

# SMALL- $x$ BEHAVIOUR OF THE NON-SINGLET AND SINGLET STRUCTURE FUNCTIONS $G_1$

B. I. Ermolaev<sup>1†</sup>, M. Greco<sup>2</sup> and S.I. Troyan<sup>3</sup>

- (1) *Ioffe Physico-Technical Institute, 194021 St.Petersburg, Russia*  
 (2) *Department of Physics and INFN, University Rome III, Rome, Italy*  
 (3) *St.Petersburg Institute of Nuclear Physics, 188300 Gatchina, Russia*  
 † *E-mail: ermolaev@pop.ioffe.rssi.ru*

## Abstract

Explicit expressions for the non-singlet and singlet structure functions  $g_1$  at the small  $x$ -region are obtained. They include the total resummation of double-logarithmic contributions and accounting for the running QCD coupling effects. We predict that asymptotically the singlet  $g_1 \sim x^{-\Delta_S}$ , with the intercept  $\Delta_S = 0.86$ , which is approximately twice larger than the non-singlet intercept  $\Delta_{NS} = 0.4$ . The impact of the initial quark and gluon densities on the sign of  $g_1$  at  $x \ll 1$  is discussed.

## Introduction

Deep inelastic scattering (DIS) is one of the basic processes for investigating the structure of hadrons. As is well known, all information about the hadrons participating into DIS comes from the hadronic tensor  $W_{\mu\nu}$ . The imaginary part of  $W_{\mu\nu}$  is proportional to the forward Compton amplitude when the deeply off-shell photon with virtuality  $q^2$  scatters off an on-shell hadron with momentum  $p$ . For the electron-hadron DIS, the spin-dependent part,  $W_{\mu\nu}^s$ , of  $W_{\mu\nu}$  is

$$W_{\mu\nu}^s = i\epsilon_{\mu\mu\lambda\rho} \frac{q_\lambda m}{pq} \left[ S_\rho g_1 + \left( S_\rho - \frac{(Sq)}{pq} p_\rho \right) g_2 \right] \approx i\epsilon_{\mu\mu\lambda\rho} \frac{q_\lambda m}{pq} \left[ S_\rho^\parallel g_1 + S_\rho^\perp (g_1 + g_2) \right] \quad (1)$$

where  $m$  is the hadron mass,  $S_\rho^\parallel$  and  $S_\rho^\perp$  are the longitudinal and transverse (with respect to the plane formed by  $p$  and  $q$ ) components of the hadron spin  $S_\rho$ ;  $g_1$  and  $g_2$  are the spin structure functions. Both of them depend on  $x = -q^2/2pq$ ,  $0 < x \leq 1$  and  $Q^2 = -q^2 > 0$ . Obviously, small  $x$  corresponds to  $s = (p+q)^2 \approx 2pq \gg Q^2$ . In this case,  $S_\rho^\parallel \approx p_\rho/m$  and therefore the part of  $W_{\mu\nu}^s$  related to  $g_1$  does not depend on  $m$ . Then if  $Q^2 \gg m^2$ , one can assume the factorization and regard  $W_{\mu\nu}^s$  as a convolution of two objects (see Fig. ). The first of them is the probability  $\Phi$  ( $\Phi = \Phi_q$  in Fig. a and  $\Phi = \Phi_g$  in Fig. b) to find a polarized parton (a quark or a gluon) in the hadron. The second one is the partonic tensor  $\tilde{W}_{\mu\nu}^s$  defined and parametrized similarly to  $W_{\mu\nu}^s$ . Whereas  $\Phi_{q,g}$  are essentially non-perturbative objects, the partonic tensor  $\tilde{W}_{\mu\nu}^s$ , i.e. the partonic structure functions  $g_1$  and  $g_2$ , can be studied within perturbative QCD. The lack of knowledge of  $\Phi$  is usually compensated by introducing initial parton distributions that are found from phenomenological considerations. On the contrary, there are regular perturbative methods for calculating the structure functions in the partonic tensor  $\tilde{W}_{\mu\nu}^s$ . The best known

