

KINETICS VERSUS HYDRODYNAMICS: GENERALIZATION OF LANDAU/COOPER-FRYE PRESCRIPTION FOR FREEZE-OUT*

YU.M. SINYUKOV, S.V. AKKELIN, I.U.A. KARPENKO

Bogolyubov Institute for Theoretical Physics
Metrolohichna str. 14b, 03680 Kiev-143, Ukraine

Y. HAMA

Instituto de Física, Universidade de São Paulo
São Paulo, SP, C.P. 66318, 05314-970, Brazil

(Received March 2, 2009)

The problem of spectra formation in hydrodynamic approach to $A + A$ collisions is considered within the Boltzmann equations. It is shown analytically and illustrated by numerical calculations that the particle momentum spectra can be presented in the Cooper–Frye form despite freeze-out is not sharp and has the finite temporal width. The latter is equal to the inverse of the particle collision rate at points $(t_\sigma(\mathbf{r}, p), \mathbf{r})$ of the maximal emission at a fixed momentum p . The set of these points forms the hypersurfaces $t_\sigma(\mathbf{r}, p)$ which strongly depend on the values of p and typically do not enclose completely the initially dense matter. This is an important difference from the standard Cooper–Frye prescription (CFp), with a common freeze-out hypersurface for all p , that affects significantly the predicted spectra. Also, the well known problem of CFp as for negative contributions to the spectra from non-space-like parts of the freeze-out hypersurface is naturally eliminated in this improved prescription.

PACS numbers: 25.75.-q, 24.10.Nz

1. Introduction

The Landau hydrodynamic approach [1] for multi-hadron production in hadronic/nuclear collisions appeared as a method that is alternative to the S -matrix one: the latter transforms the asymptotic hadronic states from

* Presented at the IV Workshop on Particle Correlations and Femtoscopy, Kraków, Poland, September 11–14, 2008.

$t = -\infty$ to $t = +\infty$ while the former deals with the space-time evolution of thermal matter, produced in the collisions. The evolution is described by using the local energy-momentum conservation laws and thermodynamic equation of state of the matter. Unlike the S -matrix formalism, the initial state in hydrodynamic approach is associated with some concrete finite time after collision when created particles reach the locally equilibrated state and so, the initial state can be described by the minimal set of parameters. As well, because system formed is quite small, the picture of continuous medium is destroyed also at likely small finite time. Since the system expands fast, the latter is not constant in configuration space but depends on the position $\mathbf{r}$ of the fluid elements: $t_\sigma(\mathbf{r})$. The set of these points $(t_\sigma(\mathbf{r}), \mathbf{r})$ in the Minkowski space forms, therefore, the hypersurface σ corresponding to the outer boundary of the applicability of hydrodynamics. The sudden freeze-out implies that the spectra are formed just on this hypersurface where, formally, an ideal fluid transforms into an ideal gas. The particle momentum spectra then can be expressed by the well-known Cooper-Frye formula [2]:

$$p^0 n(p) = p^0 \frac{d^3 N}{d^3 p} \approx \int_{\sigma} d\sigma_{\mu} p^{\mu} f_{\text{l.eq.}}(x, p). \quad (1)$$

The freeze-out hypersurface is typically associated with an isotherm. According to Landau [1] $T \approx m_{\pi} \approx 140$ MeV. In current analysis of $A + A$ collisions the corresponding temperature is in the region $T = 90\text{--}150$ MeV and is fixed, typically, from the best fit of the spectra.

Any isotherm contains usually non-space-like parts, which lead to unphysical negative contributions to spectra for the particles with momenta directed inward the system, $p_{\mu} d\sigma_{\mu} < 0$. There is no common phenomenological prescription, based on Heaviside step functions $\theta(p_{\mu} d\sigma_{\mu})$, which allows one to eliminate the negative contributions to the momentum spectra when $p^{\mu} d\sigma_{\mu} < 0$. The prescription, proposed in [3], eliminates the negative contributions in the way which preserves the number of particles in the fluid element crossing the freeze-out hypersurface. Therefore it takes into account that at the final stage the system is the only holder of emitted particles. Another prescription [4] ignores the particle number conservation considering decaying hadronic system rather as a star — practically unlimited reservoir of emitted photons/particles. The both prescriptions have a problem with momentum-energy conservation laws at freeze-out.

