AXIAL FIELD PLASMA BETATRON

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(presented by T. Ohkawa)

1. INTRODUCTION

In recent years, a great deal of interest has arisen in both the theoretical and the experimental study of neutralized electron beams. At the previous Symposium held in 1956, G. I. Budker[1] proposed a radiation cooled stationary state of neutralized relativistic electron beams and discussed how these beams could be used as strong focusing guiding fields of high energy accelerators. It has been found that all electron beams so far achieved by ordinary accelerators have too low an intensity to realize this steady state. In order to correct this deficiency, several proposals[2-6] for promising schemes have been made and some laboratories have started constructing their own machines[5, 7].

On the other hand, in the field of controlled thermonuclear fusion, there seems to be another branch of applications of relativistic electron beams. The feasibility of trapping ions in negative potentials of neutralized relativistic electron beams has been investigated by G. I. Budker[8] and G. Miyamoto and others[9]. N. C. Christophilos[10] has proposed the idea of the Astron which uses a relativistic electron layer to confine a thermonuclear plasma.

Plasma betatrons of different types have now been constructed at several laboratories, and electron beams of the order of amperes have been obtained.

Most of these machines use perpendicular magnetic fields to deflect beams into circular rings. However, it seems that a steady axial magnetic field is suitable as a guiding field for plasma betatrons. First, no requirement such as the betatron condition exists; hence a current limit such as the one suggested by Ch. Maisonnier and D. Finkelstein[11] is not set at all. Moreover, the field can act on all particles independent of their energy, so that it will be useful to trap a pre-ionized plasma, and confine delayed electrons that begin to run away or are detached from atoms some time after the accelerating field is applied.

Electrons travel in a spiral motion along the magnetic line of force and gradually drift aside, upwards or downwards, as a result of curving lines of force. The drifts may be cancelled out after many revolutions around the machine by applying a figure-eight twist to the magnetic line of force, as was originally proposed by L. Spitzer Jr.[12]. The configuration is sketched in Fig. 1.

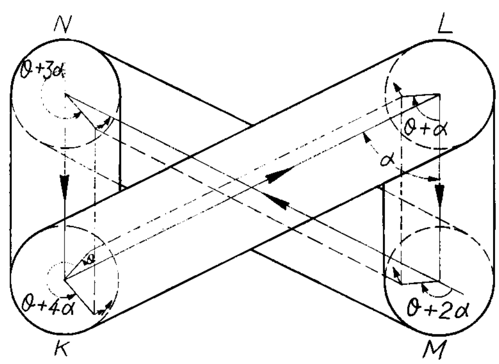

Fig. 1 The configuration of figure-eight twist.

2. CONFINEMENT OF CHARGED PARTICLES

When the magnetic field is almost uniform and, does not change appreciably in a distance of about

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one gyroradius, the motion of electrons can be adequately described by that of their guiding centres. At first we take the single particle case, assuming that the magnetic self-field is unimportant.

The drift velocity of a charged particle with mass m, charge e and parallel and transverse velocities, $v_{\parallel}$ and $v_{\perp}$ respecively, in a vacuum magnetic field is given by,

$$\underset{\sim}{v_{\alpha}} = (m/eBR)(v_{\parallel}^2 + v_{\perp}^2/2)\,\underset{\sim}{b} \tag{1}$$

where R is the radius of curvature of the magnetic line of force and $\underset{\sim}{b}$ is the binormal unit vector of the line. Therefore, the drift displacement $\underset{\sim}{dx_d}$ of the guiding centre while it is proceeded by ds along the line of force, is expressed by,

$$\underset{\sim}{dx_d} = (m/eBR)(v_{\parallel}^2 + v_{\perp}^2/2)\,\underset{\sim}{b}\,(ds/v_{\parallel}) \tag{2}$$

