Quark Elastic Scattering in Gauge Theories and Large Transverse Momentum Hadron Production

DENNIS W. DUKE

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

ABSTRACT

Quark elastic scattering in a color gauge theory of quarks and gluons is investigated as a source of large transverse momentum hadrons. Nonscaling effects due to asymptotic freedom alone are not helpful in describing the data. Elastic quark-quark form factors derived in color gauge theories do describe the data if supplemented by a phenomenological assumption which probably amounts to anti-screening of the color of quarks in a proton. Transverse momentum fluctuations of quarks in protons are quite important to understanding the normalization and shape of the theory. Patterns of scaling violations are predicted which may be compared with good future data to test the models.

I. INTRODUCTION

The production of particles at large values of transverse momentum

p_T in hadronic reactions is commonly believed to occur through the

hard scattering of constitutents within the incident hadrons. If these

constitutents are point-like quarks, then simple arguments suggest that

the invariant single particle cross section $E d^3\sigma/d^3p$ should scale as

p_T^{-4}.[1] However, at presently attainable energies the experimental data

seem to scale roughly as p_T^{-8}.[2] This p_T scaling anomaly can be understood

in the constituent-interchange-model[3] (CIM) as the scattering of

non-elementary constitutents, such as mesons in a proton. The meson

form factors then change the expected p_T^{-4} to the observed p_T^{-8}.

Recently Field and Feynman and others have formulated a model

for large p_T production that is based on quark-quark scattering and yet

avoids the p_T^{-8} anomaly by the introduction of an empirical representation

for the quark elastic scattering cross section.[4] With the use of this one

empirical function and with quark and hadron structure functions determined

essentially from lepton scattering data, the model is found to successfully

describe existing data on the large p_T production of single particles,

jets, and two-particle correlations on the same and opposite sides of the

trigger hadron.[5] Indeed, since the Field-Feynman model is so remarkably

efficient and accurate in describing the data, that model will be used as

a benchmark to be compared with the models in this paper.

If the source of large p_T particles is quark elastic scattering, then the theoretical origin of the empirical cross section used by Field-Feynman is a very interesting question. Two main approaches toward this question have been taken in the past: (1) The CIM or some variant model is correctly describing large p_T production, and the quark elastic scattering contribution is either temporarily or permanently suppressed for reasons yet to be determined. Possible reasons for this have been investigated by Cahalan, Geer, Kogut, and Susskind[6] and more recently by Cutler and Sivers[7] within the framework of asymptotically free field theories. These investigations differ substantially in their conclusions and in fact both groups give inadequate treatments of several important points. Therefore, in section III I have discussed this approach in rather careful detail, based on considerations from quantum chromodynamics (QCD) for colored quarks and gluons. Unfortunately, this whole approach ignores the question of how to interpret the success of the Field-Feynman model, and the fact that the CIM has yet to be formulated in a manner that accounts for the double-jet structure believed to have been observed in recent experiments.[2,5]

(2) Alternatively, one may insist that quark elastic scattering is the correct approach, but that various scale-breaking effects as seen, for example, in deep-inelastic electroproduction are changing p_T^{-4} to p_T^{-8}. If all the scale breaking effects are coming from the decay of the final state quarks to hadrons,[8] for example, the single jet cross section would reveal the canonical p_T^{-4} (indeed, this possiblity is still open experimentally).

Other analyses have concluded that large p_T production can be understood if the scale-breaking in the proton structure functions is parametrized with power-law quark form factors[9] or with logarithmic modifications[10] different from those predicted by asymptotic freedom. Unfortunately, the large p_T kinematic regime is largely outside the regions where the form factors in both cases were fit to electroproduction data, and therefore both analyses lack theoretical motivation. Attempts to use the logarithmic scale-breaking corrections predicted by asymptotic freedom[11] do not permit an understanding of the data (see section III).

In section IV, results for the quark elastic scattering amplitude calculated within the framework of QCD by Cornwall and Tiktopoulos[12] are used to calculate large p_T particle production. Within the freedom of a certain phenomenological assumption which probably amounts to anti-screening of the color of quarks in a proton, this model is able to mimic the Field-Feynman results rather closely. It also turns out that the effect of transverse momentum of the quarks in a proton is very significant.

The results of section IV are not theoretically firm and certainly fall short of any hoped for "justification" of the Field-Feynman approach. It is hoped, however, that these results have enough phenomenological significance to excite better theoretical consideration of the questions raised. In fact, as discussed in section V, this model does predict a certain pattern of scale-breaking effects which could possibly be detected in a careful data analysis.

