_b$ and $k = .0843(1-T_b/1.66)$; then $i_p = m_a nNE/k$. The energy at t_a is $V_0 = (nG_a+\ell)NE$. The average current in μA is $i_a = (r/1000)T_b i_p$ where i_p is in mA and r is the repetition rate in pulses/sec. The beam power in kW is $P = V_0 i_a$. The maximum difference between energy change due to SLED gain variation and beam-induced energy is (approximately) $\Delta V = V_s(T_b/2) - V_b(T_b/2)$ and the spectrum width in percent is $\Delta V' = 100\ \Delta V/V_0$.

The non-SLEDed peak current i_{pn} and energy V_n corresponding to i_a are, respectively:

$$i_{pn} = i_a/(r/1000)1.6 \quad , \quad V_n = NE - .035\ i_{pn} \ .$$

The SLED energy multiplication factor is V_0/V_n.

Table I lists values of G_a, V_0, i_p, i_a, V_n, V_0/V_n, $\Delta V'$, and P as a function of T_b. The values of G_a were obtained by programming in Fortran the gain expressions of Ref. 2. The program was written by Barbara Woo.

Compensation with Two Current Amplitudes

Any beam pulse width T_b listed in Table I can be more than doubled without exceeding ΔV. Refer to Fig. 1. To do this requires different currents for the first and second portions of the pulse. Their amplitudes are calculated as follows: At t_b, when the energy is $V_0 = (nG_b+\ell)NE$, we inject a current i_{p1} which, during t_a-t_b, induces a change in energy $V_{b1} = .0843(1-T_{b1}/1.66)i_{p1}T_{b1}$. During that time the change in energy due to SLED gain is $\Delta V_{s1} = (G_a - G_b)nNE$. Equating V_{b1} to ΔV_{s1} we obtain

TABLE I

ENERGY GAIN, SPECTRUM WIDTH, PEAK AND AVERAGE CURRENTS AS A FUNCTION OF BEAM PULSE WIDTH

T_b	G_a	V_o	i_p	i_a	V_n	V_o/V_n	ΔV'	P
μs		GeV	mA	μA	GeV		%	kW
t_2 = 2.7 μsec M = 1.441								
.03	1.435							
.08	1.421	36.94	81.8	2.36	25.86	1.43	.110	87.2
.13	1.399	36.38	106.8	5.0	25.70	1.42	.38	182
.18	1.371	35.64	134.6	8.72	25.47	1.40	.66	311
.23	1.334	34.69	166.1	13.75	25.16	1.38	1.1	477
.28	1.289	33.51	201.53	20.3	24.77	1.35	1.6	680
.33	1.234	32.08	241.6	28.7	24.26	1.32	2.4	921
.38	1.168	30.37	278.4	39.3	23.6	1.29	3.3	1194
t_2 = 3.5 μsec M = 1.606								
.03	1.603							
.08	1.593	41.42	52.65	1.17	25.91	1.60	.108	48.4
.13	1.577	40.99	75.9	2.74	25.78	1.59	.261	112
.18	1.553	40.38	102.2	5.11	25.60	1.58	.570	206
.23	1.521	39.55	132.2	8.44	25.34	1.56	1.03	334
.28	1.481	38.50	166.02	12.9	24.98	1.54	1.52	497
.33	1.431	37.21	204.14	18.7	24.53	1.52	2.2	696
.38	1.371	35.65	247.4	26.7	23.9	1.49	3.0	952
t_2 = 5 μsec M = 1.776								
.03	1.776							
.08	1.771	46.04	22.28	.321	25.96	1.77	.085	14.8
.13	1.759	45.74	44.27	1.04	25.87	1.77	.29	47.6
.18	1.740	45.25	69.19	2.24	25.73	1.76	.570	101
.23	1.714	44.56	97.3	4.03	25.51	1.75	.896	179
.28	1.679	43.65	129.3	6.56	25.21	1.73	1.36	285
.33	1.634	42.49	165.6	9.84	24.80	1.72	2.00	418
.38	1.580	41.07	206.94	14.16	24.28	1.69	2.69	586

$$i_{p1} = (G_a - G_b)nNE/.0843(1 - T_{b1}/1.66)T_{b1}$$

At t_a we inject $-i_{p1}$ and i_{p2}. During $t_2 - t_a$, the change in energy due to i_{p1}, turned on at t_b, and that due to $-i_{p1}$ and i_{p2}, turned on at t_a, is

$$V_{b2} = .0843[(T_{b2} - T_{b2}^2/1.66)(-i_{p1}) + [T_b - T_b^2/1.66 - (T_{b1} - T_{b1}^2/1.66)]i_{p1} + (T_{b2} - T_{b2}^2/1.66)i_{p2}]$$

