

# THE USE OF PLASMA WAVEGUIDES AS ACCELERATING STRUCTURES IN LINEAR ACCELERATORS \*

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At the present time a number of accelerating structures of proton and electron linear accelerators exist which, in a certain range of energy of the particles to be accelerated, and of intensities of accelerating high-frequency fields and wave-lengths, are quite effective.

A waveguide loaded with metallic discs, at a wave phase velocity  $V_\varphi > 0.5 C$ , and at an intensity of the accelerating field up to 100-200 kV/cm. and wave-lengths of the order of 10 cm., is an effective accelerating system for linear electron accelerators.

A cavity resonator with drift tubes can be used to accelerate protons to an energy of 50-100 Mev.

"Pill-box" cavity resonators with drift tubes operating on the  $\pi$  wave can also, apparently, be used in a region of higher energies up to 600 Mev.

It is convenient to use helical waveguides to accelerate protons from low energies of the order of 300-1000 Kev to energies of the order of 30-60 Mev.

All the above-listed accelerating systems, however, are not free from serious drawbacks, some of which are common to all these systems.

1. First of all it is necessary to note the inconvenient distribution of electromagnetic fields in accelerating systems. The electromagnetic fields are concentrated in volumes considerably exceeding the volume in which the beam of particles being accelerated moves. Moreover, in proton accelerators the fields outside the beam are, as a rule, considerably greater than the fields which accelerate the beam. Hence, it is necessary to develop systems in which the electromagnetic energy would be concentrated exclusively in the region where the particles are accelerated. In the case of travelling-wave accelerators a reduction of the area of the electromagnetic energy flux would result in a considerable rise of the field intensity for the given flux. In the case of standing-wave accelerators such a redistribution would increase the shunt impedance of the accelerating system. In both cases this would cause a reduc-

tion of the transverse dimensions of the accelerating systems.

2. Characteristic of the above systems, especially the disc-loaded waveguide and the divided cavity resonators, are big losses; due to large metallic surfaces in which the electromagnetic energy is dissipated. Metallic surfaces are also a cause of possible breakdown and thus limit the maximum intensity of the accelerating field to values of 100-150 kV/cm., and in cavity resonators with drift tubes, to much smaller values, viz., 30-50 kV/cm. This prompts the need to design systems in which the metallic surfaces are at a considerable distance from the regions of high field intensity, or are absent altogether.

3. Accelerating systems, with the exception of helical waveguides, have a cut-off frequency. This results in a rigid dependence of the transverse dimensions of the waveguide upon wave-length. Hence we see the necessity of developing systems without a cut-off frequency or systems with a wide pass band.

4. An important task in the development of linear accelerators is the simultaneous achievement of radial and phase stability. It is therefore necessary to develop such accelerating structures in which this would be achieved without special systems of focusing. In those cases where this is impossible, the accelerating system must offer a choice of effective parameters of the focusing system. Thus, the absence of a cut-off frequency substantially facilitates focusing.

5. The accelerating system must have low dispersion and not severe tolerances in construction.

The development of an accelerating system which would satisfy all the requirements enumerated above is a task involving great difficulties. The present paper discusses one possible accelerating system in which a number of the above-mentioned shortcomings are absent. Naturally, the application of this system meets with a number of difficulties.

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\* Besides the author, L. M. Pyatigorski and N. A. Khizhniak participated in the work of investigating plasma waveguides and interactions of charged particles with them. Part of the calculations have been done by Pakhomov, Gorbatenko, Stepanov, Tkalic and Suprunenko.