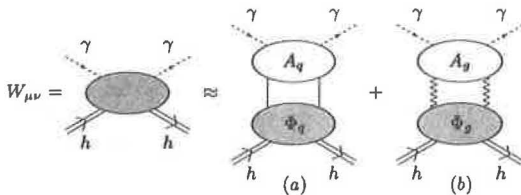


Figure 1: Representation of the hadronic tensor as the convolution of the fragmentation functions  $\Phi_{q,g}$  and the partonic tensor.

instrument to calculate the structure functions to all orders in  $\alpha_s$  is the DGLAP[1] approach. Once applied to the description of the experimental data, DGLAP provides good results[2]. The extrapolation into the small- $x$  region of DGLAP predicts an asymptotical behaviour  $\sim \exp(\sqrt{C \ln(1/x) \ln \ln Q^2})$  for all DIS structure functions (with different factors  $C$ ). However, from a theoretical point of view, such an extrapolation is rather doubtful. In particular, it neglects in a systematical way contributions  $\sim (\alpha_s \ln^2(1/x))^k$  which are small when  $x \sim 1$  but become large when  $x \ll 1$ . The total resummation of these double-logarithmic (DL) contributions made in Refs. [3] and Ref. [4] for the non-singlet ( $g_1^{NS}$ ) and singlet  $g_1$  respectively leads to the Regge (power-like) asymptotics  $g_1(g_1^{NS}) \sim (1/x)^{\Delta^{DL}} ((1/x)^{\Delta_{NS}^{DL}})$ , with  $\Delta^{DL}, \Delta_{NS}^{DL}$  being the intercepts calculated in the double-logarithmic approximation (DLA). The weakest point of Refs. [3],[4] is the fact that they keep  $\alpha_s$  fixed (at some unknown scale). It leads therefore to the intercepts  $\Delta^{DL}, \Delta_{NS}^{DL}$  explicitly depending on this unknown coupling, whereas  $\alpha_s$  is well-known to be running. The results of Refs. [3],[4] led many authors (see e.g.[5]) to suggest that the DGLAP parametrization  $\alpha_s = \alpha_s(Q^2)$  has to be used. However, according to results of Ref. [6], such a parametrization is correct only for  $x \sim 1$  and cannot be used for  $x \ll 1$ . The explicit dependence of  $\alpha_s$  suggested in Ref. [6] has been used to calculate both  $g_1^{NS}$  and  $g_1$  at small  $x$  in Refs. [7]. The present talk is based on the results obtained in those papers.

Instead of a direct study of  $g_1$ , it is more convenient to consider the forward Compton amplitude  $A$  related to  $g_1$  as follows:

$$g_1(x, Q^2) = \frac{1}{\pi} \Im_s A(s, Q^2). \quad (2)$$

We cannot use DGLAP for studying  $g_1$  or  $A$  at small  $x$  because it does not account for double-log (and single-log) contributions which are independent of  $Q^2$ . In order to account for the double-logs of both  $x$  and  $Q^2$ , we need to construct two-dimensional evolution equations that would combine the  $x$ - and  $Q^2$ - evolutions. On the other hand, these equations should sum up the contributions of the Feynman graphs involved to all orders in  $\alpha_s$ . Some of those graphs have either ultraviolet or infrared (IR) divergencies. The ultraviolet divergencies are regulated by the usual renormalization procedure. In order to regulate the IR ones, we have to introduce an IR cut-off. We use the IR cut-off  $\mu$  in the transverse momentum space for momenta  $k_i$  of all virtual quarks and gluons:

$$\mu < k_{i\perp} \quad (3)$$

where  $k_{i\perp}$  stands for the transverse (with respect to the plane formed by the external

momenta  $p$  and  $q$ ) component of  $k_i$ . This way of regulating the IR divergencies was suggested by L.N. Lipatov and used first in Ref. [19] for quark-quark scattering. Using this cut-off  $\mu$ ,  $A$  acquires a dependence on  $\mu$ . Therefore, one can evolve  $A$  with respect to  $\mu$ , constructing thereby some Infrared Evolution Equations (IREE). As  $A = A(s/\mu^2, Q^2/\mu^2)$ ,

