

Firewalls in AdS/CFT

Steven G. Avery^a and Borun D. Chowdhury^b

^a*The Institute of Mathematical Sciences,
CIT Campus, Taramani, Chennai, India 600113*

^b*Institute for Theoretical Physics, University of Amsterdam,
Science Park 904, Postbus 94485, 1090 GL Amsterdam, The Netherlands*

E-mail: steven_avery@brown.edu, borun.chowdhury@asu.edu

ABSTRACT: Several recent papers argue against firewalls by relaxing the requirement for locality outside the stretched horizon. In the firewall argument, locality essentially serves the purpose of ensuring that the degrees of freedom required for infall are those in the proximity of the black hole and not the ones in the early radiation. We make the firewall argument sharper by utilizing the AdS/CFT framework and claim that the firewall argument essentially states that the dual to a thermal state in the CFT is a firewall.

KEYWORDS: AdS-CFT Correspondence, Models of Quantum Gravity, Black Holes in String Theory, D-branes

ARXIV EPRINT: [1302.5428](https://arxiv.org/abs/1302.5428)

Contents

1	Introduction and summary	1
2	The evaporating D1-D5 system and firewalls	2
2.1	Two selected arguments against firewalls	7
3	Firewalls as duals to thermal CFT	9
A	Infall as Bell measurement	11

1 Introduction and summary

The firewall phenomenon [1] has reignited interest in the information paradox. Almheiri, Marolf, Polchinski and Sully (AMPS) have argued that the postulates of black hole complementarity as stated in [2, 3] (including an implicit assumption of smoothness of the horizon for an infalling observer) are mutually inconsistent. Explicitly, unitarity of black hole evaporation and validity of semi-classical physics outside the stretched horizon imply that observers falling through an “old” black hole horizon see intense radiation. This has fueled a passionate debate, with most papers contesting the *firewall* [4–17]. However, some papers have also come out in support [18–20], and some have been of a clarifying and commenting nature [21–30].

A line of thought that challenges the firewall phenomenon gives up the validity of local semi-classical physics outside the horizon to arbitrary distances. This argument has come to be known as $\mathcal{C} = \mathcal{A}$.¹ While the notation will become clear in the bulk of the text, the argument essentially says that the degrees of freedom inside the horizon are a subset of the degrees of freedom of the early radiation far away from the black hole. Papers proposing this view include [11, 16–18]. This is manifestly non-local, but there are good reasons to expect non-locality in quantum gravity; however, it would be interesting to quantify how much non-locality is needed and see if it is reasonable.² An attempt to put the $\mathcal{C} = \mathcal{A}$ idea on a stronger footing is made in [17] by claiming that extracting the degrees of freedom in the early Hawking radiation responsible for free infall is computationally impossible before the black hole evaporates.

Our agnosticism towards locality in quantum gravity, or rather an atheism towards it, makes the above resolution rather appealing; however, to check the reasonableness of this idea, in this article we make the firewall argument more precise by utilizing the AdS/CFT duality. We begin by looking at the evaporating D1-D5-P black string. We repeat the firewall argument in this case, noting that after the Page time the black string is highly

¹Note that $\mathcal{C} = \mathcal{A}$ is not the only possible kind of non-locality one can consider in this context, cf. [14, 15].

²For instance, reasonable might mean non-locality that can be attributed to effects within string theory.

entangled with the Hawking radiation outside. The advantage of this system is that in certain limits this can be viewed as excitations on the D1-D5 branes entangled with the radiation outside. Since the D1-D5 system flows in the infrared to a CFT, in the decoupling limit the near-horizon geometry can be viewed as a thermal state in the CFT.

Taking the next logical step, we let an arbitrary system play the role of the early radiation. In other words, we imagine coupling a source/sink to the CFT, allowing them to equilibrate and thereby become entangled, and then decouple them. Next, we couple a source to the CFT to create an infalling observer. The $\mathcal{C} = \mathcal{A}$ argument in this case would mean that the degrees of freedom of the source/sink that purifies the CFT are available to the infalling observer, allowing her free infall. Since the systems are decoupled, this seems to be a bizarre state of affairs given that we are talking about arbitrary (decoupled) systems giving universal free infall. We discuss the implications of this and suggest that the dual to a thermal state in the CFT is a firewall!

2 The evaporating D1-D5 system and firewalls

The full backreacted non-extremal D1-D5-P system in flat spacetime is a black string, whose near-horizon geometry is the BTZ black hole [31]. The full geometric solution may be viewed as interpolating between BTZ and flat spacetime. The black string evaporates by Hawking radiation and thus the recent blackhole-firewall-fuzzball debate can be embedded in this system. The advantage being that the near-horizon region is dual to the D1-D5 CFT deformed by irrelevant deformations that couple it to the flat space [32, 33]. We can then understand the implications of the firewall argument within AdS/CFT.

To begin, let us review the essential features of the firewall argument. We start with a Schwarzschild black hole formed by the collapse of matter in a pure state. The near-horizon region of the Schwarzschild black hole is the Rindler geometry. This region is separated from the asymptotic flat spacetime by a potential barrier whose exact details depend on the probe. According to black hole complementarity [2, 3], asymptotic observers describe the black hole as a hot membrane unitarily evaporating. For such an observer, the Hilbert space naturally factorizes at any time into subfactors as

$$\mathcal{H} = \mathcal{H}_{\mathcal{H}} \otimes \mathcal{H}_{\mathcal{B}} \otimes \mathcal{H}_{\mathcal{A}}, \tag{2.1}$$

where the modes populated by Hawking radiation that have escaped to flat space live in $\mathcal{H}_{\mathcal{A}}$, modes inside the barrier but outside the horizon live in $\mathcal{H}_{\mathcal{B}}$, and the Hilbert space associated with the stretched horizon degrees of freedom is denoted by $\mathcal{H}_{\mathcal{H}}$. This is depicted in figure 1a. For a freely falling observer to pass through the horizon unscathed, one requires the state to be either the Rindler or the Hartle-Hawking vacuum. Both of these have the modes across the horizon maximally entangled with each other.³ Implicitly assuming that the inside of the horizon is constructed from degrees of freedom in $\mathcal{H}_{\mathcal{H}}$ [29], AMPS conclude that free infall requires that the modes $\mathcal{B} \in \mathcal{H}_{\mathcal{B}}$ and the modes $\mathcal{C} \in \mathcal{H}_{\mathcal{H}}$ be maximally

³In fact, it is only the low-energy modes that are maximally entangled since there is a finite effective temperature. We abuse the term “maximally entangled” in this sense throughout our discussion. One can think of it as meaning maximally entangled *given* conservation of energy.