For an unbounded plasma at rest the equivalent dielectric constant is  $\epsilon = 1 - \Omega_0^2/\omega^2$ . Hence,  $\epsilon < 1$  and the phase velocity of wave propagation  $V_\varphi > C$ . For frequencies  $\omega < \Omega_0$ ,  $\epsilon$  is negative and the propagation of such waves in the plasma is impossible. If the plasma moves at a uniform velocity  $V_0$ , space-charge waves with a phase velocity  $V_\varphi \simeq V_0$  can be propagated in it, the electromagnetic field-waves in this case still being propagated with a velocity  $V_\varphi < C$ . The dispersion properties of bounded plasma are entirely different. As was first established by Schuman, electromagnetic field-waves can be propagated in a bounded plasma at rest with a phase velocity  $V_\varphi < C$ . In this connection we proposed to use the plasma waveguides as accelerating systems in linear accelerators and as a retarding system in microwave amplifiers and generators\*. To investigate the problems connected with this possibility a number of theoretical investigations have been carried out at the Physico-Technical Institute of the Ukrainian Academy of Sciences since 1952.

The results of these investigations and of Schuman's research work are given in the paper.

From an analysis of the dispersion equation of a waveguide of the plasma rod type

$$-\frac{1}{m} I_0(m_p a) K_1(ma) = \frac{\epsilon_p}{m_p} K_0(ma) I_1(m_p a), \quad (1)$$

where  $a$  is the rod radius,  $\epsilon_p = 1 - \Omega_0^2/\omega^2$ ,  $m = \frac{\omega}{c} \sqrt{1/\beta_\varphi^2 - 1}$ ,  $m_p = \frac{\omega}{c} \sqrt{1/\beta_\varphi^2 - \epsilon_p}$ , and  $I_0$ ,  $K_0$ ,  $I_1$ , and  $K_1$  are Bessel functions, it follows that :

1. The pass band for slow waves lies within the limits

$$0 \leq \omega \leq \Omega_0/\sqrt{2},$$

where  $\Omega_0$  is Langmuir's plasma frequency.

2. The phase velocity of the propagation of electromagnetic waves  $V_\varphi$  changes from  $C$  (the speed of light

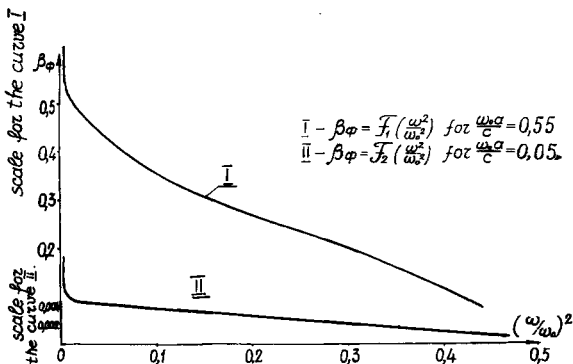


Fig. 1. Dependence of  $\beta_\varphi$  upon  $(\omega/\omega_0)^2$  in a system of coordinates at rest.

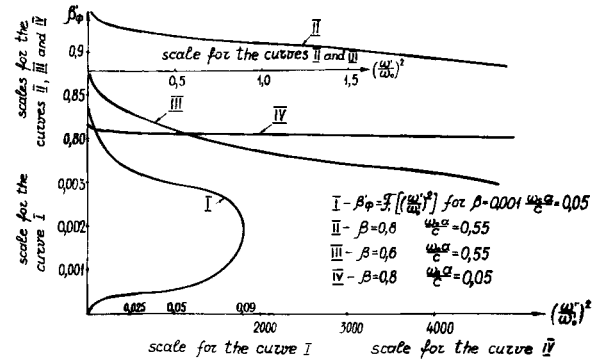


Fig. 2. Dependence of  $\beta'_\varphi$  upon  $(\omega'/\omega_0')^2$  in a moving system of coordinates.

in a vacuum) to 0 when  $\omega$  changes in the interval

$$0 \leq \omega \leq \Omega_0/\sqrt{2}$$

$\beta_\varphi$  is plotted as a function of  $(\omega/\Omega_0)^2$  on fig. 1.