The second factor can be rewritten as follows,

$$\frac{v_{\parallel}^2 + \dfrac{v_{\perp}^2}{2}}{v_{\parallel}} = (v^2 - \tfrac{1}{2}v_{\perp}^2)/\sqrt{v^2 - v_{\perp}^2} = v\left(1 + \frac{3}{8}\frac{v_{\perp}^4}{v^4} + \dots\right),$$

and the fourth order term may be neglected for axially accelerated particles, leading to the formula

$$\underset{\sim}{dx_d} = (mv/eBR)\,\underset{\sim}{b}\,ds\,. \tag{3}$$

Fig. 1 shows the side view of the torus. Sectors KL and MN are halves of a circular torus and sectors LM and NK are connecting sections. The position of the guiding centre is described by (r, θ) co-ordinates, as can be seen in the figure. While passing through the sector KL where field intensity $B(R) = B_0 R_0/R$, the particle displaces in binormal direction by the distance,

$$\int dx_d = \frac{mv}{eB_0 R_0}\cdot \pi R = \frac{mv\pi}{eB_0 R_0}\cdot R \equiv aR_0, \tag{4}$$

where R is the major radial vector and is equal to $R_0 + r\cos\theta$, and the suffix zero is referred to the quantities at the central axis.

Assuming that ratios r/R and ρ/r, where ρ is the gyroradius and is equal to mv/eB_0R_0, are small quantities of the first order, we can obtain the transformation formulas correct up to the second order (Fig. 2),

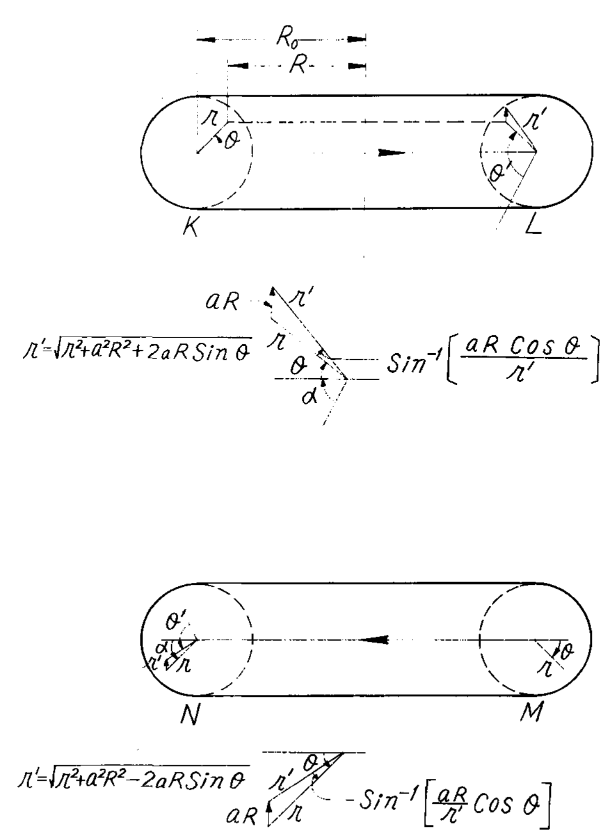

Fig. 2 The figure-eight geometry.

$$r_L = r_K + aR_0\sin\theta_K + \tfrac{1}{2}\frac{a^2R_0^2\cos^2\theta_K}{r_K} - ar_K\sin\theta_K\cos\theta_K\,, \tag{5}$$

$$\theta_L = \theta_K + \alpha + \frac{aR_0}{r_K}\cos\theta_K - \left(\frac{aR_0}{r_K}\right)^2\cos\theta_K\sin\theta_K - a\cos^2\theta_K \tag{6}$$

$$r_M = r_L\,, \tag{7}$$

$$\theta_M = \theta_L + \alpha\,, \tag{8}$$

where α is defined in Fig. 1. Similar formulas can also be derived for M to N and N to K. After repeating a transformation corresponding to one full revolution around the machine, the radial co-ordinate r_N after N turns is expressed by,

$$r_N = r_0 - \pi\rho\frac{\sin\alpha\cos\{\theta_0 + (2N-1)\alpha\}}{\cos\alpha} + \frac{\pi^2\rho^2}{r_0}A(\theta_0, N, \alpha) + \frac{\pi\rho r_0}{R_0}B(\theta_0, N, \alpha)\,, \tag{9}$$

where $A(\theta_0, N, \alpha)$ and $B(\theta_0, N, \alpha)$ are periodic factors of N of the order of unity, except when α is near the integral multiples of $\pi/2$. It can be seen from Eq. (9) that, as far as the second order approximation is concerned, the orbit of the particle is in neutral equilibrium.