$$-\mu^2 \partial A / \partial \mu^2 = \partial A / \partial \rho + \partial A / \partial y \quad (4)$$

where  $\rho = \ln(s/\mu^2)$  and  $y = \ln(Q^2/\mu^2)$ . Eq. (4) is the lhs of the IREE for  $A$ . In order to write the rhs of the IREE, it is convenient to use the Sommerfeld-Watson transform

$$A(s, Q^2) = \int_{-100}^{100} \frac{d\omega}{2\pi i} (s/\mu^2)^\omega \xi(\omega) F(\omega, Q^2) \quad (5)$$

where  $\xi(\omega)$  is the negative signature factor,  $\xi(\omega) = [1 - e^{-\pi\omega}]/2 \approx \pi\omega/2$ . It must be noted that the transform inverse to Eq. (5) involves the imaginary parts of  $A$ :

$$F(\omega, Q^2) = \frac{2}{\pi\omega} \int_0^\infty d\rho e^{-\rho\omega} \Im A(s, Q^2). \quad (6)$$

Notice that contrary to the amplitude  $A$ , the structure function  $g_1$  does not have any signature and therefore  $\xi(\omega) = 1$  when the transform (5) is applied directly to  $g_1$ .

## Infrared evolution equations for $g_1$

When the factorization depicted in Fig. is assumed, the calculation of  $g_1$  (we will not use the superscript "s" for  $g_1$  singlet, though we use the notation  $g_1^{NS}$  for the non-singlet  $g_1$ ) is reduced to calculating the Feynman graphs contributing the partonic tensor  $\tilde{W}_{\mu\nu}^{\prime s}$  depicted as the upper blobs in Fig. . Both cases, (a), when the virtual photon scatters off the nearly on-shell polarized quark, and (b), when the quark is replaced by the polarized gluon, should be taken into account. Therefore, in contrast to Eq. (2), we need to introduce two Compton amplitudes:  $A_q$  and  $A_g$  corresponding to the upper blob in Fig. a and Fig. b respectively. The subscripts "q" and "g" refer to the initial partons. Therefore,

$$g_1(x, Q^2) = g_q(x, Q^2) + g_g(x, Q^2), \quad (7)$$

where

$$g_q = \frac{1}{\pi} \Im_s A_q(s, Q^2), \quad g_g = \frac{1}{\pi} \Im_s A_g(s, Q^2) \quad (8)$$

Let us now construct the IREE for the amplitudes  $A_{q,g}$  related to  $g_1$ . To this aim, let us consider a virtual parton with minimal  $k_\perp$ . We call such a parton the softest one. If it is a gluon, its DL contribution can be factorized, i.e. its DL contribution comes from the graphs where its propagator is attached to the external lines. As the gluon propagator cannot be attached to photons, this case is absent in IREE for  $A_{q,g}$ . The second option is when the softest partons are a  $t$ -channel quark-antiquark or gluon pair. It leads us to the IREE depicted in Fig. 2. Applying the operator  $-\mu^2 \partial / \partial \mu^2$  to it, combining the result with Eq. (4) and using (5), we arrive at the following system of equations:

$$\begin{aligned} \left(\omega + \frac{\partial}{\partial y}\right) F_q(\omega, y) &= \frac{1}{8\pi^2} [F_{qq}(\omega) F_q(\omega, y) + F_{qg}(\omega) F_g(\omega, y)], \\ \left(\omega + \frac{\partial}{\partial y}\right) F_g(\omega, y) &= \frac{1}{8\pi^2} [F_{gq}(\omega) F_q(\omega, y) + F_{gg}(\omega) F_g(\omega, y)]. \end{aligned} \quad (9)$$

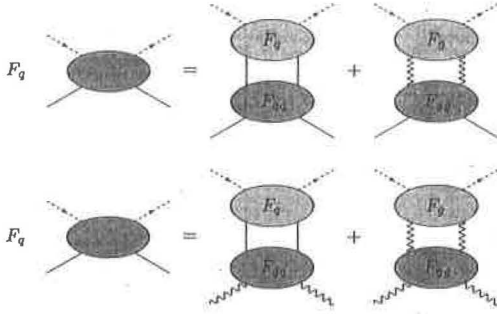


Figure 2: Infrared evolution equations for the amplitudes  $A_q, A_g$ .