But the most serious problem is the obvious conflict of the CFp with simple observation: to provide sudden transformation of the liquid to ideal gas one needs to switch suddenly cross-sections between particles from very big values to very small ones [5] that cannot happen in reality. The reason of particle liberation is another: it is the gradual change of the ratio

between the rate of system expansion and the rate of collisions, so that particles should be emitted continuously. The phenomenological model of continuous emission was proposed in Ref. [6]; another approach is the so-called hybrid models [7], where kinetic evolution is matched with hydro one at the hypersurface of hadronization. The shortcomings of these models are described in detail in [8]. But the feeling that the Landau/Cooper–Frye sudden freeze-out does not describe the real process of continuous particle emission is predominant now.

Nevertheless, in Ref. [9] it was advanced an idea of duality in hydro-kinetic approach to $A + A$ collisions: though the process of particle liberation, described by the emission function, is non-equilibrium and gradual, the observable spectra can yet be expressed by means of the Cooper–Frye prescription based on locally equilibrium distribution function. The conclusive step towards an analytical and numerical realization of this idea is done in Ref. [8] where hydro-kinetic approach is developed: it overcomes all above mentioned problems by considering the continuous dynamical freeze-out that is consistent with Boltzmann equations and conservation laws. In what follows we stick to the mainstream of arguments developed in this paper.

2. Kinetics of freeze-out in Boltzmann approach

Let us start from Boltzmann equation. It has the general form:

$$\frac{p^\mu}{p^0} \frac{\partial f_i(x, p)}{\partial x^\mu} = G_i(x, p) - L_i(x, p). \quad (2)$$

The expressions $G_i(x, p)$ and $L_i(x, p) = R_i(x, p)f_i(x, p)$ are so-called (G)ain and (L)oss terms for the particle of species i . Typically, R_i is a rate of collisions of i -th particle. Below we will omit index i , the corresponding expression can be related then, *e.g.*, to pions.

The probability $\mathcal{P}_{t \rightarrow t'}(x, p)$ for a particle to reach the point $x' = (t', \mathbf{r}')$ starting from the point $x = (t, \mathbf{r})$ without collisions is

$$\mathcal{P}_{t \rightarrow t'}(x, p) = \exp \left(- \int_t^{t'} d\bar{t} R(\bar{x}_t, p) \right), \quad (3)$$

where

$$\bar{x}_t = \left(\bar{t}, \mathbf{r} + \frac{\mathbf{p}}{p^0}(\bar{t} - t) \right).$$

In terms of this probability, the Boltzmann equation can be rewritten in the following integral form

$$f(t, \mathbf{r}, p) = f\left(t_0, \mathbf{r} - \frac{\mathbf{p}}{p^0}(t - t_0), p\right) \mathcal{P}_{t_0 \rightarrow t}\left(t_0, \mathbf{r} - \frac{\mathbf{p}}{p^0}(t - t_0), p\right) + \int_{t_0}^t G\left(\tau, \mathbf{r} - \frac{\mathbf{p}}{p^0}(t - \tau), p\right) \mathcal{P}_{\tau \rightarrow t}\left(\tau, \mathbf{r} - \frac{\mathbf{p}}{p^0}(t - \tau), p\right) d\tau. \quad (4)$$

Let us integrate the distribution (4) over the space variables to represent the particle momentum density at large enough time, $t \rightarrow \infty$, when particles in the system stop to interact. To simplify notation let us introduce the escape probability for the particle with momentum p in the point $x = (t, \mathbf{r})$ to leave system without collisions: $\mathcal{P}(t, \mathbf{r}, p) \equiv \mathcal{P}_{t \rightarrow (\tau \rightarrow \infty)}(t, \mathbf{r}, p)$. Then the result can be presented in the general form found in Ref. [5]:

$$n(t \rightarrow \infty, p) \equiv n(p) = \int d^3r f(t_0, \mathbf{r}, p) \mathcal{P}(t_0, \mathbf{r}, p) + \int d^3r \int_{t_0}^{\infty} dt' G(t', \mathbf{r}, p) \mathcal{P}(t', \mathbf{r}, p). \quad (5)$$

The first term in Eq. (5) describes the contribution to the momentum spectrum from particles that are emitted from the very initial time, while the second one describes the continuous emission with emission density $S(x, p) = G(t, \mathbf{r}, p) \mathcal{P}(t, \mathbf{r}, p)$ from 4D volume delimited by the initial and final (where particles stop to interact) 3D hypersurfaces.

