

Common gauge origin of discrete symmetries in observable sector and hidden sector

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ABSTRACT: An extra Abelian gauge symmetry is motivated in many new physics models in both supersymmetric and nonsupersymmetric cases. Such a new gauge symmetry may interact with both the observable sector and the hidden sector. We systematically investigate the most general residual discrete symmetries in both sectors from a common Abelian gauge symmetry. Those discrete symmetries can ensure the stability of the proton and the dark matter candidate. A hidden sector dark matter candidate (lightest U -parity particle or LUP) interacts with the standard model fields through the gauge boson Z' which may selectively couple to quarks or leptons only. We make a comment on the implications of the discrete symmetry and the leptonically coupling dark matter candidate, which has been highlighted recently due to the possibility of the simultaneous explanation of the DAMA and the PAMELA results. We also show how to construct the most general $U(1)$ charges for a given discrete symmetry, and discuss the relation between the $U(1)$ gauge symmetry and R -parity.

KEYWORDS: Discrete and Finite Symmetries, Beyond Standard Model, Supersymmetric Standard Model.

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1. Introduction

For many beyond standard models, discrete symmetries are invaluable ingredients to make the models phenomenologically viable. For example, in the minimal supersymmetric standard model (MSSM), R -parity [1] is usually assumed for the proton stability. R -parity also guarantees the stability of the lightest superparticle (LSP), which can be a good dark matter candidate. It is argued, however, that discrete symmetries are vulnerable to Planck scale physics unless they have a gauge origin [2]. An extra Abelian gauge symmetry is also predicted in many new physics scenarios such as superstring, extra dimension, little Higgs, and grand unification. Therefore, it would be useful to understand what discrete symmetries are allowed as a residual discrete symmetry of the extra U(1) gauge symmetry.

The first systematic study of the U(1) residual discrete symmetry in a supersymmetry (SUSY) framework was performed by Ibanez and Ross [3], where they found 3 independent generators R_N , L_N , and A_N . They studied all possible Z_2 and Z_3 discrete symmetries from a U(1), and found that R_2 (matter parity, which is equivalent to R -parity) as well as another Z_3 symmetry can be a residual discrete symmetry of the gauge symmetry, a.k.a. a discrete gauge symmetry. Complementary and general discrete symmetries (Z_N with $N > 3$) with a U(1) origin were also studied [4, 5]. In a special case where the μ -problem [6] is addressed

by a TeV scale $U(1)$, the discrete symmetries were investigated in refs. [7, 8], which allow R -parity violating $U(1)$ models without fast proton decay.

Nevertheless, these discrete symmetries concerned only the observable sector (or the MSSM sector). Many theories need exotic chiral fields for various reasons. For example, the SUSY breaking mechanism requires additional fields. Also exotic fields are often necessary to make the model anomaly free when an additional gauge symmetry is added. Even when they do not have standard model (SM) charges, such hidden sector fields may have charges under the extra $U(1)$ gauge symmetry. The SM neutral hidden sector fields can be natural dark matter candidates if they are stable.

It was shown that the same $U(1)$ symmetry that provides the discrete symmetry for the MSSM sector can also be the source of the discrete symmetry for the hidden sector simultaneously [9]. Another independent generator U_N was introduced for the hidden sector discrete symmetry. The lightest U -parity particle (LUP) from the hidden sector is stable under the U_2 (U -parity), and it was shown that the experimental constraints from the relic density and the direct detection can be satisfied in a large parameter space with the LUP dark matter candidate [10].

However, the study in ref. [9] was not completely general since the hidden sector field was assumed to be Majorana with SXX as a mass term, and only the factorizable extension $Z_{N_1}^{\text{obs}} \times Z_{N_2}^{\text{hid}}$ was exploited. In this paper, we first generalize the discussion by including the Dirac type hidden sector fields and possible nonrenormalizable mass terms. Dirac type fields allow a discrete symmetry U_N (with $N > 2$) while Majorana type fields allow only U_2 . This leads to the possibility of multiple hidden sector dark matter candidates stable due to the hidden sector discrete symmetry. We also start from the general form of the discrete symmetry taking the factorizable case as a special limit. Then we present a method to construct the most general $U(1)$ charges for a given discrete symmetry of the MSSM and hidden sector, with illustrations for specific examples. In appendix A, we discuss the $U(1)$ origin of the popular R -parity and its relation with the $U(1)$ solution of the μ -problem, which is one of the motivations to extend the supersymmetric standard model to include an extra $U(1)$. In appendix B, we discuss about the compatibility of discrete symmetries with a leptonically coupling dark matter candidate.

2. Residual discrete symmetries from the $U(1)$ gauge symmetry

In this section, we review the general discrete symmetries in the MSSM sector, which are the remnant of an Abelian gauge symmetry. Starting with a $U(1)$ gauge symmetry which is broken spontaneously by a Higgs singlet S , one is generically left with a residual discrete Z_N symmetry. In a normalization where all particles F_i of the theory have integer $U(1)$ charges $z[F_i]$, the value of N is directly determined by

$$N = |z[S]|. \quad (2.1)$$

The resulting discrete charges $q[F_i]$ of the fields F_i are then given by the mod N part of their original $U(1)$ charges

$$q[F_i] = z[F_i] \bmod N. \quad (2.2)$$

By definition the Higgs singlet S has vanishing discrete charge so that giving a vacuum expectation value (vev) to S keeps the discrete Z_N symmetry unbroken. Note that in the case with $N = 1$, we formally obtain a Z_1 which corresponds to no remnant discrete symmetry.

The possible (family-independent) discrete symmetries of the MSSM¹ which can emerge from an anomaly free $U(1)$ gauge symmetry have been identified and investigated in refs. [3–5]. Demanding Z_N invariance of the MSSM superpotential operators

$$W_\mu = \mu H_u H_d, \quad (2.3)$$

$$W_{\text{Yukawa}} = y_{jk}^D H_d Q_j D_k^c + y_{jk}^U H_u Q_j U_k^c + y_{jk}^E H_d L_j E_k^c + y_{jk}^N H_u L_j N_k^c, \quad (2.4)$$

one can express any discrete symmetry *among the MSSM particles* in terms of the two generators

$$R_N = e^{2\pi i(q_R/N)}, \quad L_N = e^{2\pi i(q_L/N)}, \quad (2.5)$$

where the charges q_R and q_L are defined in table 1. Different discrete symmetries of the observable sector are then obtained by multiplying various integer powers of these generators

$$g_N^{\text{obs}} = R_N^m L_N^p. \quad (2.6)$$

Compared to refs. [3, 4], the generator A_N , which gives nonzero discrete charge to only one of the two Higgs doublets, is omitted because its presence would forbid the μ term in eq. (2.3). As the invariance of $H_u H_d$ under Z_N requires opposite discrete charges for H_u and H_d , one can always find an equivalent set of discrete charges by adding some amount of hypercharge $y[H_i]$ such that $q'[H_i] = q[H_i] + \alpha y[H_i] = 0$ simultaneously for $i = u, d$. Thus requiring the existence of the μ term guarantees the absence of domain walls after the electroweak symmetry breaking.

