RARE MUON DECAYS

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In 1977, a group of physicists from LASL, the University of Chicago, and Stanford University[1] mounted an experiment at LAMPF to search for the decay $\mu^+ \to e^+ \gamma$. The high-intensity beams of muons available at LAMPF are particularly well suited to a search for this rare decay. This experiment did not detect the decay but did push the upper limit down to

$$\frac{\Gamma(\mu^+ \to e^+ \gamma)}{\Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_\mu)} < 1.9 \times 10^{-10} \quad ,$$

about an order of magnitude more sensitive than any previous search.

Now a new collaboration from the same three institutions has embarked on an experiment to search for the muon-number violating processes $\mu^+ \to e^+ \gamma$, $\mu^+ \to e^+ e^+ e^-$, and $\mu^+ \to e^+ \gamma\gamma$ with a large new experimental facility known as the Crystal Box. Also an extremely exciting experiment is constructed at Triumph, Canada to study the conversion process $\mu Z \to e Z$; the simple time-projection-chamber used in this experiment may be the first operating TPC in existence. The expected sensitivity is 10^{-12}.

Status of Muon Number Conservation, 1964, and Present is shown on Table I.

TABLE I

Muon Number Process	Upper Limit (1964)	1980
$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})}$	$<2.2 \times 10^{-8}$	$<1.9 \times 10^{-10}$
$\frac{\Gamma(\mu \to eee)}{\Gamma(\mu \to e\nu\bar{\nu})}$	$<1.3 \times 10^{-7}$	$<1.9 \times 10^{-9}$
$\frac{\Gamma(\mu^- Z \to e^- Z)}{\Gamma(\mu^- Z \to \nu Z')}$	$<2.4 \times 10^{-7}$	$<7 \times 10^{-11}$
$\frac{\Gamma(\mu \to e\gamma\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})}$	$<1.6 \times 10^{-5}$	$<5 \times 10^{-8}$

In this talk I will discuss the present theories which expect the lepton number to be violated, the apparatus and sensitivities of our experiment and the consequence on the results of the last $\mu \to e\gamma$ experiment at LAMPF on the mass of the Higgs.

A. Models of Rare Muon Decays:

The SU(5) group of Georgi and Glashow[2] is elegant enough to serve as a prototype for grand unified theories, in the same way as SU(2) x U(1) Weinberg model served for some time as a prototype theory of the partial unification of weak and electromagnetic interactions. At low energies the world has a $SU(3)_c \times U(1)_{em}$ symmetry. Above M_w = 87 GeV, this symmetry becomes SU(3) x SU(2) x U(1). Georgi and Glashow that the group that can simply accommodate this is SU(5). This theory contains 24 gauge vector bosons: The familiar $\gamma, W^{\pm}, Z^0, g_{i=1,\ldots,8}$, and a color triplet and weak isodoublet $\begin{pmatrix} x \\ y \end{pmatrix}_{R,W,B,}$ and their antiparticles. These latter bosons acquire their masses at the initial stage of symmetry breaking SU(5)→SU(3) x SU(2) x U(1) which occurs on the scale of 10^{14} to 10^{15} GeV.

The fermions are formed in 3 generations (e,μ,τ-generations) each of which comprises 15 left-handed helicity states and is assigned to a reducible $\bar{\underline{5}}$ - $\underline{10}$ representation of SU(5).

The breaking of SU(5) is as follows

$$SU(5) \xrightarrow[\underline{24}\ \text{adjoint}]{} SU(3)_C \times SU(2)_L \times U(1) \xrightarrow[\underline{5}\ \text{vector}]{} SU(3) \times U(1)$$

The $\underline{24}$ ϕ has a vacuum expectation value

$$<0|\phi|c> = O(10^{15}\text{GeV})\begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ 0 & & & & -3/2 \end{pmatrix}$$

and generates M_x, M_y while the $\underline{5}$ H has a VEV:

$$<0|H|0>\ O(M_w = 87\ \text{GeV})\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and generates M_f, M_w, M_Z.

One problem with this model is that $\sin^2\theta_w$ is 3σ smaller than the measurement value.

