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PLASMA FOCUSING FOR HIGH ENERGY BEAMS*

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ABSTRACT

We analyse the self-focusing effect of a relativistic electron or positron beam traversing through a thin slab of plasma in a linearized fluid theory, and show that the effect is very strong. The idea of employing this effect for a plasma lens suggested by Chen is then reviewed. Computer simulations on both thin and thick plasma lenses are presented, which show reasonable agreement with theoretical predictions. It is suggested that the particle beam precursor be replaced by a beating-laser precursor for alignment purpose such that the tolerance on random injection errors can be increased.

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I. INTRODUCTION

For future e^+e^- high energy linear colliders, one of the challenges is to increase the luminosity according to the square of the e^+e^- center of mass energy.[1] The hope lies in the increase of the repetition rate of the beam pulses, the number of particles per pulse, and the reduction of spot size of the beams at the interaction point. Recently, P. Chen[2] proposed the idea of using the self-focusing effect of a beam in a plasma for this purpose. In the study of the plasma wake field accelerator,[3] it is shown that accompanying the large acceleration gradient (from the longitudinal wake field excited by a leading charge), there is a transverse wake field with comparable strength.[4] The transverse wake field excited by a leading charge either focuses or defocuses a trailing particle depending on their relative phase in the plasma wake. When the particles are bunched into a finite volume, it is shown that the integrated transverse wake force is always focusing within the bunch.[2] As a result the bunch pinches itself to a smaller effective cross-section. This effect is suggested as a mechanism to focus the e^+e^- beams and thus to enhance the luminosity.

In this paper, we first review the self-focusing effect on a bunched beam with finite size in a plasma. Two criteria are imposed and a chain inequality is derived in order that a linear pinching force can be achieved. We then discuss in Section III the conceptual design of a thin plasma lens following Ref. 2. One novel aspect of the lens is to tailor the bunch into a "precursor" and a "main bunch" that follows behind the precursor by one quarter of a plasma wavelength. In Section IV, we present computer simulations on the focusing process through a thin lens. The results show excellent agreements with the theoretical prediction. We then turn to the discussion of a thick plasma lens suggested by D. Cline[5] based on the same physical concept. The fact that the beam particles execute substantial transverse motion during interaction with the thick plasma lens forbids simple analytic description of the beam dynamics which is both nonlinear and convolutional. We therefore relie on computer simulations in this respect in Section V.

Summary and discussion are given at the end. We suggest that there are several merits if one replace the particle beam precursor by a beating laser beam.

II. THE SELF–FOCUSING EFFECT

The longitudinal plasma wake field at point (r,ς), where $\varsigma \equiv z - ct$, is defined to be the longitudinal electric field of the beam-plasma system, i.e.

$$\vec{W}_{\parallel}(r,\varsigma) \equiv \vec{E}_{1z}(r,\varsigma) \ . \tag{1}$$

The transverse plasma wake field, on the other hand, is the Lorentz force exerting on the unit charge at (r,ς) that experiences the plasma wake and moves with velocity $\vec{\beta}' \equiv \vec{v}\,'/c \simeq 1$, i.e.

$$\vec{W}_{\perp}(r,\varsigma) \equiv \vec{E}_{1r}(r,\varsigma) + \vec{\beta}\,' \times \vec{B}_{1\phi}(r,\varsigma) \ . \tag{2}$$

Since the particles that generates the plasma wake are also assumed to be relativistic (i.e. $\beta \simeq 1$), we can write

$$\vec{E}_{1z} = (\beta\partial_\varsigma A_{1z} - \partial_\varsigma\phi_1)\hat{z} \simeq \partial_\varsigma(A_{1z} - \phi_1)\hat{z} \ , \tag{3}$$

and

$$\begin{cases} \vec{E}_{1r} = (\beta\partial_\varsigma A_{1r} - \partial_r\phi_1)\hat{r} \simeq (\partial_\varsigma A_{1r} - \partial_r\phi_1)\hat{r} \\ \vec{\beta}\,' \times \vec{B}_{1\phi} = -\beta'(\partial_\varsigma A_{1r} - \partial_r A_{1z})\hat{r} \simeq -(\partial_\varsigma A_{1r} - \partial_r A_{1z})\hat{r} \ , \end{cases} \tag{4}$$

thus the wake fields can be rewritten (with the approximation $\beta \simeq \beta' \simeq 1$) in terms of a common functional, $A_{1z} - \phi_1$:

$$\begin{cases} W_{\parallel}(r,\varsigma) = \partial_\varsigma(A_{1z} - \phi_1) \\ W_{\perp}(r,\varsigma) = \partial_r(A_{1z} - \phi_1) \ . \end{cases} \tag{5}$$

Notice that the Panofsky-Wenzel theorem,[6] $\partial_r W_{\parallel} = \partial_\varsigma W_{\perp}$, is straightforwardly satisfied.

To solve for $W_{\|}$ and $W_{\perp}$ we employ the nonrelativistic fluid theory. Assuming that the unperturbed plasma velocity v_0 be zero and the perturbed plasma density n_1 be much smaller than its unperturbed density n_0, the equation of motion and the equation of continuity can be linearized as:

$$\begin{cases} -\partial_\varsigma \vec{v}_1 = -\dfrac{e}{mc}\,\vec{E}_1 \\ -c\partial_\varsigma n_1 + n_0 \nabla\cdot\vec{v}_1 = 0\ , \end{cases} \tag{6}$$

where $\partial_z = \partial_\varsigma$ and $\partial_t = -c\partial_\varsigma$ have been used. The Maxwell's equations in the Coulomb gauge are

$$\nabla^2 \phi_1 = -4\pi\rho_1\ , \tag{7}$$

and

$$\nabla_\perp^2 \vec{A}_1 = \frac{4\pi}{c}\,\vec{J}_1 - \vec{\nabla}\partial_\varsigma\phi_1\ , \tag{8}$$

where the charge and current densities are contributed from both the plasma perturbation and the source, $\mp e\sigma(\vec{x})$, for electron $(-)$ and positron $(+)$ bunches, respectively,

$$\begin{cases} \rho_1(\vec{x}) = -en_1(\vec{x}) \mp e\sigma(\vec{x}) \\ \vec{J}_1(\vec{x}) = -en_0\vec{v}_1(\vec{x}) \mp ec\vec{c}\sigma(\vec{x})\ . \end{cases} \tag{9}$$