To determine the dispersion dependence in the case of a plasma rod moving at a uniform velocity  $V$ , it is sufficient now merely to make use of the transformations of frequency and phase velocity

$$\omega' = \frac{\omega(1 + \beta/\beta_\varphi)}{\sqrt{1 - \beta^2}}, \quad \beta'_\varphi = \frac{\beta_\varphi + \beta}{1 + \beta\beta_\varphi} \quad (2)$$

$\omega'$  and  $\beta'_\varphi = V_\varphi'/c$  are respectively the frequency and phase velocity of the wave in the laboratory reference frame;  $\omega$  and  $\beta_\varphi$  are the same quantities in a reference frame where the rod is at rest. The relationship between  $\omega$  and  $\beta_\varphi$  is defined by the dispersion eq. (1). Drawing upon (1) and (2), we obtain the dispersion dependence for a moving plasma rod

$$\beta'_\varphi = F[(\omega'/\Omega_0')^2]$$

This is plotted in fig. 2.

The case of gyrotropic plasma waveguides was examined in detail only for high magnetic fields by L.M. Pyatigorski and the general case lately by Glagolev.

An examination of fig. 2 reveals that waves with a frequency greater than  $\Omega_0/\sqrt{2}$  can also be propagated in a moving plasma rod. The phase velocity of wave propagation can be either smaller or greater than the phase velocity in a plasma rod at rest. This depends on whether the wave is propagated in or against the direction of the plasma-rod movement. As in the case of the plasma-rod at rest the pass band is not limited below.

\* As wave-guide systems electronic beams or Gabor lenses may also be used.

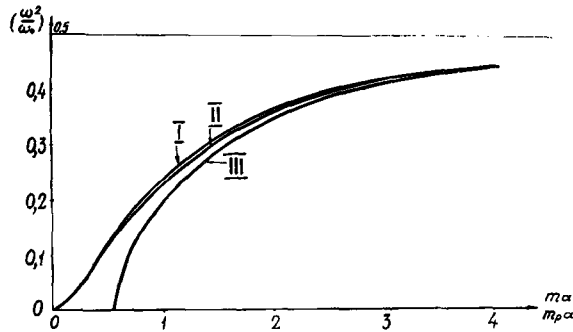


Fig. 3. I -  $(\omega/\omega_0)^2 = f_1(ma)$  for  $(\omega_0 a)/c = 0,05$   
 II -  $(\omega/\omega_0)^2 = f_2(ma)$  for  $(\omega_0 a)/c = 0,55$   
 III -  $(\omega/\omega_0)^2 = f_3(m_{pa})$  for  $(\omega_0 a)/c = 0,55$

3. For definite parameters of the system  $\Omega_0 a/c$  a considerable part of electromagnetic energy may be confined within the plasma rod. The dependence of  $ma$ ,  $m_{pa}$  inside and outside the plasma upon  $(\omega/\Omega_0)^2$  is plotted in fig. 3.

From fig. 3 it follows that even at  $ma \approx 2$ , the flux within the plasma rod is of the same order as outside the rod. It should be noted that at very high values of  $ma$ , we have a pure surface wave. In this case it is expedient to accelerate the particles moving close to the boundary of plasma. At very low values of the parameter  $ma$ , a great energy flux is concentrated outside the plasma rod.

Plasma waveguides thus have a number of distinctive features which make it convenient to use them as accelerating systems:

1. The possibility of phase-velocity variation within a wide range, starting from very low phase velocities.
2. The absence of a limiting frequency below. The upper limiting frequency is determined only by the density of the plasma and the radius of the rod  $\Omega_0 a/c$ .
3. Insofar as the electromagnetic energy is practically concentrated within the plasma waveguide, whose transverse dimensions may be small, the field intensity for a given energy flux can be materially increased.
4. There are no metallic surfaces in plasma waveguides, a fact which substantially reduces losses and the possibility of breakdown. If design considerations dictate the need to place the beam in a metallic waveguide, this can be placed in a region where the field is very low, since the dispersion properties of the system are determined by the plasma itself.
5. The foregoing results are based on a linear approximation of the theory of wave propagation in a bounded plasma. They are valid only if

$$e E \lambda / 2\pi m c^2 \beta \ll 1$$

This means that the particle-velocity change in the wave field must not exceed the phase velocity of the wave.