Another serious problem with the SU(5) model with one Higgs doublet is the prediction of the masses of fermions. In the symmetry limit:

$$m_d = m_c$$
$$m_s = m_\mu$$
$$m_b = m_\tau$$

Using renormalization group, a prediction of fermion masses can be made:

$$\left(\frac{md}{ms}\right)_{Q>>1\text{GeV}} = \left(\frac{m_e}{m_\mu}\right) \simeq \frac{1}{200}$$

and the current algebra result gives

$$\left(\frac{md}{ms}\right)_{bare} \simeq \left(\frac{f_\pi^2 m_\pi^2}{f_k^2 m_k^2}\right) \simeq \frac{1}{20}$$

This failure may either reflect the existence of a more complicated Higgs structure[3] in SU(5).

H. Georgi and D. V. Nanopolous invented a model which retains all the good predictions of SU(5), yet can calculate the $\sin^2\theta_w$ to be 0.23, and the bad mass relation above is replaced by a good mass relation $m_d m_s = m_e m_\mu$. At the same time, all the attractive predictions for mixing angles and the t-quark mass are retained. This is the O(10) model.[4]

Why am I interested in the failure of SU(5)? Because a simple way of curing it is to enlarge the Higgs structure, and an immediate consequence of it is that the lepton number does not have to be conserved. This is exactly what J. Bjorken and S. Weinberg[5] did in 1977. The motivation for lepton number non-conservation is trivial enough: the analogy between muon number and strangeness.Strangeness is automatically conserved in the color gauge theory of strong interactions, for reasons much the same as those applied to muon number. Strangeness is not conserved in weak interactions, because the unitary operators needed to diagonize the mass matrix of the charge $-\frac{1}{3}$ quark and the charge $+\frac{2}{3}$ quark are not the same. The same reasons is if ν_e and ν_μ are massless, then as unitary operator is needed to diagonalize their mass matrix, and the gauge interaction terms automatically conserve muon number.

But muon number is not automatically conserved by the interaction of leptons with the scalar bosons! Bjorken and Weinberg studied the lepton number conservation in the SU(2) x U(1) context, also before the τ-lepton was firmly established. I will sketch out what was done and I will also include the contribution of the τ in later calculations.

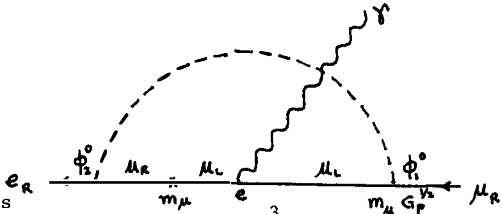
In general, assume only 2 families, the interaction between leptons and scalar bosons can be written as:

$$L_H = -g_1(\bar{\nu}_\mu \bar{\mu}-)_L \begin{pmatrix}\phi_1^+\\ \phi_1^0\end{pmatrix} \mu_R^- - g_2(\bar{\nu}_e \bar{e}-)_L \begin{pmatrix}\phi_2^+\\ \phi_2^0\end{pmatrix} \mu_R^-$$

$$- g_3(\bar{\nu}_\mu \bar{\mu}-)_L \begin{pmatrix}\phi_3^+\\ \phi_3^0\end{pmatrix} e_R^- - g_4(\bar{\nu}_e \bar{e}-)_L \begin{pmatrix}\phi_4^+\\ \phi_4^0\end{pmatrix} e_R^- + HC$$

where ϕ_i are linear combinations, not necessarily independent, of an unknown number of scalar fields of definite mass. $(g_1\langle\phi_1^0\rangle = M_\mu, g_4\langle\phi_4^0\rangle = M_e)$

If the ϕ_i are all multiplets of <u>one</u> elementary doublet, then muon number is conserved. But with more than one doublet, there is no reason why this should be the case. Then if g_2 or g_3 does not vanish and if there is a $\phi_1^0 - \phi_2^0$ or $\phi_1^0 - \phi_3^0$ mixing then the effects of virtual scalar bosons will induce physical transition between muons and electrons. They calculated the $\mu\to e\gamma$ rate in this picture.



The rate is

$$(|a|^2 + |b|^2)\,\frac{m_\mu^3}{2\pi}$$

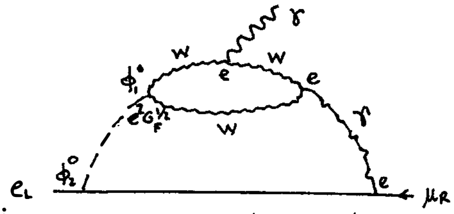
where

$$a \approx b \quad \frac{1}{8\pi^2}\,(m_\mu G_F^{1/2})^2\,\frac{m_\mu}{m_H^2}$$

since the decay rate of $\mu\to e\nu\bar{\nu}$ is $\frac{1}{192\pi^3}\, m_\mu^5\, G_F^2$ the branching ratio is

$$\frac{\mu\to e\gamma}{\mu\to e\nu\bar{\nu}} = \frac{3}{\pi^2}\left(\frac{m_\mu}{m_H}\right)^4$$