The change in energy due to SLED gain is $\Delta V_{s2} = (M-G_2)nNE$. Equating V_{b2} to ΔV_{s2} and for simplicity letting $T_{b1} = T_{b2} = T_b/2$, we obtain

$$i_{p2} = (M-G_2)nNE/.0843(T_b/2)[1-T_b/2(1.66)] + 1.2(T_b/2)/[1-T_b/2(1.66)]$$

The average current i_a is $rT_b(i_{p1} + i_{p2})/2$. V_n, V_0/V_n and P are obtained as with single amplitude compensations. Since $\Delta V(t_a+T_b/2)$ is the same as, and $V_{b2}(T_b/2)$ is not appreciably different from, the values of Table I, it is reasonable to expect that $\Delta V'$ for two current amplitudes and pulse width T_b will be the same as $\Delta V'$ for a single current amplitude and pulse width $T_b/2$.

Table II lists the parameters for 2-step compensation with beam loading.

TABLE II

ENERGY, SPECTRUM WIDTH, PEAK AND AVERAGE CURRENTS AS A FUNCTION OF BEAM PULSE WIDTH FOR TWO PEAK CURRENT AMPLITUDES DURING BEAM PULSE

T_b	G_b	G_a	V_o	i_{p1}	i_{p2}	i_a	V_n	V_o/V_n	$\Delta V'$	P
				t_2 = 2.7 1.441						
.16	1.382	1.421	35.93	158	96.9	7.3	25.5	1.41	.11	264
.26	1.311	1.399	34.1	226	146	17.5	24.94	1.37	.38	595
.36	1.198	1.371	31.1	332	215	35.5	23.8	1.31	.66	1105
				t_2 = 3.5 M = 1.606						
.16	1.562	1.593	40.6	125	65.3	4.2	25.7	1.58	.11	172
.26	1.497	1.5766	38.9	205	110	11.4	25.1	1.55	.261	443
.36	1.395	1.5529	36.3	303	175	24	24.1	1.50	.570	869
				t_2 = 5.0 M = 1.776						
.16	1.7478	1.7708	45.4	93	31.6	1.8	25.8	1.76	.08	81.7
.26	1.6927	1.7591	44	170	72	5.7	25.31	1.74	.29	251
.36	1.6015	1.7403	41.64	267	133	13	24.4	1.70	.57	540

Compensation with Beam Loading and Klystron Stagger

At any T_b, there is fixed current amplitude that negates the change in energy due to the rising SLED gain function. This current cannot be increased without degrading the spectrum width. However, it can be decreased. Then it compensates only partially for the change in SLED gain. The remaining portion of the gain increase is compensated by turning on or phase flipping a small fraction of klystrons earlier than a filling time before turnoff. Compensation with early turn-on or flipping is possible because the gain function after $t_1 + T_a$ is a (rapidly) falling function of time. This is clearly seen in Fig. 2, which shows plots of the gain function for $t_1 = 1.87$, 1.57, and 1.47 μsec, also for a turn-on time which is .2 μsec early.

The fraction of early klystrons as a function of current (below the maximum) and of beam pulse width is obtained as follows.

Let:

ℓN = number of non-SLEDed klystrons

nN = number of SLEDed klystrons with normal timing

pN = number of SLEDed klystrons with early turn-on or early phase flipping

qN = (n+p)N = total number of SLEDed klystrons, normal and early

M_2 = maximum gain of early klystrons; $M_2 = M$ if compensation is with early turn-on

m_2 = gain curve slope of early klystron right after beam turn-on, t_a

The nominal energy V_0 is (refer to Fig. 2)

$$V_0 = (G_a n + M_2 p + \ell)NE = [qG_a + p(M_2 - G_a) + \ell]NE$$

The change in energy during T_b, ΔV, is set to zero.

$$\Delta V = (m_a nNE - m_2 pNE - ki_p)T_b = 0$$

i_p is maximum when $p = 0$, in which case i_p is the same as obtained for Table I.

$$i_{pm} = qm_a NE/k = nm_a NE/k$$

p is maximum when $i_p = 0$, in which case it represents the maximum number of early klystrons.

$$p_m = qm_a/(m_a + m_2)$$

For p less than p_m, i_p, obtained by solving $\Delta V = 0$, is

$$i_p = (1 - p/p_m)i_{pm}$$

Values of i_p and V_0 as a function of the number of early klystrons pN for t_2 = 2.7 μsec, $T_b = .18$ μsec, N = 240, $\ell = 0$, are listed in Table III.