A more intuitive way of writing eq. (2.6) is obtained by defining the generator $B_N = R_N L_N$. The discrete charges q_B of the MSSM fields under B_N are related to the familiar baryon number ($\mathcal{B}$) by the hypercharge shift

$$q_B[F_i] = -\mathcal{B}[F_i] + \frac{1}{3}y[F_i]. \quad (2.7)$$

Here the hypercharge is normalized so that $y[Q] = 1$. On the other hand, the discrete charges q_L of the MSSM fields under L_N are nothing but the negative of the lepton number ($\mathcal{L}$)

$$q_L[F_i] = -\mathcal{L}[F_i]. \quad (2.8)$$

Hence, the general discrete symmetry of eq. (2.6), written in terms of B_N and L_N ,

$$Z_N^{\text{obs}} : g_N^{\text{obs}} = B_N^b L_N^\ell, \quad (2.9)$$

can be understood in terms of the well-known baryon number and lepton number, with a discrete charge

$$q = bq_B + \ell q_L \bmod N = -b\mathcal{B} - \ell\mathcal{L} + b(y/3) \bmod N. \quad (2.10)$$

¹We include 3 right-handed neutrinos N^c which do not change our argument.

The exponents in eqs. (2.6) and (2.9) are related to each other by $m = b$ and $p = b + \ell$. Specific values for b and ℓ define a Z_N symmetry of the MSSM particles for which the quantity $\mathcal{Q} = b\mathcal{B} + \ell\mathcal{L} \bmod N$ is conserved. The lightest particle with nonzero $\mathcal{Q}$ value will be stable by the discrete symmetry. The following discrete symmetries are some examples obtained for given b and ℓ values.

$(b, \ell; N)$	g_N^{obs}	$\mathcal{Q}$
$(1, 0; N)$	B_N	$\mathcal{B} \bmod N$
$(0, 1; N)$	L_N	$\mathcal{L} \bmod N$
$(1, 1; N)$	$B_N L_N$	$(\mathcal{B} + \mathcal{L}) \bmod N$
$(1, -1; N)$	$B_N L_N^{-1}$	$(\mathcal{B} - \mathcal{L}) \bmod N$

Note that with $N = 2$ the symmetry in the last line ($B_2 L_2^{-1}$) corresponds to matter parity because $(-1)^{\mathcal{B}-\mathcal{L}} = (-1)^{3(\mathcal{B}-\mathcal{L})}$ for any $\text{SU}(3)_C$ invariant term for which $\mathcal{B}$ is always an integer. As long as the spin angular momentum is conserved, matter parity is equivalent to R -parity, $R_p = (-1)^{3(\mathcal{B}-\mathcal{L})+2s}$.

The discussion so far has been completely independent of any assumptions about the origin of the discrete symmetry. Requiring that the Z_N arises as a remnant of an anomaly free $\text{U}(1)$ gauge symmetry, we have to impose the discrete anomaly conditions of ref. [3] (Note the cubic anomaly condition is disregarded [11].)

$$[\text{SU}(3)_C]^2 - \text{U}(1) : \sum_{i=\mathbf{3}, \bar{\mathbf{3}}} q_i = N \cdot \mathbf{Z}, \quad (2.11)$$

$$[\text{SU}(2)_L]^2 - \text{U}(1) : \sum_{i=\mathbf{2}} q_i = N \cdot \mathbf{Z}, \quad (2.12)$$

$$[\text{gravity}]^2 - \text{U}(1) : \sum_i q_i = \begin{cases} N \cdot \mathbf{Z} & (N = \text{odd}), \\ \frac{N}{2} \cdot \mathbf{Z} & (N = \text{even}), \end{cases} \quad (2.13)$$

where the sums run over MSSM particles only. Additional exotic fields which may or may not be singlets under the SM gauge group $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ do not contribute to these anomaly conditions as long as they acquire a mass term when $\text{U}(1)$ is broken, i.e. if they are vectorlike under the SM gauge groups, while they are not under the $\text{U}(1)$.

The consequence of eqs. (2.11)–(2.13) is that some sets of parameters $(b, \ell; N)$ correspond to Z_N symmetries which are discrete anomaly free while others are anomalous and therefore ruled out (see refs. [3–5]). For instance, the symmetries of type B_N^b automatically satisfy eqs. (2.11) and (2.13) for all N and b . However, eq. (2.12) yields the nontrivial constraint

$$\sum_{i=\mathbf{2}} q_i = b \{N_f(3q_B[Q] + q_B[L]) + N_H(q_B[H_u] + q_B[H_d])\} \quad (2.14)$$

$$= -bN_f = 0 \bmod N, \quad (2.15)$$

where N_f and N_H denote the number of families of fermions and Higgs pairs, respectively. For $N_f = 3$, we obtain only $b = 0, \pm N/3$ as allowed choices for the cyclic symmetry. Unless

symmetry		Q	U^c	D^c	L	E^c	N^c	H_u	H_d	X_b	T_b	T_b^c	meaning of q	conserved $\mathcal{Q}$
R_N	q_R	0	-1	1	0	1	-1	1	-1	0	0	0	$y/3 - (\mathcal{B} - \mathcal{L})$	$(\mathcal{B} - \mathcal{L}) \bmod N$
	$3q_R - y$	-1	1	1	3	-3	-3	0	0	0	0	0	$-3(\mathcal{B} - \mathcal{L})$	
B_N	q_B	0	-1	1	-1	2	0	1	-1	0	0	0	$y/3 - \mathcal{B}$	$\mathcal{B} \bmod N$
	$3q_B - y$	-1	1	1	0	0	0	0	0	0	0	0	$-3\mathcal{B}$	
L_N	q_L	0	0	0	-1	1	1	0	0	0	0	0	$-\mathcal{L}$	$\mathcal{L} \bmod N$
$U_{a,N}$	q_{U_a}	0	0	0	0	0	0	0	0	$-\delta_{ab}$	0	0	$-\mathcal{U}_a$	$\mathcal{U}_a \bmod N$
$U'_{a,N}$	$q_{U'_a}$	0	0	0	0	0	0	0	0	0	$-\delta_{ab}$	δ_{ab}	$-\mathcal{U}'_a$	$\mathcal{U}'_a \bmod N$
y (hypercharge)		1	-4	2	-3	6	0	3	-3	0	0	0		
$\mathcal{B}$ (baryon no.)		$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0	0	0	0	0	0	0		
$\mathcal{L}$ (lepton no.)		0	0	0	1	-1	-1	0	0	0	0	0		

Table 1: Discrete charges of R_N , B_N , L_N , $U_{a,N}$, $U'_{a,N}$ and their relation with $\mathcal{B}$, $\mathcal{L}$, $\mathcal{U}_a$, and $\mathcal{U}'_a$.

$b = 0$ (which is not a real symmetry), we are led to the symmetry $B_{|3b|}^b$ or B_3 . Similarly, the discrete anomaly free symmetries of type L_N^ℓ are only $L_{|3\ell|}^\ell$ or L_3 (unless $\ell = 0$). This conclusion does not depend on whether there are massive $SU(2)_L$ charged exotics or not since their corresponding mass term would imply vanishing contribution to the discrete anomaly condition [12].