Assuming m_H = 85 GeV, the branching ratio is 7×10^{-13}. Bjorken and Weinberg also calculated the <u>two-loop</u> graphs:



and found:

$$a \approx b \approx \frac{1}{(8\pi^2)^2}\, e^3 (m\, G_F^{1/2})(e^2 G_F^{-1/2})\,\frac{1}{m_W^2}$$

The ratio of the two calculations is

$$R = \frac{\text{one-loop}}{\text{two-loop}} \simeq \frac{m_\mu^2 G_F}{2\alpha^2}\left(\frac{m_w^2}{m_H}\right)^2 \simeq \frac{2\pi}{\alpha}\left(\frac{m_\mu}{m_H}\right)^2$$

Assuming $m_H = 85$ GeV,

$$R = \frac{1}{761}$$

Then the BR due to the two-loop calculation alone is 5.3×10^{-10}. The conclusions we can make from these calculations are:

1) If the mixing angle is small (instead of 45° maximum) these numbers are reduced.

2) If we incorporate the τ, there is an enhancement of $(m_\tau/m_\mu)^4/(m_\tau/m_\mu)^2 = 6.9 \times 10^7$ in the branching ratio.

3) The mass of the Higgs cannot be larger than 200 GeV so we can preserve perturbation calculation, then the only way to make this calculation satisfy the data including the τ lepton, is to reduce the mixing angle drastically.

This is where the 0(10) model of Georgi and Nanopolous comes in. The 0(10) model is ambidextrous and it imbeds the SU(3) color group. This grand unified theory predicts accurately the $\sin^2\theta_w(M_w)$, throw away automatically the wrong relationship between the fermions, predict the $\theta_1, \theta_2, \theta_3$ K-M angles and the mass of the t quark.

The main feature of this model is it advocates strongly the importance of the rich structure of the Higgs. These Higgs are required to constrain the mixing angles in the context of ambidextrous $SU(2)_L \times SU(2)_R \times U(1)$. In this theory, the mixing angles are related to the quark masses which cannot be implemented in an SU(2) x U(1) theory. In the remainder of this talk, I would concentrate mainly on the ordinary mass scale Higgs, meaning the Higgs of masses $\sim$ 100 GeV. These are the ones responsible for $\mu \to e\gamma$ decay. The new Higgs required by 0(10) mediate flavor changing neutral current. I will systematically study the range of the masses of these Higgs from the experimental point of view.

First they enter $K_L - K_S$ mass difference. M. Gaillard and B. Lee[6] in the context of SU(2) x U(1) calculated the mass difference to be:

$$\frac{m_L - m_S}{m_K} = \frac{G_F}{\sqrt{2}} f_K^2 \frac{\alpha}{4\pi} \frac{m_c^2}{m_w^2 \sin^2\theta_w} \sin^2\theta_c \cos^2\theta_c$$

In the 0(10) model, Cabbibo's angle is predicted to be

$$\tan^2\theta_c \simeq \frac{md}{ms}$$

The ratio of masses above is

$$G_F^2\ 2m_c^2\ (\frac{md}{ms})$$

This calculation is based on the conventional box diagram where $s\bar{d} \to ds$ goes via $W^\pm$. Since the flavor can change by means of the Higgs, the effect can be calculated (Georgi and Nanopoulos) to be:

$$\frac{4\sqrt{2}}{m_H^2}\ (md \cdot ms)$$

We do not want to upset the success of the box diagram in determining the K_L-K_S mass difference, we better demand that the new contribution due to O(10) to be smaller than the box diagram contribution. Using m_W = 95 GeV, m_c = 1.5 GeV, m_s = 0.15 GeV, we get:

$$m_H \geqslant 56 \text{ GeV.}$$

The constraint on the upper limit of the Higgs mass was established by Cabibbo et al., using the renormalization group calculation[7]. Cabibbo et al., made use of the GUT group SU(5) or O(10). The results are identical in both models. Given the unstable character of the renormalization-group equations obeyed by the Higgs self coupling λ, the requirement that λ does not blow up until the unification mass M_u is reached gives a significant upper bound to the self-coupling in the low energy region ($\sim M_W$). This bound is immediatly translated inot a bound on the Higgs meson mass. In the simplest case, where all fermions are much lighter than the W boson, they found $M_H \leqslant 170$ GeV, quite independent of the number of families.