6. If a plasma rod is placed between two metallic plates or in a cylindrical cavity resonator, the cavity resonator thus obtained will possess a number of important characteristics. Its resonant frequency at frequencies  $\omega < \Omega_0$  and  $ma > 1$  will be determined only by the parameters of the plasma rod. The limiting frequency can therefore be substantially lowered. Thanks to a high quality of the system, a resonator of this type makes it possible to concentrate the energy of a high-frequency field in the plasma rod. The dispersion dependence for a plasma cavity-resonator is of the same type as (1).

As distinct from (1), the arguments of Bessel functions  $I_0(x)$ ,  $I_1(x)$ ,  $K_0(y)$ ,  $K_1(y)$  are:

$$x = r_0 \sqrt{K_n^2 - \frac{\omega^2}{c^2} + \frac{\Omega_0^2}{c^2}}$$

$$y = r_0 \sqrt{K_n^2 - \frac{\omega^2}{c^2}} \quad K_n = \frac{n\pi}{l} \quad \text{and} \quad \omega_n = K_n C,$$

where  $l$  is the length of the resonator. In the case of  $y \ll 1$  and  $\Omega_0 \gg \omega_1 \omega_n$ , the dispersion equation takes the form

$$\xi I_n \xi - \Delta \xi = -\eta; \quad \xi = \frac{1}{2}(\gamma y)^2; \quad \Delta = \frac{z I_0(v)}{v I_1(v)};$$

$$\eta = \frac{2 I_0(v)}{v I_1(v)} \left( \frac{\gamma \Omega_0 r_0}{2C} \right)^2; \quad \gamma = 1,78; \quad v = \frac{r_0 \Omega_0}{C}$$

The quality  $Q$  of such a system was calculated in order to determine the efficiency of the plasma cavity-resonator.

In calculating  $Q$  the dielectric constant  $\epsilon$  and the conductivity  $\sigma$  are taken equal

$$\epsilon = 1 - \frac{\Omega_0^2}{\omega^2 + \nu^2_{eff}} \quad \sigma = \frac{1-\epsilon}{4\pi} \nu_{eff}$$

The  $Q$ -values for several plasma cavity-resonators are listed in Table I. Table II gives the energy losses in the waveguide\*\*. It follows from this that plasma cavity-resonators have a sufficiently high quality. This is also an indication that plasma waveguides can become a highly effective accelerating system.

7. Space-charge waves can also be utilized to accelerate charged particles. As distinct from the linear approximation, the phase velocity of space-charge wave propagation in a plasma at rest is not equal to  $C$ . It can assume values less than  $C$ . The maximum intensity of the accelerating field in this case is

$$E_{max} = p\sqrt{2} \sqrt{4\pi n + eV}$$

\* The relationship also follows from the requirement that the electron displacement in the wave field be considerably less than the wave length in the medium. Non-linear theory must therefore be applied to analyse the efficiency of plasma waveguides.

\*\* The data given in table II, I are not optimum ones and are of only an illustrative and preliminary nature.

TABLE I  
Efficiency of plasma cavity-resonator

$r_0$	1 cm.	1 cm.	1 cm.
$\eta_0$	$10^8$	$10^9$	$10^{10}$
$\omega_0$	$5,64 \cdot 10^8$	$1,78 \cdot 10^9$	$5,64 \cdot 10^9$
$\omega_H$	$9,42 \cdot 10^8$	$9,42 \cdot 10^8$	$9,42 \cdot 10^8$
$\omega$	$2,37 \cdot 10^7$	$7,49 \cdot 10^7$	$2,37 \cdot 10^8$
$\varepsilon$	-566	-566	-566
$\sigma$	$2,1 \cdot 10^5$	$1,9 \cdot 10^6$	$1,71 \cdot 10^7$
Q	5100	1700	560