3. Hidden sector discrete symmetries

We now wish to extend the concept of discrete symmetries to the hidden sector or the SM neutral particles.² To do so, we introduce the generators $U_{a,N}$ and $U'_{a,N}$ which assign nontrivial discrete charges to, respectively, the Majorana (X_b) and the Dirac (T_b , T_b^c) particles of the hidden sector while the MSSM fields remain uncharged. Note that we label hidden sector fields by indices (a , b , etc) which can refer to fields with different or identical (i.e. family) $U(1)$ charges.

The generators $U_{a,N}$, $U'_{a,N}$ as well as R_N , L_N , and B_N — extended to include the hidden sector particles — are shown in table 1. Introducing the discrete symmetry of the hidden sector

$$g_N^{\text{hid}} = U_N = \prod_{a,b} U_{a,N}^{u_a} U_{b,N}^{u'_b}, \quad (3.1)$$

the generalized discrete symmetry over the observable and the hidden sectors can be written as

$$Z_N : g_N = g_N^{\text{obs}} g_N^{\text{hid}} = B_N^b L_N^\ell U_N. \quad (3.2)$$

It is uniquely determined by the integer exponents $(b, \ell, u_a, u'_b; N)$, entailing the discrete charges

$$q = bq_B + \ell q_L + u_a q_{U_a} + u'_b q_{U'_b} \bmod N. \quad (3.3)$$

Summation over repeated indices is assumed as usual. Under the assumption that the hidden sector particles acquire a mass after the gauge symmetry $U(1)$ is broken down to

²The discrete symmetry argument does not change even if the Dirac type exotics are SM-charged.

the discrete symmetry, invariance of the bilinear terms

$$W_{\text{hidden}} = m_a X_a X_a + m'_b T_b T_b^c \quad (3.4)$$

under Z_N constrains the exponents u_a to

$$u_a = 0 \quad \text{or} \quad N/2, \quad (3.5)$$

which makes it effectively a Z_2 parity for Majorana type field (X).

Starting with an anomaly free discrete symmetry g_N^{obs} in the observable sector, the extended discrete symmetry g_N can also originate in an anomaly free $U(1)$ gauge symmetry, regardless of the chosen values for u_a and u'_b . In other words, due to the Z_N invariance of the mass terms in eq. (3.4), g_N and g_N^{obs} *jointly* either satisfy or do not satisfy the discrete anomaly conditions of eqs. (2.11)–(2.13). Now we consider the case where Z_N can be factorized into two smaller discrete symmetries.

$$U(1) \rightarrow Z_N = Z_{N_1} \times Z_{N_2}, \quad (3.6)$$

where $N = N_1 N_2$. This decomposition is only possible if N_1 and N_2 have no common prime factor, i.e. they must be coprime to each other. Let us apply this method to separate the discrete symmetries of the observable and the hidden sector. To do so, we have to assume that the exponents b and ℓ are multiples of N_2 , while u_a and u'_b are multiples of N_1 . Eq. (3.2) can then be written as

$$g_N = B_{N_1 N_2}^b L_{N_1 N_2}^\ell \prod_{a,b} U_{a,N_1 N_2}^{u_a} U_{b,N_1 N_2}^{u'_b} = B_{N_1}^{b/N_2} L_{N_1}^{\ell/N_2} \prod_{a,b} U_{a,N_2}^{u_a/N_1} U_{b,N_2}^{u'_b/N_1}. \quad (3.7)$$

This yields a $Z_{N_1}^{\text{obs}}$ symmetry in the observable sector and a $Z_{N_2}^{\text{hid}}$ in the hidden sector with charges

$$q_{Z_{N_1}}^{\text{obs}} = \left(\frac{b}{N_2} \right) q_B + \left(\frac{\ell}{N_2} \right) q_L \quad \text{mod } N_1, \quad (3.8)$$

$$q_{Z_{N_2}}^{\text{hid}} = \left(\frac{u_a}{N_1} \right) q_{U_a} + \left(\frac{u'_b}{N_1} \right) q_{U'_b} \quad \text{mod } N_2. \quad (3.9)$$

Both originate in the underlying Z_N symmetry and are conserved separately. The symmetry $Z_{N_1}^{\text{obs}}$ can be used to forbid certain processes whose external states comprise only MSSM particles. On the other hand, the $Z_{N_2}^{\text{hid}}$ symmetry can stabilize the lightest $\mathcal{U}$ charged particle, leading to a dark matter candidate in the hidden sector [10, 9].

Depending on N_2 as well as the Z_{N_2} charges $q_{Z_{N_2}}^{\text{hid}}$, there could be even more than one hidden sector particle stable due to the discrete symmetry. Assume that $N_2 = \prod_k n_k$, where all factors n_k are coprime to each other. Evidently, all but perhaps one n_k are necessarily odd. Then, the decomposition of the discrete symmetry in the hidden sector reads

$$Z_{N_2}^{\text{hid}} = Z_{n_1}^{\text{hid}} \times Z_{n_2}^{\text{hid}} \times \dots. \quad (3.10)$$

What are the charges of the particles X_a and T_b under these individual $Z_{n_k}^{\text{hid}}$? Due to the invariance of the mass term for a Majorana particle X_a , its $Z_{n_k}^{\text{hid}}$ charge must be zero for odd n_k . In the case where there is an even n_k , the particle X_a has charge

$$q_{Z_{n_k}}[X_a] = -\frac{u_a}{N_1} \cdot \frac{n_k}{N_2} = 0 \quad \text{or} \quad \frac{n_k}{2} \quad (n_k = \text{even}), \quad (3.11)$$

under $Z_{n_k}^{\text{hid}}$. For Dirac particles T_b , the $Z_{n_k}^{\text{hid}}$ charges $q_{Z_{n_k}}[T_b]$ are related by

$$-\frac{u'_b}{N_1} = \sum_k q_{Z_{n_k}}[T_b] \cdot \frac{N_2}{n_k} \bmod N_2. \quad (3.12)$$

Since all n_k are coprime to each other, the charges $q_{Z_{n_k}}[T_b]$ are uniquely fixed by the value of $-u'_b/N_1$.³ Consider for example three particles X , T_1 , T_2 , which have the Z_{60} charges $q_{Z_{60}}[X] = 30$, $q_{Z_{60}}[T_1] = 24$, $q_{Z_{60}}[T_2] = 35$, respectively. The Z_{60} symmetry breaks up into $Z_4 \times Z_3 \times Z_5$, leading to the following charges.

	$q[X]$	$q[T_1]$	$q[T_2]$	$60/n_k$
Z_4	2	0	1	15
Z_3	0	0	1	20
Z_5	0	2	0	12
Z_{60}	30	24	35	—

From eqs. (3.9) and (3.12), the Z_{60}^{hid} discrete charge for T_2 , for example, can be written as

$$q_{Z_{60}}^{\text{hid}}[T_2] = 35 = -\frac{u'_b}{N_1} \bmod 60 = 1 \cdot 15 + 1 \cdot 20 + 0 \cdot 12 \bmod 60. \quad (3.13)$$

T_2 is the only particle charged under the Z_3 symmetry. Thus it is stable. Similarly T_1 is stable because it is the only Z_5 charged particle. Finally, the symmetry Z_4 stabilizes the lighter of the two particles X and T_2 . If this is T_2 , then there is no more particle stable due to the discrete symmetry. In that way, it is possible that different Z_{n_k} symmetries stabilize the same particle.