For the case when the heaviest fermion is heavier than M_W, the situation is more complicated. Call $h(\phi^2)$ the running coupling constant of the heavy quark to the Higgs doublet ϕ, using renormalization group equations:

$$8\pi^2 \frac{dh^2}{dt^2} = (\frac{9}{4} h^2 - 16\pi\alpha_s)h^2$$

$$32\pi^2 \frac{d\lambda}{dt} = 4\lambda^2 + 12\lambda h^2 - 3\lambda(3g^2 + g'^2)$$

$$- 36h^4 + \frac{9}{4}[2g^4 + (g^2 + g'^2)]$$

where $t = \frac{Q^2}{\eta^2}$, $\eta = \langle\phi^0\rangle_0$, $\frac{g'(Q^2)}{g(Q^2)} = \tan^2\theta_W(Q^2)$

they got the upperbound for both the heavy fermion and the Higgs bosons. The requirement that $h(\phi^2)$ should not develop a singularity at a finite ϕ^2,

$$h^2(\eta^2) > \frac{64}{9} \pi\alpha_s$$

Therefore

$$M_f > 250 \text{ GeV}$$

A similar analysis can be done to the Higgs self coupling λ. The bound on $\lambda(\eta^2)$ for different values of $h^2(\eta^2)$ can be found, by numerically solving the $d\lambda/dt$ equation and require that $\lambda(\eta^2)$ be finite for Q^2 GUM. The bound on $\lambda(\eta^2)$ thus found gives a bound on the Higgs boson mass, according to:

$$M_H^2 = \frac{2}{3} \lambda(\eta^2)\eta^2$$

in the region $m_f \sim M_w$, $M_H < 170$ GeV.

We have limited the Higgs mass to be between 56 GeV and 170 GeV. This value is very consistent with perturbation calculations of weak decays.

The success of perturbation calculations forces the M_H not to be too large, commonly estimated to be less than 200 GeV or so.

I made use of the Bjorken-Weinberg one loop calculation, incorporating the τ lepton together with the μ lepton. I found the contribution of the τ dominates by a factor of 6.9×10^7. However, the mixing angle is explicitly determined by the O(10) model to be extremely small. The branching ratio of

$$B_{e\gamma} = \frac{\mu \to e\gamma}{\mu \to e\nu\bar{\nu}} = 48 \frac{\alpha}{\pi} \left(\frac{m_e}{m_H}\right)^4 \frac{m_e}{m_\mu}$$

There is no free parameter in this calculation except the Higgs mass.

$$B_{e\gamma} < \frac{5.4 \times 10^{-3}}{m_H^4}$$

The branching values corresponding to the Higgs masses are:

M_H(GeV)	$B_{e\gamma}$	Constraint
50	9×10^{-10}	$K_L - K_S$ ↓
80	1.3×10^{-10}	
91	7.9×10^{-11}	LAMPF eγ Expt. ↓
100	5.6×10^{-11}	
150	1.1×10^{-11}	
170	6.4×10^{-12}	
200	3.4×10^{-12}	↑ Higgs self coupling perturbation calculation

As we have seen before, K_L-K_s excludes the Higgs mass less than 56 GeV, Higgs self coupling excludes the mass greater than 170 GeV. The impressive limit of 1.9×10^{-10} from the recent LAMPF-Chicago-Stanford experiment [1] forces M_H > 80 GeV. The region of M_H between 80 to 170 GeV is equivalent to the $B_{e\gamma}$ between 1.9×10^{-10} and 6.4×10^{-12}.

B. The New LAMPF Experiment.

A new collaboration from the same three institutions has embarked on an experiment to search for the muon-number violating processes $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$, and $\mu^+ \to e^+\gamma\gamma$ with a large new experimental facility known as the Crystal Box.

A conceptual drawing of the apparatus is shown in Fig. 1. The basic design calls for a large solid angle modular sodium iodide detector, weighing ∿200kg, surrounding a thin target in which the muons stop and decay, a cylindrical drift chamber and trigger hodoscope counters. The approximately 400 sodium iodide modules will detect 53-MeV positrons and photons with essentially 100% efficiency, an energy resolution of ∿2 MeV (FWHM) and a timing resolution of 0.5 ns (FWHM)(1 ns = 10^{-9}). The drift chamber will record the passage of charged particles with a position resolution of ∿150 μM (FWHM) in each of eight layers. Photons will be identified by detecting energy deposited in the sodium iodide when there is no response from the drift chamber or hodoscope counters: positrons are detected by all of these systems.

