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EXPERIMENTAL STUDY OF THE THREE-BODY LEPTONIC DECAY MODES OF THE K^+ MESON (*)

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I. INTRODUCTION

We report in this paper the results of an experimental study of the three-body leptonic decay modes (K_{L3}^+) of the K^+ mesons ($K_{e3}^+ \rightarrow e^+ + \pi^0 + \nu$, $K_{\mu 3}^+ \rightarrow \mu^+ + \pi^0 + \nu$).

It is obvious from the conservation laws that all momenta and angles in a K_{L3}^+ decay at rest are determined by the specification of any two independent

variables. Pais and Treiman have pointed out that the pion momentum and the angle between pion and neutrino directions are convenient variables and have obtained the following expressions for the distribution functions in these variables (assuming pure couplings):¹⁾

Vector coupling

$$F_V(P, \theta) dP d\cos\theta = \frac{P^2 (W^2 - P^2 - m_L^2)^2}{E (W + P \cos\theta)^4} \left\{ P^2 \sin^2\theta f_V^2 + \frac{m_L^2}{M_K^2} [M_K f_V + (W + P \cos\theta) g_V]^2 \right\} dP d\cos\theta \quad (1a)$$

Scalar coupling

$$F_S(P, \theta) dP d\cos\theta = \frac{P^2 (W^2 - P^2 - m_L^2)^2}{E (W + P \cos\theta)^2} f_S^2 \cdot dP d\cos\theta \quad (1b)$$

Tensor coupling

$$F_T(P, \theta) dP d\cos\theta = \frac{P^4 (W^2 - P^2 - m_L^2)^2}{M_K^2 E (W + P \cos\theta)^4} f_T^2 [(W \cos\theta + P)^2 + m_L^2 \sin^2\theta] dP d\cos\theta. \quad (1c)$$

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Here P is the pion momentum and E is the total pion energy; θ is the angle between the pion and the neutrino; M_K is the K^+ mass; $W = M_K - E$; m_π and m_L are the pion and lepton masses respectively, and f_V , g_V , f_S , and f_T are functions ("form factors") of $q^2 = M_K^2 + m_\pi^2 - 2M_KE$, the square of the invariant four-momentum transfer. We assume time-reversal invariance and take f_V , g_V , f_S , and f_T to be real.

In the present investigation we have attempted to study the following questions:

- (1) What couplings are responsible for K_{e3}^+ and $K_{\mu3}^+$ decay ?
- (2) Are the muon and electron couplings identical; i.e., are the form factors the same ?
- (3) What can be said about the q^2 dependence of the form factors ?

The experiment was run by exposing a xenon bubble chamber to a separated K^+ beam of such momentum that the K^+ mesons were stopped near the centre of the chamber. Details of the experiment, identifica-

tion of the various modes, sample selection, etc. have been published elsewhere²⁻⁵⁾.

A. K_{e3}^+ DECAYS

1. Nature of the coupling

We have already shown that the vector coupling seems to be the only one that agrees well with our K_{e3}^+ data³⁾. This conclusion has one qualification: to rule out the scalar hypothesis, we have assumed a "gentle" q^2 dependence of the form factor f_V^e (we use superscripts, e , μ , to separate the form factors in K_{e3}^+ and $K_{\mu3}^+$ decay in those analyses where we are not specifically assuming the universality of the muon and electron couplings).

B. The q^2 dependence of f_V^e

We have represented the q^2 dependence of f_V^e by the first two terms in a series expansion

$$f_V^e(q^2) = A(1 + \lambda q^2/m_\pi^2).$$

By comparison with data from 216 events, we have obtained from a maximum-likelihood analysis

$$\lambda = 0.04 \pm 0.04.$$

C. $K_{\mu3}^+$ DECAYS

1. Nature of the coupling

To introduce the corrections due to chamber geometry, we can conveniently rewrite the various distribution functions in terms of the variables P and E_μ , where E_μ is the muon total energy:

Vector coupling

$$G_V(P, E_\mu) dP dE_\mu = \frac{2P}{E} \left\{ f_V^2 [4E_\mu(W - E_\mu) - (W^2 - P^2 - m_L^2)] + 4f_V g_V (W - E_\mu) \frac{m_L^2}{M_K} + \frac{m_L^2}{M_K^2} g_V^2 (W^2 - P^2 - m_L^2) \right\} dP dE_\mu \quad (2a)$$

Scalar coupling

$$G_S(P, E_\mu) dP dE_\mu = \frac{2P}{E} f_S^2 (W^2 - P^2 - m_L^2) dP dE_\mu \quad (2b)$$

Tensor coupling

$$G_T(P, E_\mu) dP dE_\mu = \frac{2P}{M_K^2 E} f_T^2 [P^2(W^2 - P^2 - m_L^2) - 4P^2(W - E_\mu)^2 + (W^2 + P^2 + m_L^2 - 2WE_\mu)^2] dP dE_\mu. \quad (2c)$$

We have calculated expected pion-energy distributions by integrating Eq. (2) over E_μ , taking due account of chamber geometry, with the following further assumptions:

(1) All form factors are taken to be constant.