In what follows we will use the (generalized) relaxation time approximation proposed in [5], which is the basis of the hydro-kinetic approach, described in detail in [8]. Namely, it was argued [5] that there is such a local equilibrium distribution function $f_{\text{l.eq.}}(T(x), u^\nu(x), \mu(x))$ that, in the region of not very small densities where term $G \sim S$ gives noticeable contribution to particle spectra, the function f is approximately equal to that one which would be obtained if all functions in r.h.s. of Eq. (4) are calculated by means of that function $f_{\text{l.eq.}}$. The function $f_{\text{l.eq.}}$ is determined from the local energy-momentum conservation laws based on the non-equilibrium function f in the way specified in [8]. Then, in accordance with this approach we use

$$R(x, p) \approx R_{\text{l.eq.}}(x, p), G \approx R_{\text{l.eq.}}(x, p) f_{\text{l.eq.}}(x, p). \quad (6)$$

The “relaxation time” $\tau_{\text{rel}} = 1/R_{\text{l.eq.}}$ grows with time in this method.

3. Saddle point approximation for momentum spectra

Let us generalize now the Landau/Cooper–Frye prescription (CFp) of sudden freeze-out. For this aim we apply the saddle point method to calculate the integral in the expression for spectra (5) with account of (6). To simplify notation we neglect the contribution to the spectra from hadrons which are already free at the initial thermalization time $t_0 \sim 1 \text{ fm}/c$ and thus omit the first term in (4).

To provide straightforward calculations leading to the Cooper–Frye form let us shift the spacial variables, $\mathbf{r}' = \mathbf{r} + \frac{\mathbf{p}}{p_0}(t_0 - t')$, in (5) aiming to eliminate the variable t' in the argument of the function R which is the integrand in $\mathcal{P}(t', \mathbf{r}, p)$. Then

$$n(p) \approx \int d^3r' \int_{t_0}^{\infty} dt' f_{\text{1.eq.}} \left(t', \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t' - t_0), p \right) Q(t', \mathbf{r}', p), \quad (7)$$

where

$$\begin{aligned} Q(t', \mathbf{r}', p) &= R \left(t', \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t' - t_0), p \right) \\ &\times \exp \left\{ - \int_{t'}^{\infty} R \left(s, \mathbf{r}' + \frac{\mathbf{p}}{p_0}(s - t_0), p \right) ds \right\}. \end{aligned} \quad (8)$$

Note that

$$Q(t', \mathbf{r}', p) = \frac{d}{dt'} P(t', \mathbf{r}', p), \quad (9)$$

where $P(t', \mathbf{r}', p)$ is connected with the escape probability $\mathcal{P}$:

$$P(t', \mathbf{r}', p) = \mathcal{P} \left(t', \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t' - t_0), p \right). \quad (10)$$

Therefore

$$\int_{t_0}^{\infty} dt' Q(t', \mathbf{r}', p) = 1 - \mathcal{P}(t_0, \mathbf{r}', p) \approx 1. \quad (11)$$

The saddle point $t_\sigma(\mathbf{r}, p)$ is defined by the standard conditions:

$$\begin{aligned} \left. \frac{dQ(t', \mathbf{r}', p)}{dt'} \right|_{t'=t'_\sigma} &= 0, \\ \left. \frac{d^2Q(t', \mathbf{r}', p)}{dt'^2} \right|_{t'=t'_\sigma} &< 0. \end{aligned} \quad (12)$$

Then one can get from (9), (10) the condition of the maximum of emission:

$$-\frac{p^\mu \partial_\mu R(t', \mathbf{r}, p)}{R(t'_\sigma, \mathbf{r}, p)} \Big|_{t'=t'_\sigma, \mathbf{r}=\mathbf{r}'+\frac{\mathbf{p}}{p_0}(t'_\sigma-t_0)} = p_0 R\left(t'_\sigma, \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t'_\sigma - t_0), p\right). \quad (13)$$