The three porcesses, $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$, and $\mu^+ \to e^+\gamma\gamma$ will be studied simultaneously with a sensitivity to branching ratios of about 10^{-11}. (This represents an improvement of ∿10, 100, and 5000, respectively, over present experimental limits.) Events will be selected by a hardwired processor designed to use both the analog and digital information from the detector and make a decision within 250 ns.

This speed will enable the apparatus to operate at a flux of $5 \times 10^5 \mu^+$/s and will provide an immediate suppression of accidental coincidences from the ordinary decays of several muons. The experiment will begin setting up in late 1980 with data-taking to commence by mid-1981.

If any of these processes is observed, it will be obvious evidence of the failure of the conservation of muon number. The strength of the failure will provide a great dear of information as to what is the correct model of the basic interactions. Should none of these processes be observed, this experiment will force tight constraints on many potential models and eliminate many others. If the process $\mu^+ \to e^+ \gamma$ is not observed in the Crystal Box, the collaboration plans to reconfigure the sodium iodide modules inside a large magnet and continue the search for muon-number violation with at least an order of magnitude greater sensitivity.

C. Theoretical Interpretation of Recent LAMPF Experiment

What can we say about the bound on the number of families in the O(10) model, if the LAMPF-Chicago-Stanford experiment limit of 1.9×10^{-10} is used? It turns out, in O(10), this experiment gives a very stringent limit on the number of families N more stringent than any other argument using renormalization group calculations.

For information, let us see how we can set the upper bound on N. Cabibbo et al[7] used the $SU(3) \times SU(2)_L \times U(1)$ gauge coupling equations:

$$\frac{1}{\alpha} = \frac{8}{3}\frac{1}{\alpha_u} + \frac{N}{3\pi} \ell n \frac{M_w^2}{Q^2} - \frac{11}{6\pi} \ell n \frac{M_w^2}{Q^2}$$

$$\frac{1}{\alpha_w} = \frac{1}{\alpha_u} + \frac{N}{3\pi} \ell n \frac{M_w^2}{Q^2} - \frac{11}{6\pi} \ell n \frac{M_w^2}{Q^2}$$

$$\frac{1}{\alpha_s} = \frac{1}{\alpha_u} + \frac{N}{3\pi} n \frac{M_w^2}{Q^2} - \frac{33}{6\pi \times 2} \ell n \frac{M_w^2}{Q^2}$$

where $\alpha_u \simeq 1/40$ = SU(5) unified coupling constant.

The α_u can be written as:

$$\frac{1}{\alpha_u} = \frac{3}{8}\frac{1}{\alpha} - \frac{(N - \frac{33}{16}}{3\pi} \ell n \frac{M_w^2}{Q^2}$$

and $$\ell n \frac{M_w^2}{Q^2} = \frac{2\pi}{} \frac{1}{\alpha} - \frac{8}{3\alpha_s} = 63$$

for $$\alpha_s = \frac{1}{10}, \ \alpha = \frac{1}{13}$$

Then $$\frac{1}{\alpha_u} = 6.7(9.7-N)$$

This makes $N < 10$.

However, when using $B_{e\gamma} < 1.9 \times 10^{-10}$, we get a better upper limit:

$$B_{e\gamma} < 5.4 \times 10^{-4} \left(\frac{m_L}{m_H}\right)^4$$

where m_L is the mass of the heaviest lepton in the families. Using the measured $B_{e\gamma}$ limit, we get

$$m_L > 0.024\ m_H$$

The bounds of m_H forces m_L to be between 1.2 and 4.1. The mass of the heaviest lepton is m_τ = 1.78 GeV. Colliding beam experiments at Cornell (CESR) and PETRA show most detectable heavy lepton between m_τ to 19 GeV. This makes the τ the only candidate for the heaviest lepton, thus N = 3 maximum.

D. Conclusion.

Models on rare decay of muons may come and go, but extremely sensitive experiments will continue to be performed and the programs at LAMPF, TRIUMPF and SIN to study this phenomenon.