Note that the increase in energy is small as the current is reduced to zero. This could be inferred from the expression for V_0 by noting that the change $p(M_2-G_a)$ is the product of two small numbers. The reason for the

TABLE III

CURRENT AS A FUNCTION OF NUMBER OF EARLY KLYSTRONS
FOR T_b = .18, m_a = .389, G_a = 1.371

N = 240	Early flipping M_2 = 1.39, m_2 = 1		Early turn-on M_2 = 1.441, m_2 = 2.75	
pN	i_p mA	V_o GeV	i_p mA	V_o GeV
0	134.6	35.65	134.6	35.65
10	114.5	35.67	89.3	35.72
20	94	35.69	44.1	35.80
29.7			0	35.87
30	74	35.71		
40	54	35.73		
50	34	35.75		
60	14	35.77		
67	0	35.78		

increase (however small) is that the early klystrons are operating at a higher gain at t_a.

Phase flip advance has more current points and therefore is preferred to turn-on advance. A fixed delay starting at the SB modulator trigger of $(t_2 - T_a - .4)\mu$sec and an additional delay of .4 μsec in .1 μsec steps is suitable.

Partial SLEDing

Partial SLEDing is important not only in its own right - SLEDing 3 sectors increases the beam energy by 1 GeV (10 additional stations) - but it also serves as a model for complete SLEDing and allows us to check the validity of SLED theories and assumptions. Two modes will be considered.

Mode 1. A fraction of the klystrons are SLEDed and the rest are operating normally, $n < 1$, $\ell = 240-n$. The operating parameters, normalized to the corresponding quantities when all of the accelerator is SLEDed, are given by

$$V'_0 = (n + \ell/G_a),\quad i'_p = i'_a = n,\quad P' = (n + \ell/G_a)n,\quad \Delta V' = n/(n + \ell/G_a)$$

Values of the above parameter, as a function of T_b, for 3 SLEDed sectors, n = .1, ℓ = .9, N = 240, are listed in Table IV below.

TABLE IV

ENERGY, CURRENTS, AND SPECTRUM WIDTH, 3 SECTORS SLEDed, t_2 = 2.7 μsec

T_b nsec	V_o GeV	i_p mA	i_a mA	V_n GeV	V_o/V_n	$\Delta V'$ %	P kW
.08	27.10	8.1	.12	26.00	1.04	.01	3.25
.13	27.04	10.8	.25	26.00	1.04	.05	6.75
.18	26.97	13.4	.44	25.95	1.04	.09	11.9
.23	26.87	16.7	.69	25.92	1.04	.14	17.5
.28	26.75	20.1	1.02	25.88	1.034	.20	27.2
.33	26.61	24.1	1.43	25.82	1.030	.29	37.3
.38	26.44	28.7	1.97	25.76	1.026	.38	52.0

The energy gain is approximately 4%. The energy increase is 1 GeV (10 extra stations). The spectrum is narrow even for wide beam pulses and therefore there is no need for two value current compensation. The currents, power, and spectrum width are reduced approximately by a factor of 10. Because of the reduction in spectrum width this is not a good model.

Mode 2. A fraction of the klystrons are SLEDed and the rest are off, N < 240, n = 1, ℓ = 0. The normalized operating parameters are:

$$V_0' = i_p' = i_a' = N/240, \quad P' = (N/240)^2, \quad \Delta V' = 1$$

For 3-sector SLEDed, N = 24, the quantities of Table I are reduced as follows: energy and currents by a factor of 10, power by a factor of 100, and the spectrum width stays the same. The reason for no change in the spectrum width is that $V_s(t)$, $V_b(t)$, and V_0 are all reduced by the same factor. The number of klystrons necessary for energy spread compensation, the currents and energies listed in Table III are all reduced by a factor of 10. Thus 3-sector SLEDed in Mode 2 is a good model for complete SLEDing, but is not useful for standard operation. However, if the majority of the klystrons are SLEDed this mode can be used to obtain normal or greater than normal energy values with reduced line power.

Practical SLED Operation

The instantaneous values of G(t) should be verified by measuring the energy of a narrow 20 nsec beam pulse as a function of time elapsed from the instant of RF turn-on. The instantaneous beam-induced energy should also be measured. The values of current in Tables I–IV should be recalculated using the newly determined G(t) and $V_b(t)$. However, even the new values should serve only as a guide. The actual current amplitudes should be set as follows.