If one neglects terms $\mathbf{p}^* \partial_{\mathbf{r}^*} R$ in l.h.s. and supposes that in the rest frame (marked by asterisk) of the fluid element with four-velocity $u(x)$ the collision rate, $R^*(x, p) = \frac{p_0 R(x, p)}{p^\mu u_\mu}$, does not depend on particle momentum: $R^*(x) \approx \langle v^* \sigma \rangle(x) n^*(x)$ (here $n(x)$ is particle density, σ is the particle cross-section, v is the relative velocity, $\langle \dots \rangle$ means the average over all momenta), then the conditions (13) are equivalent to the requirement that at the temporal point of maximum of the emission function the rate of collisions is equal to the rate of system expansion [8]. This is the heuristic freeze-out criterion for sudden freeze-out [10]. However, as we will demonstrate, the neglect of momentum dependence leads to quite significant errors.

To pass to the Cooper–Frye representation we use the variables which include the saddle point:

$$\mathbf{r} = \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t'_\sigma(\mathbf{r}', p) - t_0). \quad (14)$$

Then the expression for the spectrum takes the form:

$$n(p) \approx \int d^3r \left| 1 - \frac{\mathbf{p}}{p_0} \frac{\partial t_\sigma}{\partial \mathbf{r}} \right| \int_{t_0}^{\infty} dt' S(t', \mathbf{r}, p), \quad (15)$$

where the emission density in saddle point representation is ($t_\sigma \equiv (t_\sigma(\mathbf{r}, p))$)

$$\begin{aligned} S(t', \mathbf{r}, p) = & f_{\text{l.eq.}} \left(t', \mathbf{r} + \frac{\mathbf{p}}{p_0}(t' - t_\sigma), p \right) \\ & \times R(t_\sigma, \mathbf{r}, p) \mathcal{P}(t_\sigma, \mathbf{r}, p) \exp \left(-\frac{(t' - t_\sigma)^2}{2D^2(t_\sigma, \mathbf{r}, p)} \right). \end{aligned} \quad (16)$$

According to Eq. (3) $\mathcal{P}(t_\sigma, \mathbf{r}, p) = e^{-1}$, since the freeze-out zone is the region of the last collision for the particle. Then the normalization condition for Q (Q is presented by the bottom line in (16)) allows one to determine the temporal width of the emission at the point $(t_\sigma(\mathbf{r}, p), \mathbf{r}, p)$:

$$D(t_\sigma, \mathbf{r}, p) = \frac{e}{\sqrt{2\pi}} \frac{1}{R(t_\sigma, \mathbf{r}, p)} \approx \tau_{\text{rel}}(t_\sigma, \mathbf{r}, p). \quad (17)$$

Therefore if the temporal homogeneity length $\lambda(t, \mathbf{r}, p)$ of the distribution function $f_{\text{l.eq.}}$ near the 4-point $(t_\sigma(\mathbf{r}, p), \mathbf{r}, p)$ is much larger than the width

of the emission zone, $\lambda(t_\sigma, \mathbf{r}, p) \gg \tau_{\text{rel}}(t_\sigma, \mathbf{r}, p)$, then one can approximate $f_{\text{1.eq.}}(t', \mathbf{r} + \frac{\mathbf{p}}{p_0}(t' - t_\sigma), p)$ by $f_{\text{1.eq.}}(t_\sigma, \mathbf{r}, p)$ in Eq. (15) and perform integration over t' accounting for normalizing condition (11). As a result we get from (15) and (16) the momentum spectrum in a form similar to the Cooper–Frye one (1):

$$p^0 n(p) = p^0 \frac{d^3 N}{d^3 p} \approx \int_{\sigma(p)} d\sigma_\mu p^\mu f_{\text{1.eq.}}(x, p). \quad (18)$$

It is worthy to note that the representation of the spectrum through emission function (5) is the result of the integration of the total non-equilibrium distribution function $f(x, p)$, Eqs. (4), (6), over the asymptotical hypersurface in time, while the approximate representation of the spectrum, Eq. (18), uses only the local equilibrium part $f_{\text{1.eq.}}$ of the total function $f(x, p)$ at the set of points of maximal emission — at hypersurface $(t_\sigma(\mathbf{r}, p), \mathbf{r})$.

