Research Article

A Gauged Open 2-Brane String in the p-Brane Background

Fahimeh Sarvi, Majid Monemzadeh, and Salman Abarghouei Nejad

Department of Physics, University of Kashan, Kashan 87317-51167, Iran

Correspondence should be addressed to Majid Monemzadeh; monem@kashanu.ac.ir

We make a gauge theory from the Open p-brane system and map it into the Open 2-Brane one. Due to the presence of second-class constraints in this model, we encounter some problems during the procedure of quantization. In this regard, considering boundary conditions as Dirac conditions, one can drive the constrained structure of the model at first. Then, with the help of BFT formalism of constraint systems, the Open 2-Brane model is embedded into an extended phase space. For this purpose, we introduce some tensor fields to convert ungauged theory into the gauged one. This is the novel part of our research, while mostly scalar and vector fields are used to convert second-class constraints into first ones.

1. Introduction

In the scenario of point-like particles, the infinite mass leads to several infinities, which obliged physicists to developed methods of renormalization to overcome these problems. Afterwards, although string theory as a theoretical frame work provided some tools to simplify the unification of general relativity and quantum mechanics, it leads to the complicated big bang cosmology and inflation scenarios. In this theme, it is important to know how to quantize string and D-brane actions [1, 2]. As a matter of fact, this is possible using Dirac's point of view, since these actions include primary second-class constraints and actually are not gauge invariant.

As the pioneer who proposed the correlation between gauge theories and constrained systems, Dirac classified constraints into first- and second-class ones. He also implied that the existence of second-class constraints is due to the presence of some nonphysical degrees of freedom, which destroys the gauge invariance [3]. Hence, these extra degrees of freedom must be omitted or changed to physical ones to enhance gauge symmetries of the model [4]. In this regard, there are several methods available to convert second-class systems into first-class ones [3–5].

Due to the presence of gauge degrees of freedom, quantization of a first-class system is straightforward, but dealing with second-class systems gives rise to some difficulties.

Quantizing a system which includes second-class constraints is not possible due to the presence of extra coordinates in the primary phase space. Dirac solved this incompatibility by converting Poisson brackets into another kind, which are known as Dirac brackets in respect of him, today. Although his approach is serviceable in some cases, dealing with Dirac brackets is not generally an easy task. Some difficulties such as factor ordering and inverting Poisson brackets matrix of constraints may cause Dirac's approach to not be practical [6–10].

A question might have been raised which is why do not we vanish second-class constraints directly? One should say that it is not always possible to vanish second-class constraints, while they may omit some of the dynamics of the system automatically. It is more wise to convert second-class constraints into first-class ones, which leaves the dynamics of the system intact. This has been shown in some models that the embedding process does not change the dynamics of the system [4, 11–13].

To convert a second-class system to a first-class one, there have existed several methods such as gauge unfixing approach (GU) [14, 15], BFT method [16–19], and the symplectic formalism [12, 20–22].

The BFT method has been proposed to convert a gauge noninvariant system to a gauge invariant one. This method is based on embedding procedure; that is, to convert second-class constraints into first-class ones as gauge symmetries,

one should increase phase space of the primary model and redefine constrained equations in the extended phase space.

Depending on the structure of second-class constraints, the extended phase space has different forms. Sometimes, the procedure of BFT embedding is complicated and even impossible to do. It has been shown that if the Poisson bracket matrix of second-class constraints has constant components, extension of the phase space is straightforward and added correction terms to Hamiltonian will be limited in number [10].

In this research, according to constraints' algebra which leads to the constant Poisson bracket matrix, we use finite BFT approach to convert second-class constraints to first-class one, in order to obtain the gauged version of Open 2-Brane System.

The interesting and almost problematic point in the study of Open string problem in the presence of background magnetic field is that after imposing mixed boundary conditions on equations of motion, the fields' Poisson brackets are not vanished at boundaries and they are equal to a constant value which is proportional to the present background field [23–27]. This problem also leads to a contradiction when we study secondary constraints.

