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RELATIVISTIC FACTORIZED S-MATRIX IN TWO DIMENSIONS HAVING O(N) ISOTOPIC SYMMETRY

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RELATIVISTIC FACTORIZED S-MATRIX IN TWO DIMENSIONS HAVING O(N) ISOTOPIC SYMMETRY

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Релятивистская факторизованная S-матрица в двумерном пространстве-времени с изотопической симметрией O(N)

Построена самосогласованная полная S-матрица с изотопической симметрией O(N) в двумерном пространстве-времени. S-матрица факторизована и удовлетворяет требованиям аналитичности и унитарности. Приведены аргументы в пользу того, что эта S-матрица описывает рассеяние частиц в двумерной киральной O(N) модели.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Zamolodchikov A.B., Zamolodchikov Al.B. **E2 - 10857**

Relativistic Factorized S-Matrix in Two Dimensions Having O(N) Isotopic Symmetry

The factorized total S-matrix in two space-time dimensions with the isotopic O(N) symmetry is constructed. The arguments are presented that this S-matrix is the exact one of the O(N)-chiral field.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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1. INTRODUCTION

The recent progress in the study of two-dimensional quantum field theory has led to the extensive development of some models which have a remarkable property: an infinite set of conservation laws, leading to the absence of the multiple production and conservation of the set of individual momenta of particles in the scattering /1,2/. The factorization of the total S-matrix also seems to be the effect of these conservation laws /3/. The classical analog of all these models is connected with nonlinear equations completely integrable by the inverse scattering method.

The example of this type is the massive Thirring model (MTM), or, equivalently, the quantum sine-Gordon model. It turns out that, due to the simplified scattering properties of this model, all the elements of the total S-matrix /4,5,6/ and some off-shell matrix elements /7/ can be found explicitly.

In the recent paper Karowski, Thun, Truong and Weisz /8/ showed that the analyticity, unitarity and factorization equations /5,6/ of this model can be solved uniquely giving a one-parameter set of solutions, the parameter can be connected with the MTM coupling constant.

Being the model of charged fermions, MTM has the phase symmetry $\mathcal{U}(1)=O(2)$. In the present paper the factorized S-matrix with isotopic O(N) symmetry is constructed for any $N \geqslant 3$. We adopt the existence of an isovector N-plet of particles of the mass m and require the O(N)-isosymmetry of the S-matrix elements. It turns out that under these requirements the S-matrix can be determined uniquely*), without parameters, except the overall mass scale. The latter is shown in Secs.2 and 3, where we derive the explicit form of the S-matrix.

Up to the time we cannot definitely answer what two-dimensional field theory (if any) leads to this S-matrix. We have some arguments, however, that such a theory is a O(N) ($N \geqslant 3$) chiral field model described by the Lagrangian density:

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*) In this case, as well as in the MTM, the unitarity, analyticity and factorization conditions admit, of course, the arbitraryness of the CDD -type, so here we mean the uniqueness of the

$$\mathcal{L} = \frac{1}{2g_0} \sum_{i=1}^{N} (\partial_\mu n_i)^2 \qquad (1.1)$$

and the constraint

$$\sum_{i=1}^{N} n_i^2 = 1 \ . \qquad (1.2)$$

This model is O(N) symmetric, renormalizable and asymptotically free /10,11/. Infrared charge singularity in this model seems to lead to the desintegration of the goldstone vacuum and to the mass transmutation of particles /12/, which should form the O(N) multiplets in this case.

In the asymptotically free theories with the spontaneous mass transmutation the observable characteristics do not depend on the coupling constant (due to the renormalizability) /13/. We should like to mention in this connection that the S-matrix obtained in Sec.3 does not depend on free parameters.

The S-matrix obtained depends analytically on N and can be expanded in powers of 1/N. Thus,our hypothesis concerning its connection with the model (1.1) is based on the comparison of this S-matrix with the 1/N - perturbation theory results of (1.1). In Sec.4 we show that in 1/N - perturbations of (1.1) there is no particle production and the S-matrix really factorizes in the order of $1/N^2$. The two-particle matrix elements, calculated in the order 1/N do coincide with the corresponding term of the 1/N expansion of the S-matrix obtained in Sec.3.