II. KINEMATICS OF HIGH p_T SCATTERING

Treatments of the kinematics relevant to large p_T particle production are widely available,[13] so only a summary of the required formulas and notation will be given here. The invariant cross section for producing a hadron C at large p_T with momentum $\vec{p}$ in the reaction $A + B \rightarrow C + X$ is (neglecting transverse momenta)

$$E\,d^3\sigma/d^3p \;=\; \int^1 dx_a \int^1 dx_b P_{a/A}(x_a) P_{b/B}(x_b) D_{C/c}(z_C) \;\times$$

$$\frac{1}{\pi\, z_C}\, d\sigma/d\hat{t}\,(\hat{s},\,\hat{t},\,\hat{u};\, ab \rightarrow cd) \qquad . \qquad (1)$$

The process is illustrated in Fig. 1. In Eq. (1) the probability functions $P(x)$ for quarks in hadrons and $D(z)$ for hadrons in quarks[14] are taken from Field-Feynman, where they were deduced from lepton scattering data. The longitudinal fractions $x_a = p_a/p_A$, $x_b = p_b/p_B$, and $z_C = p_C/p_c$ determine the $ab \rightarrow cd$ sub-reaction kinematics through the relations $\hat{s} = x_a x_b s$, $\hat{t} = x_a t/z_C$, and $\hat{u} = x_b u/z_C$, where $s = (p_A + p_B)^2$, $t = (p_A - p_C)^2$, $u = (p_B - p_C)^2$. The conditions $z_C \leq 1$ and $\hat{s} + \hat{t} + \hat{u} = 0$ fix the lower limits of integration at $x_a^{min} = x_1/(1 - x_2)$ and $x_b^{min} = x_a x_2/(x_a - x_1)$, where $x_1 = -u/s$ and $x_2 = -t/s$. The possibility that the hadron C results from the fragmentation of the quark d is taken into account by carefully pairing the functions $P(x)$ and $D(z)$ with $d\sigma/d\hat{t}$ and $d\sigma/d\hat{u}$.[15]

Dimensional analysis and the absence of mass parameters to set a scale in Eq. (1) yields that if

$$\frac{d\sigma}{d\hat{t}} \sim \hat{s}^{-\frac{N}{2}} g\left(\frac{\hat{t}}{\hat{s}}\right) \quad ,$$

then

$$E\frac{d^3\sigma}{d^3p} \sim p_T^{-N} f(x_T, \theta_{CM})$$

where $x_T = 2p_T/\sqrt{s}$, θ_{CM} is the scattering angle of hadron C in the A + B center of momentum, and typically

$$f(x_T, \theta_{CM}) \sim (1 - x_T)^F$$

for $x_T \sim 1$ and $\theta_{CM} \sim 90^\circ$. Usually the numbers N and F are used to characterize the cross section even for $x_T < 1$.[16]

The inclusion of the effects of the transverse momentum k_T of quarks in hadrons (and to less extent, hadrons in quarks) in Eq. (1) is not trivial, but it is necessary to include these effects if any pretense of absolute normalization of the cross section is to be maintained.   For calculational purposes, two approaches are used: (a) the integrals in Eq. (1) are evaluated independent of k_T, which is fixed at some average value $<k_T>$ with the kinematical variables appropriately modified; (b) a Monte Carlo program[17] is used to perform additional k_T integrals given some model for the k_T dependence in $P(x, k_T)$ and $D(z, k_T)$. Generally, the Monte Carlo integration will give the correct answer if properly used. However, the $<k_T>$ method is relatively easy to use and also gives reasonable results, and in any event, the theoretical uncertainty

in the k_T effects (see Section IV) far exceeds the calculational uncertainty of either method. For these reasons, the $<k_T>$ method is used in this paper. The two methods have been compared and generally agree to within about (20 - 50)%.

In the $<k_T>$ method, Eq. (1) is calculated using

$$\hat{s} = x_a x_b s - 2<k_T^2> \quad ,$$

$$\hat{t} = \frac{x_a t}{z_C} + \sqrt{2}<k_T>p_T \quad ,$$

$$\hat{u} = \frac{x_b u}{z_C} + \sqrt{2}<k_T>p_T \quad ,$$

and

$$x_a^{min} = \frac{x_1 + T}{1 - x_2} \quad ,$$

$$x_b^{min} = \frac{x_a x_2 + T}{x_a - x_1}$$

where $<k_T>$ is the average transverse momentum of quarks in hadrons, and

$$T = \left(2<k_T^2> - 2\sqrt{2}<k_T>p_T\right)/s \quad .$$

If the single particle C produced at large p_T in $A + B \rightarrow C + X$ is the product of a jet, one calculates the total jet cross section from Eq. (1) by setting $z = 1$ and $D(z) = 1$, obtaining

$$E\, d^3\sigma/d^3p(\text{jet}) \;=\; \int^1 dx_a \int^1 dx_b\, P(x_a) P(x_b) \frac{1}{\pi z_C} \frac{d\sigma}{d\hat{t}}\, \delta(1 - z_C)$$

(2)

$$=\; \int^1 dx_a\, x_a\, P(x_a) x_b\, P(x_b) \frac{1}{\pi(x_a - x_1)} \frac{d\sigma}{d\hat{t}} \quad,$$

where $x_b = x_a x_2/(x_a - x_1)$ and $x_a{}^{\min}$ is unchanged. Transverse momentum

effects are added by including in the integrand a factor

$$1 + \frac{x_1}{x_a{}^2 x_2} \cdot \frac{T}{s} \quad,$$

with $x_a{}^{\min}$, $\hat{s}$, $\hat{t}$, and $\hat{u}$ modified as in the single particle case.