<u>Single Current Amplitude.</u> (1) Set the energy defining slit to V_0. (2) Inject a T_b wide current pulse at t_a. As its amplitude is increased the current spike that appears at t_a spreads toward the right. Near the final value of current a pip should appear at t_2. If the spectrum width is properly set for the pulse width used, then the current can be increased until the indicated transmitted current is constant during T_b.

<u>Two Current Amplitudes.</u> At t_b inject a current pulse of width T_b. Increase current until current spikes appear at t_b and t_a. Further increase the current until the indicated current amplitude between the spikes, between t_b and t_a, is constant. Decrease the current in the second part of the pulse until first a spike appears at t_2 and then the indicated current between t_a and t_2 is constant.

After the current for either one or two current amplitude mode has been adjusted it may be sufficient to monitor the ratio of output to input charge of the spectrum defining slit.

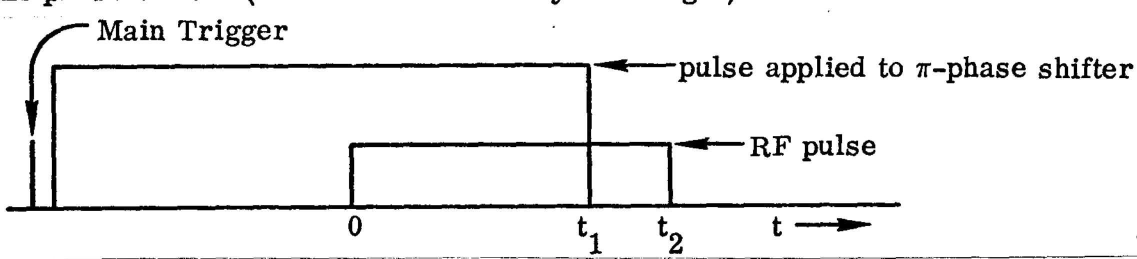
The table below lists the peak gain M as a function of t_d, the time by which t_1 is delayed from its optimum value $t_2 - T_a$. $t_d = T_a - (t_2 - t_1)$.

t_d	-.4	-.2	-.02	0	.02	.2	.4
M	1.327	1.387	1.387	1.441	1.442	1.411	1.252

Note that a 20 nsec jitter causes a change in energy of only .7%. SLED gains for t_1 delayed from its optimum value are shown in Fig. 3.

<u>Phasing</u>

If the accelerator is properly phased for normal operation it is also properly phased for SLED operation provided that the control waveform applied to the phase shifter (which is controlled by TTL logic) is as shown below:



This way the phase of the RF into the accelerator (after t_1 when it matters) is the same for SLED operation as it was for normal operation, which is the proper phase to accelerate the beam. (After t_1, the phase of the klystron output is obviously correct and the phase of the emitted field must be the same as the phase of the klystron output.) Thus to phase the accelerator for SLED operation:

1. detune cavities
2. turn off control signal to π-phase shifter
3. phase
4. tune cavities
5. turn on control signal to π-phase shifter and start SLEDing.

Compatibility with SPEAR and PEP

For SPEAR two current bursts .781 μsec apart and for PEP two current bursts 2.4 μsec apart are required.[3] The positron current normalized to 2.7 μsec pulse normal operation, i', is given by

$$i' = rbG/(360)(2)$$

where b is the number of bursts/pulse, G is the energy gain when the bursts occur. An examination of Fig. 1 shows that for SLED operation there are two identical gains at two values of t, .781 μsec apart. The normalized positron currents are given in Table V below.

TABLE V

		SPEAR					PEP				
		Normal		SLED			Normal		SLED		
t_2	r	b	i'	b	G	i'	b	i'	b	G	i'
2.7	360	2	1	2	1.15	1.15	1	.5	1	1.44	.72
3.5	278	3	1.15	2	1.3	1	2	.77	1	1.6	.8
5.0	180	4	1	2	1.5	.75	1	.5	1	1.77	.89

The value of t_1 is $t_2 - T_a - .2$ μsec.