The amplitudes  $F_q, F_g$  are related to  $A_q, A_g$  through the transform (5). The Mellin amplitudes  $F_{ik}$ , with  $i, k = q, g$ , describe the parton-parton forward scattering. They contain DL contributions to all orders in  $\alpha_s$ . We can introduce the new anomalous dimensions  $H_{ik} = (1/8\pi^2)F_{ik}$ . The way of writing the subscripts “q,g” corresponds to the DGLAP-notations. Solving this system of eqs. and using Eq. (8) leads to

$$g_q(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ C_+(\omega) e^{\Omega_+ y} + C_-(\omega) e^{\Omega_- y} \right], \quad (10)$$

$$g_g(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ C_+(\omega) \frac{X + \sqrt{R}}{2H_{gg}} e^{\Omega_+ y} + C_-(\omega) \frac{X - \sqrt{R}}{2H_{gg}} e^{\Omega_- y} \right]$$

The unknown factors  $C_\pm(\omega)$  have to be specified. All other factors in Eq. (10) can be expressed in terms of  $H_{ik}$ :

$$X = H_{gg} - H_{qq}, \quad R = (H_{gg} - H_{qq})^2 + 4H_{gg}H_{gq} \quad (11)$$

$$\Omega_\pm = \frac{1}{2} \left[ H_{qq} + H_{gg} \pm \sqrt{(H_{qq} - H_{gg})^2 + 4H_{gg}H_{gq}} \right].$$

The anomalous dimension matrix  $H_{ik}$  was calculated in Ref. [7]:

$$H_{gg} = \frac{1}{2} \left( \omega + Y + \frac{b_{qq} - b_{gg}}{Y} \right), \quad H_{qq} = \frac{1}{2} \left( \omega + Y - \frac{b_{qq} - b_{gg}}{Y} \right), \quad (12)$$

$$H_{gq} = -\frac{b_{gq}}{Y}, \quad H_{qg} = -\frac{b_{gq}}{Y}.$$

where

$$Y = -\sqrt{\left( \omega^2 - 2(b_{qq} + b_{gg}) + \sqrt{[(\omega^2 - 2(b_{qq} + b_{gg}))^2 - 4(b_{qq} - b_{gg})^2 - 16b_{qq}b_{gq}]}/2 \right)} / 2, \quad (13)$$

$$b_{ik} = a_{ik} + V_{ik}, \quad (14)$$

$$a_{qq} = \frac{A(\omega)C_F}{2\pi}, \quad a_{gg} = \frac{2A(\omega)N}{\pi}, \quad a_{gq} = -\frac{n_f A'(\omega)}{2\pi}, \quad a_{qg} = \frac{A'(\omega)C_F}{\pi}, \quad (15)$$

and

$$V_{ik} = \frac{m_{ik}}{\pi^2} D(\omega), \quad (16)$$

with

$$m_{qq} = \frac{C_F}{2N}, \quad m_{gg} = -2N^2, \quad m_{qg} = n_f \frac{N}{2}, \quad m_{gq} = -NC_F. \quad (17)$$

We have used here the notations  $C_F = 4/3$ ,  $N = 3$  and  $n_f = 4$ . The factors  $A$  and  $D$  account for running  $\alpha_s$ . They are given by the following expressions:

$$A(\omega) = \frac{1}{b} \left[ \frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho + \eta)^2 + \pi^2} \right], \quad (18)$$

$$D(\omega) = \frac{1}{2b^2} \int_0^\infty d\rho e^{-\omega\rho} \ln((\rho + \eta)/\eta) \left[ \frac{\rho + \eta}{(\rho + \eta)^2 + \pi^2} + \frac{1}{\rho + \eta} \right] \quad (19)$$