The idea of considering boundary conditions as constraints of a physical system was proposed by the late 1990s [28–30]. In fact, boundary conditions in the background magnetic field are first-order equations of time variable, whereas the Euler-Lagrange equation is a second-order one, which means that the acceleration determines the dynamics of the motion. Hence, boundary conditions in Lagrangian equations are called acceleration-free equations [24–27, 30]. So, these objects do not play any role in the dynamics of the system. Instead, they set up some identities in the phase space which satisfy Dirac's constrained conditions. Also, the difference between constraints, obtained from boundary conditions equations and other constraints, could be evident in Hamiltonian approach [30].

By the idea of boundary conditions as Dirac conditions, we study the constrained structure of bosonic p-brane systems [31–33]. First of all, we simplify the action of this theory, based on existed symmetries of the model. Then, constrained structure of the model with $p = 2$, that is, bosonic Open p-brane system, is obtained and the consistency condition of constraints is checked in order to obtain secondary constraints. Considering Poisson brackets of constraints, we prove the second-class nature of them.

2. Action of Open p-Brane

D branes are particular types of branes. They are places in the space where strings are located. D_p brane is stretched out in P spatial dimensions which sweeps out a $(P + 1)$ dimensional world volume [34, 35]. The action studied in this paper is the bosonic part of a classical action for an Open p-brane ending on a D_q-brane [33, 36, 37]. To simplify the model, in this procedure we avoid probing other aspects such as terms related to fermionic parts of the action as much as possible, and we only consider the aspects of action which impose mixed boundary conditions in order to decrease the

generality of our research. This means that we ignore all additional couplings to D_p-brane which are available in the context of string theory, by setting all backgrounds to zero. Some more general models are studied at [38–42].

The action of such a system can be written as

$$
\begin{aligned}
&S \\
&= -\frac{1}{4\pi\acute{\alpha}} \int_{\Sigma} d^{p+1}\zeta \sqrt{-h} \left[G_{\mu\nu} h^{\alpha\beta} \frac{\partial X^{\mu}}{\partial \zeta^{\alpha}} \frac{\partial X_{\mu}}{\partial \zeta^{\beta}} + (p-1) \right],
\end{aligned} \tag{1}
$$

where the values of μ vary from zero to D. Also, $\alpha, \beta = \{0, 1, \ldots, p\}$ and $(p - 1)$ is the cosmological term.

This model has a large surface of global symmetries, which are determined by the world volume and the target space. To study a simpler model, we consider the dynamics of fields in the flat target space

$$
G_{\mu\nu} = \eta_{\mu\nu}. \tag{2}
$$

Also, we let $\acute{\alpha}$ be a constant numerical parameter. Moreover, the world volume has diffeomorphisms and scaling invariance. These symmetries can be fixed by determining a metric on it. We consider this metric as $h_{\alpha\beta} = (-, +, +, \ldots, +)$.

The canonical momentum and the Hamiltonian are defined as follows:

$$
\begin{aligned}
\Pi_{\mu} &= \partial_{\tau} X^{\mu}, \\
H &= \frac{1}{2} \int \frac{d^{p}\zeta}{(2\pi)^{p}} \left[(\Pi_i)^2 + (\Pi_a)^2 + \sum_{k=1}^{p} (\partial_k X^k)^2 \right. \\
&\quad \left. + \sum_{k=1}^{p} (\partial_k X^a)^2 + (p - 1) \right],
\end{aligned} \tag{3}
$$

where $\partial_k = \partial/\partial \sigma^k$, $i = 0, 1, \ldots, q$, and $a = q + 1, \ldots, D$.

It is evident that, in directions orthogonal to D branes, the boundary condition is the form of Dirichlet boundary condition and toward D-brane is the form of Neumann boundary condition. Assuming $q \geq p$, that is, the dimension of D-brane being less than or equal to the space dimension, we can only have Neumann boundary condition. Dynamic boundary points with Neumann boundary conditions are as follows:

$$
\partial_k X^i (0) = \partial_k X^i (\pi) = 0. \tag{4}
$$

Obviously these equations are at most first-class derivatives with respect to the time and are called acceleration-free equations. As we mentioned before, such equations do not play any roles in present object dynamics of the theory and they only limit available phase spaces; thus they satisfy Dirac constrained condition.