Now we take into consideration the effect of magnetic self field, but the field is assumed again to be smaller than the external magnetic field by a factor $\alpha \ll 1$. The drift velocity $\underset{\sim}{v_d}$, when a volume current is present, is given by,

$$\underset{\sim}{v_d} = \frac{m}{2e}(v^2 - v_\perp^2/2)\frac{\underset{\sim}{B} \times \nabla B^2}{B^4} + \frac{mv_\parallel^2}{e}\frac{\underset{\sim}{B} \times (\text{rot}\underset{\sim}{B} \times \underset{\sim}{B})}{B^4} \qquad (10)$$

An infinitesimal displacement $\underset{\sim}{dv_g}$ of the guiding centre drifting aside with the velocity $\underset{\sim}{v_d}$ from the line of force is

$$\underset{\sim}{dr_g} = \underset{\sim}{dr_m} + \underset{\sim}{v_d}\, ds_m/v_\parallel \qquad (11)$$

where $\underset{\sim}{dr_m}$ is the infinitesimal vector along the line of force satisfying,

$$\frac{h_1 dx_{m1}}{B_1} = \frac{h_2 dx_{m2}}{B_2} = \frac{h_3 dx_{m3}}{B_3}, \qquad (12)$$

in a reference frame with its diagonal metric (h_1^2, h_2^2, h_3^2). From Eqs. (11) and (12), follows the equation of the guiding centre,

$$\frac{h_1 dx_{g1}}{B_1 + Bv_{d1}/v_\parallel} = \frac{h_2 dx_{g2}}{B_2 + Bv_{d2}/v_\parallel} = \frac{h_3 dx_{g3}}{B_3 + Bv_{d3}/v_\parallel}. \qquad (13)$$

Taking a co-ordinate system with $h_1 = 1$, $h_2 = r$ and $h_3 = 1 + \kappa r \cos\theta$, that is a polar co-ordinate system based on a circular ring of radius $1/\kappa$, the magnetic field can be expanded in Fourier series,

$$B_r = (1 + \kappa r \cos\theta)^{-1} B_{oe} \alpha\kappa a \sum_{n=1}^{\infty} a_n^*(r) \sin n\theta,$$

$$B_\theta = (1 + \kappa r \cos\theta)^{-1} B_{oe} \alpha \left\{ c_0(r) + \kappa a \sum_{n=1}^{\infty} c_n^*(r) \cos n\theta \right\},$$

$$B_z = (1 + \kappa r \cos\theta)^{-1} B_{oe} \left\{ 1 + \alpha b_0(r) + \alpha\kappa a \sum_{n=1}^{\infty} b_n^*(r) \cos n\theta \right\}, \qquad (14)$$

where $a^*(r)$ and $c^*(r)$ are related to each other by

$$\frac{d}{dr}(ra^*(r)) = nc_n^*(r) \qquad (15)$$

$c_0(r)$, $b_0(r)$, $a^*(r)$, $b^*(r)$ and $c^*(r)$ are functions of r of the order of unity and a represents the beam radius. It may be noted here that the coefficients of the higher order in the series are smaller than those of the lowest order by a factor κa.

In reality the magnetic field can be calculated from the motions of charged particles and the latter can be derived from the former, hence it may be possible to solve them in a self-consistent way. However, in this paper, we only pursue the orbits of charged particles in a magnetic field given by Eq. (14) and investigate whether they are able to be confined or not.