with  $\eta = \ln(\mu^2/\Lambda_{QCD}^2)$  and  $b = (33 - 2n_f)/12\pi$ .  $A'$  is defined as  $A$  with the  $\pi^2$  term dropped out. Now we can specify the coefficients  $C_\pm$  of Eq. (10). When  $Q^2 = \mu^2$ ,

$$g_q = \tilde{\Delta}q(x_0), \quad g_g = \tilde{\Delta}g(x_0) \quad (20)$$

where  $\tilde{\Delta}q(x_0)$  and  $\tilde{\Delta}g(x_0)$  are the input distributions of the polarized partons at  $x_0 = \mu^2/s$ . They do not depend on  $Q^2$ . Using Eq. (20) allows to express  $C_\pm(\omega)$  in terms of  $\Delta q(\omega)$  and  $\Delta g(\omega)$ , which are related to  $\tilde{\Delta}q(x_0)$  and  $\tilde{\Delta}g(x_0)$  through the ordinary Mellin transform. Indeed,

$$C_+ + C_- = \Delta q, \quad C_+ \frac{X + \sqrt{R}}{2H_{gg}} + C_- \frac{X - \sqrt{R}}{2H_{gg}} = \Delta g. \quad (21)$$

This leads to the following expressions for  $g_q$  and  $g_g$ :

$$g_q(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ \left( A^{(-)} \Delta q + B \Delta g \right) e^{\Omega+\nu} + \left( A^{(+)} \Delta q - B \Delta g \right) e^{\Omega-\nu} \right], \quad (22)$$

$$g_g(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ \left( E \Delta q + A^{(+)} \Delta g \right) e^{\Omega+\nu} + \left( -E \Delta q + A^{(-)} \Delta g \right) e^{\Omega-\nu} \right] \quad (23)$$

with

$$A^{(\pm)} = \left( \frac{1}{2} \pm \frac{X}{2\sqrt{R}} \right), \quad B = \frac{H_{gg}}{\sqrt{R}}, \quad E = \frac{H_{gq}}{\sqrt{R}}. \quad (24)$$

Eqs. (22), (23) express  $g_1$  in terms of the parton distributions  $\Delta q(\omega)$  and  $\Delta g(\omega)$ . However, they are related to the distributions  $\tilde{\Delta}q(x_0)$  and  $\tilde{\Delta}g(x_0)$  at very low  $x$ :  $x_0 \approx \mu^2/s \ll 1$ . Therefore, they scarcely can be found from experimental data. It is much more useful to express  $g_q, g_g$  in terms of the initial parton densities  $\bar{\delta}q$  and  $\bar{\delta}g$  defined at  $x \sim 1$ . We can do it, using the evolution of  $\tilde{\Delta}q(x_0)$ ,  $\tilde{\Delta}g(x_0)$  with respect to  $s$ . Indeed, the  $s$ -evolution of  $\bar{\delta}q, \bar{\delta}g$  from  $s \approx \mu^2$  to  $s \gg \mu^2$  at fixed  $Q^2$  ( $Q^2 = \mu^2$ ) is equivalent to their  $x$ -evolution from  $x \sim 1$  to  $x \ll 1$ . In the  $\omega$ -space, the system of IREE for the parton distributions looks quite similar to Eqs. (9). However, the eqs for  $\Delta q, \Delta g$  are algebraic because they do not depend on  $Q^2$ :

$$\begin{aligned}\Delta q(\omega) &= \langle e_q^2 \rangle / 2 \delta q(\omega) + (1/\omega) [H_{qq}(\omega)\Delta q(\omega) + H_{qg}(\omega)\Delta g(\omega)] , \\ \Delta g(\omega) &= \langle e_q^2 \rangle / 2 \hat{\delta} g(\omega) + (1/\omega) [H_{gq}(\omega)\Delta q(\omega) + H_{gg}(\omega)\Delta g(\omega)] .\end{aligned}\quad (25)$$

where  $\langle e_q^2 \rangle$  is the sum of the quark electric charges ( $\langle e_q^2 \rangle = 10/9$  for  $n_f = 4$ ),  $\delta q$  is the sum of the initial quark and antiquark densities and  $\hat{\delta} g \equiv -(A'(\omega)/2\pi\omega^2)\delta g$  is the starting point of the evolution of the gluon density  $\delta g$ . It corresponds to Fig. b where the upper blob is substituted by the quark box. Solving Eqs. (25), we obtain:

$$\Delta q = \langle e_q^2 \rangle / 2 \frac{[\omega(\omega - H_{gg})\delta q + \omega H_{qg}\hat{\delta} g]}{[\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{qg}H_{gq})]} , \quad (26)$$

$$\Delta g = \langle e_q^2 \rangle / 2 \frac{[\omega H_{gq}\delta q + \omega(\omega - H_{qq})\hat{\delta} g]}{[\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{qg}H_{gq})]} . \quad (27)$$

Then Eqs. (22,23,26,27) express  $g_1$  in terms of the initial parton densities  $\delta q, \delta g$ .

When we put  $H_{qg} = H_{gq} = H_{gg} = 0$  and do not sum over  $e_q$ , we arrive at the expression for the non-singlet structure function  $g_1^{NS}$ : Obviously, in this case  $A^{(+)} = B = E = \Omega_- = 0$ ,  $A^{(-)} = 1$ ,  $\Omega_+ = H_{qq}$ . However, the nonsinglet anomalous dimension  $H_{qq}$  should be calculated in the limit  $b_{gg} = b_{qg} = b_{gq} = 0$ . We denote such  $H_{qq} \equiv H^{NS}$ . The explicit expression for it is:

$$H^{NS} = (1/2) \left[ \omega - \sqrt{\omega^2 - 4b_{qq}} \right] . \quad (28)$$

Therefore, we arrive at

$$g_1^{NS} = \frac{e_q^2}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left( \frac{\omega \delta q}{\omega - H^{NS}} \right) (1/x)^\omega (Q^2/\mu^2)^{H^{NS}} . \quad (29)$$

### Small- $x$ asymptotics for $g_1$

When  $x \rightarrow 0$  and  $Q^2 \gg \mu^2$ , one can neglect contributions with  $\Omega_-$  in Eqs. (10). As is known,  $g_1 \sim (1/x)^{\omega_0}$  at  $x \rightarrow 0$ , with  $\omega_0$  being the position of the leading singularity of the integrand of  $g_1$ . According to Eqs. (12), the leading singularity,  $\omega^{NS}$  for  $g_1^{NS}$  is the rightmost root of the equation

$$\omega^2 - 4b_{qq} = 0 \quad (30)$$

while the leading singularity,  $\omega_0$  for  $g_1$  is the rithmost root of

$$\omega^4 - 4(b_{qq} + b_{gg})\omega^2 + 16(b_{qq}b_{gg} - b_{qg}b_{gq}) = 0 . \quad (31)$$

In our approach, all factors  $b_{ik}$  depend on  $\eta = \ln(\mu^2/\Lambda_{QCD})$ , so the roots of Eqs. (30,31) also depend on  $\eta$ . This dependence is plotted in Fig. 3 for  $\omega^{NS}$  and in Fig. 4 for  $\omega_0$ . Both the curve in Fig. 3 and the curve 1 in Fig. 4 have a maximum. We denote this maximum as the intercept. Therefore,

$$g_1^{NS} \sim (e_q^2/2)\delta q \left( \frac{1}{x} \right)^{\Delta_{NS}} \left( \frac{Q^2}{\mu^2} \right)^{\Delta_{NS}/2} , \quad g_1 \sim (1/2)[Z_1\delta q + Z_2\delta g] \left( \frac{1}{x} \right)^{\Delta_S} \left( \frac{Q^2}{\mu^2} \right)^{\Delta_S/2} , \quad (32)$$

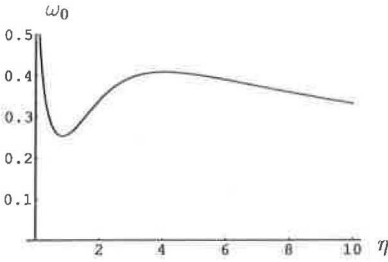


Figure 3: Dependence of the intercept  $\omega_0$  on infrared cut-off  $\eta = \ln(\mu^2/\Lambda_{QCD})$  for  $g_1^{NS}$ .