3. Constraint Structure of Open 2-Brane System

It is important to know how to quantize string and D-brane action. The alternative method is solving all of the constraints

of the system at first. So, in this section and as an example, we analyse Open 2-Brane system. As we know, a 2-Brane is surrounded by a three-dimensional hyper surface. The Neumann boundary conditions are imposed at the end of Open 2-Brane [33],

$$\partial_1 X^i(0) = \partial_1 X^i(\pi),$$
$$\partial_2 X^i(0) = \partial_2 X^i(\pi).$$
(5)

The boundary conditions can be rewritten in terms of normal 2-Brane modes

$$^{(1)}\phi_m^{i(0)} = \sum_n n X_{nm}^i \approx 0,$$
$$^{(1)}\overline{\phi}_m^{i(0)} = \sum_n (-1)^n n X_{nm}^i \approx 0,$$
$$^{(2)}\phi_n^{i(0)} = \sum_m m X_{nm}^i \approx 0,$$
$$^{(2)}\overline{\phi}_n^{i(0)} = \sum_m (-1)^m m X_{nm}^i \approx 0,$$
(6)

where $\approx$ means weak equality.

The canonical Hamiltonian can be written as

$$H = \frac{1}{2} \sum_n \sum_m \eta_{ij} \left[P_{nm}^i P_{(-n)(-m)}^j \right.$$
$$\left. + \left(n^2 + m^2\right) X_{nm}^i X_{(-n)(-m)}^j \right] + \frac{1}{2}$$
$$\cdot \sum_n \sum_m \eta_{ab} \left[P_{nm}^a P_{(-n)(-m)}^b \right.$$
$$\left. + \left(n^2 + m^2\right) X_{nm}^a X_{(-n)(-m)}^b \right].$$
(7)

Hence, primary constraints must be held all the time; we set the complete time derivative of first-class constraints equal to zero. In this manner, the consistency procedure gives the set of secondary constraint [33]

$$^{(1)}\Psi_m^{i(0)} = \left\{ H, {}^{(1)}\phi_m^{i(0)} \right\}_{PB} = \sum_n n P_{nm}^i \approx 0,$$
$$^{(1)}\overline{\Psi}_m^{i(0)} = \left\{ H, {}^{(1)}\overline{\phi}^{i(0)} \right\}_{PB} = \sum_n (-1)^n n P_{nm}^i \approx 0,$$
$$^{(2)}\Psi_m^{i(0)} = \left\{ H, {}^{(2)}\phi_m^{i(0)} \right\}_{PB} = \sum_n m P_{nm}^i \approx 0,$$
$$^{(2)}\overline{\Psi}^{i(0)} = \left\{ H, {}^{(2)}\overline{\phi}^{i(0)} \right\}_{PB} = \sum_n (-1)^m m P_{nm}^i \approx 0.$$
(8)

By further imposing the time consistency of the secondary constraints, new constraints will be obtained as follows:

$$^{(1)}\phi_m^{i(1)} = \left\{ H, {}^{(1)}\Psi_m^{i(0)} \right\}_{PB} = -\sum_n n \left(n^2 + m^2\right) X_{nm}^i \approx 0,$$
$$^{(1)}\overline{\phi}_m^{i(1)} = \left\{ H, {}^{(1)}\overline{\Psi}_m^{i(0)} \right\}_{PB} = -\sum_n (-1)^n n \left(n^2 + m^2\right) X_{nm}^i$$
$$\approx 0,$$
$$^{(2)}\phi_n^{i(1)} = \left\{ H, {}^{(2)}\Psi_m^{i(0)} \right\}_{PB} = -\sum_m m \left(n^2 + m^2\right) X_{nm}^i \approx 0,$$
$$^{(2)}\overline{\phi}_m^{i(1)} = \left\{ H, {}^{(2)}\overline{\Psi}^{i(0)} \right\}_{PB}$$
$$= -\sum_n (-1)^m n \left(n^2 + m^2\right) X_{nm}^i \approx 0.$$
(9)