In the case of circular torus, radial co-ordinate r can be worked out regarding κa and $\delta = \rho/a$ as small parameters of the first order,

$$r\ (\theta) = r_0 + x_1(\theta) + x_2(\theta),$$

$$x_1(\theta) = \frac{\kappa a}{c_0(r_0)} \cdot r_0 \sum_{n=1}^{\infty} u_n(r_0)\{-\cos n\theta + \cos n\theta_0\},$$

$$x_2(\theta) = \sum_{n=1}^{\infty} q_n(r_0)\{-\cos n\theta + \cos n\theta_0\}, \qquad (16)$$

with $q_n(r_0)$ given by,

$$q_n(r_0) = p_n(r_0) + (\kappa a)^2 \frac{a^2}{n} \cdot \frac{du_n(r_0)}{dr_0} \cdot \left\{ \sum_{k=1}^{\infty} u_k(r_0) \frac{1}{k} \cos k\theta_0 \right\}$$

$$- \frac{\kappa a \cdot \alpha a}{n} \left[\sum_{k=0}^{n} u_k(r_0) W_{n-k}(r_0) + \tfrac{1}{2} \sum_{k=0}^{\infty} \left\{ u_{n+k}(r_0) W_k(r_0) - u_k(r_0) W_{n+k}(r_0) \right\} \right]$$

$$- \frac{(\kappa a)^2 a^2}{n} \left[(1 - \delta_{n1}) \sum_{k=0}^{n-2} \frac{du_{k+1}(r_0)}{dr_0} u_{n-k+1}{}^{(r_0)} + \tfrac{1}{2} \sum_{k=0}^{\infty} \left\{ \frac{du_{n+k+1}{}^{(r_0)}}{dr_0} u_{k+1}{}^{(r_0)} - \frac{du_{k+1}{}^{(r_0)}}{dr_0} u_{n+k+1}(r_0) \right\} \right],$$

and

$$\begin{aligned} u_n(r_0) &= (r_0/c_0(r_0)a)\{a_n^*(r_0)-(\delta/\alpha)A_1\delta_{n1}\}\,, \\ W_n(r_0) &= c_0(r_0)^{-1}\{-c_0(r_0)b_0(r_0)\delta_{n0}+(\delta/\alpha)ab_0'(r_0)(A_1-A_2)\delta_{n0} \\ &\qquad +(\kappa r_0/\alpha)c_0(r_0)\delta_{n1}-(\kappa a\delta/\alpha^2)A_1\delta_{n1}+(1-\delta_{n0})(\kappa a/\alpha)c_n^*(r_0)\}\,, \\ p_n(r_0) &= \{\kappa a r_0/c_0(r_0)\}\{-(\alpha/n)\,b_0(r_0)a_n^*(r_0)+(\delta a/r_0)(A_1-A_2)b_n^*(r_0) \\ &\qquad +(\tfrac{1}{2})(\kappa r_0/n)(a_{n+1}^*(r_0)+(1-\delta_{n1})a_{n-1}^*(r_0)) \\ &\qquad +(-Q_1+1)\delta A_1 b_0(r_0)\delta_{n1}+(\tfrac{1}{4})(\delta/\alpha)\kappa r_0 Q_2 A_1\delta_{n2}\}\,. \end{aligned} \tag{17}$$

where A_1, A_2, Q_1 and Q_2 have a connection with the transverse—to—parallel ratio of the particle velocity and are constants of the order of unity, and δn_1 etc. are Kronecker deltas. It results from this equation that there exists no monotonous increase or decrease in radial position in the present approximation, while the particle goes around the machine. Higher order terms may be divergent, convergent or oscillating. But even if they are divergent, no marked change in radial co-ordinate appears as long as the product of the number of revolutions N and $(\kappa a)^i\ \delta^j\ \alpha^k$, with $i+j+k=3$, is smaller than unity. For example, taking these parameters equal to 0.1, N can amount to one thousand turns.