The comparison of the ultraviolet asymptotics of the S - matrix of Sec.3 with the results of the ordinary g - perturbations of the model (1.1) is another possible check. Although in such perturbation theory one deals with N-1 - component multiplet of goldstone particles instead of the N - component multiplet of the massive particles and,hence, faces the infrared divergencies, one may suppose that the contribution of ultraviolet logarithms

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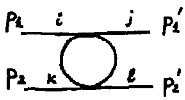
"minimum" solution, i.e., the solution with the minimum set of singularities (see Sec.3).

of the perturbation theory into the scattering amplitudes gives the correct asymptotics of this amplitudes (at least up to g^2). The comparison with the perturbation theory is performed till g^2 in Sec.4. The result also confirms our hypothesis.

2. ANALYTISITY, UNITARITY AND FACTORIZATION EQUATIONS FOR THE O(N) - SYMMETRIC S-MATRIX

Consider the O(N) isovector N-plet of particles of the mass m. The S-matrix element of the 2→2 scattering can be taken in the form

$${}_{ik}S_{j\ell} =$$



$$= \delta(p_1-p_1')\delta(p_2-p_2')\left[\delta_{ik}\delta_{j\ell}\,\sigma_1(s) + \delta_{ij}\delta_{k\ell}\,\sigma_2(s) + \delta_{i\ell}\delta_{jk}\,\sigma_3(s)\right], \qquad (2.1)$$

where $s=(p_1+p_2)^2$. Further it will be convenient to use the rapidities θ_a instead of the momenta p_a^μ:

$$p_a^0 = m\,\mathrm{ch}\,\theta_a \quad ; \quad p_a^1 = m\,\mathrm{sh}\,\theta_a . \qquad (2.2)$$

Then σ_1, σ_2 and σ_3 will be the functions of the rapidity differences of the initial particles $\theta=|\theta_1-\theta_2|$, which is simply connected with s:

$$s = 2m^2(1+\mathrm{ch}\,\theta) . \qquad (2.2a)$$

Note that under the transformation (2.2a) the threshold points $s=0$ and $s=4m^2$ of the functions $\sigma(s)$ (which are the square-root branching points due to the two-particle unitarity) become the nonbranching points of $\sigma(\theta)$. So σ_1, σ_2 and σ_3 are the meromorphic functions θ.

The two-particle unitarity conditions and the crossing-symmetry relations of the two-particle S-matrix (2.1) can be represented as the functional equations

$$\sigma_2(\theta)\sigma_2(-\theta) + \sigma_3(\theta)\sigma_3(-\theta) = 1 \qquad (2.3a)$$

$$\sigma_2(\theta)\sigma_3(-\theta) + \sigma_2(-\theta)\sigma_3(\theta) = 0 \qquad (2.3b)$$

$$[N\sigma_1(\theta)+\sigma_2(\theta)+\sigma_3(\theta)][N\sigma_1(-\theta)+\sigma_2(-\theta)+\sigma_3(-\theta)] = 1 \tag{2.3c}$$

and

$$\sigma_2(\theta) = \sigma_2(i\pi-\theta) \tag{2.4a}$$

$$\sigma_3(\theta) = \sigma_1(i\pi-\theta)\,. \tag{2.4b}$$

The equations (2.4) and (2.3) do not determine the functions $\sigma(\theta)$. In addition to unitarity and analyticity let us require the factorization of the multiparticle S-matrix.

The factorization means the special structure of the multiparticle S-matrix: the multiparticle S-matrix elements are the sums of terms, each being the product of the two-particle S-matrix elements, as if the multiparticle scattering would be the consequence of two-particle collisions /14,15,5,6/.

The factorized S-matrix can be represented by the simple algebraic construction /5/, which in our case consists of N types of special noncommutative symbols $A_i(\theta)$; $i = 1,2,\ldots\ldots\ldots N$, each symbol corresponding to certain component of the isovector multiplet. The asymptotic states of the scattering theory should be identified with the products of this symbols, each symbol $A_i(\theta_a)$ corresponding to the particle with rapidity θ_a in the state. We identify the in(out)-states with the products in which all symbols are arranged in the order of decreasing (increasing) θ. Any in-state can be reordered in terms of out-states by means of the commutation rules

$$A_i(\theta_1)A_j(\theta_2) = \delta_{ij}\,\sigma_1(\theta_{12})\sum_{k=1}^{N} A_k(\theta_2)A_k(\theta_1) + \tag{2.5}$$

$$+\,\sigma_2(\theta_{12})\,A_j(\theta_2)A_i(\theta_1)+\sigma_3(\theta_{12})\,A_i(\theta_2)A_j(\theta_1)\,,\quad \theta_{12}=\theta_1-\theta_2$$

which correspond to the two-particle S-matrix (2.1). The algebra (2.5) represents the factorized total S-matrix.