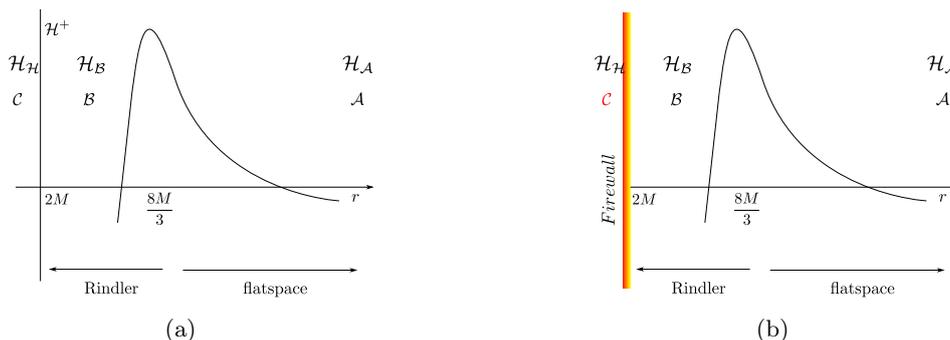


Figure 1. In (a) a Schwarzschild black hole with the effective potential for a minimally coupled scalar is shown. The asymptotic flat space, its associated Hilbert space \mathcal{H}_A and modes living in it \mathcal{A} are on the outside of the potential barrier. The near-horizon region, its associated Hilbert space \mathcal{H}_B and associated modes \mathcal{B} are between the horizon and the barrier. Finally there is the region inside the horizon which is not accessible in Schwarzschild coordinates but is in Kruskal coordinates (for instance). Black hole complementarity posits that the experiences of an asymptotic and an infalling observer are complementary. The inside of the black hole is replaced by a stretched horizon for the asymptotic observer. Assuming the inside and stretched horizon Hilbert spaces to be isomorphic we denote it by \mathcal{H}_H and the associated degrees of freedom by \mathcal{C} . Free infall requires \mathcal{B} and \mathcal{C} to be maximally entangled. In (b) the firewall is shown which is supposed arise for old black holes because the entanglement structure of \mathcal{A} and \mathcal{B} preclude maximal entanglement between \mathcal{B} and \mathcal{C} .

entangled with each other. But after the black hole has evaporated away half its entropy (i.e. after the Page time [34–36]), \mathcal{B} has to be maximally entangled with the early radiation $\mathcal{A} \in \mathcal{H}_A$ in order to ensure unitarity. The monogamy of entanglement then precludes \mathcal{B} from being maximally entangled with \mathcal{C} . (This is basically the contrapositive of Mathur’s theorem against small corrections to the evaporation process restoring unitarity [37, 38].) AMPS emphasize that this means an infalling observer cannot freely pass through the horizon after the Page time. Largely agreeing with AMPS, Susskind argues for firewalls in a slightly different way in [20]. After the Page time, the system inside the potential barrier is maximally entangled with the outside system and thus the system inside the barrier cannot be split into two maximally entangled parts (viz. \mathcal{B} and \mathcal{C}); this implies that there is no space inside the horizon. There is another argument advanced by AMPS: once more than half the entropy of the black hole has been radiated away, the radiation is the bigger part of the full system, which is in a pure state. The infalling observer can then perform a very non-local and fine-grained, but viable, measurement on the early radiation to project the state at the horizon to a state that is not the Unruh vacuum. Since the horizon is highly red-shifted, any state other than the Unruh vacuum or the Hartle-Hawking vacuum will be very hot for the infalling observer and hence this conjectured phenomenon has been dubbed a firewall.

We now turn to framing the AMPS argument in the D1-D5 system. Consider type IIB compactified on $S^1 \times T^4$ with the volume of S^1 given by $2\pi R$ and the volume of T^4 given by $(2\pi)^4 V$. The torus is taken to be string size. We wrap n_1 D1 branes on S^1 and n_5 D5 branes on $S^1 \times T^4$. This system has a ground state degeneracy of $2\sqrt{2}\pi\sqrt{n_1 n_5}$ [39] which

is accounted for by the Lunin-Mathur geometries [40, 41]. We may think of starting with any one of these states/geometries.

We can then make a black string by throwing in matter in the form of closed strings into these geometries. We take the initial closed string state to be pure. In general any energy above extremality excites all kinds of branes and anti-branes and momenta in all possible directions consistent with equipartition of energy [42]. By taking the size of S^1 to be much larger than T^4 , however, the momentum along the S^1 becomes much lighter than any other charges ensuring that it is preferentially excited. It is in this limit that the near-horizon region becomes AdS_3 [43]. For simplicity, we consider the extra matter coming in with no net momentum along the S^1 , so that it then excites equal numbers of left and right movers. The metric for this simplified system is⁴

$$ds^2 = \frac{1}{\sqrt{H_1 H_5}} \left(- \left(1 - \frac{4Q_p}{r^2} \right) dt^2 + dy^2 \right) + \sqrt{H_1 H_5} \left(\frac{dr^2}{1 - \frac{4Q_p}{r^2}} + r^2 d\Omega_3^2 \right) + \sqrt{\frac{H_1}{H_5}} \sum_{i=1}^4 dx_i^2 \tag{2.2}$$

where

$$H_{1,5} = 1 + \frac{Q_{1,5}}{r^2} \tag{2.3}$$

and

$$Q_1 = n_1 g_s l_s^2, \quad Q_5 = n_5 \frac{g_s l_s^6}{V}, \quad Q_P = n_p \frac{g_s^2 l_s^8}{R^2 V}; \tag{2.4}$$

n_p is the left and right momentum measured in units of R^{-1} . The ADM mass of the black string is

$$M_{ADM} = \frac{\pi}{4G_5} [Q_1 + Q_5 + 2Q_p]. \tag{2.5}$$

where $16\pi G_{10} = (2\pi)^7 g_s^2 l_s^8$ and $G_5 = \frac{G_{10}}{(2\pi)^5 R V}$. The core region of this geometry is $BTZ \times S^3 \times T^4$ with $R_{AdS_3} = R_{S^3} = \sqrt{Q_1 Q_5}$, which is obtained by zooming into the region $r^2 \ll Q_1, Q_5$. This core region is separated from the asymptotically flat region by probe-dependent potential barriers as shown in figure 2. The horizon, near-horizon, and asymptotically flat Hilbert spaces are again represented by $\mathcal{H}_H, \mathcal{H}_B$ and \mathcal{H}_A , respectively. The core region is dual to a 1 + 1-dimensional $\mathcal{N} = (4, 4)$ CFT, whose Hilbert space, according to the AdS/CFT duality [46], is dual to $\mathcal{H}_H \otimes \mathcal{H}_B$. Note the similarity of figures 1a and 2 but also note that in the Schwarzschild case there is no decoupling limit, and no obvious gauge-gravity duality.

The left and right sectors of the D1-D5 CFT are on equal footing for our simplified system, with left and right moving momenta, entropy, and temperature:

$$P_{L,R} = \frac{n_p}{R}, \tag{2.6}$$

$$S_{L,R} = 2\pi \sqrt{n_1 n_5 n_p}, \tag{2.7}$$

$$T_{L,R} = \frac{1}{R} \sqrt{\frac{n_p}{n_1 n_5}}. \tag{2.8}$$

⁴For the most general solution, see for example [44, 45].