$$T = 10^4 \quad l = 100 \text{ cm.}$$

$$\varepsilon' = \frac{x}{y} \frac{I_0(x) K_1(y)}{I_1(x) K_0(y)}; \quad \begin{aligned} x &= x_1 r_0 = r_0 \sqrt{K_n^2 - \frac{\omega^2}{c^2} + \frac{\omega_0^2}{c^2}}, \quad \varepsilon' = -\varepsilon \\ y &= x_2 r_0 = r_0 \sqrt{K_n^2 - \frac{\omega^2}{c^2}}, \quad K_n = \frac{n\pi}{l} \end{aligned}$$

$$Q = \frac{\omega}{8n\sigma} \frac{\varepsilon^0 [I_0^2(x) - I_1^2(x)] + \frac{\varepsilon^0(K_n^2 + \varepsilon K^2)}{x_1^2} [I_1^2(x) - I_0(x)I_2(x)] + \frac{I_0^2(x)}{K_0^2(y)} [K_1^2(y) - K_0^2(y) + \frac{K_n^2 + K^2}{x_2^2} (K_0(y)K_2(y) - K_1^2(y))]}{I_0^2(x) - I_1^2(x) + \frac{K_n^2}{x_1^2} [I_1^2(x) - I_0(x)I_2(x)]}$$

$$\varepsilon^0 = 2 - \varepsilon, \quad \omega_0 = \sqrt{\frac{4\pi n_0 e^2}{m}}, \quad \varepsilon = 1 - \frac{\omega_0^2}{\omega^2 + v^2 ef}$$

$$v_{ef} = v_{efm} + v_{efl}$$

$$v_{efl} = \frac{5,5 \text{ Ni}}{T^{3/2}} \ln \left( 220 \frac{T}{N^{1/3}} \right)$$

$$v_{efm} = 1,7 \cdot 10^{11} \frac{\text{Nm}}{2,7 \cdot 10^{19}} \sqrt{\frac{T}{300}}$$

TABLE II  
Energy losses in plasma wave-guide

$x = y$	1	2	3	4
$\omega_0$	$1,5 \cdot 10^9$	$1,5 \cdot 10^9$	$1,5 \cdot 10^9$	$1,5 \cdot 10^9$
$n_0$	$7 \cdot 10^8$	$7 \cdot 10^8$	$7 \cdot 10^8$	$7 \cdot 10^8$
Kz	1	2	3	4
$\omega$	$0,74 \cdot 10^9$	$0,9 \cdot 10^9$	$0,96 \cdot 10^9$	$0,99 \cdot 10^9$
$\sigma$	$10^4$	$0,66 \cdot 10^4$	$0,58 \cdot 10^4$	$0,55 \cdot 10^4$
D( $\omega/m$ )	320	530	1700	6700

$$D = \frac{\sigma E_0^2 I_0^2}{2} \left\{ I_0^2(x) - I_1^2(x) + \frac{Kz^2}{x_1^2} [I_1^2(x) - I_0(x)I_2(x)] \right\}$$

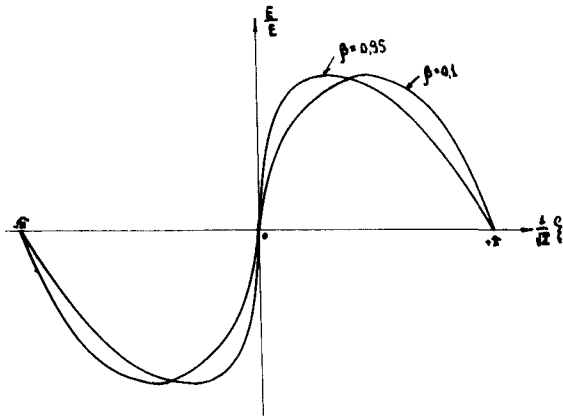


Fig. 4.

The parameter  $p$  varies within the limits  $0 \leq p \leq 1$ .  $eV = mV_\phi^2/2$ ;  $V_\phi$  is the wave phase velocity.