The Jacoby identities of algebra (2.5) give us the functional equations for σ_1 , σ_2 and σ_3 . The factorization property forces these identities necessarily, so we shall refer to them as the factorization equations.

The factorization equations have the simple meaning. Consider, for example, the collision of three particles with rapidities $\theta_1 > \theta_2 > \theta_3$. In the infinite past they have spatial coordinates $x_1 < x_2 < x_3$ .The particles collide with each other subsequently in the interaction region, the succession of the collisions depending on the initial positions of particles, as is shown in Fig. 1a),b).

In quantum mechanics both these possibilities give two parts of the same outgoing wave. The conservation of the set of momenta implies the monochromacy of this wave, hence, the outgoing waves of processes in Fig.1 a), b) should be coherent. The Jacoby identities of the algebra (2.5) ensure this coherency.

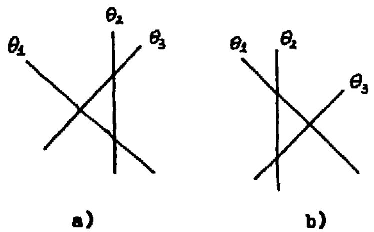


Fig.1.

One obtains the factorization equations rearranging the product of three symbols $A_i(\theta_1)A_j(\theta_2)A_k(\theta_3)$ in two possible successions and requiring the results to be equal. The number and the form of identities turns out to be different for the cases N=2 and N $\geqslant$ 3. For N=2 the factorization equations are given in /5,6,8/ and their solution is the sine-Gordon S-matrix. For the case N $\geqslant$ 3 they acquire the form:

$$\sigma_2\sigma_3\sigma_3 + \sigma_3\sigma_3\sigma_2 = \sigma_3\sigma_2\sigma_3 \qquad (2.6a)$$

$$\sigma_2\sigma_1\sigma_1 + \sigma_3\sigma_2\sigma_1 = \sigma_3\sigma_1\sigma_2 \qquad (2.6b)$$

$$N\sigma_1\sigma_3\sigma_1 + \sigma_1\sigma_3\sigma_2 + \sigma_1\sigma_3\sigma_3 + \sigma_1\sigma_2\sigma_1 + \sigma_2\sigma_3\sigma_1 + \qquad (2.6c)$$
$$+ \sigma_3\sigma_3\sigma_1 + \sigma_1\sigma_1\sigma_1 = \sigma_3\sigma_1\sigma_3 ,$$

where the first, second and third σ in each term are functions of θ , $\theta+\theta'$ and θ' , respectively.

3. SOLUTION OF THE UNITARITY, ANALYTICITY AND FACTORIZATION EQUATIONS

In terms of the ratio $h(\theta)=\dfrac{\sigma_2(\theta)}{\sigma_3(\theta)}$ equation (2.6a) takes the form:

$$h(\theta)+h(\theta')=h(\theta+\theta') , \tag{3.1}$$

i.e.,

$$\sigma_3(\theta)=\frac{-i\lambda}{\theta}\,\sigma_2(\theta) , \tag{3.2}$$

where λ is a certain parameter. Crossing equations (2.4) lead to

$$\sigma_1(\theta)=\frac{-i\lambda}{i\pi-\theta}\,\sigma_2(\theta) . \tag{3.3}$$

Note that (3.2) and (3.3) satisfy equations (2.3b) and (2.6b) identically. It is notable also that after substitution (3.2) and (3.3) equations (2.3c) and (2.6c) lead to the same algebraic equation for the parameter λ, which has, except trivial $\lambda = 0$, the unique solution

$$\lambda=\frac{2\pi}{N-2} . \tag{3.4}$$

The rest equation (2.3a) acquires the form:

$$\sigma_2(\theta)\,\sigma_2(-\theta)=\frac{\theta^2}{\theta^2+\lambda^2} . \tag{3.5}$$

Eqs. (3.5) and (2.4a) form the system for $\sigma_2(\theta)$.