4. Generalized Cooper–Frye prescription

Now let us summarize the conditions when the Landau/Cooper–Frye form for sudden freeze-out can be used. They are the following:

- (i) For each momentum $\mathbf{p}$, there is a region of $\mathbf{r}$ where the emission function as well as the function Q , Eq. (8), have a clear maximum. The temporal width of the emission D , defined by Eq. (16), which is found to be equal to the relaxation time (inverse of collision rate), should be smaller than the corresponding temporal homogeneity length of the distribution function: $\lambda(t_\sigma, \mathbf{r}, p) \gg D(t_\sigma, \mathbf{r}, p) \simeq \tau_{\text{rel}}(t_\sigma, \mathbf{r}, p)$.
- (ii) The contribution to the spectrum from the residual region of $\mathbf{r}$, where the saddle point method (Gaussian approximation (16) and/or condition $\tau_{\text{rel}} \ll \lambda$) is violated, does not affect essentially the particle momentum density.

If these conditions are satisfied, then the momentum spectra can be presented in the Cooper–Frye form *despite the fact that actually it is not sudden freeze-out and the decoupling region has a finite temporal width* $\tau_{\text{rel}}(t_\sigma, \mathbf{r}, p)$.

The analytical results as for the temporal width of the spectra agree remarkably with the numerical calculations of pion emission function within hydro-kinetic model (HKM) [8]. For example, near the point of maximum, $\tau = 16.5$ fm/c, $r = 0, p_T = 0.2$ GeV, the “experimental” temporal width D_{HKM} obtained by numerical solution of the complete hydro-kinetic equations is $D_{\text{HKM}} \approx 4.95$ fm (see Fig. 1, top). Our theoretical estimate is

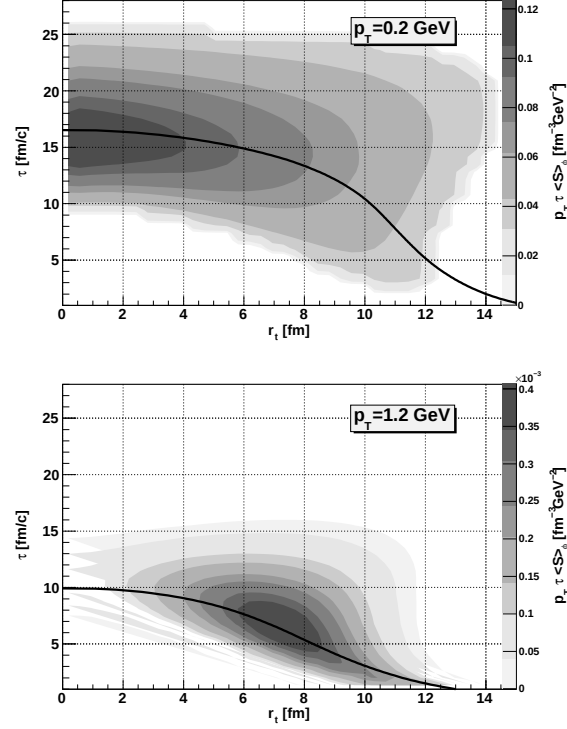


Fig. 1. The pion emission function for different p_T in hydro-kinetic model (HKM) [8]. The isotherms of 80 MeV (top) and 135 MeV (bottom) are superimposed.

$D = \frac{e}{\sqrt{2\pi}R} \approx 5.00$ fm, since the rate of collisions in this phase-space point is $R(\tau_\sigma(\mathbf{r}, p) = 16.5 \text{ fm}/c, \mathbf{r} = 0, p_T = 0.2 \text{ GeV}, p_L = 0) \approx 0.217 \text{ c}/\text{fm}$.

It is worthy to emphasize that such a generalized Cooper–Frye representation is related to *freeze-out hypersurfaces that depend on the momentum $\mathbf{p}$ and typically do not enclose the initially dense matter*. In Fig. 1, one can see the structure of the emission domains for different p_T in HKM [8] for initially (at $\tau = 1 \text{ fm}/c$) Gaussian energy density profile with $\epsilon_{\max} = 6 \text{ GeV}/\text{fm}^3$. The maximal emission regions for different p_T are crossed by isotherms with different temperatures: 80 MeV for low momenta and 135 MeV for high ones. This is completely reflected in the concave structure of the transverse momentum spectrum as one can see in Fig. 2.