Continuing this procedure, we will obtain the complete chain structure of constraints of the model [4]. Hence, we will have

$$^{(1)}\phi_m^{i(k)} = (-1)^k \sum_n n \left(n^2 + m^2\right)^k X_{nm}^i \approx 0,$$
$$^{(1)}\overline{\phi}_m^{i(k)} = (-1)^k \sum_n (-1)^n n \left(n^2 + m^2\right)^k X_{nm}^i \approx 0,$$
$$^{(2)}\phi_n^{i(k)} = (-1)^k \sum_m m \left(n^2 + m^2\right)^k X_{nm}^i \approx 0,$$
$$^{(2)}\overline{\phi}_n^{i(k)} = (-1)^k \sum_m (-1)^m n \left(n^2 + m^2\right)^k X_{nm}^i \approx 0,$$
$$^{(1)}\Psi_m^{i(k)} = (-1)^k \sum_n n \left(n^2 + m^2\right)^k P_{nm}^i \approx 0,$$
$$^{(1)}\overline{\Psi}_m^{i(k)} = (-1)^k \sum_n (-1)^n n \left(n^2 + m^2\right)^k P_{nm}^i \approx 0,$$
$$^{(2)}\Psi_n^{i(k)} = (-1)^k \sum_m m \left(n^2 + m^2\right)^k P_{nm}^i \approx 0,$$
$$^{(2)}\overline{\Psi}_n^{i(k)} = (-1)^k \sum_m (-1)^m n \left(n^2 + m^2\right)^k P_{nm}^i \approx 0.$$
(10)

Summing over ks and noting the fact that the result must be equal to zero, these constraints make a close set. Hence, we may find the following sets of equations of constraints which obviously are independent from k:

$$^{(1)}\chi_{nm}^i = X_{nm}^i - X_{(-n)m}^i \approx 0,$$
$$^{(2)}\chi_{nm}^i = X_{nm}^i - X_{n(-m)}^i \approx 0,$$
$$^{(1)}\varphi_{nm}^i = P_{nm}^i - P_{(-n)m}^i \approx 0,$$
$$^{(2)}\varphi_{nm}^i = P_{nm}^i - P_{n(-m)}^i \approx 0.$$
(11)

4. Gauging the Model

The emergence of second-class constraints in the Noninvariant Open 2-Brane model is due to the presence of a broken

gauge symmetry. As we mentioned before, the gauging process will be done via the BFT method. This method is based on extending the phase space of the original model by introducing some auxiliary variables.

Considering a pure second-class system where its dynamics is defined by the Hamiltonian $H^{(0)}$ in the phase space with coordinates (p, q) and a set of second-class constraints Θ_α in which $\alpha = \{1, \ldots, m\}$, we will have the Poisson bracket matrix as the following invertible matrix:

$$\Delta_{\alpha\beta} = \left\{ \Theta_\alpha, \Theta_\beta \right\}. \tag{12}$$

For our model, the gauge structure constructing Poisson bracket matrix of second-class constraint is constant matrix as follows:

$$\Delta = \begin{pmatrix} 0 & 0 & 2\eta^{ij} & \eta^{ij} \\ 0 & 0 & \eta^{ij} & 2\eta^{ij} \\ -2\eta^{ij} & -\eta^{ij} & 0 & 0 \\ -\eta^{ij} & -2\eta^{ij} & 0 & 0 \end{pmatrix}. \tag{13}$$

Now, we enlarge the phase space by the variable ξ^α which satisfies the following algebra [10, 16–19]:

$$\begin{aligned} \left\{ q_i, p_i \right\} &= \delta_{ij}, \\ \left\{ q_i, q_j \right\} &= \left\{ p_i, p_j \right\} = 0, \\ \left\{ \xi_\alpha, q_i \right\} &= \left\{ \xi_\alpha, p_i \right\} = 0, \\ \left\{ \xi_\alpha, \xi_\beta \right\} &= \omega_{\alpha\beta}, \end{aligned} \tag{14}$$

where $\omega_{\alpha\beta}$ is an antisymmetric invertible matrix. The first-class constraints in the extended phase space $(p, q) \oplus \xi$ are defined by

$$\tau_\alpha = \tau_\alpha (p, q, \xi), \quad \alpha = 1, 2, \ldots, m, \tag{15}$$

which satisfy the following boundary condition:

$$\tau_\alpha^{(0)} (p, q, 0) = \Theta_\alpha (p, q). \tag{16}$$

For special cases where the matrix $\Delta_{\alpha\beta}$ is symplectic or constant, a systematic method is invented which proposes the following choices [10]:

$$\begin{aligned} \omega &= -\Delta, \\ \Omega &= 1. \end{aligned} \tag{17}$$

These choices lead to the following finite order embedding formalism for constraints:

$$\tau_\alpha = \tau_\alpha^{(0)} + \xi^\alpha. \tag{18}$$

Another important point is that, in order to enlarge the phase space in string theory, we should introduce tensor fields, in contrast with other theories that scaler and vector fields must be added. So the new set of constraints will be

$$\begin{aligned} {}^{(1)}\tau_{nm}^i &= {}^{(1)}\chi_{nm}^i + {}^{(1)}\xi_{nm}^i, \\ {}^{(2)}\tau_{nm}^i &= {}^{(2)}\chi_{nm}^i + {}^{(2)}\xi_{nm}^i, \\ {}^{(3)}\tau_{nm}^i &= {}^{(1)}\varphi_{nm}^i + {}^{(3)}\xi_{nm}^i, \\ {}^{(4)}\tau_{nm}^i &= {}^{(2)}\varphi_{nm}^i + {}^{(4)}\xi_{nm}^i. \end{aligned} \tag{19}$$

The generators of Hamiltonian correction terms [10, 16–19] will be

$$\begin{aligned} G_1^{(0)} &= \left\{ X_{nm}^i - X_{(-n)m}^i, H^{(0)} \right\} \\ &= 2 \left(P_{(-n)(-m)}^i - P_{n(-m)}^i \right), \\ G_2^{(0)} &= \left\{ X_{nm}^i - X_{n(-m)}^i, H^{(0)} \right\} \\ &= 2 \left(P_{(-n)(-m)}^i - P_{(-n)m}^i \right), \\ G_3^{(0)} &= \left\{ P_{nm}^i - P_{(-n)m}^i, H^{(0)} \right\} \\ &= 2 \left(n^2 + m^2 \right) \left(X_{(-n)(-m)}^i - X_{n(-m)}^i \right), \\ G_4^{(0)} &= \left\{ P_{nm}^i - P_{n(-m)}^i, H^{(0)} \right\} \\ &= 2 \left(n^2 + m^2 \right) \left(X_{(-n)(-m)}^i - X_{(-n)m}^i \right). \end{aligned} \tag{20}$$

Hence, first-order Hamiltonian is obtained as

$$\begin{aligned} \widetilde{H}^{(1)} &= -2\eta^{ij} \Big[\left(P_{(-n)(-m)}^i - P_{n(-m)}^i \right) \left(2^{(3)}\xi_{nm}^i + {}^{(4)}\xi_{nm}^i \right) \\ &\quad - \left(P_{(-n)(-m)}^i + P_{(-n)m}^i \right) \left(2^{(4)}\xi_{nm}^i + {}^{(3)}\xi_{nm}^i \right) \\ &\quad + 2 \left(n^2 + m^2 \right) \left(X_{(-n)(-m)}^i - X_{n(-m)}^i \right) \\ &\quad \cdot \left(2^{(1)}\xi_{nm}^i + {}^{(2)}\xi_{nm}^i \right) + \left(X_{(-n)(-m)}^i - X_{(-n)m}^i \right) \\ &\quad \cdot \left({}^{(1)}\xi_{nm}^i + 2^{(2)}\xi_{nm}^i \right) \Big], \end{aligned} \tag{21}$$

and the generators in the second correction term of Hamiltonian are investigated as

$$\begin{aligned} G_1^{(1)} &= \left\{ X_{nm}^i - X_{(-n)m}^i, \widetilde{H}^{(1)} \right\} = -6^{(3)}\xi_{(-n)(-m)}^i \\ &\quad + 6^{(3)}\xi_{n(-m)}^i + 2^{(3)}\xi_{(-n)m}^i - 2^{(3)}\xi_{nm}^i + 4^{(4)}\xi_{(-n)m}^i \\ &\quad - 2^{(4)}\xi_{nm}^i, \end{aligned}$$

$$G_2^{(1)} = \left\{ X_{nm}^i - X_{n(-m)}^i, \widetilde{H}^{(1)} \right\} = -6^{(3)}\xi_{(-n)(-m)}^i$$

$$+ 4^{(3)}\xi_{n(-m)}^i + 2^{(3)}\xi_{(-n)m}^i + 2^{(3)}\xi_{nm}^i + 3^{(4)}\xi_{(-n)(-m)}^i$$