It is clear that these equations permit σ_2 to be multiplied by any $2\pi i$ - periodic meromorphic function which is real on the imaginary axis and satisfies identities

$$\begin{aligned} f(\theta)\,f(-\theta)&=1\\ f(\theta)&=f(i\pi-\theta) . \end{aligned} \tag{3.6}$$

Therefore, the general solution having singularities on the imaginary axis only has the form:

$$\sigma_2(\theta) = \left[\prod_{k=1}^{L} \frac{Sh\,\theta + i\,Sin\,\alpha_k}{Sh\,\theta - i\,Sin\,\alpha_k} \right] \sigma_2^{(0)}(\theta) \, , \tag{3.7}$$

where α_k are real numbers and $\sigma_2^{(0)}$ is the "minimum" solution of (3.5) and (2.4a), i.e., the solution with the minimum set of singularities in the θ plane:

$$\sigma_2^{(0)}(\theta) = Q(\theta)\,Q(i\pi-\theta) \, , \tag{3.8}$$

where

$$Q(\theta) = \frac{\Gamma(\Delta - i\frac{\theta}{2\pi})\,\Gamma(1/2 - i\frac{\theta}{2\pi})}{\Gamma(-i\frac{\theta}{2\pi})\,\Gamma(1/2 + \Delta - i\frac{\theta}{2\pi})} \tag{3.9}$$

and

$$\Delta = \frac{\lambda}{2\pi} = \frac{1}{N-2} \, . \tag{3.10}$$

In principle, all the solutions (3.7) are permitted. However, the solution $\sigma_2 = \sigma_2^{(0)}$ is the only one, which does not lead to the isospin degeneracy of the spectrum *). This solution does not display any poles on the physical sheet of the S-plane, i.e., isovector particles cannot produce any bound states.

Note that in the case N=3, i.e., Δ =1 expression (3.8) is reduced to

$$\sigma_2^{(0)}(\theta) = \frac{\theta(i\pi-\theta)}{(2\pi i-\theta)(i\pi+\theta)} \, , \quad N=3 \, . \tag{3.11}$$

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*) The other remarkable solution contains single CDD pole $\alpha_1 = 2\pi\Delta$ Contrary to $\sigma_2^{(0)}$, this solution corresponds to the attractive interaction and seems to be the exact S-matrix of the fundamental fermions of the Gross-Neveu model /13,16/. The arguments will be published elsewhere.

4. THE COMPARISON OF THE FACTORIZED S-MATRIX WITH THE 1/N - EXPANSION OF THE MODEL (1.1)

It is convenient to develop the 1/N expansion of the model (1.1) in the following way /17/. The generating functional for the Green functions of the $n_i(x)$ field can be written in the form:

$$Z[J_i] = I[J_i]/I[0],$$

$$I[J_i] = \int \prod_x d\omega \prod_i dn_i \exp\left\{ i \int d^2x \left[\mathcal{L}'[n_i, \omega] + J_i(x) n_i(x) \right] \right\}, \tag{4.1}$$

where

$$\mathcal{L}'[n_i, \omega] = \frac{1}{2g_0} \left[(\partial_\mu n_i)^2 - \omega n_i^2 \right] + \frac{\omega(x)}{2g_0} . \tag{4.2}$$

The n_i - integration in (4.1) can be performed explicitly and leads to $Z[J_i] = I'[J_i]/I'[0]$,

$$I'[J_i] = \int \prod_x d\omega \exp\left\{ i S_{eff}[\omega] + \right. \tag{4.3}$$

$$\left. + i \int J_i(x) J_i(x') G(x, x'|\omega) d^2x d^2x' \right\},$$

where

$$S_{eff}[\omega] = i \frac{N}{2} \operatorname{tr} \ln\left(\partial_\mu^2 - \omega(x) \right) + \int \frac{\omega(x)}{2g_0} d^2x , \tag{4.4}$$

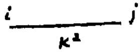
and $G(x, x'|\omega)$ - the Green function of the operator $\partial_\mu^2 - \omega(x)$. The perturbative calculation of the integral (4.3) leads to the 1/N expansion of the model (1.1). The stationary phase point of the integral (4.3) $\omega(x) = m^2 = \Lambda^2 \exp(-\frac{4\pi}{Ng_0})$ should be taken into account, so functionals S_{eff} and $G(x, x'|\omega)$ should be expanded in $\omega' = \omega - m^2$ rather than in ω.