Notice that in defining the current this way, we have neglected the transverse current in the bunch. This approximation is valid only if the transverse motions of the bunch particles are negligibly small during the beam-plasma interaction, which is the case for a thin plasma lens as we shall discuss in Sections III and IV. Combining the fluid equation and the Poisson equation (Eq. (7)), with the help of $\vec{E}_1 = -\nabla\phi_1 + \partial_\varsigma\vec{A}_1$, we obtain the equation for the plasma density perturbation due to the beam:

$$\partial_\varsigma^2 n_1 + k_p^2 n_1 = \mp k_p^2 \sigma(\vec{x})\ , \tag{10}$$

where the plasma wave number $k_p \equiv \omega_p/c = (4\pi e^2 n_0/mc^2)^{1/2}$, and the signs in the source term are associated with electron beam $(-)$ and positron beam $(+)$, respectively.

Now we assume the separation of variables in $\sigma(\vec{x})$ and that the bunch is confined to the region $\varsigma \leq 0$ and $r \leq a$:

$$\sigma(\vec{x}) = \rho_b f(r) g(\varsigma) \ , \quad a \leq r, \quad \varsigma \leq 0 \ . \tag{11}$$

Then

$$n_1 = \begin{cases} \pm k_p f(r) \rho_b \int_\varsigma^\infty d\varsigma' g(\varsigma') \sin k_p(\varsigma' - \varsigma) \equiv \pm \rho_b f(r) G(\varsigma) \ , & \varsigma \leq 0 \\ 0 \ , & \varsigma > 0 \end{cases} \tag{12}$$

for electron and positron beams, respectively.

Next we apply ∂_ς to Eq. (8),

$$\nabla_\perp^2 \partial_\varsigma \vec{A}_1 = -\frac{4\pi}{c} \ \partial_\varsigma \vec{J}_1 - \vec{\nabla} \partial_\varsigma^2 \phi_1 \quad . \tag{13}$$

Evoking Eqs. (6) and (9) we have

$$\nabla_\perp^2 \partial_\varsigma \vec{A}_1 = k_p^2 (\partial_\varsigma \vec{A}_1 - \nabla \phi_1) \pm \frac{4\pi e \rho_b}{c} \ \vec{c} \ f(r) \ \partial_\varsigma \ g(\varsigma) - \nabla \partial_\varsigma^2 \phi_1 \quad . \tag{14}$$

Concentrating on the z-component and removing the common ∂_ς we obtain

$$(\nabla_\perp^2 - k_p^2) A_{1z} = -(\partial_\varsigma^2 + k_p^2) \phi_1 \pm 4\pi e \rho_b \ f(r) \ g(\varsigma) \quad , \tag{15}$$

which is equivalent to what was obtained before.[7]

Since A_{1z} and ϕ_1 always appear together in the expressions for the wake fields, we do not need to know them separately. Notice that $\partial_\varsigma^2 = \nabla^2 - \nabla_\perp^2$, we can rewrite the above expression as

$$(\nabla_\perp^2 - k_p^2)(A_{1z} - \phi_1) = -4\pi e n_1 \quad . \tag{16}$$

This is the Master Equation for the plasma wake fields excited by either an electron beam or a positron beam. With n_1 given in Eq. (12), we get

$$A_{1z} - \phi_1 = \mp \frac{4\pi e \rho_b}{k_p^2} \, G(\varsigma) \left\{ k_p^2 \int_0^r r' dr' f(r') I_0(k_p r') K_0(k_p r) \right.$$
$$\left. + \, k_p^2 \int_r^\infty r' dr' f(r') I_0(k_p r) K_0(k_p r') \right\} \tag{17}$$
$$\equiv \mp \frac{4\pi e \rho_b}{k_p^2} \, G(\varsigma) F(r) \quad .$$

Therefore the wake fields can be simply expressed as

$$\begin{cases} W_{\parallel} = \mp \dfrac{4\pi e \rho_b}{k_p^2} \, \partial_\varsigma G(\varsigma) F(r), \\ \\ W_{\perp} = \mp \dfrac{4\pi e \rho_b}{k_p^2} \, G(\varsigma) \partial_r F(r), \end{cases} \tag{18}$$

for electron $(-)$ and positron $(+)$ beams, respectively.

Consider the following density distribution for a "standard" electron or positron bunch (see Fig. 1) in most of the presently existing accelerators:

$$\sigma(\vec{x}) = \rho_b f(r) g(\varsigma) = \rho_b \left(1 - \frac{r^2}{a^2}\right) \left(1 - \frac{(\varsigma + b)^2}{b^2}\right) \quad , \tag{19}$$

where $0 \leq r \leq a$ and $-2b \leq \varsigma \leq 0$. The parabolic profiles in both r and ς directions are introduced to approximate the Gaussian profiles. The constant, ρ, can be related to the total number of particles N in the bunch:

$$\rho_b = \frac{3N}{2\pi a^2 b} \quad . \tag{20}$$

With this density distribution, it is straightforward to find that within the bunch,

$$
\begin{aligned}
A_{1z} - \phi_1 = \mp \frac{8\pi e \rho_b}{k_p^2} & \left[I_0(k_p r) K_2(k_p a) + \frac{1}{2}\left(1 - \frac{r^2}{a^2}\right) - \frac{2}{k_p^2 a^2} \right] \\
& \times \left[\left(1 - \frac{(\varsigma + b)^2}{b^2}\right) + \frac{2}{k_p b} \sin k_p \varsigma + \frac{2}{k_p^2 b^2} (1 - \cos k_p \varsigma) \right] .
\end{aligned} \tag{21}
$$

The corresponding transverse wake field within the bunch is thus

$$
\begin{aligned}
W_\perp = \mp \frac{8\pi e \rho_b}{k_p} & \left[I_1(k_p r) K_2(k_p a) - \frac{r}{k_p a^2} \right] \\
& \times \left[\left(1 - \frac{(\varsigma + b)^2}{b^2}\right) + \frac{2}{k_p b} \sin k_p \varsigma + \frac{2}{k_p^2 b^2} (1 - \cos k_p \varsigma) \right] .
\end{aligned} \tag{22}
$$

Notice that the transverse force is exerting on the like particles in the same bunch, i.e. $\mathcal{F}_\perp = \mp e W_\perp$, thus $\mathcal{F}_\perp$ has the same sign for both electron and positron bunches. Furthermore, it can be verified that $G(\varsigma)$ in this case is always positive definite, so the force is always focusing.