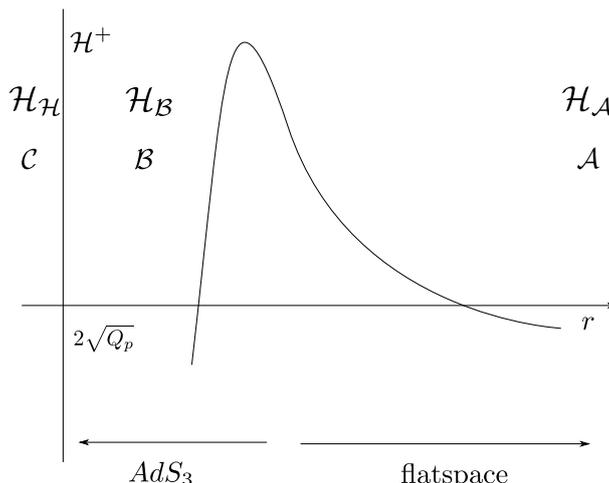


Figure 2. The D1-D5-P black string with the effective potential barrier separating the flat space from the near horizon BTZ region. The Hilbert spaces and associated degrees of freedom have the same interpretation as in figure 1. For the traditional horizon with free infall, \mathcal{B} and \mathcal{C} have to be maximally entangled.

We are working in the dilute gas limit, $Q_p \ll Q_1, Q_5$, when the evaporation rate obtained from the bulk and the D1-D5 field theory match [47–51],

$$\Gamma = 2\pi^2 Q_1 Q_5 \frac{\pi\omega}{2} \frac{1}{e^{\omega/2T_L} - 1} \frac{1}{e^{\omega/2T_R} - 1} \frac{d^4 k}{(2\pi)^4}. \quad (2.9)$$

For our system, this relation gives

$$\frac{dn_p}{dt} \propto -g_s^2 l_s^4 \frac{1}{R^5} \frac{n_p^3}{(n_1 n_5)^2}. \quad (2.10)$$

After time

$$t_{\text{Page}} \propto \frac{R^5 (n_1 n_5)^2}{g_s^2 l_s^4 n_p^2}, \quad (2.11)$$

the system evaporates away half its entropy.⁵

This may be interpreted as (a strong coupling version of) the process shown in figure 3. Closed strings hit a stack of D1-D5 branes and become open strings on them. Fractionation of the branes [52, 53] and the world-volume interactions cause the open strings to break up into many lower energy open strings [54–57]. It is the coarse-grained entropy of these excitations which account for the entropy of the D1-D5 CFT (2.7). With time these open strings collide and leave the D-branes as closed strings and this process is interpreted as the dual of Hawking radiation.

The firewall argument can be made for the D1-D5 system as follows. After the Page time (2.11), the brane system is maximally entangled with the radiation outside. Following the line of reasoning in [20], the core region dual to the brane system now has a firewall

⁵By taking $n_p \gg n_1 n_5$ we can ignore the entropy coming from ground state degeneracy of the D1-D5 system.

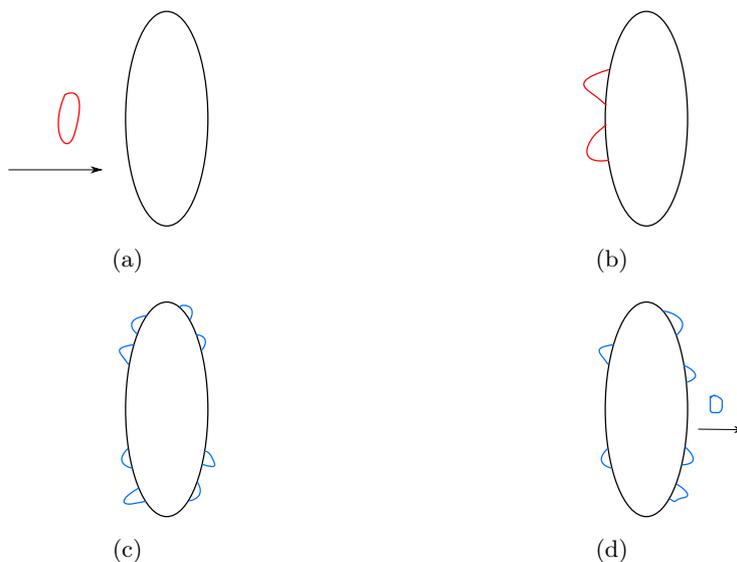


Figure 3. Closed strings in a pure state hitting a stack of D-branes in (a) become open strings on the D-branes in (b). These open strings break into many lower energy open strings due to interactions on the D-branes in (c). These lower energy open strings then collide with each other and are emitted as closed strings because of the time reversal of the process (a) in (d). Since it is entropically unfavorable for all low-energy open strings to find one another at the same time so the same closed string as in (a) is not generically emitted. An effective arrow of time thus emerges.

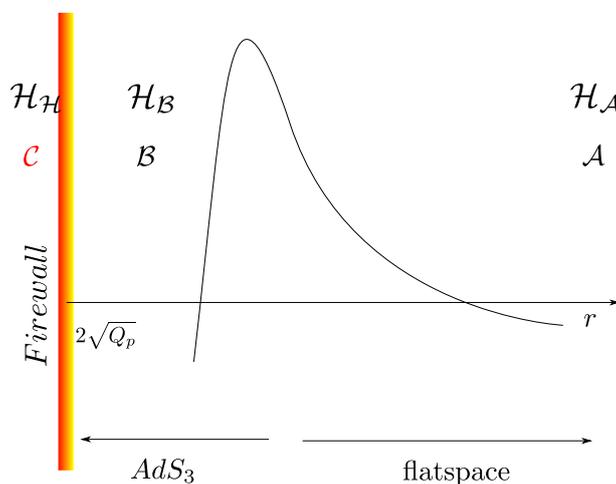


Figure 4. The firewall for the D1-D5-P system is shown. The argument works just like that for the Schwarzschild case. The entanglement structure between \mathcal{A} and \mathcal{B} required by unitarity at late times precludes the entanglement structure between \mathcal{B} and \mathcal{C} required for free infall. The added advantage in looking at the D1-D5-P system is that the near-horizon region is supposed to be dual to a CFT. We see that after the Page time the CFT is maximally entangled with early radiation \mathcal{A} .

instead of a harmless horizon. The picture before the Page time is shown in figure 2, and the picture after the Page time according to the firewall argument is shown in figure 4.

The argument for the D1-D5-P black string runs just like the one for Schwarzschild black hole. What do we gain by casting the firewall argument this way? The new feature here is that after Page time the near-horizon region of this system is dual to the D1-D5 CFT in a thermal state. Thus, essentially the firewall argument says that the dual to a thermal state⁶ is the firewall!

At this juncture, it is worth emphasizing a few points. First, it is obvious that this same argument applies to other incarnations of AdS/CFT duality with an explicit brane construction. Furthermore, let us note that there are several closely related physical scenarios to keep in mind:

- the near-horizon region of a very young black string,
- the near-horizon region of the post-Page time black string,
- the dual CFT at finite temperature,
- the CFT maximally entangled with a second CFT,
- and the CFT maximally entangled with some arbitrary heat bath.

All except for the first case are described by a thermal state in the CFT. The first case is described by a (pure) typical state, which for many purposes can be approximated by a thermal state. Thus, according to the standard AdS/CFT dictionary, all except possibly the first example, are expected to describe the same asymptotically AdS geometry, which we just argued has a firewall. To escape this conclusion, one must conjecture a generalization of superselection sectors proposed in [58] as we discuss in the next section. Alternatively, if one finds an evasion to the firewall argument, it may be that none have a firewall; however, we want to emphasize that the evasion better work for all of the above cases. Let us point out that if firewalls form at the Page time and one does not postulate superselection sectors, then observers may freely fall through the horizon only in the first case, which is dual to a pure state in the CFT. It is amusing to note that this is the opposite of the conclusion one might draw from [59–61].