It is convenient to use in calculations the following diagram technique which contains:

1) the ω' field propagator

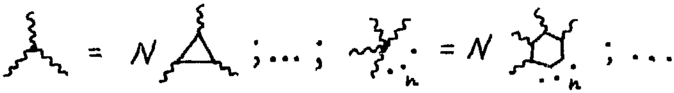
$$\mathcal{D}(k^2) = \overset{k}{\sim\!\sim\!\sim\!\sim} = \frac{i}{N\phi(k^2)} , \tag{4.5}$$

2) the n_i propagator

$$G_{ij}(k^2) = \quad$$  $$= \frac{i\,\delta_{ij}}{k^2 - m^2 + i\varepsilon} \,, \qquad (4.6)$$

3) and vertices

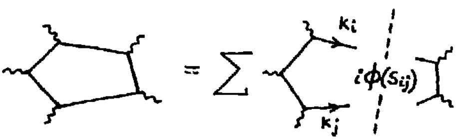
$$= i\,\delta_{ij} \;; \qquad (4.7)$$



In this technique the closed loops on n_i - field lines should not be drawn (they are already taken into account in (4.7)). In (4.5)

$$i\,\phi(k^2) = \frac{1}{(2\pi)^2}\int \frac{d^2p}{(p^2-m^2+i\varepsilon)((p+k)^2-m^2+i\varepsilon)} \,. \qquad (4.8)$$

The calculation of loops in (4.7) can be made explicitly by means of the following "cutting rule" /18/[*]. The arbitrary loop is the sum of terms, each corresponding to any division of the loop through two lines.



$$; \qquad (4.9)$$

$$s_{ij} = (k_i + k_j)^2$$

The momenta k_i and k_j are determined by the condition $k_i^2 = k_j^2 = m^2$. The contribution of each division is equal to the production of two trees in both sides of dashed line in (4.9) by the function $i\,\phi(s_{ij})$. At s_{ij} fixed the equations $k_i^2 = k_j^2 = m^2$ have two solutions ($k_i \leftrightarrow k_j$), both should be taken into account in (4.9).

Consider the $2 \to 4$ amplitude (Fig.2) in the order of $1/N^2$.

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*) An analogous result for the arbitrary fermion loop has been obtained in /19/.

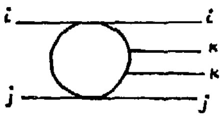


Fig.2.

For the sake of simplicity we shall concentrate on the case $i \neq j \neq k$. This amplitude is given by the sum of diagrams in Fig.3. Using (4.9) one can replace the diagram in Fig.3g) by the sum of the loop divisions.

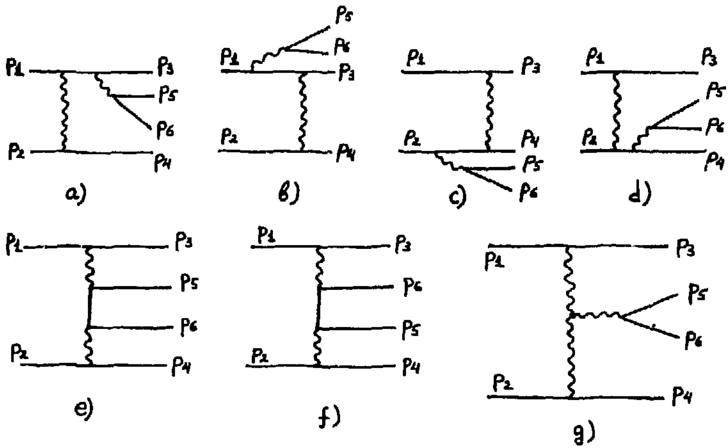


Fig.3.

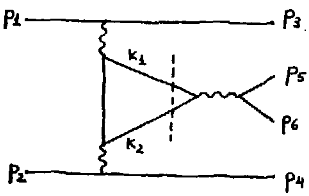


Fig.4.

Consider, for example, the division in Fig.4. Two solutions of $k_1^2 = k_2^2 = m^2$ are $k_1 = p_5$; $k_2 = p_6$ and $k_1 = p_6$, $k_2 = p_5$. The factor $i\phi(S_{56})$ in this division is the reciprocal wavy line with an opposite sign. Therefore the division in Fig.4 cancells out diagrams in Fig.3 e),f). It is easy to check up that other possible divisions of the triangle in Fig.4 cancell out diagrams in Fig.3 a),b),c),d).

The cases $i = j$, $j = k$ and so on contain more diagrams, however, one can check up that the same cancellation takes place in all these cases too.

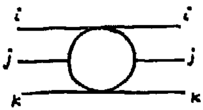


Fig.5.

Now let us turn to the process $3 \to 3$ (Fig.5) and consider again the case $i \neq j \neq k$. In the order of $1/N$ the matrix element contains disconnected diagrams only, the kinematics ensuring the conservation of the momenta set. In the order of $1/N^2$ we have 7 connected diagrams listed in Fig. 6.