An interesting case corresponds to the situation where $k_p r \leq k_p a \ll 1$. In this limit

$$
I_1(k_p r) K_2(k_p a) - \frac{r}{k_p a^2} \simeq -\frac{1}{4} k_p r \quad , \tag{23}
$$

and we have a focusing force which is linear in r:

$$
\mathcal{F}_\perp \simeq - \left[\frac{3 e^2 N}{a^2 b} G(\varsigma) \right] r \quad . \tag{24}
$$

The requirement that the focusing force be linear in r, i.e. that $k_p a \ll 1$, can be rewritten as

$$
n_0 \ll \frac{1}{4\pi r_e a^2} \quad , \tag{25}
$$

where r_e is the classical electron radius. On the other hand, self-consistency in the linearized fluid theory that we employed requires that $n_0 \gg n_1$. Combining these two conditions we arrive at a chain inequality which the system must satisfy:

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \rho_b\, f(r) G(\varsigma) \geq \rho_b\, G(\varsigma) \quad . \tag{26}$$

This inequality puts a constraint on the bunch length $2b$. Physically, this is a condition imposing on the relative densities between the plasma and the bunch.

In the specific case of a standard bunch, the inequality reads

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{3N}{2\pi a^2 b} \left[\left(1 - \frac{(\varsigma + b)^2}{b^2} \right) + \frac{2}{k_p b} \sin k_p b + \frac{2}{k_p^2 b^2} (1 - \cos k_p b) \right] \quad . \tag{27}$$

For the present SLAC parameters where $N = 5 \times 10^{10}$ and $b = 1$ mm, this condition is hard to satisfy. However, with slight modifications of the SLAC parameters, the inequality can be easily satisfied in the following two cases.

Case A, Round Beam Limit:

Assuming $k_p b \ll 1$, then the inequality becomes

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{3N}{2\pi a^2 b} \left(-\frac{1}{3} \frac{k_p^3 \varsigma^3}{k_p b} \right) , \qquad -2b \leq \varsigma \leq 0 \, . \tag{28}$$

Notice that the maximum on the right hand side occurs at $\varsigma = -2b$, where the bunch ends. Thus the inequality is further specified to be

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{4N k_p^2 b}{\pi a^2} \quad . \tag{29}$$

Together with the previous assumption that $k_p a \ll 1$, this is thus a situation where $a \approx b$, and the beam has roughly the same size in both directions.

When the condition Eq. (29) is satisfied, the transverse force on a particle at (r, ς) within either an electron bunch or a positron bunch is

$$\mathcal{F}_\perp(r, \varsigma) = \mp e W_\perp \simeq \frac{e^2 k_p^2 N}{a^2 b^2}\, \varsigma^3 r \quad , \qquad k_p b \ll 1 \, , \tag{30}$$

which is always focusing ($\because$ $\varsigma \leq 0$) towards the axis of symmetry of the beam. In this case the focusing force is maximum at the tail of the bunch.

Case B, Long Beam Limit:

If N is, for instance, one order of magnitude less than the present SLAC parameter while b remains the same, then the inequality can be straightforwardly satisfied for all $k_p b$. An interesting situation in this case is when $k_p b \gg 1$. Therefore $b \gg \lambda_p/2\pi \gg a$, and we have a long bunch where the longitudinal extent is much larger than the transverse extent. When this is satisfied, the focusing force is

$$\mathcal{F}_\perp(r,\varsigma) \simeq \frac{3e^2N}{a^2b}\left(1 - \frac{(\varsigma+b)^2}{b^2}\right)r \ , \qquad k_p b \gg 1 \ . \tag{31}$$

We see that the maximum of the force is at the mid-point along the bunch.

In either case the focusing force is very strong. For comparison, consider minimal departures from the SLAC parameters: In case A if $N = 1 \times 10^9$, $a = b = 100\ \mu$m, and $k_p \simeq 6 \times 10^{-3}\ \mu\mathrm{m}^{-1}$, the field gradient $G \sim 173$ KG/cm at the mid-point along the bunch. In case B if $N = 5 \times 10^9$, and $a = 100\ \mu$m, $b = 1$ mm, we find the corresponding $G \sim 720$ KG/cm. In contrast, typical iron magnets ($G \sim 5$ KG/cm) and superconducting magnets ($G \sim 10$ KG/cm) are about $1 \sim 2$ orders of magnitude weaker. Notice that in the case of plasmas, the focusing force is governed by the densities of the beam and the plasma. By properly arranging the densities, the field gradient can be still larger.

Physically, this self-focusing effect arises because the electrons in the plasma are either expelled (for the case of interacting with an electron bunch) or pulled (for the case of interacting with a positron bunch) by the leading particles in the bunch, while on this time scale the ions in the plasma are essentially stationary. As a result the trailing particles in the same bunch experience an attractive force due to the access charges in plasma within the volume of the bunch. Large self-pinching of the beam is thus induced. This effect has been observed in computer simulations.[8]

III. A THIN PLASMA LENS

Although the self-focusing field gradients that we showed in the previous section are high, they unfortunately suffer strong ς dependence in both the round beam limit and the long beam limit for standard bunches. For the purpose of a plasma lens, it is desirable to have a self-focusing force which is independent of particle's longitudinal position in a bunch. To achieve this it is necessary to tailor the charge distribution of the bunch.