The shape of the field and the mode of its variation in the direction of wave propagation may vary within a wide range (fig. 4). In particular, the derivative  $\partial E_z / \partial z \sim v$  which determines the frequencies of phase oscillations may, unlike the usual linear approximation, be made large.

Density distribution is sharply different from that in the linear case. In place of the relatively smooth sinusoidal law, we may obtain (considering the space charge) very high values of alternating density components (fig. 5). This factor does not increase the field intensity, since the field is  $E_z = f \rho dz$ , but it can be utilized when it is necessary to obtain a high charge density.

Non-relativistic non-linear one-dimensional plasma is considered in the papers of Bohm and Gross<sup>1</sup>; Akhiezer and Liubarski; Akhiezer, Liubarski and Fainberg<sup>2</sup>. A number of two-dimensional cases have been dealt with by the author.

8. In linear accelerators with a constant or slowly changing synchronous phase, the simultaneous attainment of radial and phase stability without special focusing devices is known to be impossible.\* This, however, has been proved only for the case of axially symmetric fields, for which the relation  $\text{div } \mathbf{E} = 0$  holds true. In the case of anisotropic or gyrotropic media  $\text{div } \mathbf{E} \neq 0$ . Therefore we may expect that in definite conditions simultaneous radial and phase stability is possible in this case. This possibility will become all the more apparent if it is noted that in the case of an anisotropic medium the electric field components are  $E_z = E_0 I_0(k_1 r)$

$$E_r = i \frac{\epsilon_z}{\epsilon_r} \cdot E_0 \frac{K_3}{K_1} \cdot I_1(Kr)$$

$$K_1 = K^2 \epsilon_z - \epsilon_z / \epsilon_r \cdot K_3^2$$

As pointed out by the author of the present paper, in the case of  $\epsilon_z / \epsilon_r < 1$ , the radial defocusing forces can be substantially reduced, while in the case of  $\epsilon_z / \epsilon_r < 0$  radial defocusing will be replaced by radial focusing simultaneously with phase focusing. Laminar plasma, plasma placed in a magnetic field, and a waveguide with a laminated dielectric for frequencies  $\omega < \omega_0$  (the critical waveguide frequency without a dielectric of the same radius) can be used as an anisotropic medium with negative values of  $\epsilon_z$  or  $\epsilon_r$ . A more detailed examination of the focusing effect was carried out for laminar plasma waveguide\*\*.

In the case of an arbitrary field distribution it may be demonstrated that to achieve radial stability, the quantity determining the frequency of radial oscillations must satisfy the condition

$$\left. \frac{\epsilon_z E_z}{VT} \right|_0^L - \frac{1}{V^2 T} (1 - \epsilon \beta^2) \frac{\partial \Delta w}{\partial t_0} > 0.$$

Since the necessary condition is

$$\partial \Delta w / \partial t_0 > 0,$$

radial focusing will be achieved if  $1 - \epsilon \beta^2 < 0$ . In waveguides containing a dielectric this condition can be observed. In the case of an anisotropic medium,  $\text{div } \mathbf{E} \neq 0$ ; therefore the condition of radial stability assumes the form

$$\left. \frac{\epsilon_z}{\epsilon_r} \frac{e}{VT} E_z \right|_0^L - \frac{1}{V^2 T} (1 - \epsilon_r \beta^2) \frac{\epsilon_z}{\epsilon_r} \frac{\partial \Delta w}{\partial t_0} > 0.$$

Consequently, simultaneous radial and phase stability can be achieved if  $\epsilon_z$  and  $\epsilon_r$  have different signs, or if

$$\epsilon_r \beta^2 > 1$$

For a vacuum similar relations were obtained by McMillan<sup>3</sup>. In the case of an arbitrary axially symmetric anisotropic medium the frequencies of radial and phase

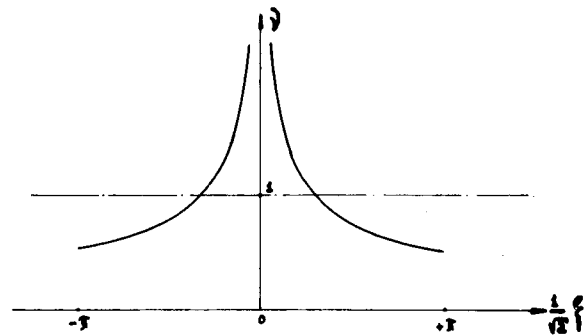


Fig. 5.