Choice of RF Pulse Width: 3.5 vs. 5 μsec

Assume that operation with r = 360 (2.7/3.5) = 278 pps is possible. (One possibility would be to operate at r = 360 for 15 μsec, 5,400 pulses, and off for 4.5 μsec, 1620 pulses. This way the problem due to the interference which occurs at submultiples of 360 will be eliminated and the average power rating of the modulators, substations, etc., will not be exceeded.) A pulse width of 3.5 μsec is preferred to 5 μsec for the following reasons:

1. The beam power is greater by approximately a factor of 2.
2. Does not require new pulse transformers.
3. Present time delay between master trigger and RF turn-on is adequate.
4. The increase in gain for going from 2.7 to 3.5, a .8 μsec increase in pulse width, is 11% and going from 3.5 to 5, an additional 1.5 μsec increase in pulse width, is also 11%.
5. The improvement in duty cycle for the non-SLED mode is 16% for t_2=3.5 μsec and 22% for t_2 = 5.0 μsec.
6. The SB's, the high power klystrons, the accelerator section loads work better with t_2 = 3.5 μsec then with t_2 = 5 μsec.

SLED gain curves for t_2 = 3.5 μsec are shown in Fig. 4.

Conclusion

It was shown that for a 2.7, 3.5, and 5 μsec RF pulse a .2 μsec beam pulse with a .5% spectrum width, and with average currents of 9, 5, 2 mA, respectively, is possible. If the current has two values within the pulse a .4 μsec beam pulse with average currents of 35, 24, and 13 mA, respectively, is possible. The duty cycle is decreased by a factor of 4, 3, and 8, respectively. The average powers are 1100, 870, and 540, respectively. Thus for a 2.7 and 3.5 μsec RF pulse the conversion or RF to beam energy is about as efficient as normal operation. The maximum average beam current and beam power for standard operation are 40 μA and 900 kW, respectively. They are limited by beam breakup.

If the average power of the modulators and/or substations can be increased by 30% or if operation at other than a submultiple of 360 is possible, then a 3.5 μsec RF pulse is optimum.

It was shown how to operate partially or completely in the SLED mode. The results expected when 3 sectors are SLEDed were given. If operation with 3 sectors is as predicted then we may have confidence in the predictions for fully SLEDed operation.

Acknowledgement

The author wishes to thank H. Hogg and G. Loew for valuable discussions.

References

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2. Z. D. Farkas et al., "SLED: A Method of Doubling SLAC's Energy," Proc. IX Int. Conf. on High Energy Accelerators, SLAC, Stanford, Calif., 1974, p. 576 (SLAC-PUB-1453).
3. G. A. Loew, SLAC-TN-75-2 (February 1975).