III. THE QCD BORN TERM AND ASYMPTOTIC FREEDOM

A. The QCD Born Term

If the dominant hard scattering sub-process in large p_T single

particle production is quark-quark scattering, then one should expect[18]

that the QCD Born term due to color octet gluon exchange (OGX),

$$\frac{d\sigma}{d\hat{t}} \;=\; 2\pi\alpha^2 \cdot \frac{2}{9}\left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{s}^2 \hat{t}^2}\right) \quad,$$

(3)

will dominate Eq. (3). If the final quarks c and d have the same flavor,

the interference term

$$\frac{d\sigma}{d\hat{t}}(\text{int.}) \;=\; \frac{4\pi\alpha^2}{9}\left(-\frac{2}{3}\frac{1}{\hat{t}\hat{u}}\right)$$

is included in the following calculations. Simple quark-quark scattering

via single gluon exchange cannot be directly relevant to the presently

observed large p_T single particle production, since Eq. (3) predicts

that $E d^3\sigma/d^3p$ scales as p_T^{-4}, and experimentally the scaling behavior

p_T^{-8} is observed. In fact, Eq. (3) with $\alpha = 2$ (typical of a strong coupling

constant) yields a cross section that exceeds the Field-Feynman benchmark

by an order of magnitude at $p_T = 4$ GeV/c and $\sqrt{s} = 52.8$ (see Fig. 2).

There is an obvious resolution of this dilemma[6]: asymptotic freedom

(AF) corrections might be suppressing the OGX contribution below the

level of the observed cross section, thus unmasking the contributions of

other hard scattering sub-processes (e.g., the CIM-like qM → qM) which

naturally scale as p_T^{-8}. If this point of view is correct, then it is of

some interest to determine when OGX with AF corrections will exceed

extrapolations of the current data. Therefore, the effects of asymptotic

freedom on the three components of Eq. (1): $P(x)$, $D(z)$, and $d\sigma/d\hat{t}$ must

be considered in turn.

B. Asymptotic Freedom Effects on $P(x)$

The structure functions $P(x)$ of quarks in hadrons used to calculate

the OGX cross section are those determined in Field-Feynman by analyzing

deep-inelastic electron and neutrino scattering. It is now widely believed

that the Bjorken scaling violations observed in deep-inelastic electroproduction

are accounted for by the predictions of non-Abelian gauge theories of colored

quarks and gluons.[11] A detailed analysis of the electro- and neutrinoproduction

data permits evaluation of $F_2(x, Q^2) = xP(x, Q^2)$ given $F_2(x, Q_0^2)$, where Q_0^2 is some small value (e.g., $Q_0^2 \sim 4$ GeV^2), via the relation

$$\frac{\partial \ln F_2(x, Q^2)}{\partial \ln Q^2} \simeq d(x)$$

and the approximation that $\ln F_2(x, Q^2)$ varies linearly with $\ln Q^2$ at fixed x.[19] Thus

$$\frac{\ln F_2(x, Q^2) - \ln F_2(x, Q_0^2)}{\ln Q^2 - \ln Q_0^2} \simeq d(x) \quad . \tag{4}$$

The function $d(x)$ evaluated by Fox with $Q_0^2 = 4$ GeV^2 and $Q^2 = 2500$ GeV^2 is shown in Fig. 3. Note that $F_2(x, Q^2)$ decreases for large x and increases for small x as Q^2 increases. Qualitatively, this behavior is explained in very elegant terms by the physical picture of Kogut and Susskind[20]: at $Q^2 \sim Q_0^2$, the photon probes structures in the proton of size $R_0^2 \sim 1/Q_0^2$. As Q^2 increases to $Q_1^2 > Q_0^2$, the photon tends to break up the structures of size R_0^2 and momentum fraction x_0 into sub-structures of size $R_1^2 \sim 1/Q_1^2$ each having some momentum fraction $x_1 < x_0$. Thus as Q^2 increases, the quark population decreases at large x and increases at small x.

To the extent that such a picture describes (even heuristically) the physical processes behind the asymptotic freedom corrections to $P(x, Q^2)$, one expects that a similar picture will hold if the photon probe is replaced

by a gluon probe, as appropriate in large p_T particle production which

results from quark-quark scattering via gluon exchange ($Q^2 = -\hat{t}$ is typically

10 - 300 GeV^2 in the region of currently measured data). However, it

is not at all obvious that the function d(x) obtained from the electroproduction

data is the appropriate function to use. This is because a photon probes

the charge density of a proton, while the gluon probes the color density,

and the two densities may not be identical. More technically, the AF

corrections to electroproduction are obtained by evaluating the anomalous

dimensions of the operator product expansion of two electromagnetic

(color singlet) currents $J_\mu^{e.m.}(x) J_\nu^{e.m.}(0)$, under the specific assumption

that the symmetry group of the currents commutes with the color gauge

group.[11] A similar calculation using color octet currents has not yet

appeared (to my knowledge). Therefore, the function d(x) in Eq. (4)

will be used henceforth, with the understanding that there may be large

theoretical uncertainty in the application. In the actual calculations I

have used d(x) for valence quarks, and $|d(x)|$ for sea quarks. Since

the sea quarks do not contribute significantly for $x \gtrsim 0.2$, the prescription

for that region is actually irrelevant.