The important point in this discussion is that *a single* $U(1)$ gauge symmetry can effectively give rise to more than one discrete symmetry. One part of it might be used to forbid unwanted processes involving the MSSM fields only, while other parts lead to stable hidden sector particles, i.e. multiple dark matter candidates.⁴ This setup is schematically sketched in figure 1. The discussion here is basically a generalization of that of ref. [9], which dealt with only the Majorana case with a specific SXX mass term.

An example of the purely hidden sector discrete symmetry in the non-SUSY case can be found in ref. [13], where an additional $U(1)$ was introduced to explain the neutrino mass and dark matter simultaneously.

³If there were a second charge assignment $\tilde{q}_{Z_{n_k}}[T_b]$ for the same value of $-u'_b/N_1$, the sum $\sum_k (q_{Z_{n_k}}[T_b] - \tilde{q}_{Z_{n_k}}[T_b])/n_k$ would have to be integer. This however is only possible for $q_{Z_{n_k}}[T_b] - \tilde{q}_{Z_{n_k}}[T_b] = 0$.

⁴Of course, we can have multiple dark matter candidates from the MSSM sector and hidden sector for $Z_N = R_2 \times U_3$, for example, which can provide the LSP dark matter (stable under R -parity) and the Dirac type hidden sector dark matter (stable under U_3).

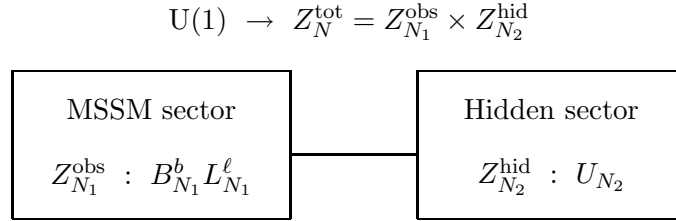


Figure 1: A unified picture of a single U(1) gauge symmetry that provides the discrete symmetries for the observable sector and the hidden sector.

4. General U(1) charges

Having discussed the most general Z_N symmetries that can arise from a U(1) gauge symmetry, we now want to derive the most general U(1) charges within our setup. Including the possibility that the superpotential terms of eqs. (2.3), (2.4), and (3.4) originate from higher-dimensional operators, the underlying theory before U(1) breaking generally includes the following terms⁵

$$\hat{W}_\mu = \hat{\mu} \left(\frac{S}{M} \right)^p S H_u H_d, \quad (4.1)$$

$$\begin{aligned} \hat{W}_{\text{Yukawa}} = & \hat{y}_{jk}^D \left(\frac{S}{M} \right)^{\tilde{d}} H_d Q_j D_k^c + \hat{y}_{jk}^U \left(\frac{S}{M} \right)^{\tilde{u}} H_u Q_j U_k^c + \\ & + \hat{y}_{jk}^E \left(\frac{S}{M} \right)^{\tilde{e}} H_d L_j E_k^c + \hat{y}_{jk}^N \left(\frac{S}{M} \right)^{\tilde{n}} H_u L_j N_k^c, \end{aligned} \quad (4.2)$$

$$\hat{W}_{\text{hidden}} = \hat{m}_a \left(\frac{S}{M} \right)^{\tilde{x}_a} S X_a X_a + \hat{m}'_b \left(\frac{S}{M} \right)^{\tilde{t}_b} S T_b T_b^c, \quad (4.3)$$

where we assume generation independent integer exponents with $0 \leq \tilde{d}, \tilde{u}, \tilde{e}, \tilde{n}$ and $-1 \leq p, \tilde{x}_a, \tilde{t}_b$. M is some high mass scale (e.g. M_{GUT} or M_{Pl}) at which new physics generates the nonrenormalizable operators. Note that $\hat{\mu}$, $\hat{m}_a$, and $\hat{m}'_b$ are dimensionless parameters.

These terms yield severe constraints on the allowed U(1) charges of the chiral matter fields. We find

$$Y_S : (1 + p)z[S] + z[H_u] + z[H_d] = 0, \quad (4.4)$$

$$Y_D : z[H_d] + z[Q] + z[D^c] + \tilde{d}z[S] = 0, \quad (4.5)$$

$$Y_U : z[H_u] + z[Q] + z[U^c] + \tilde{u}z[S] = 0, \quad (4.6)$$

$$Y_E : z[H_d] + z[L] + z[E^c] + \tilde{e}z[S] = 0, \quad (4.7)$$

$$Y_N : z[H_u] + z[L] + z[N^c] + \tilde{n}z[S] = 0, \quad (4.8)$$

$$Y_{X_a} : (1 + \tilde{x}_a)z[S] + 2z[X_a] = 0, \quad (4.9)$$

$$Y_{T_b} : (1 + \tilde{t}_b)z[S] + z[T_b] + z[T_b^c] = 0. \quad (4.10)$$

⁵In addition to the factors $\left(\frac{S}{M}\right)$ one could also have powers of $\left(\frac{H_u H_d}{M^2}\right)$ multiplying the effective superpotential terms. For the sake of clarity, we omit this possibility.

From this we obtain the general solution of U(1) charges in terms the continuous real parameters $\alpha, \beta, \gamma, \delta, \tau_b$

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_u] \\ z[H_d] \\ z[S] \\ z[X_a] \\ z[T_b] \\ z[T_b^c] \end{pmatrix} = \frac{\alpha}{3} \begin{pmatrix} 1 \\ -1 \\ -1 \\ -3 \\ 3 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \frac{\beta}{6} \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 6 \\ 0 \\ 3 \\ -3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \frac{\gamma}{3N_f} \begin{pmatrix} N_H(1+p) \\ -3N_f\tilde{u} + (3N_f - N_H)(1+p) \\ -3N_f\tilde{d} - N_H(1+p) \\ 0 \\ -3N_f\tilde{e} \\ -3N_f\tilde{n} + 3N_f(1+p) \\ -3N_f(1+p) \\ 0 \\ 3N_f \\ -3N_f(1+\tilde{x}_a)/2 \\ 0 \\ -3N_f(1+\tilde{t}_b) \end{pmatrix} - \frac{\delta}{3N_f} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\tau_b \\ \tau_b \end{pmatrix}. \quad (4.11)$$

In writing eq. (4.11), we have chosen a specific basis in which the first basis vector (corresponding to the parameter α) is $\mathcal{B} - \mathcal{L}$, the second (corresponding to β) is hypercharge. The parameters τ_b are related to the exponents u'_b of the Z_N symmetry by

$$u'_b = \tau_b \bmod N. \quad (4.12)$$

Furthermore, our basis is suitable to discuss the $[\text{SU}(2)_L]^2 - \text{U}(1)$ anomaly condition easily. From

$$A_{221'} : N_f(3z[Q] + z[L]) + N_H(z[H_d] + z[H_u]) + A_{221'}^{\text{exotic}} = 0 \quad (4.13)$$

we see that the parameters α, β, γ , and τ_b do not enter the anomaly condition. Plugging in the U(1) charges of eq. (4.11), we obtain

$$\delta = A_{221'}^{\text{exotic}}. \quad (4.14)$$

In the case where there are no exotic states which are charged under $\text{SU}(2)_L$, the parameter δ must therefore vanish due to the $[\text{SU}(2)_L]^2 - \text{U}(1)$ anomaly condition. Of course, to be free from gauge anomaly, the other anomaly conditions should also be satisfied with a specified particle spectrum. To be as general as possible we do not consider these full gauge anomaly conditions in this paper. However, see refs. [14–17, 7, 18] for some examples.