The main point of this article is to make the preceding more precise and to address some of the arguments against firewalls in light of this idea. We return to this after reviewing these arguments.

2.1 Two selected arguments against firewalls

While there have been many arguments against firewalls, as noted in the Introduction, we review two that are especially relevant to this article.

Papadodimas-Raju conjecture. In [11], Papadodimas and Raju argue that in the context of AdS/CFT, infall is captured in the semi-classical limit by n -point functions with $n \ll N$, the latter being the central charge of the CFT. It is claimed the Hilbert space factorizes into a coarse-grained and fine-grained part

$$\mathcal{H} = \mathcal{H}_{\text{coarse}} \otimes \mathcal{H}_{\text{fine}}. \quad (2.12)$$

⁶Due to leakage over the barrier the CFT has a physical cutoff; it is the state in the cutoff theory that is thermal. We believe this does not change the basic argument.

Since Papadodimas and Raju consider a large black hole in AdS that does not evaporate, there is only what we refer to as \mathcal{B} and \mathcal{C} above, satisfying

$$\mathcal{B} \in \mathcal{H}_{\text{coarse}}, \quad \mathcal{C} \in \mathcal{H}_{\text{fine}}. \quad (2.13)$$

In fact since there is no evaporation, there is no radiation, and

$$\mathcal{H}_{\text{coarse}} = \mathcal{H}_{\mathcal{B}}, \quad \mathcal{H}_{\text{fine}} = \mathcal{H}_{\mathcal{H}}; \quad (2.14)$$

the fine-grained degrees of freedom account for the horizon degrees of freedom.

Even though the large black hole does not evaporate Papadodimas and Raju conjecture that even when part of the fine-grained space has evaporated away from the horizon, the degrees of freedom responsible for free fall inside the horizon are the same as those in the radiation outside the horizon. In other words, they claim that for an evaporating black hole

$$\mathcal{H}_{\text{fine}} = \mathcal{H}_{\mathcal{H}} \otimes \mathcal{H}_{\mathcal{A}} \quad (2.15)$$

and

$$\mathcal{C} \in \mathcal{H}_{\mathcal{H}} \otimes \mathcal{H}_{\mathcal{A}}, \quad (2.16)$$

so \mathcal{C} can be found in either the fine-grained degrees of freedom localized at the horizon or in the radiation spread over a typical distance t_{Page} away from it. As the black hole evaporates, it is increasingly found in the radiation. This opinion has also been suggested by others [16–18] and is probably shared by many others in unpublished form. This idea has come to be known as $\mathcal{C} = \mathcal{A}$. This bypasses the firewall argument, which uses strong subadditivity, by claiming that \mathcal{A} , \mathcal{B} and \mathcal{C} are not independent systems. This still leaves the possibility that the infalling observer (or someone else) may perform detailed experiments on \mathcal{A} to spoil infall and indeed such a possibility is acknowledged by Papadodimas and Raju, but they claim that such an experiment is very hard to perform so generically there will be free infall.

Harlow-Hayden conjecture. In [17], Harlow and Hayden argue that the measurement on \mathcal{A} that AMPS showed would render the horizon a firewall, takes a time $t \sim e^{M^2}$ while the time for the black hole to evaporate completely is $t \sim M^3$. They conclude that it is not possible for an infalling observer to perform the AMPS measurement before falling into the black hole. Acknowledging that the argument using strong subadditivity does not actually require the infalling observer to perform the measurement on the early radiation, they propose the following criteria for breakdown of effective field theory:

Two spacelike-separated low-energy observables which are not both computationally accessible to some single observer do not need to be realized even approximately as distinct and commuting operators on the same Hilbert space.

This computational complementarity proposal implies that the computational complexity of measuring \mathcal{A} in the way AMPS propose means that it is possible that \mathcal{C} has support on $\mathcal{H}_{\mathcal{A}}$, as was suggested in [11, 16, 18], even though they are spacelike-separated.⁷

⁷Harlow and Hayden go on to claim that a stronger form of complementarity is also consistent with their conjecture: the infalling observer’s quantum mechanics may not be embeddable in that of the outside

3 Firewalls as duals to thermal CFT

In the previous section, we explain how an evaporating brane system becomes maximally entangled with the radiation outside. The firewall argument can be made for this system, and it implies that in the core region of the geometry, the part that is dual to the low-energy limit of the branes, there is a firewall.

However, we also review some rebuttals to the firewall argument, which state that the degrees of freedom required for free infall, \mathcal{C} , do not only come from the horizon degrees of freedom contained in $\mathcal{H}_{\mathcal{H}}$ but may have support in the radiation degrees of freedom in $\mathcal{H}_{\mathcal{A}}$. This involves a certain degree of non-locality. The required non-locality has been acknowledged in [11, 17], but it is tolerated by saying we do not know enough about quantum gravity to rule it out. Harlow and Hayden support their claim by noting that computational complexity suggests that verification of the non-locality is not possible.

Let us understand what role locality plays in the firewall argument. Locality mandates that the degrees of freedom required for free infall, \mathcal{B} and \mathcal{C} , are both present in the vicinity of the horizon since that is where infall is taking place. While one's lack of understanding of quantum gravity allows one to postulate that \mathcal{C} may be present in \mathcal{A} , we can make the puzzle sharper.

As we discuss above, the near-horizon region for the D1-D5 system is BTZ so we can imagine decoupling the near-horizon region by making S^1 much larger than all scales in the problem after the Page time. Alternatively, rather than using the Hawking radiation to thermalize the branes, we can directly start with a CFT in a thermal state. We can imagine that it was thermalized by coupling it to a large source/sink with a Hilbert space $\mathcal{H}_{\mathcal{S}}$ that acted as a heat bath. After equilibrium is attained, we then decouple $\mathcal{H}_{\mathcal{S}}$. Thus the state of the full system is

$$|\Psi\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\beta E/2} |E\rangle_{\mathcal{H}_{\text{CFT}}} \otimes |E\rangle_{\mathcal{H}_{\mathcal{S}}} \tag{3.1}$$

where $Z = \sum_E e^{-\beta E}$ and $|E\rangle_{\mathcal{H}_{\text{CFT}}}$ are states of the CFT and $|E\rangle_{\mathcal{H}_{\mathcal{S}}}$ are states of the heat bath. What is the dual to this thermal state?

In discussions, we have found many people claim that the dual of a thermal state in the CFT is the eternal AdS black hole based on arguments in [62].⁸ This is not correct. In [62], it is proposed that the dual description of maximally extended eternal AdS black holes with two boundaries involves two CFTs living on the boundaries in an entangled state resembling (3.1) (with the Hilbert spaces being those of the two boundary CFTs).

observer's. Namely, $\mathcal{H}_{\mathcal{A}}$ may not be part of the infalling observer's Hilbert space and then \mathcal{C} may be maximally entangled with \mathcal{B} for the infalling observer despite \mathcal{B} being maximally entangled with \mathcal{A} for the asymptotic observer. While we do not directly address the stronger form of complementarity in this paper, we would like to point out that it does not seem likely that $\mathcal{H}_{\mathcal{A}}$ may be missing from the infalling observer's quantum mechanics completely since the early radiation would backreact and influence the geodesic of the infalling observer.