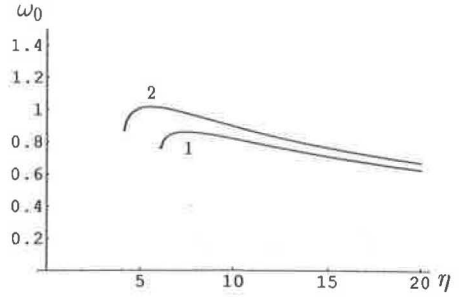


Figure 4: Dependence on  $\eta$  of the rightmost root of Eq. (31),  $\omega_0$ . Curve 2 corresponds to the case when gluon contributions only are taken into account; curve 1 is the result of accounting for both gluon and quark contributions.

and we find for the intercepts

$$\Delta_{NS} \approx 0.4, \quad \Delta_S \approx 0.86 \quad (33)$$

and  $Z_1 = -1.2$ ,  $Z_2 = -0.08$ . This implies that  $g_1^{NS}$  is positive when  $x \rightarrow 0$  whereas  $g_1$  can be either positive or negative, depending on the relation between  $\delta q$  and  $\delta g$ . In particular,  $g_1$  is positive when

$$15 \delta q + \delta g < 0. \quad (34)$$

otherwise it is negative. In other words, the sign of  $g_1$  at small  $x$  can be positive if the initial gluon density is negative and large.

## Conclusion

The total resummation of the most singular ( $\sim \alpha_s^n/\omega^{2n+1}$ ) terms in the expressions for the anomalous dimensions and the coefficient functions leads to the expressions of Eqs. (7,22,23,29) for the singlet and the non-singlet structure functions  $g_1$ . It guarantees the Regge (power-like) behaviour (32) of  $g_1$ ,  $g_1^{NS}$  when  $x \rightarrow 0$ , with the intercepts given by Eq. (33). The intercepts  $\Delta_{NS}, \Delta_S$  are of course obtained with the running QCD coupling effects taken into account. The value of the non-singlet intercept  $\Delta_{NS} = 0.4$  is now confirmed by several independent analyses [10] of experimental data. The value  $\Delta_S = 0.86$  of the singlet intercept is in a good agreement with the estimate  $\Delta_S = 0.88$  obtained in Ref. [11] from analysis of the HERMES data.

Another interesting point to discuss is the sign of these structure functions. Eq. (29) states that  $g_1^{NS}$  is positive both at  $x \sim 1$  and at  $x \ll 1$ . The situation concerning the singlet  $g_1$  is more involved: being positive at  $x \sim 1$ , the singlet  $g_1$  can remain positive at  $x \ll 1$  only if the initial parton densities obey Eq. (34), otherwise it becomes negative.

## Acknowledgement

The work is supported by grants POCTI/FNU/49523/2002 and RSGSS-1124.2003.2 One of us (B.I Ermolaev) is grateful to the Organizing Committee of the Workshop for their hospitality.

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## Discussion

**Q.** (A.Efremov, JINR, Dubna): Is  $\alpha_s$  in your formulas for  $\Delta$  constant or running?

**A:**  $\alpha_s$  is running in all my equations including the equation for the intercept. It is the key point of our approach: Quite often small- $x$  evolution equations operate with fixed  $\alpha_s$  (e.g. LO BFKL for the Pomeron; my papers with Ryskin/Bartels about  $g_1$ ), they obtain expressions for the intercepts explicitly depending on  $\alpha_s$ . This  $\alpha_s$  is just a symbol, the scale (the argument) is unknown. After that, the authors try to find some argumentation for fixing the scale. Contrary to this approach, in our equations  $\alpha_s$  is running in every vertex of every Feynman graph contributing to  $g_1$ . So, our expressions for the singlet and non-singlet intercepts account for the running QCD coupling.

**Q.** (A.Sidorov, JINR, Dubna): Could you fix a sign of your asymptotic expressions for  $g_1$ ?

**A:** Yes, I can fix it. The sign is negative.