Note that eq. (4.11) is a generalization of the discussion presented in refs. [7, 8] where $\delta = p = \tilde{u} = \tilde{d} = \tilde{e} = 0$. This general charge assignment is consistent with the following well-known fact: assuming (i) Yukawa couplings with $\tilde{u} = \tilde{d} = \tilde{e} = \tilde{n} = 0$, (ii) no SM-charged particles other than quarks and leptons, (iii) vanishing of the mixed anomalies $[\text{SU}(3)_C]^2 - \text{U}(1)$ (yielding $p = -1$, see discussion in ref. [7], for example) and $[\text{SU}(2)_L]^2 - \text{U}(1)$ (yielding $\delta = 0$), the most general generation independent U(1) which can be defined on the quarks and leptons is a superposition of $\text{U}(1)_{B-L}$ and $\text{U}(1)_Y$, the first and the second basis vector of eq. (4.11) (see also refs. [19, 20]). Relaxing these conditions would allow different U(1) symmetries.

Disregarding τ_b , the parameters α , β , γ , and δ can be written in terms of the U(1) charges as

$$\alpha = z[H_d] - z[L], \quad \beta = -2z[H_d], \quad \gamma = z[S], \quad \delta = -N_f(3z[Q] + z[L]) + N_H(1+p)z[S]. \quad (4.15)$$

In a normalization in which all U(1) charges are integer, the above four parameters (as well as τ_b) are automatically also integer. Note that the contribution of δ can be absorbed effectively in the number of Higgs doublet pairs. However, it is not guaranteed in general that N_H^{eff} would remain integer.

Eq. (4.11) is useful to obtain general U(1) charges in various limits. For example, assuming $\tilde{u} = \tilde{d} = \tilde{e} = \tilde{n} = 0$, the quark-phobic case ($z[Q] = z[U^c] = z[D^c] = 0$) requires $p = -1$, $\beta = 0$, $\delta = N_f\alpha$.⁶ The lepton charges in this case are then given by

$$z[L] = -\alpha, \quad z[E^c] = \alpha, \quad z[N^c] = \alpha. \quad (4.16)$$

The lepto-phobic case ($z[L] = z[E^c] = 0$) requires $\alpha = 0$, $\beta = 0$. The quark charges in this case are then given by

$$z[Q] = \frac{N_H(1+p)\gamma - \delta}{3N_f}, \quad z[U^c] = \frac{(3N_f - N_H)(1+p)\gamma + \delta}{3N_f}, \quad z[D^c] = -\frac{N_H(1+p)\gamma - \delta}{3N_f}. \quad (4.17)$$

Depending on value of p , we can categorize the models. Especially the $p = 0$ case can solve the μ -problem by generating the effective μ parameter as

$$\mu = \hat{\mu} \langle S \rangle. \quad (4.18)$$

This is one of the most interesting cases for phenomenology, since the new gauge boson Z' and the exotic colored particles which are necessary to cancel the $[\text{SU}(3)]_C^2 - \text{U}(1)$ anomaly, are at the μ (TeV) scale, which can be explored by the LHC. A TeV scale Z' has implications also in cosmology such as providing a venue so that the right-handed sneutrino LSP dark matter candidate or the LUP dark matter candidate can be a thermal dark matter candidate through the Z' resonance [21, 10]. See ref. [22] for a review of this model. It might appear that this type of U(1) cannot have matter parity (R -parity) as its residual discrete symmetry, but there are ways to achieve this (see appendix A).

5. Construction of the U(1) charges for a given discrete symmetry

We discuss how to construct the most general U(1) charges, which have a given discrete symmetry as its residual symmetry. The SM-charged exotics are highly model-dependent and they may be obtained by scanning (see e.g. refs. [7, 8]). Here, we limit ourselves only to the MSSM particles and the SM-singlet exotics (X , T). The specific discrete symmetries we want to cover in this paper are listed in table 2. An overall sign change does not affect the discrete symmetry.

⁶See appendix B for further discussion related to DAMA/PAMELA results.

symmetry	Q	U^c	D^c	L	E^c	N^c	H_u	H_d	X_c	T_d	T_d^c	meaning of q
B_3	0	-1	1	-1	2	0	1	-1	0	0	0	$y/3 - \mathcal{B}$
$B_3 \times \prod_{a,b} U_{a,2}^{u_a} U_{b,2}^{u'_b}$	0	-2	2	-2	4	0	2	-2	$-3u_c$	$-3u'_d$	$3u'_d$	$2y/3 - 2\mathcal{B} - 3u_a\mathcal{A}_a - 3u'_b\mathcal{A}'_b$
$B_3 \times R_2$	0	1	-1	-2	1	3	-1	1	0	0	0	$-y/3 + \mathcal{B} - 3\mathcal{L}$

Table 2: The discrete charges of B_3 , $B_3 \times \prod_{a,b} U_{a,2}^{u_a} U_{b,2}^{u'_b}$, and $B_3 \times R_2$. Since $R_N = B_N L_N$, the latter symmetry can be expressed as $B_3 \times R_2 = B_6^5 L_6^3 = B_6^{-1} L_6^3$ from which one can easily calculate the discrete charges in terms of $\mathcal{B}$, $\mathcal{L}$, and y .

The general U(1) charges, before any discrete symmetry is assumed, are given in eq. (4.11). Integer normalization is achieved through the coefficient α , β , γ , and δ . Then, N of Z_N is determined by $z[S]$ fixing also the parameter $\gamma = N$ as shown in eq. (4.15). Since invariance under a hypercharge transformation is implicitly assumed throughout the paper, the hypercharge column (with coefficient β) of eq. (4.11) has no effect on the discrete symmetry. However, in order to obtain integer U(1) charges, β must be chosen in a particular way.

As a general procedure, we suggest the following:

- (i) Take $\gamma = N$ of Z_N .
- (ii) Identify some terms which are allowed by the given discrete symmetry as well as the SM gauge group.
- (iii) Extract an additional condition about the U(1) charges from these allowed terms (MSSM sector only).
- (iv) Using this additional relation, obtain the U(1) charges from eq. (4.11), the most general U(1) charge assignments before imposing any particular discrete symmetry.
- (v) Require the U(1) charges to be integer.

The resulting set of equation is the most general U(1) solution that contains the given discrete symmetry, up to arbitrary hypercharge shift and scaling. We illustrate our method on three examples: B_3 , $B_3 \times U_2$, and $B_3 \times R_2$.

5.1 U(1) $\rightarrow B_3$

Here, we will consider only the MSSM sector disregarding the hidden sector fields (X , T , T^c).

- (i) B_3 dictates $\gamma = 3$.
- (ii) To figure out the most general U(1) charge assignment that contains B_3 , use the fact that B_3 allows additional terms such as LLE^c , LQD^c , and LH_d . These terms can be written in a general form in the spirit of section 4. For example, LLE^c can be written as

$$\left(\frac{S}{M}\right)^n LLE^c, \quad (5.1)$$

where n is an integer.