Employing a technique developed earlier[7,9] based on the convolution theorem in Laplace transforms, one is able to find the desirable charge distribution which generates a constant transverse wake field. To be explicit, for a given charge distribution in the bunch, $\rho_b g(\varsigma) f(r)$, we have (c.f. Eq. (18))

$$\begin{aligned} W_\perp &= \mp \frac{4\pi e \rho_b}{k_p^2} \, G(\varsigma) \partial_r F(r) \\ &\equiv \mp \alpha \int_\varsigma^\infty d\varsigma' g(\varsigma') \sin k_p(\varsigma' - \varsigma) \quad . \end{aligned} \tag{32}$$

From the convolution theorem $g(\varsigma)$ can be obtained for the wanted $W_\perp$ by an inverse Laplace transform, i.e.

$$g(\varsigma) = \frac{1}{2\pi \alpha i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{\mathcal{L}\{W_\perp\}}{\mathcal{L}\{\sin\ k_p \varsigma\}} \, e^{s\varsigma} ds \ , \tag{33}$$

where $\mathcal{L}\{\ \}$ indicates a Laplace transform. One of the possible arrangements for a constant $W_\perp$ is the following (see Fig. 2):

$$g(\varsigma) = k_p^{-1} \delta(\varsigma) + \theta\left(\varsigma + b + \frac{\pi}{2k_p}\right) - \theta\left(\varsigma + \frac{\pi}{2k_p}\right) , \tag{34}$$

where θ's are the step functions. We see that there are two components in the tailored bunch: an infinitely thin disk, and a "cylinder" with length b which follows behind the disk by one quarter of a plasma wavelength. The transverse density distributions are, however, the same for both components. Under this arrangement, the thin disk serves as a precursor which generates a transverse wake field that varies as a sine function. The transverse wake field reaches its maximum at $\lambda_p/4$ behind the precursor where the main bunch starts. The wake field generated by the main bunch partially balances the sinusoidal wake field excited by the precursor and gives rise to a net constant transverse wake along the bunch (see Fig. 2).

The total charge distribution in this arrangement is therefore

$$\sigma(\vec{x}) = \rho_b \left(1 - \frac{r^2}{a^2}\right) \left[k_p^{-1}\delta(\varsigma) + \theta\left(\varsigma + b + \frac{\pi}{2k_p}\right) - \theta\left(\varsigma + \frac{\pi}{2k_p}\right)\right] , \tag{35}$$

where

$$\rho_b = \frac{2Nk_p}{\pi a^2(1 + k_p b)} .$$

The self-focusing force along the main bunch is now independent of ς:

$$\mathcal{F}_\perp = -\frac{4Ne^2k_p r}{a^2(1 + k_p b)} . \tag{36}$$

Note, however, that the precursor experiences no focusing force. The inequality in this case is

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{2Nk_p}{\pi a^2(1 + k_p b)} , \tag{37}$$

where the denominator on the right hand side is associated with the ratio of charges between the precursor and the main bunch, i.e.

$$\frac{Q_{\text{precursor}}}{Q_{\text{bunch}}} = \frac{1}{k_p b} . \tag{38}$$

Assuming that such tailored two-component bunches can be prepared in future e^+e^- linear colliders, consider now the following construction of a plasma lens for final focus (see Fig. 3): At distance s down stream from the e^+e^- interaction point, a nonrelativistic, neutral plasma jet (pulsed or continuous) streams in the direction transverse to the beam pipe. The jet speed is chosen such that the plasma which has been perturbed by an incoming tailored bunch can move out of the region before the next bunch enters. Assuming a repetition rate of the e^+e^- bunches in a future linear collider to be $10^3 \sim 10^4$ Hz,[1] and the range of the beam-plasma interaction to be $10 \sim 100$ μm transversely, then the jet speed is supposed to be $1 \sim 100$ cm/sec. Thus the plasma is practically stationary during the transient time of an ultra-relativistic bunch, and all previous formulas are applicable to this situation. The plasma density is chosen such that the tailored e^+e^- bunches would focus to their minimum sizes in distance s after traversing through the plasmas.

In such a plasma lens, the "focusing strength" is

$$K = \frac{\mathcal{F}_\perp / r}{\gamma m c^2} = \frac{4 N k_p r_e}{\gamma a^2(s)(1 + k_p b)} , \tag{39}$$

where $a(s)$ is the radius of the bunch at the lens, and γmc^2 is the energy of the particles in the bunch. The field gradient of the plasma lens is

$$G = \frac{1}{r}\, W_\perp = \frac{4 N e k_p}{a^2(s)(1 + k_p b)} . \tag{40}$$

The focal length s can be determined if the emittance ϵ_n and the β-function at the interaction point, β^*, are given:

$$a^2(s) = \frac{\epsilon_n}{\gamma}\, \beta^* \left(1 + \frac{s^2}{\beta^{*2}}\right) . \tag{41}$$

Once s is known, one can evaluate the thickness of the plasma lens (in the direction of the beam pipe) via the thin lens formula,

$$\ell = \frac{1}{K s} . \tag{42}$$

This set of parameters, together with the plasma wave number k_p (or plasma density n_0) completes the conceptual design of the plasma lens.

As a numerical example, we take a nonoptimized set of parameters of a 5 TeV + 5 TeV e^+e^- linear collider discussed by Richter[1]:

Each of the colliding e^+ and e^- bunches has 4.1×10^8 particles. At the interaction point the longitudinal and transverse sizes of the bunch are $\sigma_z = 3.4 \times 10^{-3}$ mm and $\sigma_{r_0} = 2.0 \times 10^{-3}$ μm, respectively. The normalized emittance is assumed to be 4×10^{-8} m-rad, and the β-function at the interaction point is $\beta^* = 1$ mm.

For our purpose, we should tailor the bunches, and match their parameters to a reasonable set of plasma parameters such that the inequality in Eq. (37) can be satisfied. We certainly do not need the bunch to have the same transverse dimension σ_{r_0} at the plasma lens, this is actually the reason for the focus. In order not to make the focal length too long we choose $a(s)$ to be about four orders of magnitude larger than σ_{r_0}. However in order that the beam density b satisfies Eq. (37) with a given plasma density n_0, one cannot choose b to be as small as σ_z given above. Let us therefore tailor the bunch such that $b = 100$ μm and $a(s) = 3$ μm and choose the plasma density to be $n_0 = 10^{18}$ cm^{-3}, such that $k_p \simeq (1/5)$ μm^{-1}.