$$- 2^{(4)}\xi_{n(-m)}^i - 9^{(4)}\xi_{(-n)m}^i + 2^{(4)}\xi_{nm}^i,$$

$$G_3^{(1)} = \left\{ P_{nm}^i - P_{(-n)m}^i, \widetilde{H}^{(1)} \right\} = \left(n^2 + m^2 \right)$$

$$\cdot \left[10^{(1)}\xi_{(-n)(-m)}^i - 6^{(1)}\xi_{n(-m)}^i - 2^{(1)}\xi_{(-n)m}^i + 2^{(1)}\xi_{nm}^i \right.$$

$$\left. + 8^{(2)}\xi_{(-n)(-m)}^i - 8^{(2)}\xi_{n(-m)}^i - 4^{(2)}\xi_{(-n)m}^i + 2^{(2)}\xi_{nm}^i \right],$$

$$G_4^{(1)} = \left\{ P_{nm}^i - P_{n(-m)}^i, \widetilde{H}^{(1)} \right\} = \left(n^2 + m^2 \right) \left[8^{(1)}\xi_{(-n)(-m)}^i \right.$$

$$- 8^{(1)}\xi_{(-n)m}^i - 4^{(1)}\xi_{n(-m)}^i + 10^{(2)}\xi_{(-n)(-m)}^i$$

$$\left. - 2^{(2)}\xi_{(-n)(-m)}^i - 10^{(2)}\xi_{(-n)m}^i - 2^{(2)}\xi_{nm}^i \right].$$

$$(22)$$

Hence, the second-order correction of Hamiltonian is obtained as

$$\widetilde{H}^{(2)} = \frac{-1}{2}\eta^{ij} \left[\left(2^{(3)}\xi_{nm}^i + {}^{(4)}\xi_{nm}^i \right) G_1^{(1)} \right.$$

$$- \left(2^{(1)}\xi_{nm}^i + {}^{(2)}\xi_{nm}^i \right) G_3^{(1)} + \left(2^{(4)}\xi_{nm}^i + {}^{(3)}\xi_{nm}^i \right) G_2^{(1)} \qquad (23)$$

$$\left. - \left({}^{(1)}\xi_{nm}^i + 2^{(2)}\xi_{nm}^i \right) G_2^{(1)} \right].$$

It is easy to see that $G_\alpha^{(n)}$ vanished for $n \geq 2$. This leads to the truncated series for Hamiltonian for $n \geq 2$

$$H_T = H^{(0)} + \widetilde{H}^{(1)} + \widetilde{H}^{(2)}. \qquad (24)$$

Thus, by introducing some tensor fields, first-class constraints of second ones would be obtained and the Hamiltonian investigated in such away will be fully gauged in the extended phase space.

5. Conclusion

String theory is an extremely good theory to describe the world. So, quantizing string and D-brane action is an interesting topic to work on. An appropriate approach to do such an investigation is the formalism of constrained systems.

In this survey, first, we extracted the constrained structure of the Open 2-Brane System in the flat space. Because the boundary conditions are usually relations between fields and their derivatives, we considered the Neumann boundary conditions as Dirac constraints which are not consequences of a singular Lagrangian. In other words, the momenta are not independent functions of velocities which is a new feature in the context of constrained systems. Finally, we got four second-class constraints. As we know, first-class constraints are the generators of gauge transformations, while the advent of second-class constraints restricts the system to a smaller submanifold of the phase space in which a Poisson structure can be recognized. This means that gauge symmetries are broken.

We desire to have a quantized theory, but quantization of second-class system is nontrivial and is more difficult in comparison with first-class one. Difficulties such as factor ordering problem or noninvertible nature of some Poisson brackets matrices of constraint may arise dealing with second-class systems. Also, the construction of a BRST charge is only possible for first-class systems. More importantly, the usual quantization method like canonical quantization and path integral approach is only used for first-class constrained systems. Since, physical theories tend to be gauged theories, and the presence of second-class constraints is against this assumption, we convert the Noninvariant Open 2-Brane System to a first-class one by means of BFT method.

As we mentioned, for special cases when the Δ-matrix has constant elements, the finite order BFT embedding can be applied. This is the reason that we only studied the bosonic part of a classical action for an Open p-brane ending on a D_q-brane and avoid additional couplings to D_p-brane by setting all background fields to zero.