C. Asymptotic Freedom Effects on D(z)

The structure functions D(z) of hadrons in quarks do not share the

same firm theoretical underpinning as the functions P(x). However, both

heuristic arguments[21] and explicit model field theory calculations[22]

indicate that it may not be unreasonable to assume that the asymptotic

freedom corrections to $D(z, Q^2)$ are the same as those applied to $P(x, Q^2)$,

especially for $z \sim 1$, which is the only region of importance for large p_T

single particle production. The arguments are based on the Gribov-

Lipatov reciprocity relation[23]

$$D(z) \simeq xP(x) \quad ,$$

which should be approximately true in lowest-order perturbation theory

and for $z \sim x \sim 1$. In particular, the model calculations support the view

that the anomalous dimensions γ_N in electroproduction and $\tilde{\gamma}_N$ in e^+e^-

annihilation should be about equal for $\bar{\alpha}$, the (running) coupling constant,

small and N large. Since the large N anomalous dimensions control the

large N moments of the structure functions and hence the behavior near

$z \sim x \sim 1$, we conclude that a good guess is to use the function $d(x)$ in

Eq. (4) to compute the asymptotic freedom corrections to the structure

functions $D(z)$. Of course, the same color octet/singlet ambiguities that

afflict $P(x)$ also apply to $D(z)$.

D. Asymptotic Freedom Effects on $d\sigma/d\hat{t}$

If the quark-quark scattering were truly a short-distance dominated

process, then it might be appropriate to use the running coupling constant[24]

$\bar{\alpha}(Q^2)$ in $d\sigma/d\hat{t}$. This could be the case if, for example, the quarks were

far off-mass-shell. In section IV an alternate and more plausible point

of view involving theoretically derived fixed-angle form factors[12] is

presented. However, for the sake of argument and completeness, the

effects of the running coupling constant will be pursued here. The reader

should keep in mind that there are serious theoretical reservations concerning

the use of $\bar{\alpha}(Q^2)$ in the present context.

The running coupling constant for four flavors and three colors is

$$\bar{\alpha}(Q^2) = \frac{\alpha_0}{1 + \frac{25}{12\pi}\alpha_0 \ln\frac{Q^2}{\mu^2}}$$

$$= \frac{12\pi}{25 \ln\frac{Q^2}{\Lambda^2}} \quad ,$$

where the second relation defines Λ and eliminates the redundant parameter

α_0. Neglect of the charm flavor changes $12\pi/25 \to 4\pi/9$ and is insignificant

here. Estimates of the renormalization point Λ vary; I have used the

value $\Lambda^2 = (500 \text{ MeV/c})^2$ favored by Fox[19] and others as a reasonable

value.

E. Gluon Contributions

The gluon momentum distribution in the proton cannot be measured

directly and not much is known about it. It is possible that the gluons

carry as much as 50% of the proton momentum. In order to set an upper

limit on the gluon scattering contribution to large p_T production, I have

fixed the gluon distribution in the most optimistic yet reasonable way:

$$P(x) = 2.5 \, x^{-1}(1 - x)^4 \quad ,$$

with the normalization chosen to give the gluons about 50% of the proton momentum. Including the contributions from (gluon) + q → (gluon) + q and the accompanying crossed-channel reactions,[25] the total contribution from gluons still amounts to only a few percent of the quark elastic scattering contribution and is therefore ignored in the following analysis.

F. Comparison with Data

I have calculated the cross section for pp → π^0 X at $\sqrt{s}$ = 52.8 and θ_{CM} = 90° due to color octet gluon exchange (OGX), applying successively the asymptotic freedom corrections to $P(x, Q^2)$, $D(z, Q^2)$, and $\bar{\alpha}(Q^2)$, with α = 2 until the last step. The results are displayed in Fig. 4, and compared to the Field-Feynman benchmark. Several points of interest may be noted:

(1) In the region of existing experimental measurements ($p_T \lesssim 6 - 8$ GeV/c, $x_T \lesssim 0.3 - 0.5$), the average values $<x_a>$ and $<x_b>$ in Eq. (1) are about 0.1 to 0.4, and therefore the asymptotic freedom corrections to $P(x_a, Q^2)$ and $P(x_b, Q^2)$ are not very large. On the other hand, $<z_c> \sim 0.7 - 0.9$, and the AF corrections to $D(z, Q^2)$ are relatively large. The corrections to the effective exponents N and F defined by

$$E\frac{d^3\sigma}{d^3p} \sim p_T^{-N}(1 - x_T)^F$$

can be understood semi-quantitatively:

$$\langle Q^2 \rangle \simeq -\frac{\langle x_a \rangle t}{\langle z_C \rangle}$$

$$\simeq \frac{2\langle x_a \rangle}{x_T \langle z_C \rangle} p_T^2 \quad ,$$

$$\sim (7 - 5x_T) p_T^2 \quad ,$$

for $\theta_{CM} = 90°$. Thus

$$E\frac{d^3\sigma}{d^3 p} \simeq \left(E\frac{d^3\sigma}{d^3 p}\right)_{OGX} \cdot \left((7 - 5x_T) p_T^2\right)^\delta \quad ,$$

where

$$\delta = d(\langle x_a \rangle) + d(\langle x_b \rangle) + d(\langle z_C \rangle)$$

and typically

$$-0.75 \lesssim \delta \lesssim -0.25 \quad .$$

Then $N \rightarrow 4 - 2\delta$ and F also increases. Precise results are shown in Fig. 4b. The p_T, x_T factorization is now only approximate and N and F depend weakly on both p_T and x_T: at fixed x_T F increases and N decreases as p_T increases. The cross section for OGX with AF corrections to $P(x, Q^2)$ and $D(z, Q^2)$ still exceeds the Field-Feynman benchmark.