(2) The ratio g_V^μ/f_V^μ is chosen, on the assumption of the identity of muon and electron coupling strengths, from the measured ratio of $K_{\mu 3}^+$ and $K_{e 3}^+$ decay rates²⁾, 0.96 ± 0.16 . Because the rates are quadratically related to the f_V and g_V , this ratio is satisfied by two values of g_V/f_V , namely 0.5 ± 0.4 and -4.8 ± 0.4 .

In Fig. 1 we show the experimental pion momentum spectrum and compare it with the scalar, tensor and both vector distributions calculated as described above. If one makes a χ^2 test (see Table I) of the experimental data with these various theoretical distributions based on constant form factors, one finds that the most likely coupling is vector (with

TABLE I

Comparison of experimental $K_{\mu 3}^+\pi^0$ momentum distribution with various couplings having constant form factors

Coupling	χ^2 Probability (%)
Vector, $g_V/f_V = +0.5$	45
Vector, $g_V/f_V = -4.8$	<0.1
Scalar	<0.1
Tensor	9

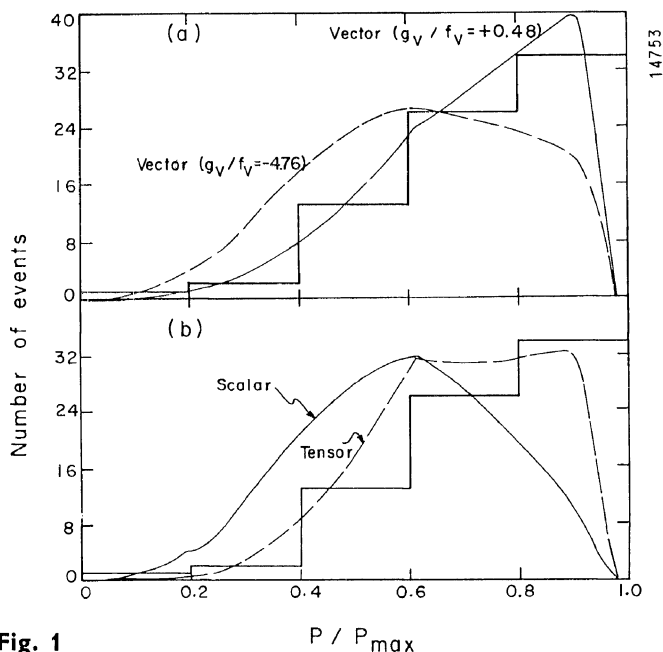


Fig. 1

$g_V/f_V = +0.5$), although tensor is not ruled out. The scalar coupling, as well as the vector coupling with $g_V/f_V = -4.8$, does not fit the data at all.

2. Energy Dependence of f_V^μ and g_V^μ

Our data are insufficient to permit significant conclusions on the energy dependence of f_V^μ and g_V^μ . We have, however, compared our results with the predictions of the conserved current theory⁶⁾:

$$\frac{g_V}{f_V} = -\frac{M_K W}{W^2 - P^2} = -\frac{M_K W}{q^2}.$$

In this theory the ratio g_V/f_V has a very strong energy dependence due to the pole at $q = 0$, which causes a marked peak in the pion spectrum at high energy. There is less than a 2% chance that the observed $K_{\mu 3}^+$ spectra are correctly described by this theory. Moreover, if $\mu-e$ universality is assumed, the ratio of $K_{\mu 3}^+$ and $K_{e 3}^+$ decay rates is predicted to be:

$$\frac{R(K_{\mu 3}^+)}{R(K_{e 3}^+)} = 0.38.$$

This is to be compared with the observed ratio²⁾: 0.96 ± 0.15 .

We conclude that our experiment is in substantial disagreement with this prediction. On the other hand, more sophisticated models⁷⁾ based on assumptions about partially conserved currents lead to much less violent energy dependences which are compatible with our data.

