Comments on Simpson's Rule Correction and SSC Cell Asymmetries

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It has been recently proposed[1] that the trim correction coils in the SSC dipoles for the correction of systematic multipole components be replaced by lumped correcting elements placed at the center (C) of the SSC half cells as well as in the spool pieces near the F and D quadrupoles. The strengths per half cell of the elements are given to first order by Simpson's Rule for three-point integration $(f_F : f_C : f_D) = (\frac{1}{6} : \frac{4}{6} : \frac{1}{6})$. First order tune shifts are correctable by two orders of magnitude in this scheme.

The initial evaluations of correction were calculated using an idealized lattice with zero-length quads and correction elements. The correctors were placed precisely at the center and ends of the half-cells. The finite size of dipole magnet ends and connectors was also ignored. Third-order accuracy in Simpson's Rule is obtained by the symmetry.

The currently planned SSC cell differs somewhat from the symmetric cell. Rather than having separate correction elements symmetrically placed on both sides of the quadrupoles, a single correction spool piece is placed on one side of the quads. The SSC half cell has six dipoles, and a slot for the center corrector can be placed between dipoles in the middle of the dipole series. Because of the asymmetric placement of the spool piece, the center corrector slot will not be precisely at the center of the half cell, but will be displaced by a distance δ from the center.

However, the reflection symmetry of the betatron functions about the quadrupoles ensures that the correction retains the basic symmetry necessary for third-order accuracy when averaged over the full cell (see figure). Averaging over a full cell, we find that the correction per half cell is equivalent to a four-corrector system with correctors placed at $z = -L_B/2, -\delta, \delta,$ $L_B/2$ with respect to the half-cell center. (see figure) L_B is the total dipole

length in the half cell.

The relative strengths of the correctors is perturbed somewhat from the Simpson's Rule values. Requiring accurate parabolic integration implies

$$f_C = \frac{2}{3} \frac{1}{1 - (\frac{2\delta}{L_B})^2} \tag{1}$$

For a typical value of $2\delta/L_B = 0.06$, f_C is modified from 2/3 to 0.669075. The strengths of the F and D correctors are reduced correspondingly from $f_D = f_F = 1/3$ to 0.330925. The correction ability is not diminished with these modifications.

In the actual SSC cell the F and D correctors are not exactly at the ends of the dipoles but, because of their finite lengths, are a couple of meters past the ends of the dipoles on average. However, if the reflection symmetry over a full cell is considered, the dipoles in the following half cell overlap the corrector positions. On average the dipole ends and corrector positions can actually overlap precisely. This is actually an advantage over a symmetrized cell where the end correctors are necessarily centered a finite length beyond the dipoles and an error is necessarily introduced into the correction. In the unsymmetric cell this error can be canceled in the full-cell average.

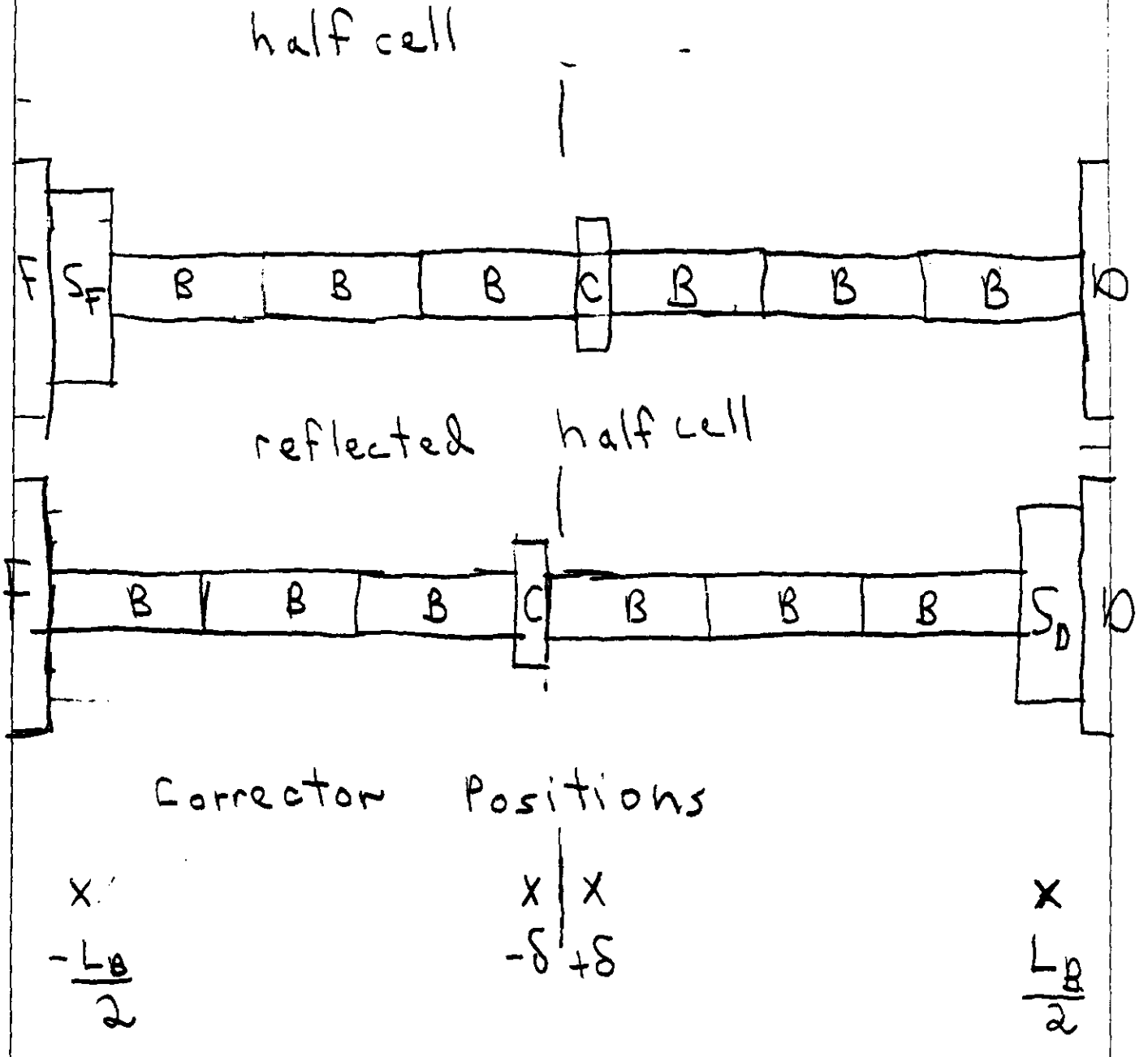
As discussed elsewhere,[2] adding a center corrector reduces second order sextupole induced tune shifts by a factor of four from the end-correctors only case. This improvement is simply due to the increase in the number of correction points and does not depend on cell symmetry. That improvement would be the same for the slightly unsymmetric SSC cell as for the symmetric case.

In summary, the small asymmetries in an actual SSC cell do not endanger the accuracy of the Simpson's Rule correction. The relative strengths of the correctors should be modified following equation 1. Exact symmetrization of the SSC half-cells is not required or even preferable.

References

1. D. Neuffer, SSC-N-339, May 19, 1987.
2. D. Neuffer, SSC-132, June, 1987.

-Half-cell Representation of Full-cell with Simpson's Rule Correction



Corrector Positions

x x x

$-\frac{L_B}{2}$ $-\delta$ $+\delta$ $\frac{L_B}{2}$