(2) Adding the running coupling constant $\overline{\alpha}(Q^2)$ into the calculation

depresses the OGX cross section below the Field-Feynman calculation

and increases N by about one unit while decreasing F a similar amount.

Unlike other recent estimates,[7] we cannot conclude that observations at

larger p_T at the ISR will reveal the OGX contribution, since we see that

N increases with x_T and keeps the OGX cross section below Field-Feynman

However, for $\sqrt{s}$ = 52.8 GeV^2 and $p_T \sim 4 - 8$ GeV/c, the OGX cross

section is only a factor 2 - 3 below the Field-Feynman model, and such

a factor can easily be made up by considering $<k_T>$ smearing effects

(see section IV).

(3) Figure 5 shows the angular distributions for OGX and Field-Feynman

for p_T = 4.0 GeV/c. We see that a search for OGX contributions in data

will benefit somewhat by looking away from $\theta_{CM} = 90^{\circ}$. This peripherality

in θ_{CM} is only qualitatively sensitive to the AF corrections and should

be quite reliable.

Any conclusions based on this model must be weighted by the large

theoretical uncertainties involved: (a) are the AF corrections to $P(x, Q^2)$

determined with a photon probe applicable when a gluon probe is used,

(b) are the same corrections applicable to the quark decay functions $D(z)$,

(c) is the running coupling constant $\overline{\alpha}(Q^2)$ applicable when short-distance

effects do not obviously dominate, and (d) how important are $<k_T>$ smearing

effects? Within the context of the model defined above, we have seen that

OGX will not dominate the Field-Feynman benchmark calculation until

the next generation of accelerators is constructed. With colliding beams

of (250×250) GeV/c momentum, the OGX cross section (with $P(x, Q^2)$,

$D(z, Q^2)$, $\bar{\alpha}(Q^2)$) included) for pp $\rightarrow \pi^0 X$ at $90°$ is 7.5×10^{-37} cm^2/GeV^2

at p_T = 25 GeV/c compared to 1.1×10^{-38} cm^2/GeV^2 for Field-Feynman.

The single jet cross sections are about 100 times larger. OGX cross

sections this large are likely to seriously impair searches for $W^{\pm}$ and Z^0

production at large p_T.[26]

The Field-Feynman model assumes that the dominant hard subprocess

in large p_T production is quark-quark scattering, and agreement with

experiment is obtained by lumping a good deal of ignorance into an empirical

$d\sigma/d\hat{t}$ in Eq. (1). One might have hoped that by accounting for asymptotic

freedom corrections as completely as possible, agreement between OGX

and the data might be obtained. Apparently this is not the case. Therefore,

we will turn to another approach still within the framework of QCD to

account for the data.

IV. QUARK-QUARK SCATTERING IN QCD

A. QCD Form Factors for Quark-Quark Scattering

Recently Cornwall and Tiktopoulos[12] (CT) and others[27] have calculated

the amplitude for quark-quark scattering at large fixed angle and $\hat{s}$, $\hat{t}$, $\hat{u} \gg m^2$,

where m^2 is some mass scale, say $m^2 \lesssim 1$ GeV^2. The CT result is that

in the leading log approximation the perturbation theory results exponentiate,

giving

$$\frac{d\sigma}{d\hat{t}} = \left(\frac{d\sigma}{d\hat{t}}\right)_B F^4(\hat{t}) \qquad ,$$

where $(d\sigma/dt)_B$ is the Born approximation (Eq. (3)), and $F(\hat{t})$ is the quark-quark

color singlet form factor

$$F(\hat{t}) = \exp\left(-\frac{\alpha}{4\pi} c_F \ln^2\left(-\frac{\hat{t}}{\mu^2}\right)\right) \qquad (4)$$

or

$$F(\hat{t}) = \exp\left(-\frac{\alpha}{2\pi} c_F \ln^2\left(-\frac{\hat{t}}{m^2}\right)\right) \qquad , \qquad (5)$$

with $c_F = 4/3$ for SU(3) color. Equation (4) results from calculating with

the quarks on-mass-shell and giving the gluons a mass μ, while Eq. (5)

results from allowing the quarks to be off-mass-shell ($p_i^2 = m^2 \neq m^2_{quark}$)

while keeping the gluons massless. Presumably Eq. (5) is relevant for

the present problem, and it will be used here. Whether the arguments in

the logs are $\hat{s}$, $\hat{t}$, or $\hat{u}$ is irrelevant in the fixed-angle limit with all

variables asymptotically large. I have used $\ln (-\hat{t}/m^2)$ in all calculations

at $\theta_{CM} = 90^o$.