The focusing strength in the case is (c.f. Eq. (39))

$$K \simeq 4.8 \times 10^{-2} \text{ cm}^{-2} \quad , \tag{43}$$

and the focusing field gradient is

$$G \simeq 830 \text{ MG/cm} \quad . \tag{44}$$

To evaluate the focal length, notice that the transverse size is reduced from $a(s)$ to σ_{r_0}, so from Eq. (41) we have

$$s \simeq 4.8 \text{ m} \quad , \tag{45}$$

and the plasma lens thickness is

$$\ell \simeq 23 \text{ cm} \quad , \tag{46}$$

which is consistent with the thin lens assumption (i.e. $s \gg \ell$) and justifies our approximation on neglecting the transverse current within the beam (c.f. Eq. (9)) during the beam-plasma interaction time.

IV. SIMULATION ON THIN LENS

Computer simulations on the focusing process based on the idea discussed above were performed using the $2+\frac{1}{2}$ dimensional $(z, r, \theta;\ v_z, v_r)$ electromagnetic, particle in cell code "ISIS".[11] The initial direction of propagation of the beam is chosen to be along the z-axis, and the system is axi-symmetric.

The parameters of the system are as follows: The Lorentz factor for the incoming beam is $\gamma = 20$, the charge distribution is Gaussian in the radial direction with one standard derivation $k_p a = 0.29$, and the length of the main bunch is $k_p b = 3.14$; the ratio of peak beam density along z-axis over the lens plasma density is $\rho_b/n_0 = 0.33$, while the lens thickness is $k_p \ell = 6.0$. The total number of particles in the beam is 2000, and in the lens it is 10 particles per cell.

In the simulation, we place the lens at a distance $\frac{1}{2}\lambda_p$ from the left hand boundary. the precursor and the main bunch are injected into the system at $\omega_p t = 0$. Figures 4 and 5 are taken at $\omega_p t = 37.5$ when the out-going main bunch reaches its minimum size. Figure 4(a) shows an instantaneous density as a function of radius at the mid-point along the main bunch. At $\omega_p t = 37.5$, the beam radius has been reduced to about $\frac{1}{10}$ of the initial radius. Figure 4(b) is a three dimensional plot of the beam density at the same instant as functions of both r and z. The front half of the bunch has already reached the minimum cross section and has started to disperse, while the tail half of the bunch is just reaching the minimum size at this time. The detailed shape of the bunch can be clearly seen from Fig. 5.

To check our simulation result with the theory described in the previous section, we rewrite Eq. (39) as

$$K = \frac{k_p^2}{2\gamma} \frac{\rho_b}{n_0} \quad , \tag{47}$$

where $n_0 = k_p^2/4\pi r_e$ is used. Therefore Eq. (42), i.e. the thin lens formula, becomes

$$k_p s \cdot k_p \ell = 2\gamma \cdot \frac{n_0}{\rho_b} \quad . \tag{48}$$

From Fig. 5 the focal point appears to be at $k_p L \simeq 30$. Since the front surface of the lens is $\frac{1}{2} k_p \lambda_p = \pi$ from the left hand boundary, and the lens thickness is $k_p \ell = 6$, we infer that the distance from the middle of the lens to the focal point is $k_p s \simeq 23$. Pluging $\gamma = 20$ and $\rho_b/n_0 = 0.33$, and the above values of $k_p \ell$ and $k_p s$ into Eq. (48), the formula is satisfied within 15%. The descripancy is obviously due to the fact that our lens is not thin enough for such a low γ beam.

V. A THICK PLASMA LENS

D. Cline suggests[5] that the idea of plasma focusing discussed above may be applied to enhance the luminosity by placing a thick slab of plasma at the e^+e^- interaction point. In contrast to the thin lens idea, a thick lens would interact with a particle beam substantially enough that the beam dynamics is unavoidably both nonlinear and convolutional. The focusing effect in this case is difficult to be analysed theoretically. We therefore resort to computer simulations for this purpose.

A simulation on the thick plasma lens was performed using the $2 + \frac{1}{2}$ dimensional $(x, y, z;\ v_y, v_z)$ electromagnetic, particle in cell code "WAVE".[12] One of the two full dimensions (z, v_z) is used for beam's longitudinal propagation, and the other (y, v_y) for one of the two transverse degrees of freedom. Without detailed knowledge of the beam dynamics, we naively repeat similar composition of the beam. Namely, we still construct a precursor and a main bunch with a spatial separation of $\frac{1}{4}\lambda_p$, except that the transverse density distribution of both components are chosen to be uniform in y direction. The initial beam energy is $\gamma = 40$, the beam width in y direction is $k_p a = 2$, and the length of the main bunch is $k_p b = 10$. The precursor in this case, however, is not infinitely thin but has a thickness $k_p \delta = 0.2$. The finite thickness of the precursor is actually a more realistic situation.

In Fig. 6 we show an instantaneous real space diagram of the beam-plasma system on the y-z plane at time $\omega_p t = 32.5$. We see that by this time the main bunch has pinched itself into a smaller cross section, but not uniformly. An envelope in its longitudinal profile has developed where the waist appears at $\frac{1}{2}$ plasma wavelength from the head of the main bunch. At the mean time, the precursor is basically not pinched, as expected, other than some minor pinching effect due to its finite thickness. The gradual development of the mismatch of cross sections between the unpinched precursor and the pinched main bunch during long distance of beam-plasma interaction is actually the cause for the profile envelope in the main bunch.

It is obvious that once the beam comes out of the thick lens after the envelope has been developed, there would be huge aberration. Therefore it is not desirable to invoke the thick lens idea as a final focusing mechanism. However, notice that at the maximum cross section along the envelope, the beam width has reduced to $\sim \frac{1}{2}$ of its initial value, while at the waist the reduction rate is as much as $\sim \frac{1}{10}$. Since the luminosity scales as the inverse of the cross sectional area, which in our case varies from $\sim \frac{1}{4}$ to $\sim \frac{1}{100}$ of the initial value, the luminosity would be greatly enhanced if the e^+e^- beams collide inside the thick plasma lens.