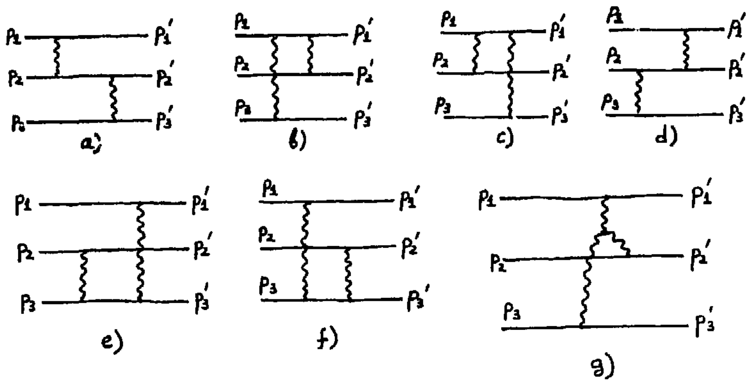
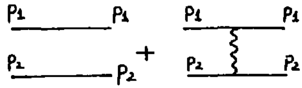


Fig.6.

It can be easily checked up, that if all the intermediate propagators in diagrams in Figs. 6 a-f) are nonsingular, different divisions of the diagram in Fig.6g) cancell the other diagrams in the same manner, as in the previous example. Mass-shell singularities of diagrams in Fig.6 a-f) require more detailed analyses. For example, if $p_1' \to p_3$; $p_2' \to p_1$; $p_3' \to p_2$ diagrams in Figs.6c), 6d) and 6f) get into mass-shell poles. It can be shown, however, that the principal parts of these three diagrams cancell each other, and one remains with some regular function and terms with mass-shell δ - functions. The diagram 6g) cannot cancell the latter, being nonsingular in this region (all the momenta transfered are space-like). Finally we are left with δ - function terms only, the δ - functions ensuring the factorised structure of the S-matrix element in Fig.5.

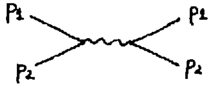
Using the technique (4.5),(4.6),(4.7) one can calculate two particle S-matrix elements. In the order 1/N they are

$\sigma_2(\theta) =$ 

$$= 1 - \frac{2\pi i}{N \,\mathrm{Sh}\,\theta} \,, \tag{4.10a}$$

$\sigma_3(\theta) =$ 

$$= - \frac{2\pi i}{N\theta} \,, \tag{4.10b}$$

$\sigma_1(\theta) =$ 

$$= - \frac{2\pi i}{N(i\pi - \theta)} \,. \tag{4.10c}$$

Expressions (4.10a-c) do coincide with the first terms in 1/N expansion of (3.8),(3.2) and (3.3).

Another possible expansion check of the S-matrix obtained is worth mentioning. Adopting the S-matrix (3.8), (3.2) and (3.3) to correspond to some renormalizable and asymptotically free field theory, one can expand matrix elements which are the functions of

$$\ln \frac{s}{m^2} = \ln \frac{s}{\mu^2} + \int^{g(\mu)} \frac{dg}{\beta(g)} \tag{4.11}$$

in the asymptotic series in powers of $g(\mu)$. Taking the first term /10/ of the Gell-Mann-Low function of the model (1.1)

$$\beta(g) = - \frac{N-2}{4\pi} g^2 + O(g^3) \tag{4.12}$$

one gets up to g^2 ($g \equiv g(\mu)$)

$$\begin{aligned} \sigma_2(s) &= 1 - i\, g^2/8 + O(g^3) \,, \\ \sigma_3(s) &= -i\, g/2 + i\, \frac{N-2}{8\pi} g^2 \ln \frac{s}{\mu^2} + O(g^3) \,, \end{aligned} \tag{4.13}$$

$$\sigma_1(s) = i\,\frac{g}{2} - i\,\frac{N-2}{8\pi}\,g^2 \ln\frac{s}{\mu^2} - \frac{N-2}{8}\,g^2 + O(g^3).$$

In Eqs.(4.13) the asymptotics $S \to \infty$ is written down and the power terms in S are dropped.

The usual g - perturbations of (1.1) are based on the goldstone vacuum and therefore lead in two-dimensions to infrared divergencies. However, one can obtain the asymptotics of the scattering amplitudes, calculating the ultraviolet logarithms of the scattering amplitudes of goldstone particles (to circumvent the infrared difficulties one can impure formally the mass of the goldstone particle). Calculations are straightforward and the result coincides with (4.13).

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