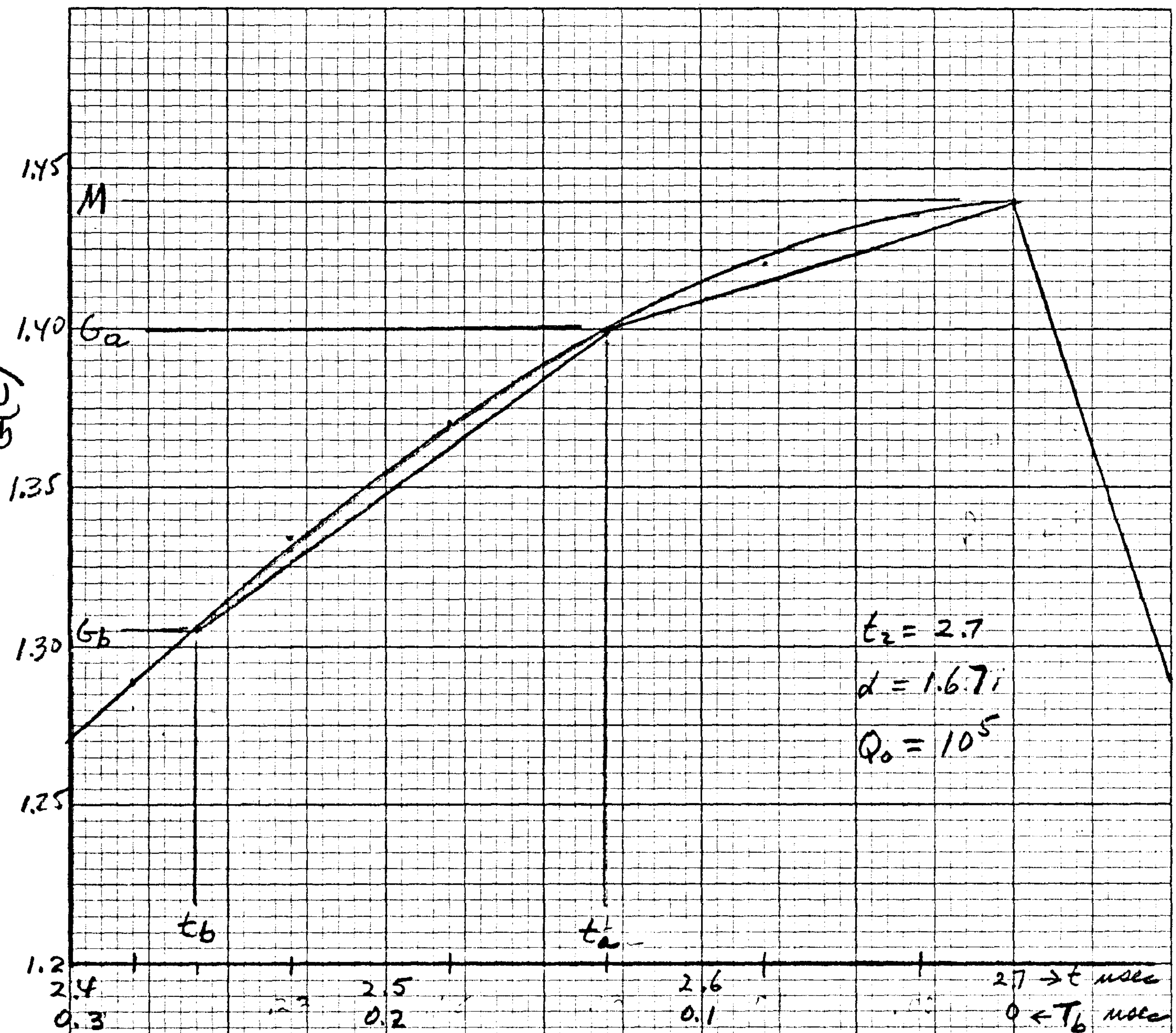

FIG 1 SLED GAIN G(t) VS TIME

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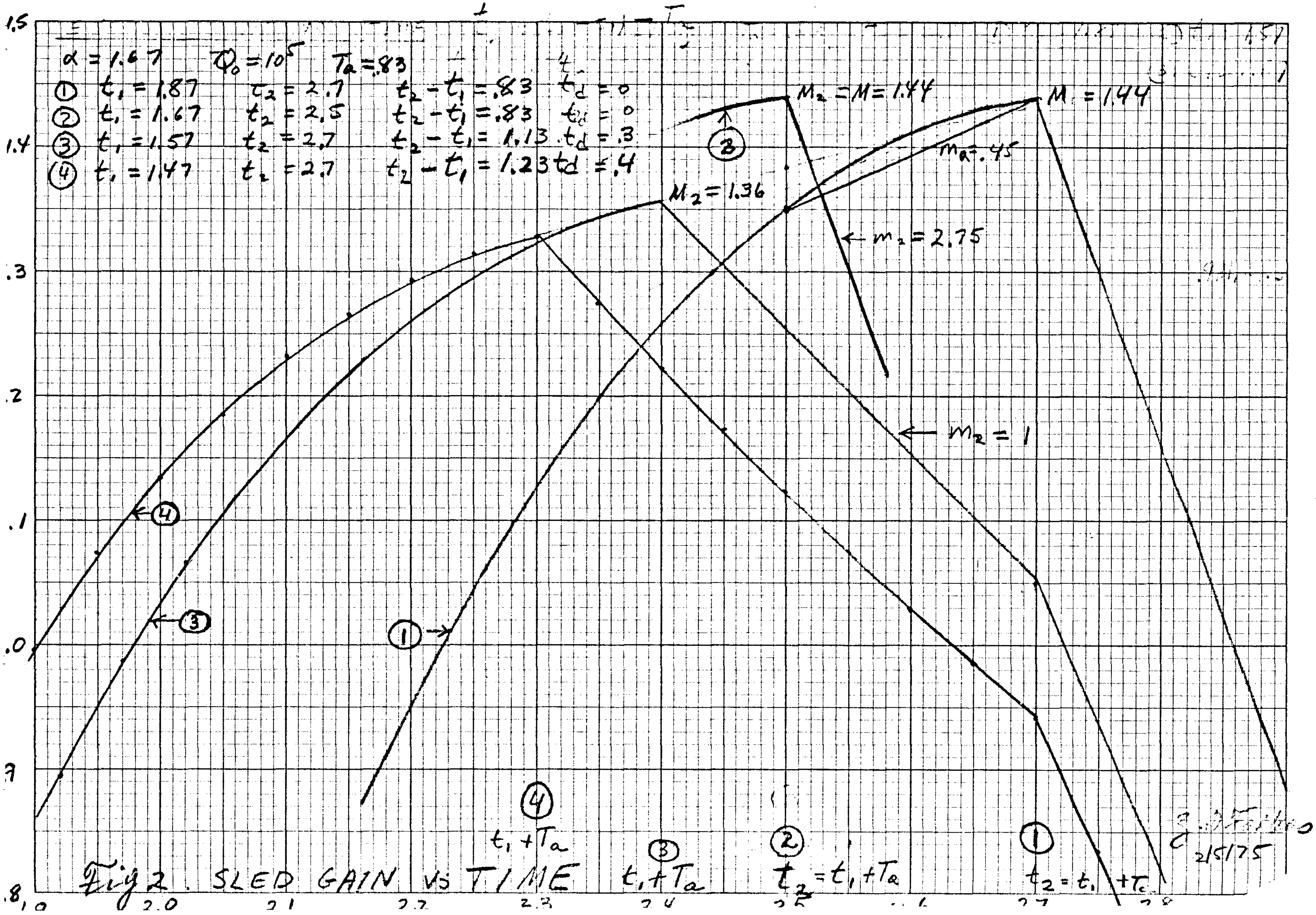