VI. DISCUSSION

The phenomena of the self-focusing effect of a relativistic electron or positron beam traversing through a thin slab of plasma was described in a linearized fluid theory, and the effect was shown to be very strong. Computer simulation of a thin plasma lens incorporating a "tailored" beam shows reasonable agreement with theoretical predictions. A variant of this idea, namely, a thick plasma lens was also examined via computer simulation. It is shown that the thick lens concept is not appropriate for the purpose of final focus in linear colliders because of the non-uniform pinching force. It may, however, still be useful for final enhancement of luminosity at the interaction point of a collider where two high energy beams collide. It would also help reducing the dispersion between two electron beams when an e^-e^- linear collider is considered.

One concern of these plasma lenses is that the collision events between the beam particles and the plasma particles (in the high energy physics sense) would compete with the "true" events between the two high energy beams. There are two types of these "background" events in this regard: the beam-electron events and the beam-ion events. The former case has been analysed by Chen,[2] which shows that even though the background noise ratio is high, the absolute event rate is very low at high energies. Furthermore, the center of mass energy of such events is $\sqrt{2\gamma}\, mc^2$ rather than $2\gamma mc^2$ for the true events. This makes the background events easily to be distinguished from the true events. More importantly, since this beam-electron collision is a stationary target event, the product would mostly go down the beam pipe without hitting the detector. As for the beam-ion events, the products can in principle come out at large angles thus indeed compete with the true event signals. A detailed analysis of this case still awaits further efforts.

The most serious challenge to the idea of a plasma lens is its tolerance on the random error of beams' injection positions. In the conventional focusing magnets where the axis of symmetry is predefined and therefore can be prealigned, any off-set of the beam injection position will be reduced at the focal point by the same rate as the beam spot size. On the other hand, the axis of symmetry of a plasma lens, with the beam tailored into a precursor and a main bunch, is defined by the beam itself. When the off-set is random, any misalignment between the two incoming beams cannot be reduced proportionally at the interaction point. The idea that the final spot sizes of e^+e^- beam should be reduced to ängstrom dimensions for future colliders therefore puts a severe constraint on the tolerance of the alignment errors.

A remedy to this draw back is to replace the particle beam precursor by a beating laser pulse as suggested in the Plasma Beat-Wave Accelerator.[13] The beating lasers, with proper arrangements, will excite a plasma wave identical to that excited by a disk particle bunch.[14] The only necessary modification when invoking the beating lasers is that the positron bunch should be injected at $(n + \frac{3}{4})\lambda_p$ behind the laser precursor whereas the electron bunch still remains to be $(n + \frac{1}{4})\lambda_p$ behind its own laser precursor. The laser precursors from opposite sides of the interaction point can then also serve for the alignment. Studies of this concept are now in progress and will be reported in another paper.

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FIGURE CAPTIONS

Figure 1: A 'standard' electron bunch traversing a plasma. The radius and half-length of the bunch are a and b, respectively. The parabolic dashed curves within the bunch indicate its longitudinal density distribution. For the plasma, the squares represent ions and circles represent electrons.

Figure 2: The density $g(\varsigma)$ and the wake function $G(\varsigma)$ as functions of distance for a 'tailored' bunch. A thin-disk precursor is followed by a constant (longitudinal) density main bunch by one quarter of a plasma wavelength.

Figure 3: A conceptual design of a plasma lens. The plasma (in squares and circles) is ejected from a pipe perpendicular to the beam pipe, and absorbed by a low pressure pipe across the beam's trajectory. The speed of the plasma, $v_\perp$, only needs to be large enough such that a chain of incoming bunches can experience fresh plasmas.

Figure 4: (a) Normalized beam density as a function of radius at time $\omega_p t = 37.5$. The cross section of the beam is taken at the mid-point of the main bunch. (b) A three dimensional plot of the density profile at the same time.

Figure 5: A two dimensional real space plot of the beam profile at $\omega_p t = 37.5$. The front half of the bunch has passed the focal point and started to disperse.

Figure 6: A real space plot of the beam-plasma system in a thick lens at $\omega_p t = 32.5$. The main bunch has developed a "gold fish" like envelope due to the mismatch between the precursor and the main bunch.