Finally, in the last section we found correction terms for constraints and Hamiltonian in the extended phase space. All in all, one can easily check that first-class constraints and Hamiltonian of Open 2-Brane System represent a gauge invariant theory.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

Acknowledgments

References

[1] E. Witten, "Perturbative gauge theory as a string theory in twistor space," *Communications in Mathematical Physics*, vol. 252, no. 1, pp. 189–258, 2004.

[2] T. Azeyanagi, M. Hanada, H. Kawai, and Y. Matsuo, "World-sheet analysis of gauge/gravity dualities," *Nuclear Physics B*, vol. 816, no. 1-2, pp. 278–292, 2009.

[3] P. A. M. Dirac, *Lectures on Quantum Mechanics*, Belfer Graduate School of Science, New York, NY, USA, 1964.

[4] S. Abarghouei Nejad, M. Dehghani, and M. Monemzadeh, "Lagrange multiplier and Wess-Zumino variable as large extra dimensions in the torus universe," http://arxiv.org/abs/1508.01866.

[5] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems*, Princeton University Press, Princeton, NJ, USA, 1992.

[6] M. Sadegh and A. Shirzad, "Constraint structure of the three dimensional massive gravity," *Physical Review D*, vol. 83, no. 8, Article ID 084040, 2011.

[7] C. Becchi, A. Rouet, and R. Stora, "Renormalization of gauge theories," *Annals of Physics*, vol. 98, no. 2, pp. 287–321, 1976.

[8] I. V. Tyutin, *Lebedev*, vol. 39, 1975.

[9] T. Kugo and I. Ojima, "Local covariant operator formalism of non-abelian gauge theories and quark confinement problem,"

Progress of Theoretical Physics Supplement, vol. 66, pp. 1–130, 1979.

[10] M. Monemzadeh and A. Shirzad, "Finite order BFFT method," *International Journal of Modern Physics A*, vol. 18, no. 30, pp. 5613–5625, 2003.

[11] S. Abarghouei Nejad, M. Dehghani, and M. Monemzadeh, "Fundamental symmetries of the modified anyonic particle," http://arxiv.org/abs/1507.07890.

[12] S. Abarghouei Nejad, M. Dehghani, and M. Monemzadeh, "Gauging the relativisticparticle model on the noncommutative plane," http://arxiv.org/abs/1507.02329.

[13] M. Dehghani, M. Mardaani, M. Monemzadeh, and S. Abarghouei Nejad, "First class models from linear and nonlinear second class constraints," *Modern Physics Letters A*, vol. 30, no. 38, Article ID 1550199, 2015.

[14] S. Das and S. Ghosh, "Gauge-invariant extension of linearized horava gravity," *Modern Physics Letters A*, vol. 26, no. 37, pp. 2793–2801, 2011.

[15] J. Ananias Neto, "An improved gauge unfixing formalism and the abelian pure chern simons theory," *Brazilian Journal of Physics*, vol. 37, no. 3, pp. 1106–1110, 2007.

[16] I. A. Batalin and E. S. Fradkin, "Operational quantization of dynamical systems subject to second class constraints," *Nuclear Physics B*, vol. 279, no. 3-4, pp. 514–528, 1987.

[17] I. A. Batalin and I. V. Tyutin, "Existence theorem for the effective gauge algebra in the generalized canonical formalism with abelian conversion of second-class constraints," *International Journal of Modern Physics A*, vol. 6, no. 18, p. 3255, 1991.

[18] A. Shirzad and M. Monemzadeh, "The BFT method with chain structure," *Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics*, vol. 584, no. 1-2, pp. 220–224, 2004.

[19] M. Monemzadeh and A. Shirzad, "Batalin-fradkin-tyutin method for mixed constrained systems and chern-simons theory," *Physical Review D*, vol. 72, no. 4, Article ID 045004, 2005.

[20] L. Faddeev and R. Jackiw, "Hamiltonian reduction of unconstrained and constrained systems," *Physical Review Letters*, vol. 60, no. 17, pp. 1692–1694, 1988.

[21] N. M. J. Woodhouse, *Geometric Quantization*, Clarendon Press, Oxford, UK, 1980.