When a figure-eight twist is applied, it may be found also that r is the oscillating function of θ, as long as the change in θ during one full revolution is not near an integral multiple of 2π.

The beam current may be restricted by the so-called resonance phenomena. If the locus of a guiding centre forms a closed loop, small disturbances, for example irregularities of the magnetic field, may push the particle aside and it will finally be lost. In one revolution θ increases by an amount of $\tau_o - \tau_s(r)$ where τ_o is the rotational angle when self field is zero and $\tau_s(r)$ is the rotational angle due to the self field and nearly equal to $B_\theta(r)\,2\pi R/B_{0e}r$. From this, the condition that line of force closes after N revolutions, that is the resonance condition, results. The off-resonance condition is,

$$\{\tau_o-\tau_s(r)\}N \neq 2\pi n, \text{ for arbitrary r, N and n,} \tag{18}$$

Equation (18) may be replaced by the following,

$$\min_{n,N}\left|\tau_0-\frac{2\pi n}{N}\right| > \frac{2\pi R_0}{B_{0e}}\max_r\left|\frac{B_\theta(r)}{r}\right|, \tag{19}$$

and this gives an upper limit to the beam current. However, the relation looks very severe because, by taking a pair of suitable values of n and N, the left hand side can be made small arbitrarily. Practically, cases with the lower order of N seem to be important. Taking $N=1$ and assuming a surface distribution of current of beam radius a, we obtain,

$$\min_n\left|\tau_0-2\pi n\right| > \frac{2\pi R_0}{B_{0e}}\cdot\frac{B_\theta(a)}{a}, \tag{20}$$

which is equivalent to the Kruskal limit[13] in the stellarators.

Whether this limit restricts absolutely the maximum current is not quite clear. For smaller disturbances and higher accelerating field, the current may increase beyond the limit, because $\tau_s(r)$ is dependent on the $B_\theta(r)$, hence the circulating current.

3. EXPERIMENTAL MODEL

The parameters of the model machine which is under construction are as follows (Fig. 3). The circumference and inner diameter of the vacuum vessel are about 300 cm and 8 cm respectively. The characteristic angle α is chosen near 60°. A confining magnetic field up to 20 000 G is produced in the torus by condenser discharge, and electrons are

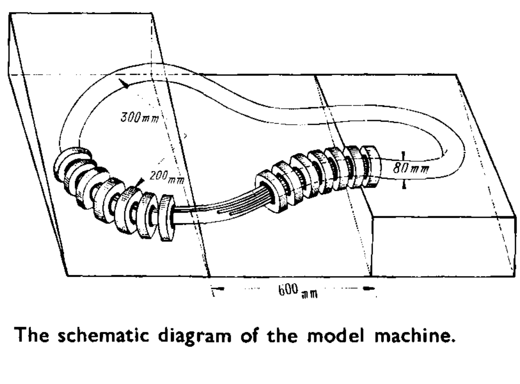

Fig. 3 The schematic diagram of the model machine.

accelerated in this field to an energy of a few MeV. Taking 3 cm as an effective radius of the beam, the limiting current of the beam due to the resonance is about 6 000 A. The current corresponding to $\alpha = 0.1$ is found to be 30 000 A.

The magnitude of the accelerating electric field is related to the condition of electron runaway, and also to whether the Kruskal limit can be cleared or not. A one turn coil is wound onto the torus composed of several cables connected in parallel. The coil is expected to produce electric fields of 100 to 200 V/cm along the axis. Acceleration is achieved in several microseconds, and a crow-bar switch or critical damping resistance will lead to a stationary state of electron beams.

Inert gas introduced into the vacuum vessel at a pressure of 10^{-4} to 10^{-5} mm Hg may be pre-ionized by electrons injected from electron guns [14].

Measurement of X-rays, visible rays and magnetic field will give the information on the behaviour of the electron beam circulating in the machine. Microwave techniques will also be used.

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(*) See note on reports, p. 696.