Cornwall and Tiktopoulos have also suggested, on the basis of

renormalization group arguments, that Eqs. (4) and (5) could be modified to

$$F(t) = \exp\left(-\frac{c_F}{8\pi^2 b} \ln\left(-\frac{t}{\mu^2}\right) \ln \ln \left(-\frac{t}{\mu'^2}\right)\right) \qquad (4')$$

or

$$F(t) = \exp\left(-\frac{c_F}{4\pi^2 b} \ln\left(-\frac{t}{m^2}\right) \ln \ln \left(-\frac{t}{m'^2}\right)\right) , \qquad (5')$$

where b is calculated from the asymptotic freedom β function[24]

$\beta(g) = -b\,g^3 + O(g^5)$. For four flavors and three colors $b = 25/48\pi^2$.

Clearly, Eq. (5') is much more speculative than Eq. (5).

We will henceforth assume that Eq. (5) (or Eq. (5')) correctly

describes large p_T quark-quark scattering and proceed to build a model

based on these form factors. In Figs. 6 and 7 the curves labelled CT I

and CT II are based on Eq. (5) and Eq. (5'), respectively. Asymptotic

freedom corrections to $P(x, Q^2)$ and $D(z, Q^2)$ are included as described

in section III, and the bare coupling constant $\alpha = 2$. Note that the running

coupling constant $\bar{\alpha}(Q^2)$ is not used. The parameter m^2 is hard to determine

theoretically and is taken as $m^2 = 1$ GeV^2 in the calculations. It will be

shown in section B below that considerations of $<k_T>$ of quarks in hadrons

indicates that the quarks are off-mass-shell an amount of order 1 GeV^2.

The value $m'^2 = 1$ GeV^2 is taken in Eq. (5').

Clearly both models are a disaster as they stand. In particular,
the effective exponents N and F are much larger than the Field-Feynman
benchmark values. The remaining part of this section consists of a
phenomenological demonstration that the model CT I can be modified in
a reasonable way and agreement with data obtained. The same successful
demonstration is also possible with model CT II, but since Eq. (5') is
much less firmly established theoretically, I have decided to neglect
the model CT II in the following.

As the first step toward rescuing model CT I, we must modify the
form factor $F(\hat{t})$ with a simple parameter δ in the exponential,

$$F(t) = \exp\left(-\frac{\alpha}{2\pi}\delta\, c_F \ln^2\left(-\frac{t}{m^2}\right)\right) \quad .$$

Figures 6 and 7 show that the value $\delta = 0.1$ gives much improved values
for F and especially N, the exponent controlling the p_T dependence.
Theoretical motivation for the parameter δ will be discussed in section C
below; we turn now to a discussion of $<k_T>$ smearing effects, as these
effects must surely be present and are quite important.

B. $<k_T>$ Smearing Effects

Since quarks are confined to a region of radius R $\sim$ 1 fermi, they
must have a minimum $<k_T^2> \sim (400$ MeV/c$)^2$ just from the uncertainty

principle. However, when the proton is probed with a large Q^2 beam,

the quark transverse position is determined with a resolution of order

$1/Q^2$, and $<k_T^2>$ for quarks must increase proportional to Q^2. This is

the "Heisenberg microscope" effect.[28] In asymptotically free theories,

it is probably true[29] that

$$<k_T^2> \sim \bar{\alpha}(Q^2) \cdot Q^2 \quad . \tag{6}$$

It is not easy to determine $<k_T>_{q/h}$ for quarks in hadrons directly

from experiment. The most direct method is by measuring $<k_T>_{\mu\mu}$

of high mass μ-pairs. If the μ-pairs are produced by the Drell-Yan

mechanism, then

$$<k_T>_{q/h} \sim \frac{1}{\sqrt{2}} <k_T>_{\mu\mu} \quad .$$

Presently available μ-pair data[30] can be parametrized[31] as

$$<k_T^2>_{\mu\mu} \simeq 0.6 + 0.09 \ Q^2 \quad ,$$

yielding

$$<k_T^2>_{q/h} \simeq 0.20 + 0.14 \ \bar{\alpha}(Q^2) \cdot Q^2 \tag{7}$$

if the form in Eq. (6) is used. Equation (7) should be useful for $Q^2 > 1 - 2$ GeV^2.

It is interesting that this value of $<k_T>_{q/h}$ predicts the observed value of

$R = \sigma_L/\sigma_T \simeq 0.18$ measured in electroproduction.[31] The value of $<k_T>_{q/h}$

as given by Eq. (7) is shown in Fig. 8. Note that the rate of increase of

$<k_T^2>$ decreases as Q^2 increases.[29]

There is also considerable indirect evidence from ordinary hadron

interactions that $<k_T>_{q/h}$ is larger than one might naively expect based

on pion production, where $<k_T>_\pi \sim 330$ MeV/c. These ideas are discussed

in detail by Levin and Ryskin.[32]

Naturally the whole basis of the parton model is destroyed if

$<k_T^2>/Q^2$ is not a small number. Fortunately, the values implied by

Eq. (7) for that ratio do not contradict the impulse approximation. Furthermore,

we see that it is not unreasonable to use $m^2 \sim 1$ GeV^2 as the amount that

the quarks are off-mass-shell for the large Q^2 values $10 - 300$ (GeV/c)^2

of interest.