If a part of the hypersurface $t_\sigma(\mathbf{r}, p)$ is non-space-like and corresponds to the maximum of the emission of particles with momentum $\mathbf{p}$, directed outward the system, the same part of the hypersurface cannot correspond to the maximal emission for particles with momentum directed inward the

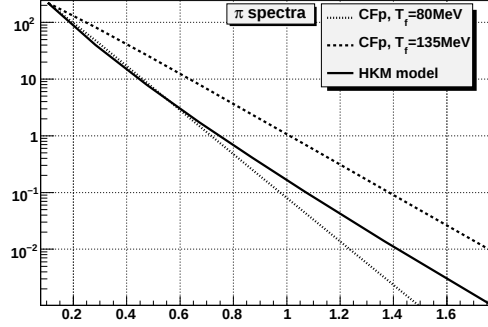


Fig. 2. Transverse momentum spectrum of π^- in HKM, compared with the sudden freeze-out ones at temperatures of 80 and 160 MeV with arbitrary normalizations.

system. It is clear that the emission function at these points is close to zero for such particles. Even formally, in the Gaussian approximation (16) for Q , validated in the region of its maximal value, the integral $\int_{t_\sigma}^{\infty} ds R(s, \mathbf{r} + \frac{\mathbf{p}}{p^0}(s - t_\sigma(\mathbf{r}, p), p) \gg 1$, if particle world line crosses almost the whole system. The latter results in $Q \rightarrow 0$ and, therefore, completely destroys the saddle-point approximation (12) for Q and then the Cooper–Frye form (18) for spectra. Recall that if a particle crosses some non-space-like part of the hypersurface σ moving inward the system, this corresponds to the condition $p^\mu d\sigma_\mu < 0$ [3]. Hence the value $p^\mu d\sigma_\mu(p)$ in the generalized Cooper–Frye formula (18) should be *always* positive: $p^\mu d\sigma_\mu(p) > 0$ across the hypersurface where fairly sharp maximum of emission of particles with momentum $\mathbf{p}$ is situated; and so requirement $p^\mu d\sigma_\mu(p) > 0$ is a necessary condition for $t_\sigma(\mathbf{r}, p)$ to be a true hypersurface of the maximal emission. It means that hypersurfaces of maximal emission for a given momentum $\mathbf{p}$ may be open in the space-time, not enclosing the high-density matter at the initial time t_0 , and different for different $\mathbf{p}$. All this is illustrated in Fig. 3, where the structure of particle emission domain is shown for two groups of particles. In the first one, the momentum is directed as the radius vector to the point of particle localization (they move outward the system), in the second one — in opposite direction (they move inward). The points of maximum for different p_T , where Cooper–Frye form can be applied, do not overlap. The calculations have been done in HKM [8].

Therefore, there are no negative contributions to the particle momentum density from non-space-like sectors of the freeze-out hypersurface, that is a well known shortcoming of the Cooper–Frye prescription [3, 4]; the negative contributions could appear only as a result of utilization of improper freeze-

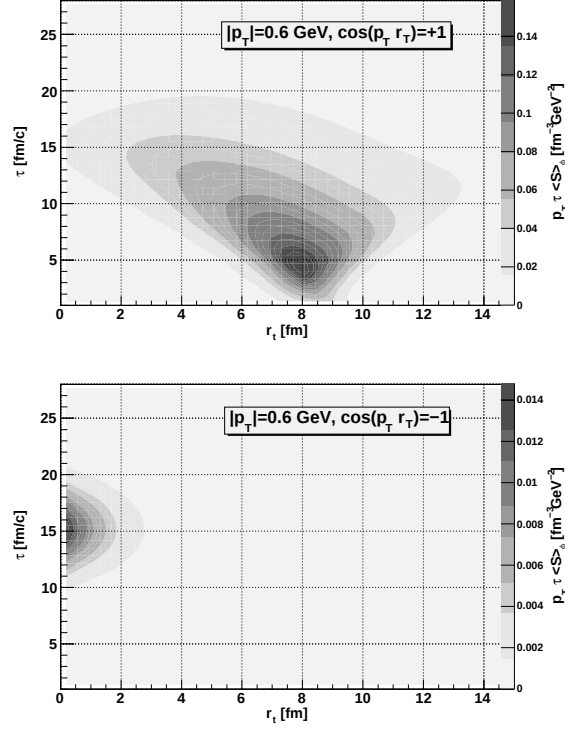


Fig. 3. The emission function in HKM for particles with momentum directed along the radius vector at the emission points (left) and for those ones in the opposite direction to the radius vector (right).