[22] J. Ananias Neto, C. Neves, and W. Oliveira, "Gauging the SU(2) Skyrme model," *Physical Review D*, vol. 63, no. 8, Article ID 085018, 2001.

[23] A. Abouelsaood, C. G. Callan, C. R. Nappi, and S. A. Yost, "Open strings in background gauge fields," *Nuclear Physics B*, vol. 280, pp. 599–624, 1987.

[24] A. Shirzad, A. Bakhshi, and Y. Koohsarian, "Symplectic quantization of massive bosonic string in background B-field," *Modern Physics Letters A*, vol. 27, no. 13, Article ID 1250073, 2012.

[25] M. M. Sheikh-Jabbari and A. Shirzad, "Boundary conditions as Dirac constraints," *The European Physical Journal C—Particles and Fields*, vol. 19, no. 2, pp. 383–390, 2001.

[26] F. Ardalan, H. Arfaei, and M. M. Sheikh-Jabbari, "Dirac quantization of open strings and noncommutativity in branes," *Nuclear Physics B*, vol. 576, no. 1–3, pp. 578–596, 2000.

[27] F. Loran, "Comment on 'Constraint quantization of open string in background B field and noncommutative D-brane'," *Physics Letters B*, vol. 544, no. 1-2, pp. 199–201, 2002.

[28] W. T. Kim and J. J. Oh, "Noncommutative open strings from Dirac quantization," *Modern Physics Letters A. Particles and Fields, Gravitation, Cosmology, Nuclear Physics*, vol. 15, no. 26, pp. 1597–1604, 2000.

[29] T. Lee, "Canonical quantization of open string and noncommutative geometry," *Physical Review D*, vol. 62, no. 2, Article ID 024022, 2000.

[30] M. Dehghani and A. Shirzad, "The reduced phase space of an open string in the background B-field," *The European Physical Journal C*, vol. 48, no. 1, pp. 315–325, 2006.

[31] D. Fiorenza, H. Sati, and U. Schreiber, "Super-Lie n-algebra extensions, higher WZW models and super-p-branes with tensor multiplet fields," *International Journal of Geometric Methods in Modern Physics*, vol. 12, no. 2, Article ID 1550018, 35 pages, 2015.

[32] T. Lee, "Open superstring and noncommutative geometry," *Physics Letters B*, vol. 483, no. 1–3, pp. 277–283, 2000.

[33] C. X. Yu and Y. C. Huang, "Dirac quantization of open p-brane," *Physics Letters B*, vol. 647, no. 1, pp. 49–55, 2007.

[34] B. Zwiebach, *A First Course in String Theory*, Cambridge University Press, Cambridge, UK, 2004.

[35] J. Polchinski, *String Theory*, Cambridge University Press, Cambridge, UK, 1998.

[36] R. Banerjee, P. Mukherjee, and A. Saha, "Bosonic P-brane and Arnowitt-Deser-Misner decomposition," *Physical Review D*, vol. 72, no. 6, Article ID 066015, 2005.

[37] G. T. Horowitz and L. Susskind, "Bosonic M theory," *Journal of Mathematical Physics*, vol. 42, no. 7, pp. 3152–3160, 2001.

[38] C.-X. Yu, C. Huang, P. Zhang, and Y.-C. Huang, "First and second quantization theories of open p-brane and their spectra," *Physics Letters B*, vol. 697, no. 4, pp. 378–384, 2011.

[39] Y.-C. Huang and C.-X. Yu, "Quantization and spectrum of an open 2-brane," *Physical Review D*, vol. 75, no. 4, Article ID 044011, 2007.

[40] M. A. Wasay and D. F. Zeng, "An inconsistency in the spectrum of bosonic open 2-brane," *Advances in High Energy Physics*, vol. 2015, Article ID 360356, 4 pages, 2015.

[41] A. Escalante, "Canonical covariant formalism for Dirac-Nambu-Goto bosonic p-branes and the GAUss-Bonnet topological term in string theory," *International Journal of Modern Physics A*, vol. 21, no. 26, pp. 5275–5284, 2006.

[42] M. Wasay Abdul, Y.-C. Huang, and D.-f. Zeng, "Quantization and spectrum of RNS supersymmetric open 2-brane," *Nuclear Physics B*, vol. 892, pp. 353–363, 2015.

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