IV. IDENTITY OF THE MUON AND ELECTRON COUPLING STRENGTHS

Since we have shown above that the experimental evidence favors vector coupling for both $K_{e 3}^+$ and $K_{\mu 3}^+$ decay, we are now ready to test the actual identity of the coupling by comparing f_V^μ and f_V^e (remembering that g_V^e cannot be observed, and hence no comparison of g_V^μ and g_V^e is possible). Again we assume the constancy of the f_V 's, noting that this assumption is compatible with our measurement of λ discussed in Section II. We then note that:

(a) the $K_{e 3}^+$ rate determines f_V^e

(b) the $K_{\mu 3}^+$ rate determines a relation between f_V^μ and g_V^μ

(c) the study of the $K_{\mu 3}^+$ decay (P, E_μ) distribution provides another relation between f_V^μ and g_V^μ .

Thus from (b) and (c), f_V^μ can be determined and compared to f_V^e obtained from (a). The result is that $f_V^\mu/f_V^e = 1.09 \pm 0.15$, in good agreement with the expectations.

V. FURTHER DISCUSSION

We have already noted in Sec. III that if universality of the muon and the electron is assumed, the measured $K_{e 3}^+$ and $K_{\mu 3}^+$ rates lead to two values of g_V/f_V , namely $+0.5$ and -4.8 . On the other hand, the study of the (P, E_μ) distribution in $K_{\mu 3}^+$ decay leads to

$$\frac{g_V}{f_V} = +0.3 \pm 0.5,$$

thus ruling out the large negative g_V/f_V possibility.

This result, already clear from Fig. 1, disagrees with the published results of Dobbs *et al.*⁸⁾ Various theoretical models predict values of g_V/f_V from -0.2 to -0.9 ⁷⁾. These are in reasonable agreement with the conclusion from the $K_{\mu 3}^+(P, E_\mu)$ distribution and a little lower (i.e., more negative) than the result from the ratio of $K_{\mu 3}^+$ and $K_{e 3}^+$ rates. However, in view of the quoted statistical errors, as well as conceivable systematic biases in determining the $K_{\mu 3}^+$ rate, we interpret our data as being fully compatible with the predictions of these models.

To make the line of reasoning more apparent, we have assumed throughout our discussion that f_V and g_V are constant, which is in agreement with our data. A more complete (but more complicated) analysis, allowing additional parameters to describe the energy dependence of f_V and g_V , has been published elsewhere^{4, 5)}. The conclusions drawn in this case are substantially the same as those of the present paper, except that one can no longer rigorously exclude the possibility of scalar and tensor couplings for $K_{\mu 3}^+$ decay.

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DISCUSSION

BLUDMAN: Are there additional statistics on this branching ratio $K_{\mu 3}/K_{e 3}$?

VAN DE WALLE: No. The value we have used in doing this calculation was 0.96 ± 0.16 and this is the value published by the Berkeley Xenon Bubble Chamber Group some time ago. There are no more statistics available at present.

BLUDMAN: The comment I wanted to make was that the g_V/f_V ratio as measured in your experiment and at Pennsylvania

are both inconsistent with the Goldberger-Treiman-type calculation based upon a dominant $K-\pi$ state at 880 MeV. Only if the ratio $K_{\mu 3}/K_{e 3}$ were to go down to something like 0.7 would the spectrum be so interpretable.

VAN DE WALLE: The value 0.7 is *not excluded* by the actual branching ratio result. It falls within 2σ of the measured value (which corresponds to a probability level of *more than 5%*). However the authors feel that if anything could happen to their

numbers as a consequence of un-detected systematic errors, it would be in the direction of making this ratio lower.

TREIMAN: How do you understand the discrepancy between your results and the Pennsylvania experiment? Could it be that there is an energy-dependence of g_V ?

VAN DE WALLE: The difference between our results is an experimental fact, not explainable by means of a peculiar theoretical situation. We are supposed to have measured the same physical facts and whatever particular situation one chooses, it should have manifested itself in *both* experiments in exactly the same way.

As for possible experimental causes of the discrepancy between our results, I would just like to give two arguments which, I believe, favour our experiment. First of all, the Pennsylvania Group has studied the muon spectrum which in itself is a less sensitive tool to look at the different couplings; it is the π^0 -spectrum which contains most of the information about the nature of the couplings involved and which is most sensitive to different g_V/f_V -ratios. In our experiment we looked at *both* the μ and the π^0 spectra.

Secondly, the Pennsylvania Group equipment was not able to detect muons with $E_{\text{kin}} < 50$ MeV, as a consequence of which they were obliged to use the emulsion data in order to normalize their distribution.