Fig 2. SLED GAIN vs TIME

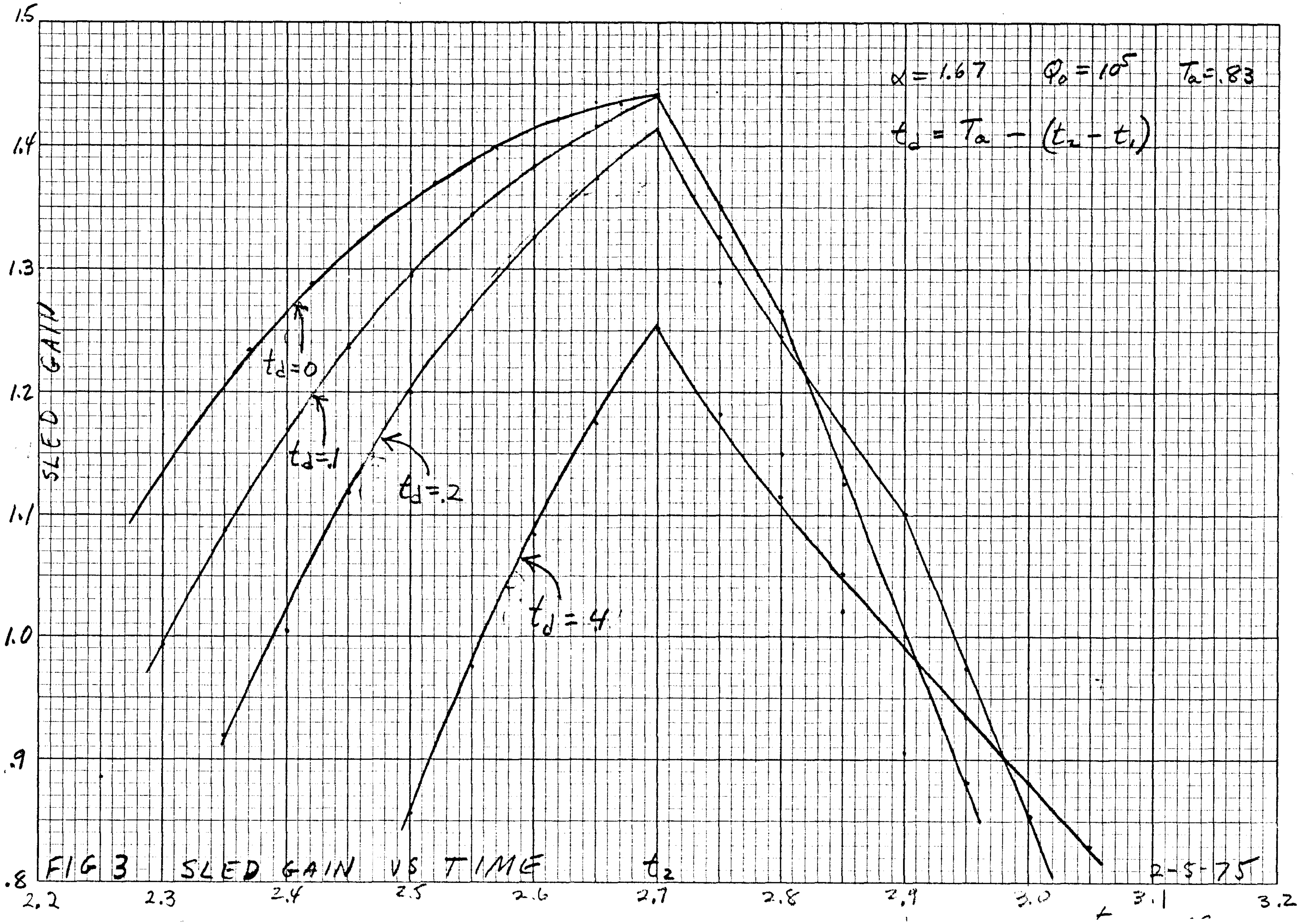