The relevance of the $<k_T>$ considerations results from the so-called

"trigger bias" effect in large p_T production.[33] This effect means that

it is easier for a quark to gain transverse momentum by a $<k_T>$ fluctuation

than from a hard scattering off another quark, since the latter process

is damped by p_T^{-8}. Therefore, as discussed in section II, I have included

$<k_T>$ effects by calculating the integrals in Eq. (1) assuming the quarks

have transverse momenta in the trigger direction of exactly $<k_T>$ as

given by Eq. (7).[34] The transverse momenta of hadrons in quarks contributes

a relatively small effect ($<k_T> \sim 330$ MeV/c) and is neglected here.

Recently, Landshoff has suggested on the basis of the covariant

parton model that $<k_T>$ might increase as x increases.[35] Unfortunately,

the suggestion has not been formulated in a manner that does not introduce

further parameters undetermined by data, so I have not pursued the matter

further. It is interesting to note, however, that physically the $<k_T(x)>$

phenomenon is caused by the quark becoming more off-mass-shell as

x increases,[35,36] and we will see below that the parameter δ = 0.2 can

be accounted for in this way (among others).

The result of calculating the CT I model with δ = 0.1 and $<k_T>$

given by Eq. (7) is shown in Figs. 9 - 11. The agreement with experiment

is remarkable.

C. Motivation for the Parameter δ

If the model CT I (δ, $<k_T>$) is to be believed, then we must account

physically for the value $\delta \simeq 0.1$. I can suggest at least three reasons to

account for this: (1) The form factors $\hat{F}(t)$ in Eqs. (4) and (5) are supposed

to be correct for the scattering of free colored quarks by colored gluons.

This accounts for the group theory factor c_F in the exponential. As stressed

by Cornwall and Tiktopoulos,[12] the scattering of color-singlet objects

is not suppressed: c_F = 0 in that case. It is possible that gluon bremsstrahlung

is partially screening the color of the quarks which are bound in the color

singlet proton, thus creating an effective $c_F' \sim \delta c_F$. Whether or not this

is reasonable probably depends on the mechanism for color neutralization

and confinement (in particular, the time scale involved). A consequence

of this line of argument is that the effective number of colors N_C seen

by the gluons in this kinematic regime is $N_C \simeq 1.14$. In some sense the

nearness of this number to unity indicates the consistency of using the

color-singlet anomalous dimensions to calculate the AF corrections to

$P(x, Q^2)$ and $D(x, Q^2)$.

Unfortunately I have no argument to estimate δ on the basis of color anti-screening considerations. However, the empirical value $\delta = 0.1$ suggests an intriguing conjecture that δ be identified with the running coupling constant $\bar{\alpha}(Q^2)$, as discussed next.

(2) The same leading log analysis that yields Eqs. (4) and (5) has an interesting infrared (as opposed to fixed angle) limit that has been suggested as a signal for quark confinement.[12] Since it is known that there is no signal for confinement order by order in perturbation theory,[37] it has been suggested that the leading log analysis may be a method of transcending perturbation theory.[12,38] Recent attempts to adapt renormalization group arguments to the infrared region have resulted in the suggestion that the coupling constant α in the exponentiated one-loop form factor integral might in fact be the running coupling constant that couples the internal loop gluons to the quarks.[39] Thus we might conjecture that a similar effect is occuring in the fixed angle regime, yielding

$$F(Q^2) = \exp\left(-\frac{\bar{\alpha}(Q^2)}{2\pi} \, c_F \, \ln^2\left(\frac{Q^2}{m^2}\right)\right) \quad , \tag{8}$$

with the difference that $\bar{\alpha}$ is evaluated at Q^2. This conjecture resembles the renormalization group argument of CT that resulted in Eqs. (4', 5'), except for (phenomenologically crucial) numerical differences. In any event the theoretical motivation of Eq. (8) is presently far from clear. It is possibly related to the color anti-screening argument in the previous

paragraph. Calculations using this conjecture and $m^2 = 1$ GeV^2 are

displayed in Figs. 10, 11. Better agreement with the data is easily obtained

by small adjustments of m^2 and the bare coupling constant α.

(3) The value for m^2 in Eq. (5) is particularly hard to guess in a

reasonable way. The value $m^2 = 1$ GeV^2 has been used above since

$<k_T>$ considerations indicate that the quark is off-mass-shell an amount

of order 1 GeV^2. Landshoff and Polkinghorne[36] have suggested that m^2

can be determined by simple kinematics in the covariant parton model.

They find

$$-m^2 = \frac{xs' + k_T^2}{(1-x)} - xM^2 \quad , \tag{9}$$

where M is the proton mass and s' is the squared invariant mass of the

debris left behind when the proton emits the quark with momentum fraction

x (and before confining forces begin to act). As before, k_T is the transverse

momentum of the single quark. The value of s' is somewhat arbitrary,

but if I take the value s' $= 4$ GeV^2 used by Landshoff[35] and the values

given in Eq. (7) for $<k_T^2>$, then the results shown in Fig. 12 for $-m^2$

are obtained. Using Eq. (9) for m^2 in Eq. (5) yields the results shown

in Figs. 10, 11. Once again it is clear that better agreement can be

obtained by small adjustments of the parameters.