⁸Ref. [62] proposes that the Lorentzian eternal AdS_{d+1} black hole is dual to two decouples CFTs on $S^{d-1} \times R$ in the highly entangled thermofield double state (3.1). In this paper we assume this is true. For arguments refuting this proposal see [63].

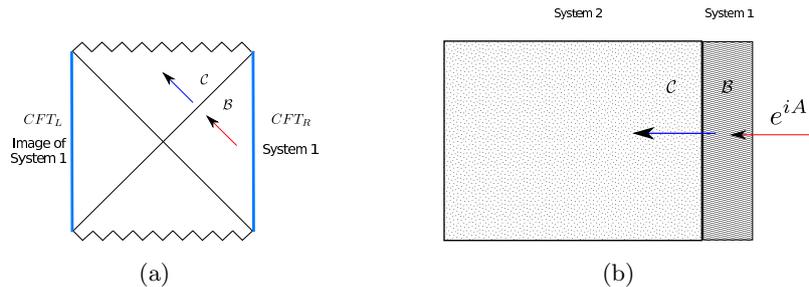


Figure 5. (a) The eternal AdS black hole is dual to two CFTs entangled in a certain way. An excitation on the right side representing an infalling observer requires degrees of freedom associated with the left CFT to move past the horizon. In (b) this system is realized as System 1 being in contact and thermal equilibrium with a bigger System 2.

As discussed in [58], one can create an infalling observer close to the right boundary of the geometry by acting on CFT_R with a unitary operator e^{iA} . Describing the evolution of the observer past the horizon requires the degrees of freedom of CFT_L . This is shown in figure 5. One realizes the setup in the following way. The CFT_R may describe some System 1 that is in thermal equilibrium and in contact with a bigger System 2. Assuming it is described by a conformal theory, the part of the bigger system that purifies the smaller system may play the role of CFT_L . Note that an excitation created in System 1 will eventually leak into System 2.

The situation we are interested in is subtly different. We are asking what is the bulk dual of *one* copy of the CFT, which is in a thermal state. We simply do not have the other CFT's degrees of freedom that are necessary for free infall. The equivalent of Papadodimas-Raju and Harlow-Hayden argument would be that the degrees of freedom of \mathcal{H}_S (which is the equivalent of \mathcal{H}_A for the evaporating branes) nevertheless come into play. However, given that we have decoupled the source/sink from the CFT and that the source/sink may not be a CFT or have a viable holographic description this seems rather implausible. The reason we say it's implausible is that the crossing of a horizon involves a Bell measurement (a joint measurement) on the modes on either side. This means the observer crossing the horizon interacts with both sets of modes. See appendix A for more details. It seems rather implausible that an observer living on the CFT system would still be able to access the degrees of freedom of \mathcal{H}_S in order to do a joint measurement, irrespective of the properties of the latter system and independent of the coupling between the two systems. Said differently the evolution of a perturbation created with support on the CFT beyond the horizon depends not only on how the CFT is entangled with some other system but also on the Hamiltonian of the combined system. We thus suggest that for generic System 2 the infalling observer hits a firewall. This is shown in figure 6.⁹

⁹We should point out a possible loophole in the above reasoning. Joint measurement assumes the apparatus is coupled to both the systems during the decoherence process. Since the apparatus in this context lives on the CFT system we are inclined to say a joint measurement is not possible in the absence of a coupling between the two systems. However, the theory of decoherence is not very well understood particularly in the context of AdS/CFT. Without the same it is hard to completely rule out a reconciliation

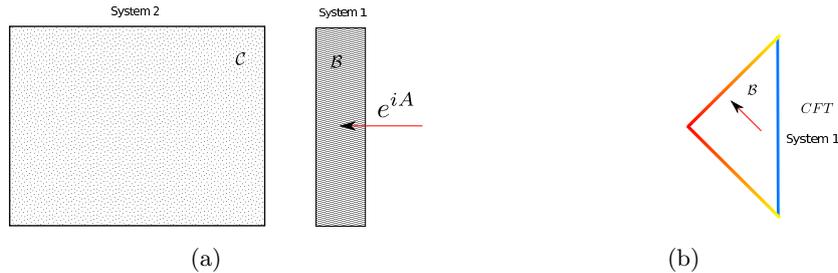


Figure 6. We consider a CFT in a thermal state living on Hilbert space \mathcal{H}_{CFT} . We can imagine it was thermalized by a sink/source with degrees of freedom living on \mathcal{H}_S . We take the two systems to be decoupled. Like the situation shown in figure 5 we can consider infall of an observer coming from the right. However, unlike the situation in figure 5, the degrees of freedom of \mathcal{H}_S are not available and there is a firewall at the horizon.

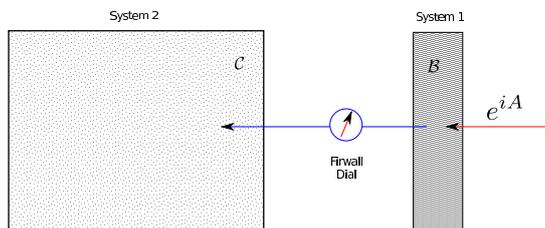


Figure 7. How much and in what way the degrees of freedom purifying the thermal CFT are available for free infall is dependent on the full theory. We can also phrase this as free infall depends on which superselection sector quantum gravity lives in.

The two cases — free infall as illustrated in figure 5 and hitting a firewall as illustrated in figure 6 — are two extremes. The answer to what is the dual to a thermal CFT seems beyond the information in the one CFT. This can be seen as a generalization of superselection sectors discussed in [58]. How much of the early radiation is available to act as the other copy of CFT for the stack of evaporating D-branes is a question that can be rephrased as which superselection sector quantum gravity is in. A question that seems can only be answered by knowing the full theory of quantum gravity or by jumping into a black hole.

Acknowledgments

We would like to thank Jan de Boer, Bartek Czech, Daniel Harlow, Samir Mathur, Shiraz Minwalla, Kyriakos Papadodimas, Ashoke Sen and Erik Verlinde for helpful discussions. The work of BDC was supported by the ERC Advanced Grant 268088-EMERGRAV.

A Infall as Bell measurement

Immediately after AMPS’s firewall paper [1], one of us proposed that the correct way to analyse the situation would be think of the observer as part of the complete system and of the bulk and boundary measurement processes.

measurements as coming from decoherence between the observer (or her apparatus) and the rest of the system [22]. This appendix is based on the same theme and on the talk [64].