\* In the general case of an arbitrarily shaped field this assumption was proved by A. I. Akhiezer and G. Ia. Liubarski (1948) and E. M. McMillan<sup>3</sup>.

\*\* This case was examined by the author together with N. A. Khizhniak. The case of a waveguide loaded with a dielectric was considered by N. A. Khizhniak.

oscillations are respectively

$$\Omega_r^2 = -\frac{\pi e E_0}{m \beta \lambda} \frac{\epsilon_z}{\epsilon_r} \sin \psi_s (1-\beta^2)^{1/2} (1-\beta^2 \epsilon_r)$$

$$\Omega_q^2 = \frac{2\pi e E_0}{m \beta \lambda} \sin \psi_s (1-\beta^2)^{3/2}$$

Therefore if  $\epsilon_z$  and  $\epsilon_r$  have different signs, the requirement  $\Omega_r^2 > 0$  and  $\Omega_q^2 > 0$  may be satisfied simultaneously and simultaneous radial and phase stability ensured in this way. In the specific case of a laminar plasma waveguide the effective values of the dielectric constants  $\epsilon_z$  and  $\epsilon_r$  — provided the repetition length  $L \gg \beta \phi \lambda$  — are equal to

$$\epsilon_z = \frac{1 - \frac{\Omega_0^2}{\omega^2}}{1 - \frac{a}{L} \frac{\Omega_0^2}{\omega^2}}; \quad \epsilon_r = 1 - \frac{b}{L} \frac{\Omega_0^2}{\omega^2},$$

where  $L = a + b$ ;  $a$  is the distance between plasma laminae, and  $b$  is the thickness of the plasma laminae,

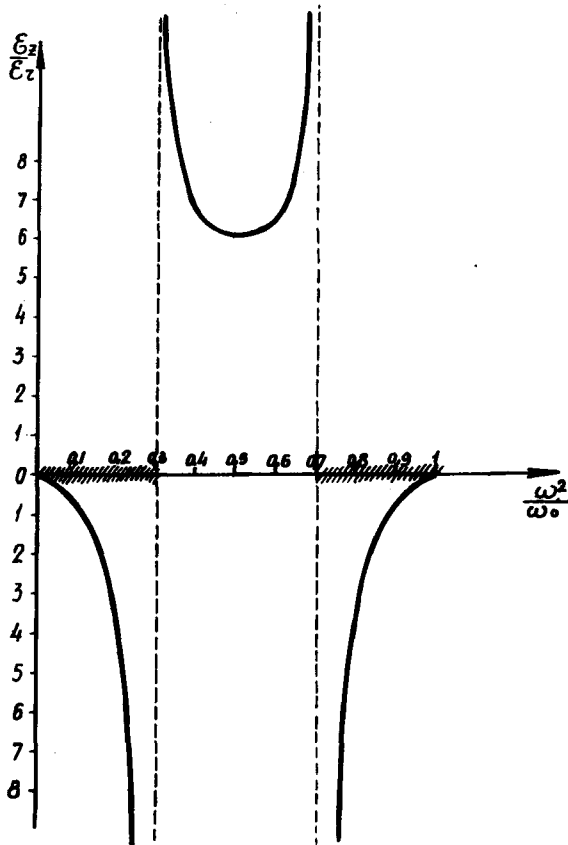


Fig. 6. Dependence of  $\epsilon_z/\epsilon_r$  upon  $\omega^2/\Omega_0^2$  for  $b/L = 0.6$ . The shaded region corresponds to simultaneous radial and phase stability.