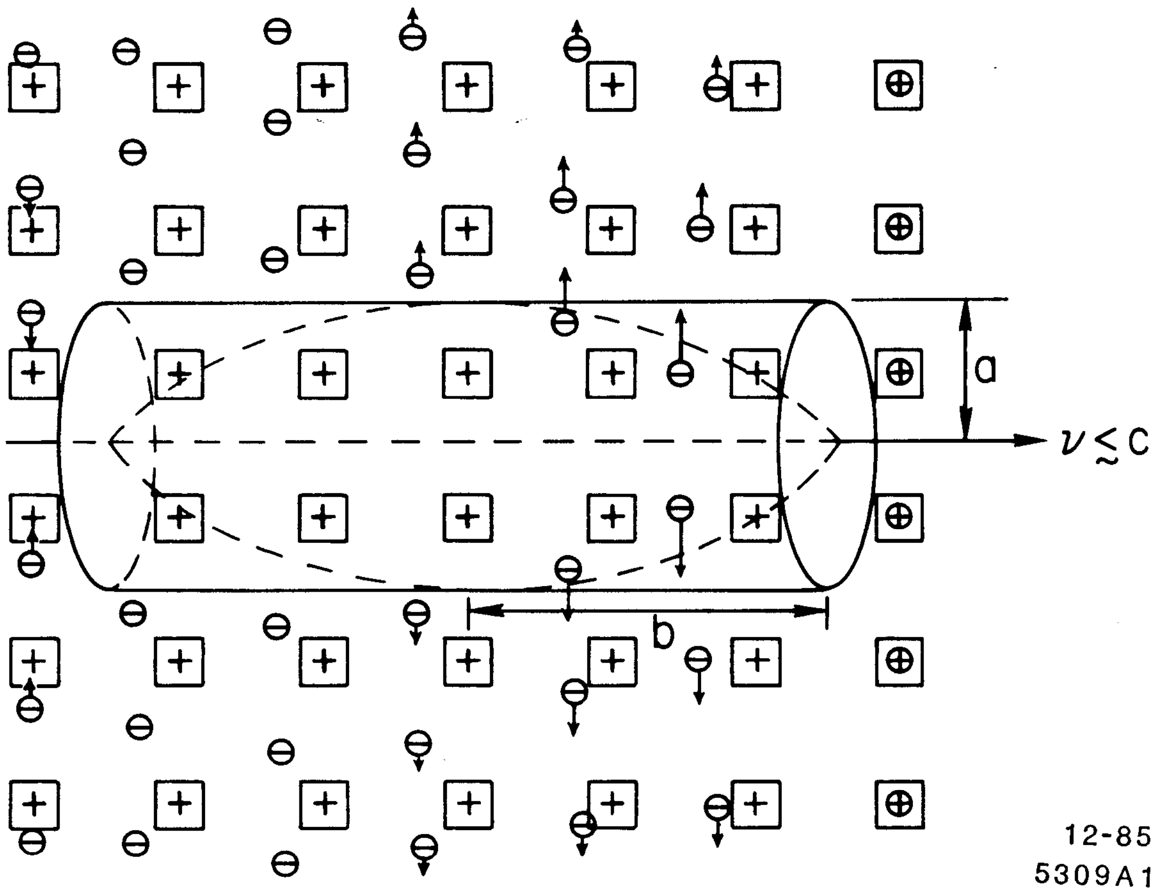


Fig. 1

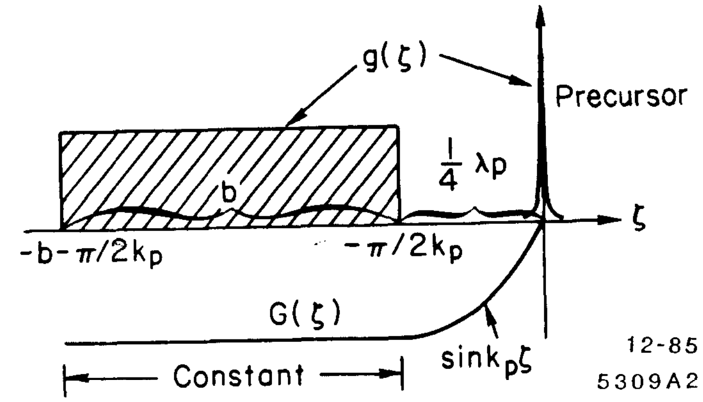

g(ζ)
Precursor
$\frac{1}{4}\lambda p$
b
ζ
$-b-\pi/2k_p$
$-\pi/2k_p$
G(ζ)
$\sin k_p\zeta$
Constant
12-85
5309A2

Fig. 2

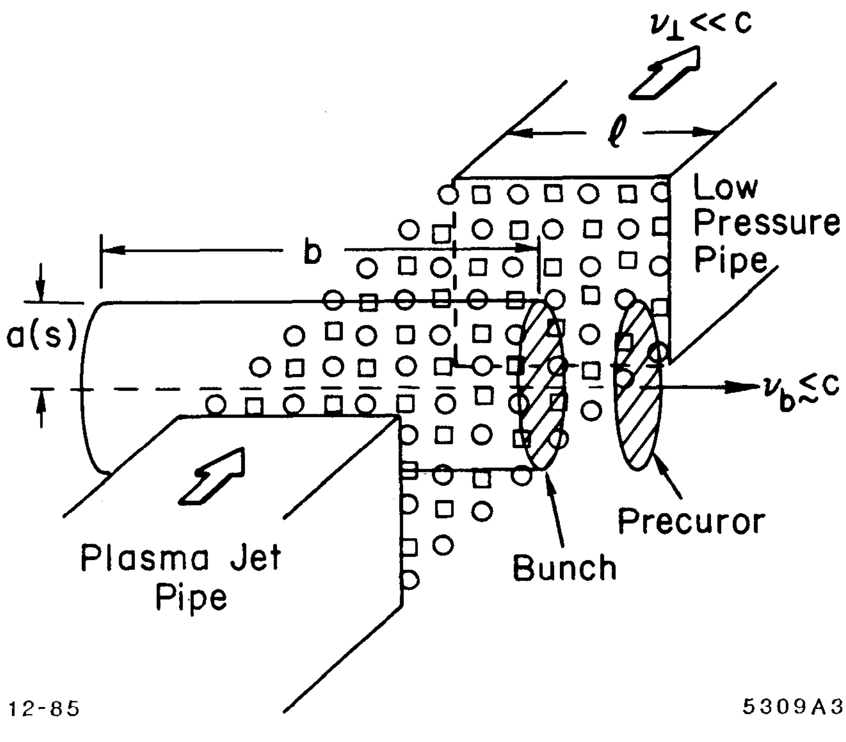
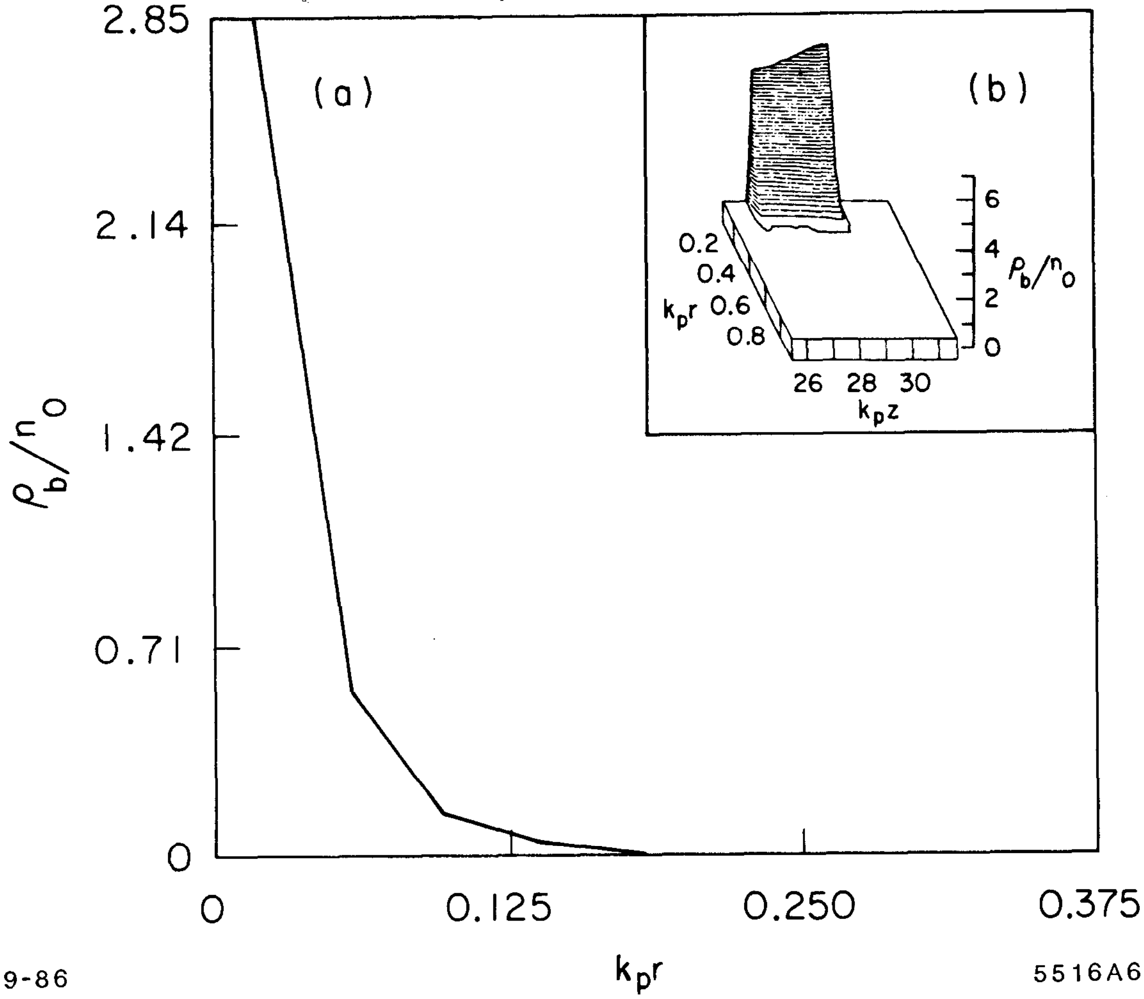