(iii) This gives another condition on the U(1) charge assignment,

$$nz[S] + 2z[L] + z[E^c] = 0, \quad (5.2)$$

fixing the parameter α in eq. (4.11),

$$\alpha = \gamma(n - \tilde{e}) = 3(n - \tilde{e}). \quad (5.3)$$

(iv) Then the general solution for the MSSM part of the B_3 case can be written as

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_u] \\ z[H_d] \\ z[S] \end{pmatrix} = \frac{\beta}{6} \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 6 \\ 0 \\ 3 \\ -3 \\ 0 \end{pmatrix} + \begin{pmatrix} -\tilde{e} + n + \frac{N_H(1+p)}{N_f} \\ \tilde{e} - 3\tilde{u} - n + 3(1+p) - \frac{N_H(1+p)}{N_f} \\ \tilde{e} - 3\tilde{d} - n - \frac{N_H(1+p)}{N_f} \\ 3\tilde{e} - 3n \\ -6\tilde{e} + 3n \\ -3\tilde{e} - 3\tilde{n} + 3n + 3(1+p) \\ -3(1+p) \\ 0 \\ 3 \end{pmatrix} - \frac{\delta}{3N_f} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (5.4)$$

with two free parameters.

(v) Now the U(1) charges should all be integers. Regarding the first component of eq. (5.4), we therefore demand $z[Q] \equiv I_Q \in \mathbf{Z}$. This yields

$$\beta = 6(I_Q + \tilde{e} - n) - 2 \cdot \frac{3N_H(1+p) - \delta}{N_f}, \quad (5.5)$$

and eq. (5.4) takes the form

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_u] \\ z[H_d] \\ z[S] \end{pmatrix} = I_Q \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 6 \\ 0 \\ 3 \\ -3 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ -\tilde{u} + (n - \tilde{e}) + (1+p) \\ -\tilde{d} - (n - \tilde{e}) \\ 0 \\ -n \\ -\tilde{n} + (n - \tilde{e}) + (1+p) \\ -(n - \tilde{e}) - (1+p) \\ (n - \tilde{e}) \\ 1 \end{pmatrix} + \frac{3N_H(1+p) - \delta}{N_f} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \\ -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}. \quad (5.6)$$

Due to the requirement that all charges should be integer, the coefficient of the last column, $\frac{3N_H(1+p) - \delta}{N_f}$, must be an integer. As already mentioned, the hypercharge column (I_Q) makes no difference in fixing the discrete symmetry. The second column (with a coefficient of 3) cannot give any net discrete charges for a Z_3 symmetry. So the

last column carries all the information about the discrete symmetry. Its coefficient must be integer, but not a multiple of 3 to yield a Z_3 symmetry, i.e.

$$\frac{3N_H(1+p)-\delta}{N_f} = 3 \cdot \mathbf{Z} \pm 1 . \quad (5.7)$$

If eq. (5.7) is satisfied, the discrete charges can be easily read off from eq. (5.6) by disregarding the hypercharge column and then applying eq. (2.2) to the remaining two vectors. The result is B_3 , which becomes evident by comparing the third column to the charges of B_3 in table 2. Note that the discrete symmetry is independent of n and $\tilde{u}, \tilde{d}, \tilde{e}, \tilde{n}$. It is also independent of p as long as eq. (5.7) is satisfied for a given δ .

Using eq. (4.15), we can rewrite the condition of eq. (5.7) as

$$3z[Q] + z[L] = 3 \cdot \mathbf{Z} \pm 1 , \quad (5.8)$$

which forbids the operator $QQQL$ effectively. This shows that B_3 arises automatically as a residual discrete symmetry of the $U(1)$ if we require both:

1. presence of an $(S/M)^n LLE^c$ term (or any effective renormalizable $\mathcal{L}$ violating term),
2. absence of an $(S/M)^m QQQL$ term (for any integer m).

Assuming $\delta = 0$, $N_H = 1$, and $N_f = 3$, the second requirement is equivalent to the requirement of the presence of an effective μ term $(S/M)^p SH_u H_d$ with $p = 3 \cdot \mathbf{Z}$ or $3 \cdot \mathbf{Z} + 1$. The $p = 0$ case with an effective μ term $(SH_u H_d)$ can belong to this category. In the MSSM-like case with an original μ term $(H_u H_d)$, i.e. $p = -1$ case, we need nonvanishing contributions from $SU(2)_L$ exotics ($\delta \neq 0$) in order to have B_3 as a residual discrete symmetry.

The discrete charges are given by $q = q_B = -\mathcal{B} + y/3 \bmod 3$. Since the hypercharge is conserved by itself, the quantity which is conserved by B_3 is $\mathcal{B} \bmod 3$, dictating the selection rule

$$\Delta \mathcal{B} = 0 \bmod 3 . \quad (5.9)$$

Hence, proton decay ($\Delta B = 1$) and neutron-antineutron oscillation ($\Delta B = 2$) are absolutely forbidden by the selection rule of B_3 [23].

Unless R -parity is separately imposed, this is an R -parity violating model. The violation of R -parity implies distinguishable phenomenology. See refs. [24–29] for some implications of the R -parity violation, for example. The proton is still protected by B_3 even better than by R -parity [30]. The dark matter issue still needs to be addressed.

5.2 $U(1) \rightarrow Z_6 = B_3 \times U_2$

Here we will consider the B_3 symmetry for the MSSM sector, augmented with U -parity ($U_2 = \prod_{a,b} U_{a,2}^{u_a} U_{b,2}'^{u_b'}$) for the hidden sector.⁷

- (i) Z_6 fixes $\gamma = 6$.

⁷See ref. [9] for a special case of only Majorana hidden sector fields.

(ii) In the MSSM sector, the LLE^c term can be written as

$$\left(\frac{S}{M}\right)^n LLE^c, \quad (5.10)$$

where n is an integer.

In the hidden sector, the $X_a X_a$ and $T_b T_b^c$ terms read

$$\left(\frac{S}{M}\right)^{\tilde{x}_a} S X_a X_a, \quad \left(\frac{S}{M}\right)^{\tilde{t}_b} S T_b T_b^c, \quad (5.11)$$

where $\tilde{x}_a, \tilde{t}_b$ are integers.

(iii) For the MSSM sector we obtain

$$nz[S] + 2z[L] + z[E^c] = 0, \quad (5.12)$$

yielding the condition

$$\alpha = \gamma(n - \tilde{e}) = 6(n - \tilde{e}). \quad (5.13)$$

The hidden sector mass terms in eq. (5.11) do not give any additional constraints on the general solution because we already used these to derive eq. (4.11).