while  $\Omega_0^2 = 4\pi e^2 n/m$  is plasma frequency. Fig. 6 gives graphs for the regions of simultaneous radial and phase stability. The regions of simultaneous radial and phase stability are shaded. The dispersion equation for a laminar plasma wave guide in the case of a plasma rod is

$$\sqrt{\epsilon_z \epsilon_r} \sqrt{\frac{1-\beta^2}{\epsilon_r \beta^2 - 1}} \frac{J_1\left(\frac{KR}{\beta} \sqrt{\frac{\epsilon_z}{\epsilon_r} \sqrt{\epsilon_r \beta^2 - 1}}\right)}{J_0\left(\frac{KR}{\beta} \sqrt{\frac{\epsilon_z}{\epsilon_r} \sqrt{\epsilon_r \beta^2 - 1}}\right)} = \frac{K_1 \left[ \frac{KR}{\beta} \sqrt{1-\beta^2} \right]}{K_0 \left[ \frac{KR}{\beta} \sqrt{1-\beta^2} \right]}$$

For a metallic waveguide completely filled with laminar plasma,

$$K^2 \left( \epsilon_z - \frac{\epsilon_z}{\epsilon_r} \frac{1}{\beta^2} \right) = \left( \frac{\alpha i}{R} \right)^2$$

These numerical calculations show that in regions of simultaneous radial and phase stability, the phase velocity may be smaller than the phase velocity of light in vacuum  $V_\phi < C$ . This points to the possibility of simultaneously attaining radial and phase stability and the convenience of focusing plasma waveguides and cavity resonators as accelerating systems in linear accelerators.

In connection with the possibility of using plasma waveguides as accelerating systems in linear accelerators and as retarding systems in microwave amplifiers and generators, the problem of determining the energy losses of charged particles travelling in such waveguides arises.

In the case of the uniform motion of a charged particle through the plasma rod there are two types of losses: polarization losses and losses due to Vavilov-Čerenkov radiation:

$$\left( \frac{d\epsilon}{dx} \right)_{\text{cer}} = -\frac{2q^2 \omega^3}{av \Omega_0^2} (1-\beta^2)^{1/2} \frac{[1-\beta^2 \epsilon(\omega)]^{3/2}}{2-\beta^2 \epsilon} I_0^{-1}(k_1 a) I_1^{-1}(ka)$$

$$\frac{k_1 I_0(k_1 a)}{I_1(k_1 a)} = -\frac{\epsilon k_2 K_0(k_2 a)}{K_1(k_2 a)}$$

$$k_1 = \frac{|\omega|}{v} \sqrt{1-\beta^2 \epsilon(\omega)}; \quad k_2 = \frac{|\omega|}{v} \sqrt{1-\beta^2};$$

$$\epsilon = 1 - \frac{\Omega_0^2}{\omega^2}$$

When a charged particle moves in a laminar plasma waveguide and the repetition length is  $L \ll \beta \lambda$ , there are not

only polarization losses, but also a Čerenkov radiation of the type observed in anisotropic media. The polarization losses in this case are equal to

$$\left(\frac{d\varepsilon}{dz}\right)_{\text{polar}} = \frac{4\pi n e^4}{m v^2} \frac{b}{L} \ln \frac{Kmb}{7,4}$$

The losses due to Čerenkov radiation are

$$\left(\frac{d\varepsilon}{dz}\right)_{\text{cer}} = \frac{2\pi n e^4}{m v^2} \frac{b}{L} \ln \left( \frac{L}{b} \frac{k^2 v^2}{\Omega_0^2} \right)$$

If the repetition length  $L$  is comparable with  $\beta\lambda$  there is a parametric Čerenkov effect\*.

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\* The losses of a particle in laminar media form the subject of a paper by Ia. B. Fainberg and N. A. Khizhniak.