Consider massless fields in 1 + 1 dimensions. The equations of motion split the fields into left and right movers. We consider only the left movers and the right movers behave the same. It can be shown (see [65] for example) that the Minkowski vacuum can be expressed in terms of Rindler modes as

$$|0_M\rangle = \frac{1}{\sqrt{\prod Z_\lambda}} \prod_\lambda e^{\tanh\theta_\lambda b_{\lambda,R}^\dagger b_{\lambda,L}^\dagger} |0_R\rangle |0_L\rangle \tag{A.1}$$

where $Z_\lambda = \text{Tr}[e^{-2\pi\lambda/a}]$, $\tanh\theta_\lambda = e^{-\pi\lambda/a}$ where a is the acceleration of the Rindler observer. Different modes given by different λ decouple and we can focus on the vacuum for a particular λ

$$|0_{M,\lambda}\rangle = \frac{1}{\sqrt{Z_\lambda}} \sum \tanh^n \theta_\lambda |n_{\lambda,R}\rangle |n_{\lambda,L}\rangle. \tag{A.2}$$

Note that if we consider the high temperature limit and restrict to fermionic modes then the above truncates to

$$|0_{M,\lambda}\rangle = \frac{1}{\sqrt{2}} (|0_{\lambda,R}\rangle |0_{\lambda,L}\rangle + |1_{\lambda,R}\rangle |1_{\lambda,L}\rangle) \tag{A.3}$$

and we can simplify our analysis by just considering qubits. The right moving observer will encounter left moving modes localised inside and outside the horizon and will find the state as the vacuum only if together they are in the state (A.3).

There is a simple generalization of the Minkowski vacuum state (A.3) which is a maximally entangled state between the two subsystems. One can write down four such orthogonal states

$$\begin{aligned} |\varphi_1\rangle &:= \frac{1}{\sqrt{2}} (|\hat{0}\rangle |0\rangle + |\hat{1}\rangle |1\rangle), \\ |\varphi_2\rangle &:= \frac{1}{\sqrt{2}} (|\hat{0}\rangle |0\rangle - |\hat{1}\rangle |1\rangle), \\ |\varphi_3\rangle &:= \frac{1}{\sqrt{2}} (|\hat{0}\rangle |1\rangle + |\hat{1}\rangle |0\rangle), \\ |\varphi_4\rangle &:= \frac{1}{\sqrt{2}} (|\hat{0}\rangle |1\rangle - |\hat{1}\rangle |0\rangle), \end{aligned} \tag{A.4}$$

and these are referred to as Bell states. The $|\hat{0}\rangle$ and $|\hat{1}\rangle$ are eigenstates of $\hat{\sigma}_z$ and similarly $|0\rangle$ and $|1\rangle$ are eigenstates of σ_z . Observe that in a simplified qubit model the Minkowski state corresponds to the first Bell state.

The reduced density matrix of the hatted and the unhatted systems for all four states are

$$\hat{\rho} = \frac{1}{2} (|\hat{0}\rangle \langle \hat{0}| + |\hat{1}\rangle \langle \hat{1}|), \quad \rho = \frac{1}{2} (|0\rangle \langle 0| + |1\rangle \langle 1|) \tag{A.5}$$

which means that Charlie with access to only one of the systems (i.e. with access to operators $\hat{I} \otimes \sigma_x$, $\hat{I} \otimes \sigma_y$ and $\hat{I} \otimes \sigma_z$) will get identical response from all four states and will be unable to distinguish between them. This does not, however, mean that the four states

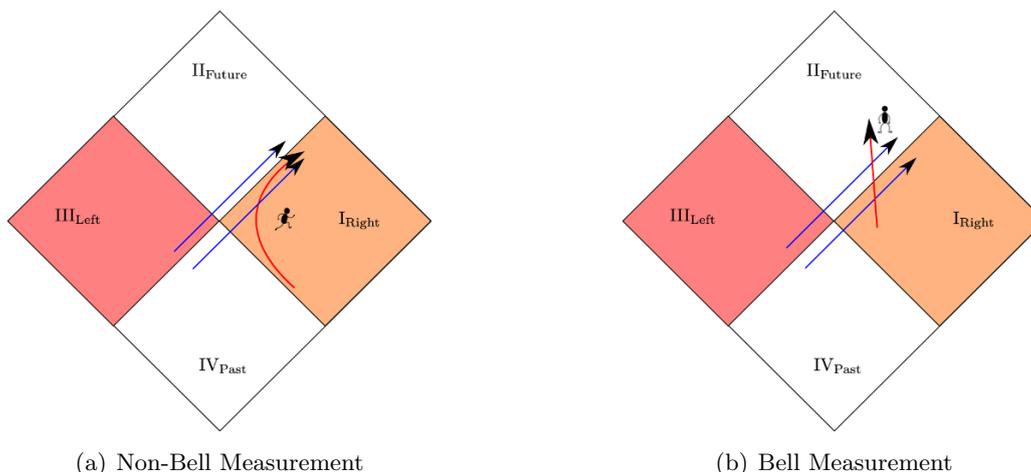


Figure 8. An accelerating observer only intersects the modes of the right wedge so can only do non-Bell measurements. These do not measure the actual state of the system but instead collapse the system into a different state. An inertial observer on the other hand intersects both modes and thus can perform a Bell measurement to verify that the full state is the Minkowski vacuum.

are indistinguishable. These states are eigenstates of the operators $\hat{\sigma}_x \otimes \sigma_x$, $\hat{\sigma}_y \otimes \sigma_y$ and $\hat{\sigma}_z \otimes \sigma_z$. The eigenvalues are shown in the table below. Thus, an observer, Alice, can

state	$\hat{\sigma}_x \otimes \sigma_x$	$\hat{\sigma}_y \otimes \sigma_y$	$\hat{\sigma}_z \otimes \sigma_z$
$ \varphi_1\rangle$	+1	-1	+1
$ \varphi_2\rangle$	-1	+1	+1
$ \varphi_3\rangle$	+1	+1	-1
$ \varphi_4\rangle$	-1	-1	-1

distinguish between the four states by measuring the expectation value of any of the two operators, say $\hat{\sigma}_x \otimes \sigma_x$ and $\hat{\sigma}_z \otimes \sigma_z$. This is called a Bell measurement.

In light of this, our previous comment about a right moving observer finding the left movers in the vacuum only if they are in the state (A.3) can be restated in the following way. Accelerating observers who stay inside the Rindler wedge have access to only half the system can only perform non-Bell measurements and cannot tell if the full state is the Minkowski vacuum or any other state that leaves the right wedge density matrix the same (see figure 8(a)). However, inertial observers can measure the full state of the system and in fact do so while crossing the horizon. They can thus tell if the full state is the Minkowski vacuum or some other state. Thus inertial observers perform Bell measurements. This is shown in figure 8(b).

Now consider a CFT in a thermal state that we call system A. One can always find another system B (which may or may not be a CFT) which together with the CFT is in a pure state. Now one can consider boundary-Alice living on system A. It is widely believed that system A which is a CFT captures the bulk *at least* outside the horizon and we will assume that. Similarly, system B can *approximately* capture the bulk physics external to

the horizon on the other side with the approximation getting better the more system B's dynamics can be described by a CFT.