(iv,v) Demanding $I_Q \equiv z[Q]$ to be an integer, β is given by

$$\beta = 6(I_Q + 2\tilde{e} - 2n) - \frac{2}{N_f} (6N_H(1+p) - \delta), \quad (5.14)$$

and eq. (4.11) takes the form

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_u] \\ z[H_d] \\ z[S] \\ z[X_a] \\ z[T_b] \\ z[T_b^c] \end{pmatrix} = I_Q \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 6 \\ 0 \\ 3 \\ -3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + 6 \begin{pmatrix} 0 \\ -\tilde{u} + (n - \tilde{e}) + (1+p) \\ -\tilde{d} - (n - \tilde{e}) \\ 0 \\ -n \\ -\tilde{n} + (n - \tilde{e}) + (1+p) \\ -(n - \tilde{e}) - (1+p) \\ (n - \tilde{e}) \\ 1 \\ 0 \\ 0 \\ -(1 + \tilde{t}_b) \end{pmatrix} + \frac{6N_H(1+p) - \delta}{N_f} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \\ -2 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -3(1 + \tilde{x}_a) \\ -\tau_b \\ \tau_b \end{pmatrix}. \quad (5.15)$$

Again the first two columns have no effect on the discrete symmetry, which is therefore only determined by the coefficient of the third vector, $\frac{6N_H(1+p) - \delta}{N_f}$, as well as the parameters $\tilde{x}_a$ and τ_b . The former defines the discrete charges of the MSSM fields

while the latter two fix those of the hidden sector particles. As we are looking for the case with B_3 among the MSSM fields, we must require (see table 2)

$$\frac{6N_H(1+p) - \delta}{N_f} = 6 \cdot \mathbf{Z} \pm 2. \quad (5.16)$$

On the other hand, the Z_2 symmetry (U -parity) of the hidden sector necessitates

$$\tau_b = 3 \cdot \mathbf{Z}, \quad (5.17)$$

whereas $\tilde{x}_a$ remains unconstrained. It is worth noting that the discrete symmetry is independent of $\tilde{t}_b$.

With only one Majorana X and one Dirac T particle in the hidden sector, one can have three different nontrivial scenarios:

- X is odd and T is even under U -parity, i.e. $u = 1$ and $u' = 0$. This requires $\tilde{x} = 2 \cdot \mathbf{Z}$ and $\tau = 6 \cdot \mathbf{Z}$. Reversely, if $(\frac{S}{M})^{\tilde{x}} SXX$ with $\tilde{x} = 0, 2, 4, \dots$ exists, the hidden field X automatically has odd U -parity.
- X is even and T is odd under U -parity, i.e. $u = 0$ and $u' = 1$. Such a situation requires $\tilde{x} = 2 \cdot \mathbf{Z} + 1$ and $\tau = 6 \cdot \mathbf{Z} + 3$. The exponent $\tilde{t}$ in the mass term $(\frac{S}{M})^{\tilde{t}} TT^c$ does not enter the discussion of the discrete symmetry.
- X is odd and T is odd under U -parity, i.e. $u = 1$ and $u' = 1$. Here we need $\tilde{x} = 2 \cdot \mathbf{Z}$ and $\tau = 6 \cdot \mathbf{Z} + 3$. In this case, the lighter of the two particles will be stable due to U -parity.

Using eq. (4.15), we can rewrite the condition of eq. (5.16) as

$$3z[Q] + z[L] = 6 \cdot \mathbf{Z} \pm 2, \quad (5.18)$$

which forbids the operator $QQQL$ effectively. Therefore, a symmetry of type $B_3 \times \prod_a U_{a,2}^{u_a}$ arises automatically as a residual discrete symmetry of the $U(1)$ if we require:

1. presence of an $(S/M)^n LLE^c$ term (or any effective renormalizable $\mathcal{L}$ violating term),
2. absence of an $(S/M)^m QQQ L$ term (for any integer m),
3. presence of an $(\frac{S}{M})^{\tilde{x}_a} SX_a X_a$ term with $\tilde{x}_a = 0, 2, 4, \dots$, resulting in X_a being odd under U -parity.

Unfortunately, the case where T_b has odd U -parity cannot be discussed in terms of requiring the presence or absence of some effective operators as discussed above.

The discrete charges are given by $q = 2q_B + 3u_a q_{U_a} + 3u'_b q_{U'_b} \bmod 6 = -2\mathcal{B} + 2y/3 - 3u_a \mathcal{U}_a - 3u'_b \mathcal{U}'_b \bmod 6$. Since the hypercharge is conserved by itself, the quantity which is conserved by B_3 is $\mathcal{B} \bmod 3$ and the one conserved by U -parity is $u_a \mathcal{U}_a + u'_b \mathcal{U}'_b \bmod 2$, dictating the selection rules

$$\Delta \mathcal{B} = 0 \bmod 3, \quad \Delta(u_a \mathcal{U}_a + u'_b \mathcal{U}'_b) = 0 \bmod 2, \quad (5.19)$$

which prevents the proton and the LUP from decaying. Therefore, R -parity is not necessary to address the stability of the proton and the dark matter candidate.

5.3 $U(1) \rightarrow Z_6 = B_3 \times R_2$

Here we will consider the $B_3 \times R_2$ symmetry for the MSSM sector without any hidden sector fields.⁸ As we can check with table 2, this symmetry allows $(\frac{S}{M})^n (H_u L)^2$ which can provide an additional condition. Applying the general procedure, we find

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_u] \\ z[H_d] \\ z[S] \end{pmatrix} = I_Q \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 6 \\ 0 \\ 3 \\ -3 \\ 0 \end{pmatrix} + 6 \begin{pmatrix} 0 \\ -\tilde{u} \\ -\tilde{d} + (1+p) \\ 0 \\ -\tilde{e} + (1+p) \\ -\tilde{n} \\ 0 \\ -(1+p) \\ 1 \end{pmatrix} + \frac{6N_H(1+p) - \delta}{N_f} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \\ -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + 3n \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}. \quad (5.20)$$

To have the $B_3 \times R_2$, the last two vectors should results in the discrete charges of table 2. Then we need

$$\frac{6N_H(1+p) - \delta}{N_f} = 6 \cdot \mathbf{Z} \mp 2, \quad 3n = 6 \cdot \mathbf{Z} + 3, \quad (5.21)$$

where the second equation requires n to be an odd integer.

6. Summary and conclusions

In this paper, we systematically studied the residual discrete symmetry of an extra Abelian gauge symmetry, which may interact with both the MSSM sector and the hidden sector. Despite a common gauge origin, the discrete symmetry can have important implications separately for the observable and the hidden sector, such as the stability of the proton *and* dark matter. We provided the most general framework to discuss such a symmetry including Majorana type and Dirac type hidden sector fields.

We also argued how to construct the most general $U(1)$ symmetry for the MSSM sector and hidden sector for a given discrete symmetry, illustrating our procedure for several examples. Our results should be useful for $U(1)$ model building. For example, in order to make sure the proton and the Majorana hidden sector dark matter candidate are stable in the absence of R -parity, one can, in a minimal framework with $\delta = 0$, $N_H = 1$, and $N_f = 3$, just require (i) $SH_u H_d$ (i.e. the effective μ term that solves the μ -problem with the $U(1)$ gauge symmetry), (ii) LLE^c (a renormalizable $\mathcal{L}$ violating term), and (iii) SXX (a mass term for the Majorana hidden sector field X). Then, B_3 and U -parity are automatically invoked in the MSSM and the hidden sector, respectively, as a residual discrete symmetry of the common $U(1)$ gauge symmetry (in the form of $Z_6 = B_3 \times U_2$). Their selection rules ensure absolute stability of the proton and the LUP dark matter.

In appendix A, we investigated the cases in which the $U(1)$ gauge symmetry that solves the μ -problem can contain matter parity (equivalent to R -parity) as a residual discrete

⁸See ref. [4, 31] for details about this symmetry.

symmetry. This can provide a useful framework for R -parity conserving U(1) extended supersymmetric models, without imposing a separate R -parity.