out hypersurface that roughly ignores its momentum dependence and so is common for all $\mathbf{p}$. If, anyhow, such a common hypersurface will be used, *e.g.* as the hypersurface of the maximal particle number emission (integrated over $\mathbf{p}$), there is no possibility to justify the approximate expression for momentum spectra similar to Eq. (18).

5. Conclusions

Our analysis and numerical calculations show that the widely used phenomenological Landau/Cooper–Frye prescription for calculation of pion (or other particle) spectrum is too rough if the freeze-out hypersurface is considered as common for all momenta of pions. The Cooper–Frye formula, however, could be applied in generalized form accounting for direct momentum dependence of the freeze-out hypersurface $\sigma(p)$; the latter corresponds to the maximum of emission function $S(t_\sigma(\mathbf{r}, p), \mathbf{r}, p)$ at fixed momentum

$\mathbf{p}$ in an appropriate region of $\mathbf{r}$. If such a hypersurface $\sigma(p)$ is found, the condition of applicability of the Cooper–Frye formula *for given* $\mathbf{p}$ is that the width of the maximum, which in the simple cases — *e.g.*, for one component system or at domination of elastic scatterings — is just the relaxation time (inverse of collision rate), should be smaller than the corresponding temporal homogeneity length of the distribution function.

This work was supported in part by FAPESP (Brazil), under the contract numbers 2004/10619-9 and 2008/55658-2; the Ukrainian-French grant (programme DNIPRO), Agreement with MESU No. M/4-2009; the Bilateral award DLR (Germany) — MESU (Ukraine) for the UKR 06/008 Project, Agreement No. M/26-2008. The research was carried out within the scope of the ERG — European Research Group (GDRE): Heavy ions at ultrarelativistic energies.

REFERENCES

- [1] L.D. Landau, *Izv. Akad. Nauk SSSR, Ser. Fiz.* **17**, 51 (1953).
- [2] F. Cooper, G. Frye, *Phys. Rev.* **D10**, 186 (1974).
- [3] Yu.M. Sinyukov, *Z. Phys.* **C43**, 401 (1989).
- [4] K.A. Bugaev, *Nucl. Phys.* **A606**, 559 (1996).
- [5] Yu.M. Sinyukov, S.V. Akkelin, Y. Hama, *Phys. Rev. Lett.* **89**, 052301 (2002).
- [6] F. Grassi, Y. Hama, T. Kodama, *Phys. Lett.* **B355**, 9 (1995); *Z. Phys.* **C73**, 153 (1996).
- [7] S.A. Bass, A. Dumitru, *Phys. Rev.* **C61**, 064909 (2000); D. Teaney, J. Lauret, E.V. Shuryak, *Phys. Rev. Lett.* **86**, 4783 (2001); [arXiv:nucl-th/0110037](#); T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, *Phys. Lett.* **B636**, 299 (2006); C. Nonaka, S.A. Bass, *Phys. Rev.* **C75**, 014902 (2007).
- [8] S.V. Akkelin, Y. Hama, Iu.A. Karpenko, Yu.M. Sinyukov, *Phys. Rev.* **C78**, 034906 (2008).
- [9] S.V. Akkelin, M.S. Borysova, Yu.M. Sinyukov, *Acta Phys. Hung. A: Heavy Ion Physics* **22**, 165 (2005).
- [10] J. Bondorf, S. Garpman, J. Zimanyi, *Nucl. Phys.* **A296**, 320 (1978); K.S. Lee, U. Heinz, E. Schnedermann, *Z. Phys.* **C48**, 525 (1990); F.S. Navarra, M.C. Nemes, U. Ornik, S. Paiva, *Phys. Rev.* **C45**, R2552 (1992); E. Schnedermann, U. Heinz, *Phys. Rev.* **C47**, 1738 (1993).