According to AdS/CFT, boundary-Alice's interaction with the other degrees of freedom of system A in principle describes bulk-Alice hurtling towards the horizon. However, when bulk-Alice crosses the horizon, she is performing a joint measurement on the degrees of freedom on the two sides of the horizon. The degrees of freedom inside the horizon can be traced to modes from the other exterior region and thus are associated to degrees of freedom of system B. Thus, bulk-Alice crossing the horizon performs a joint measurement on the bulk modes associated to boundary systems A and B. Boundary-Alice on the other hand lives only on system A and cannot perform a joint measurement on the degrees of freedom of both system A and system B. This tension between the measurements accessible to bulk-Alice and boundary-Alice may suggest that (a) system A and system B cannot capture the physics behind the horizon, or (b) the bulk physics needs modification at the horizon scale. However, since the theory of decoherence and measurements is not fully developed, especially in the context of AdS/CFT, we cannot fully rule out some surprise which can resolve the aforementioned tension between the bulk and the boundary point of view.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, *Black Holes: Complementarity or Firewalls?*, *JHEP* **02** (2013) 062 [[arXiv:1207.3123](https://arxiv.org/abs/1207.3123)] [[INSPIRE](https://inspirehep.net/literature/1207312)].
- [2] L. Susskind, L. Thorlacius and J. Uglum, *The Stretched horizon and black hole complementarity*, *Phys. Rev. D* **48** (1993) 3743 [[hep-th/9306069](https://arxiv.org/abs/hep-th/9306069)] [[INSPIRE](https://inspirehep.net/literature/31112)].
- [3] L. Susskind and L. Thorlacius, *Gedanken experiments involving black holes*, *Phys. Rev. D* **49** (1994) 966 [[hep-th/9308100](https://arxiv.org/abs/hep-th/9308100)] [[INSPIRE](https://inspirehep.net/literature/31112)].
- [4] Y. Nomura, J. Varela and S.J. Weinberg, *Complementarity Endures: No Firewall for an Infalling Observer*, *JHEP* **03** (2013) 059 [[arXiv:1207.6626](https://arxiv.org/abs/1207.6626)] [[INSPIRE](https://inspirehep.net/literature/1207662)].
- [5] S.D. Mathur and D. Turton, *Comments on black holes I: The possibility of complementarity*, *JHEP* **01** (2014) 034 [[arXiv:1208.2005](https://arxiv.org/abs/1208.2005)] [[INSPIRE](https://inspirehep.net/literature/1208200)].
- [6] T. Banks and W. Fischler, *Holographic Space-Time Does Not Predict Firewalls*, [arXiv:1208.4757](https://arxiv.org/abs/1208.4757) [[INSPIRE](https://inspirehep.net/literature/1208475)].
- [7] R. Brustein, *Origin of the blackhole information paradox*, *Fortsch. Phys.* **62** (2014) 255 [[arXiv:1209.2686](https://arxiv.org/abs/1209.2686)] [[INSPIRE](https://inspirehep.net/literature/1209268)].
- [8] S.G. Avery, B.D. Chowdhury and A. Puhm, *Unitarity and fuzzball complementarity: 'Alice fuzzes but may not even know it!'*, *JHEP* **09** (2013) 012 [[arXiv:1210.6996](https://arxiv.org/abs/1210.6996)] [[INSPIRE](https://inspirehep.net/literature/1210699)].
- [9] K. Larjo, D.A. Lowe and L. Thorlacius, *Black holes without firewalls*, *Phys. Rev. D* **87** (2013) 104018 [[arXiv:1211.4620](https://arxiv.org/abs/1211.4620)] [[INSPIRE](https://inspirehep.net/literature/1211462)].

- [10] S.K. Rama, *Remarks on Black Hole Evolution a la Firewalls and Fuzzballs*, [arXiv:1211.5645](#) [INSPIRE].
- [11] K. Papadodimas and S. Raju, *An Infalling Observer in AdS/CFT*, *JHEP* **10** (2013) 212 [[arXiv:1211.6767](#)] [INSPIRE].
- [12] Y. Nomura and J. Varela, *A Note on (No) Firewalls: The Entropy Argument*, *JHEP* **07** (2013) 124 [[arXiv:1211.7033](#)] [INSPIRE].
- [13] S.D.H. Hsu, *Macroscopic superpositions and black hole unitarity*, [arXiv:1302.0451](#) [INSPIRE].
- [14] S.B. Giddings, *Nonviolent nonlocality*, *Phys. Rev. D* **88** (2013) 064023 [[arXiv:1211.7070](#)] [INSPIRE].
- [15] S.B. Giddings, *Nonviolent information transfer from black holes: A field theory parametrization*, *Phys. Rev. D* **88** (2013) 024018 [[arXiv:1302.2613](#)] [INSPIRE].
- [16] T. Jacobson, *Boundary unitarity and the black hole information paradox*, *Int. J. Mod. Phys. D* **22** (2013) 1342002 [[arXiv:1212.6944](#)] [INSPIRE].
- [17] D. Harlow and P. Hayden, *Quantum Computation vs. Firewalls*, *JHEP* **06** (2013) 085 [[arXiv:1301.4504](#)] [INSPIRE].
- [18] R. Bousso, *Complementarity Is Not Enough*, *Phys. Rev. D* **87** (2013) 124023 [[arXiv:1207.5192](#)] [INSPIRE].
- [19] L. Susskind, *Singularities, Firewalls and Complementarity*, [arXiv:1208.3445](#) [INSPIRE].
- [20] L. Susskind, *The Transfer of Entanglement: The Case for Firewalls*, [arXiv:1210.2098](#) [INSPIRE].
- [21] S. Hossenfelder, *Comment on the black hole firewall*, [arXiv:1210.5317](#) [INSPIRE].
- [22] B.D. Chowdhury and A. Puhm, *Is Alice burning or fuzzing?*, *Phys. Rev. D* **88** (2013) 063509 [[arXiv:1208.2026](#)] [INSPIRE].
- [23] I. Bena, A. Puhm and B. Verhocke, *Non-extremal Black Hole Microstates: Fuzzballs of Fire or Fuzzballs of Fuzz?*, *JHEP* **12** (2012) 014 [[arXiv:1208.3468](#)] [INSPIRE].
- [24] A. Giveon and N. Itzhaki, *String Theory Versus Black Hole Complementarity*, *JHEP* **12** (2012) 094 [[arXiv:1208.3930](#)] [INSPIRE].
- [25] A. Ori, *Firewall or smooth horizon?*, [arXiv:1208.6480](#) [INSPIRE].
- [26] D.-i. Hwang, B.-H. Lee and D.-h. Yeom, *Is the firewall consistent?: Gedanken experiments on black hole complementarity and firewall proposal*, *JCAP* **01** (2013) 005 [[arXiv:1210.6733](#)] [INSPIRE].
- [27] D.N. Page, *Hyper-Entropic Gravitational Fireballs (Gfireballs) with Firewalls*, *JCAP* **04** (2013) 037 [[arXiv:1211.6734](#)] [INSPIRE].
- [28] M. Saravani, N. Afshordi and R.B. Mann, *Empty Black Holes, Firewalls and the Origin of Bekenstein-Hawking Entropy*, [arXiv:1212.4176](#) [INSPIRE].
- [29] L. Susskind, *Black Hole Complementarity and the Harlow-Hayden Conjecture*, [arXiv:1301.4505](#) [INSPIRE].
- [30] D.N. Page, *Time Dependence of Hawking Radiation Entropy*, *JCAP* **09** (2013) 028 [[arXiv:1301.4995](#)] [INSPIRE].