In appendix B, we made a comment on the relation between the discrete symmetry and the leptonically interacting LUP dark matter candidate, which has been recently focused on due to the possibility of the simultaneous explanation of the DAMA modulation and the PAMELA results.

A. U(1) gauge origin of R -parity and the μ -problem solution

In this appendix, we investigate the conditions under which the R_2 matter parity (equivalent to the R -parity) can emerge as a residual discrete symmetry of the extra U(1) gauge symmetry. The general U(1) charges of the MSSM sector and the hidden sector are given in eq. (4.11). In order to unveil the discrete symmetry, let us introduce a new parameter β' which is related to the original parameters $\alpha, \beta, \gamma, \delta$ by

$$\frac{\beta}{6} = \frac{\beta'}{6} - \frac{\alpha}{3} - \frac{\gamma}{3N_f} N_H(1+p) + \frac{\delta}{3N_f}. \quad (\text{A.1})$$

Using this definition, eq. (4.11) can be rewritten to separate the columns into those which do and which do not affect the discrete symmetry (in the R_N and B_N basis) among the MSSM fields. In the following, we do not consider the the hidden sector fields, which are irrelevant to our discussion.

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_u] \\ z[H_d] \\ z[S] \end{pmatrix} = -\alpha \underbrace{\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 1 \\ -1 \\ 1 \\ -1 \\ 0 \end{pmatrix}}_{q_R} - \frac{\gamma N_H(1+p) - \delta}{N_f} \underbrace{\begin{pmatrix} 0 \\ -1 \\ 1 \\ -1 \\ 2 \\ 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}}_{q_B} + \frac{\beta'}{6} \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 6 \\ 0 \\ 3 \\ -3 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ -\tilde{u} + (1+p) \\ -\tilde{d} \\ 0 \\ -\tilde{e} \\ -\tilde{n} + (1+p) \\ -(1+p) \\ 0 \\ 1 \end{pmatrix}. \quad (\text{A.2})$$

As mentioned before, the hypercharge (third) column does not influence the discrete symmetry at all since we require hypercharge shift invariance. The fourth column has no effect on the discrete symmetry among the MSSM fields because their contributions to the U(1) charges are integer multiples of $\gamma = z[S] = N$. Therefore only the first and the second column define the discrete symmetry among the MSSM fields. Comparing the entries of both vectors with the discrete charges of table 1, we see that the first column corresponds to q_R and the second to q_B . Hence, the type of Z_N^{obs} symmetry depends only on the coefficients of these two vectors, namely on α and $\frac{\gamma N_H(1+p) - \delta}{N_f}$. Our assumption of integer U(1) charges requires both to be integer. Note that $\frac{\beta'}{6}$ is necessarily also integer and can be replaced by I_Q (see step (v) of the examples in section 5).

In order to have a pure R_N symmetry, the coefficient of q_R should be $\pm 1 \bmod N$ and the coefficient of q_B must vanish $\bmod N$, i.e.

$$\alpha = \gamma \cdot \mathbf{Z} \pm 1, \quad \frac{\gamma N_H(1+p) - \delta}{N_f} = \gamma \cdot \mathbf{Z}. \quad (\text{A.3})$$

With $p = -1$, $\delta = 0$, and $\tilde{u} = \tilde{d} = \tilde{e} = \tilde{n} = 0$, we have only a mixtures of the $U(1)_{B-L}$ and $U(1)_Y$. It always has the pure R_N symmetry as a residual discrete symmetry as long as $\alpha = \gamma \cdot \mathbf{Z} \pm 1$. With $\gamma = 2$, we obtain R_2 parity. Relaxing $\tilde{u} = \tilde{d} = \tilde{e} = \tilde{n} = 0$ does not change the discrete symmetry.

In order to solve the μ -problem with a TeV scale $U(1)$ gauge symmetry, however, we should take $p = 0$ (see section 4). With $N_f = 3$, eq. (A.3) can be written as

$$\delta = \gamma(N_H - 3 \cdot \mathbf{Z}). \quad (\text{A.4})$$

Therefore we need additional $SU(2)_L$ exotic fields in the form of $N_H = 3 \cdot \mathbf{Z}$ generations of Higgs pairs or some exotic doublet contribution δ , in order to have $Z_N^{\text{obs}} = R_N$ while solving the μ -problem with a common $U(1)$ gauge symmetry.

However, it should be mentioned that there is another way for the $U(1)$ to be a solution to the μ -problem while having matter parity as a residual discrete symmetry, which may not require additional $SU(2)_L$ charged particles. If the total discrete symmetry in the MSSM sector has R_2 as a *part* of it, i.e. $Z_N^{\text{obs}} = R_2 \times Z_{N/2}$ (where 2 and $N/2$ are coprime), both R_2 and $Z_{N/2}$ will be conserved independently. For instance, consider $B_3 \times R_2$ as the $U(1)$ residual discrete symmetry as in section 5.3. With $N_f = 3$ and $N_H = 1$, and no $SU(2)_L$ exotics ($\delta = 0$), eq. (5.21) gives $1 + p = 3 \cdot \mathbf{Z} \mp 1$, which allows $p = 0$ to solve the μ -problem. Hence, one $U(1)$ gauge symmetry can be the common source of the μ -problem solution as well as R -parity.

B. Leptonically coupling dark matter

It is worth to note that various coupling limits are still compatible with discrete symmetries. For instance, consider the $B_3 \times U_2$ we studied in section 5.2. The quark-phobic case ($z[Q] = z[U^c] = z[D^c] = 0$) requires $\delta = 6(N_f(\tilde{d} - \tilde{e} + n) + N_H(1+p))$ and $\tilde{u} + \tilde{d} = (1+p)$, and $I_Q = 0$. The lepton charges in this case, up to arbitrary scaling, are

$$z[L] = -6(\tilde{d} - \tilde{e} + n), \quad z[E^c] = 6(2\tilde{d} - 2\tilde{e} + n), \quad z[N^c] = 6(-\tilde{e} - \tilde{n} + n + (1+p)). \quad (\text{B.1})$$

In particular, a dark matter candidate that interacts with only leptons has been paid good attention since it may be able to explain the DAMA annual modulation without making conflict with other direct detection experiments [32]. This kind of dark matter would be consistent with the property that can naturally explain the PAMELA results (for example, see ref. [33]). PAMELA showed a significant positron excess [34] but no deviation in the proton/antiproton data [35]. Since the LUP dark matter, which is stable under the U -parity, interacts with the gauge boson Z' of the $U(1)$ gauge symmetry, the quark-phobic

case can satisfy this property. See ref. [36] for an illustration how such a dark matter can explain the DAMA and PAMELA results.⁹

As $B_3 \times U_2$ is compatible with the quark-phobic case, the existence of such a dark matter may not only explain the DAMA and PAMELA results but also suggests why the proton and dark matter are stable without introducing separate parities. This scenario may be tested, for example, by the precise measurement of Z' coupling to leptons and comparison with eq. (B.1).

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⁹There are other scenarios to explain the PAMELA data with an extra $U(1)$ gauge symmetry. For example, see ref. [37] for an illustration how a hidden $U(1)$ gauge symmetry, which interacts with the SM particles via kinetic mixing can explain such a signature.

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