$\nu_\perp << c$
ℓ
Low
Pressure
Pipe
b
a(s)
$\nu_b \lesssim c$
Precuror
Bunch
Plasma Jet
Pipe
12-85
5309A3

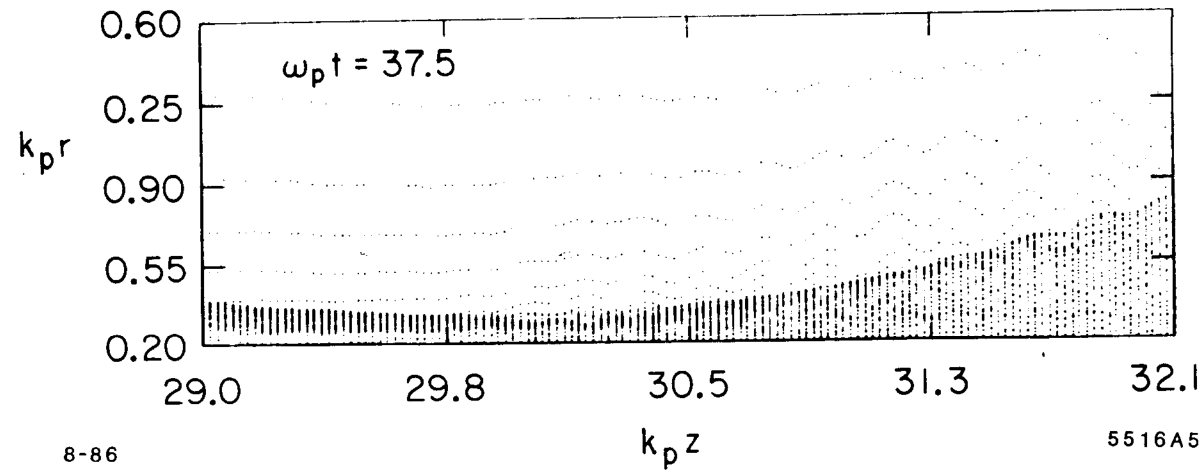
Fig. 3



9-86

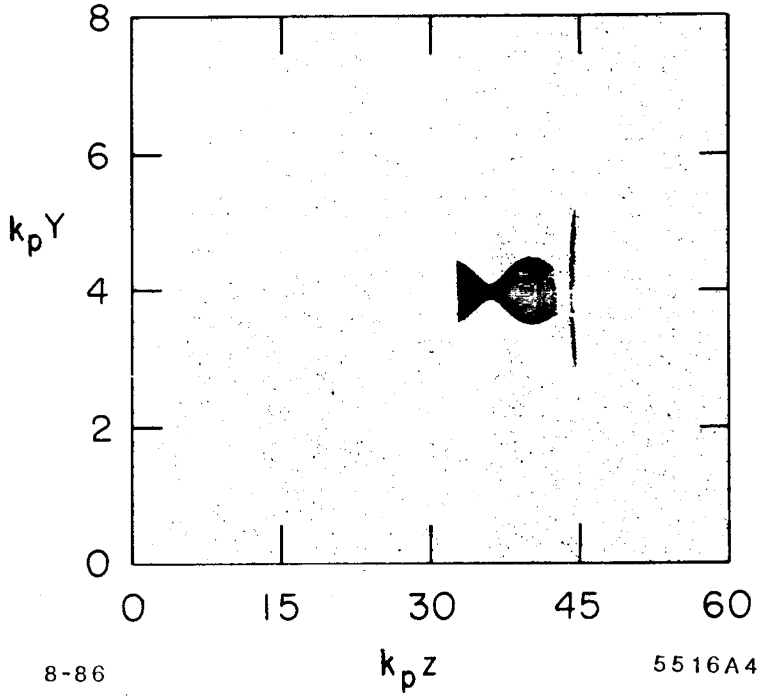
5516A6

Fig. 4



0.60
0.25
0.90
0.55
0.20
$k_p r$
$\omega_p t$ = 37.5
29.0
29.8
30.5
31.3
32.1
$k_p z$
8-86
5516A5

Fig. 5



8
6
4
2
0
$k_p Y$
0
15
30
45
60
$k_p z$
8-86
5516A4

Fig. 6