The Collaborators Are:

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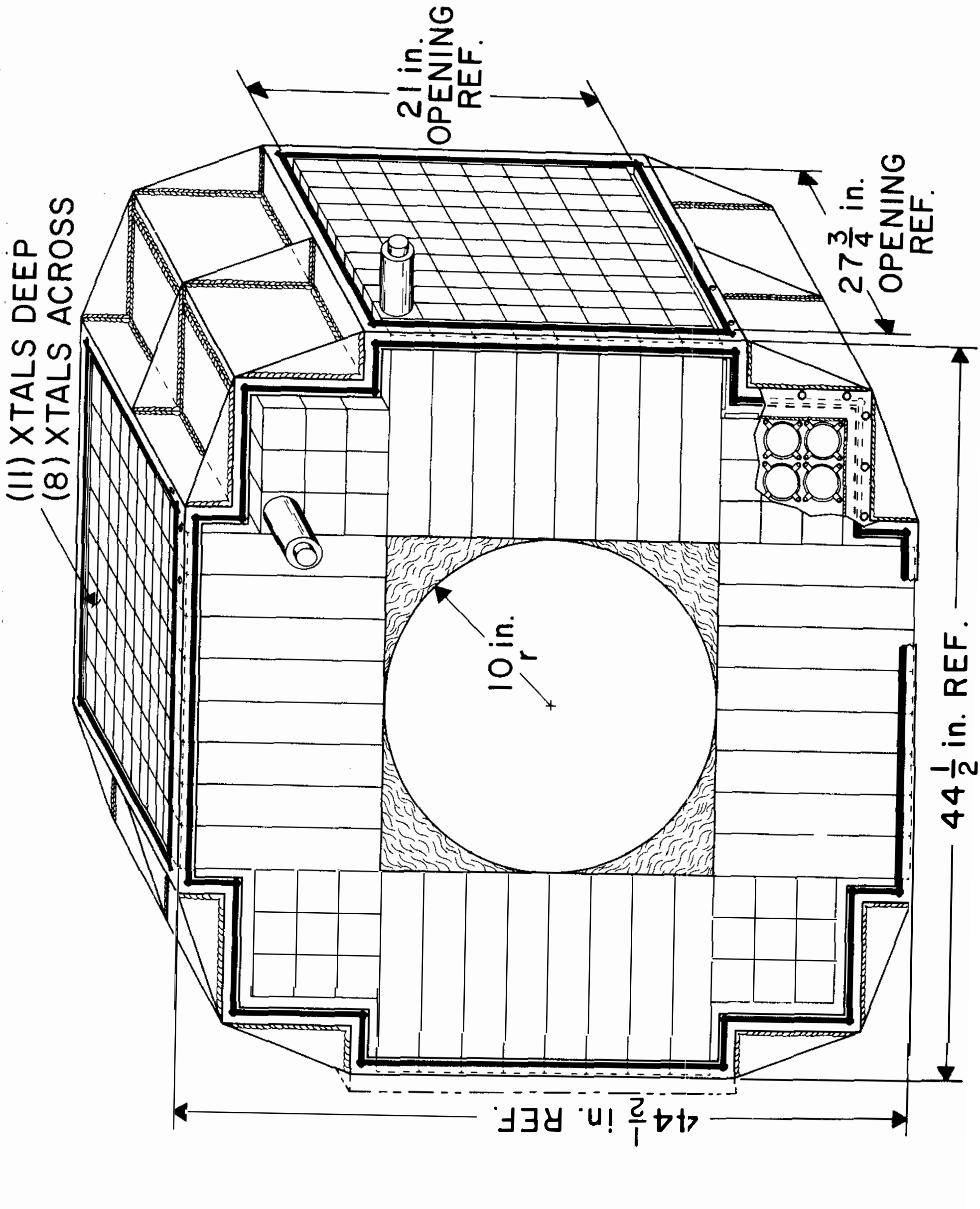
REFERENCES

1. J. D. Bowman, et al, Phys. Rev. Letters 42, 556 (1979).
2. H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974).
3. H. Georgi and C. Jarskog, Harvard University preprint HUTP-79/A026(1979)
4. H. Georgi and D. V.Nanopoulos, Harvard preprint HUTP-79/A001 (1979).
5. J. Bjorken and S. Weinberg, Phys. Rev. Lett. 38, 622 (1977).
6. M. K. Gaillard and B. M. Lee, Phys. Rev. D10, 897 (1974).
7. N. Cabibbo, L. Maiani, G. Parisi, and P. Petronzo, Nucl. Phys.B158, 295 (1979).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(8) XTALS ACROSS
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27 3/4 in. OPENING REF.
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44 1/2 in. REF.
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