- [31] M. Bañados, C. Teitelboim and J. Zanelli, *The Black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [[hep-th/9204099](#)] [[INSPIRE](#)].
- [32] J.R. David, G. Mandal and S.R. Wadia, *Absorption and Hawking radiation of minimal and fixed scalars and AdS/CFT correspondence*, *Nucl. Phys. B* **544** (1999) 590 [[hep-th/9808168](#)] [[INSPIRE](#)].
- [33] S.G. Avery, B.D. Chowdhury and S.D. Mathur, *Emission from the D1D5 CFT*, *JHEP* **10** (2009) 065 [[arXiv:0906.2015](#)] [[INSPIRE](#)].
- [34] D.N. Page, *Average entropy of a subsystem*, *Phys. Rev. Lett.* **71** (1993) 1291 [[gr-qc/9305007](#)] [[INSPIRE](#)].
- [35] D.N. Page, *Black hole information*, [hep-th/9305040](#) [[INSPIRE](#)].
- [36] S. Sen, *Average entropy of a subsystem*, *Phys. Rev. Lett.* **77** (1996) 1 [[hep-th/9601132](#)] [[INSPIRE](#)].
- [37] S.D. Mathur, *The Information paradox: A Pedagogical introduction*, *Class. Quant. Grav.* **26** (2009) 224001 [[arXiv:0909.1038](#)] [[INSPIRE](#)].
- [38] S.G. Avery, *Qubit Models of Black Hole Evaporation*, *JHEP* **01** (2013) 176 [[arXiv:1109.2911](#)] [[INSPIRE](#)].
- [39] V. Balasubramanian, P. Kraus and M. Shigemori, *Massless black holes and black rings as effective geometries of the D1 – D5 system*, *Class. Quant. Grav.* **22** (2005) 4803 [[hep-th/0508110](#)] [[INSPIRE](#)].
- [40] O. Lunin and S.D. Mathur, *Statistical interpretation of Bekenstein entropy for systems with a stretched horizon*, *Phys. Rev. Lett.* **88** (2002) 211303 [[hep-th/0202072](#)] [[INSPIRE](#)].
- [41] V.S. Rychkov, *D1 – D5 black hole microstate counting from supergravity*, *JHEP* **01** (2006) 063 [[hep-th/0512053](#)] [[INSPIRE](#)].
- [42] B.D. Chowdhury and S.D. Mathur, *Fractional Brane State in the Early Universe*, *Class. Quant. Grav.* **24** (2007) 2689 [[hep-th/0611330](#)] [[INSPIRE](#)].
- [43] F. Larsen and E.J. Martinec, *U(1) charges and moduli in the D1 – D5 system*, *JHEP* **06** (1999) 019 [[hep-th/9905064](#)] [[INSPIRE](#)].
- [44] S. Giusto, S.D. Mathur and A. Saxena, *Dual geometries for a set of 3-charge microstates*, *Nucl. Phys. B* **701** (2004) 357 [[hep-th/0405017](#)] [[INSPIRE](#)].
- [45] M. Cvetič and D. Youm, *General rotating five-dimensional black holes of toroidally compactified heterotic string*, *Nucl. Phys. B* **476** (1996) 118 [[hep-th/9603100](#)] [[INSPIRE](#)].
- [46] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [[hep-th/9905111](#)] [[INSPIRE](#)].
- [47] C.G. Callan and J.M. Maldacena, *D-brane approach to black hole quantum mechanics*, *Nucl. Phys. B* **472** (1996) 591 [[hep-th/9602043](#)] [[INSPIRE](#)].
- [48] A. Dhar, G. Mandal and S.R. Wadia, *Absorption versus decay of black holes in string theory and T symmetry*, *Phys. Lett. B* **388** (1996) 51 [[hep-th/9605234](#)] [[INSPIRE](#)].
- [49] S.R. Das and S.D. Mathur, *Comparing decay rates for black holes and D-branes*, *Nucl. Phys. B* **478** (1996) 561 [[hep-th/9606185](#)] [[INSPIRE](#)].
- [50] S.R. Das and S.D. Mathur, *Interactions involving D-branes*, *Nucl. Phys. B* **482** (1996) 153 [[hep-th/9607149](#)] [[INSPIRE](#)].

- [51] J.M. Maldacena and A. Strominger, *Black hole grey body factors and D-brane spectroscopy*, *Phys. Rev. D* **55** (1997) 861 [[hep-th/9609026](#)] [[INSPIRE](#)].
- [52] R. Dijkgraaf, G.W. Moore, E.P. Verlinde and H.L. Verlinde, *Elliptic genera of symmetric products and second quantized strings*, *Commun. Math. Phys.* **185** (1997) 197 [[hep-th/9608096](#)] [[INSPIRE](#)].
- [53] S.D. Mathur, *The Quantum structure of black holes*, *Class. Quant. Grav.* **23** (2006) R115 [[hep-th/0510180](#)] [[INSPIRE](#)].
- [54] S.G. Avery and B.D. Chowdhury, *Intertwining Relations for the Deformed D1D5 CFT*, *JHEP* **05** (2011) 025 [[arXiv:1007.2202](#)] [[INSPIRE](#)].
- [55] S.G. Avery, B.D. Chowdhury and S.D. Mathur, *Deforming the D1D5 CFT away from the orbifold point*, *JHEP* **06** (2010) 031 [[arXiv:1002.3132](#)] [[INSPIRE](#)].
- [56] S.G. Avery, B.D. Chowdhury and S.D. Mathur, *Excitations in the deformed D1D5 CFT*, *JHEP* **06** (2010) 032 [[arXiv:1003.2746](#)] [[INSPIRE](#)].
- [57] C.T. Asplund and S.G. Avery, *Evolution of Entanglement Entropy in the D1 – D5 Brane System*, *Phys. Rev. D* **84** (2011) 124053 [[arXiv:1108.2510](#)] [[INSPIRE](#)].
- [58] D. Marolf and A.C. Wall, *Eternal Black Holes and Superselection in AdS/CFT*, *Class. Quant. Grav.* **30** (2013) 025001 [[arXiv:1210.3590](#)] [[INSPIRE](#)].
- [59] M. Van Raamsdonk, *Building up spacetime with quantum entanglement*, *Gen. Rel. Grav.* **42** (2010) 2323 [[arXiv:1005.3035](#)] [[INSPIRE](#)].
- [60] B. Czech, J.L. Karczmarek, F. Nogueira and M. Van Raamsdonk, *Rindler Quantum Gravity*, *Class. Quant. Grav.* **29** (2012) 235025 [[arXiv:1206.1323](#)] [[INSPIRE](#)].
- [61] B. Czech, J.L. Karczmarek, F. Nogueira and M. Van Raamsdonk, *The Gravity Dual of a Density Matrix*, *Class. Quant. Grav.* **29** (2012) 155009 [[arXiv:1204.1330](#)] [[INSPIRE](#)].
- [62] J.M. Maldacena, *Eternal black holes in anti-de Sitter*, *JHEP* **04** (2003) 021 [[hep-th/0106112](#)] [[INSPIRE](#)].
- [63] S.G. Avery and B.D. Chowdhury, *No Holography for Eternal AdS Black Holes*, [arXiv:1312.3346](#) [[INSPIRE](#)].
- [64] B.D. Chowdhury, *What do bell measurements have to do with (the lack of) infall?*, in *Black Holes in String Theory* workshop, Ann Arbor, Oct. 7–18 (2013).
- [65] R. Parentani, *The Energy momentum tensor in Fulling-Rindler vacuum*, *Class. Quant. Grav.* **10** (1993) 1409 [[hep-th/9303062](#)] [[INSPIRE](#)].