FIG 3 SLED GAIN VS TIME

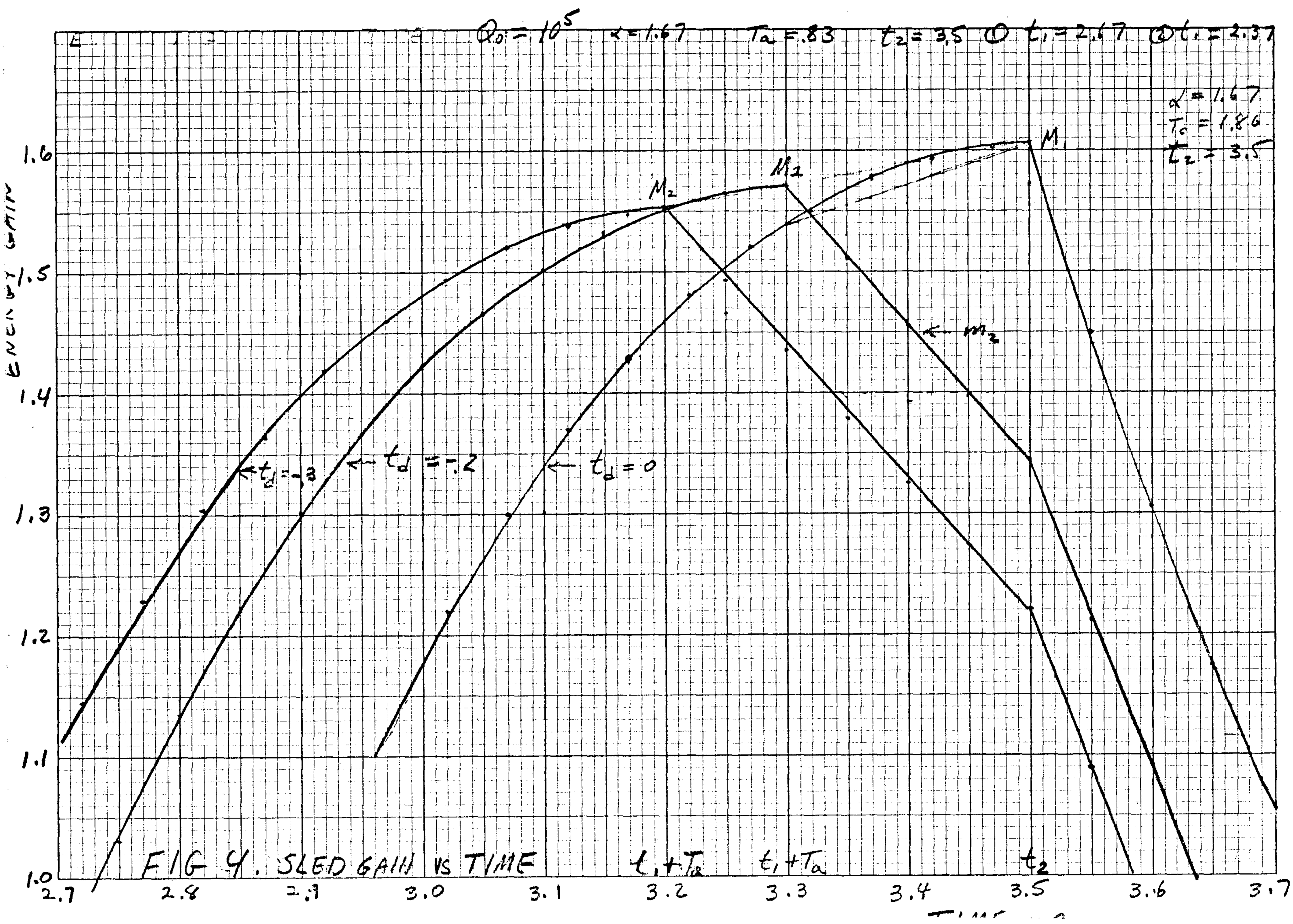

FIG 4. SLED GAIN VS TIME