D. Angular Distributions and Regge Behavior in QCD

Past attempts to identify the anomalous p_T behavior in large p_T production with simple form factor modifications to quark-quark scattering were plagued with incorrect peripheral angular dependences.[4] Indeed, the present analysis suffers the same problem if naively extended away from 90°.

The resolution of the problem is to note that the Cornwall-Tiktopoulos results apply when the invariants $\hat{s}$, $\hat{t}$, $\hat{u}$ grow large together. Calculation of the angular dependence at fixed p_T is correctly done using the formula for $F(\hat{t})$ given by Tyburski,[40]

$$F(t) = \exp\left(-\frac{\alpha}{2\pi}c'_F \frac{(-\hat{t}+\mu^2)}{\sqrt{\Delta}} \ln\left(\frac{-\hat{t}+\sqrt{\Delta}}{\hat{t}+\sqrt{\Delta}}\right) \ln\left(\frac{\hat{s}}{s_0}\right)\right) , \qquad (10)$$

where $\Delta = \hat{t}^2 - 4\hat{t}m^2$, and a slight modification in the group theory factor calculated by Tyburski has been implemented so that Eq. (10) agrees with Eq. (5) in the large fixed angle regime for $m^2 = \mu^2 = s_0/2$ and $\hat{s}$, $\hat{t}$, $\hat{u}$ asymptotically large. Angular distributions using Eqs. (5) and (10) with $\delta = 0.1$ in both are compared with the Field-Feynman result in Fig. 13. Once again the agreement is remarkable.

V. PREDICTED PATTERN OF SCALING VIOLATION

The variations of the effective exponents N and F shown in Fig. 11 indicate a definite pattern of scaling violation for the CT(δ, $<k_T>$) model: N is a function of x_T, F is a function of p_T, and both N and F increase as p_T increases at fixed x_T. Examples of the predicted scaling violations are presented in Fig. 14, where the lines for fixed p_T are solid for $13.4 \leq \sqrt{s} \leq 63$ GeV^2 and dotted otherwise, and the lines for fixed $\sqrt{s}$ are solid for $2 \leq p_T \leq 8$ (GeV/c)^2 and dotted otherwise. Clearly, the particular quantitative values shown in the figures are not reliable, but the qualitative pattern is reliable. It is doubtful whether any meaningful comparison can be made with present data.

VI. CONCLUSIONS AND SUMMARY

The effects of asymptotic freedom and color gauge symmetry on the production of large p_T particles by quark-quark scattering have been systematically investigated in an attempt to understand the striking phenomenological success of the Field-Feynman model. Apparently, asymptotic freedom effects on the proton structure functions P(x, Q^2) are not particularly important in regions of current experimental accessibility ($0.1 \lesssim x_T \lesssim 0.4$). Asymptotic freedom effects on the quark structure functions D(z, Q^2) are potentially very significant; unfortunately, equally significant theoretical uncertainties are also present. Similarly, the use of the running coupling constant $\bar{\alpha}(Q^2)$ is also discussed; however, in the

context of wide angle quark elastic scattering, the use of $\bar{\alpha}(Q^2)$ is almost

certainly incorrect: the quarks are simply not far off-mass-shell. Using

the most optimistic estimates that are still reasonable, gluon contributions

to large p_T production are found to be negligible.

Altogether, these results based only on asymptotic freedom considerations

lead to rather unsatisfying conclusions: with a bare coupling constant of

order unity, it is hard to understand why OGX is not experimentally visible;

using $\bar{\alpha}(Q^2)$, which is theoretically dubious, OGX will not be seen until

$\sqrt{s}$ exceeds several hundred GeV^2, and the success of Field-Feynman is

still not understood.

We have seen that the use of form factors derived from color gauge

theories by Cornwall and Tiktopoulos does open the possiblity of understanding

existing data in terms of quark-quark scattering. The viability of this

approach rests on a plausible but unproven phenomenological assumption

that probably amounts to partial. anti-screening of the color of quarks

in protons when probed by color gluons. It is also possible that a simple

kinematic effect proposed by Landshoff and Polkinghorne is responsible.

In either case, it is quite important to consider the transverse momentum

fluctuations of quarks in protons if any pretense of absolute normalization

is to be maintained. It is clearly of great interest to search for the pattern

of scaling violations predicted by these models.

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FIGURE CAPTIONS

Fig. 1:   Illustration of the constituent hard subscattering process.

Constituents a and b of hadrons A and B scatter through a

large angle, followed by the fragmentation of the constituent

c into hadrons, one of which (C) is detected.

Fig. 2:   Invariant cross section for octet gluon exchange (—— OGX)

with coupling constant $\alpha = 2$ compared with the Field-Feynman

model (--- F-F), which accurately represents the data.

Fig. 3:   The function $d(x) = \Delta \ln F_x(x, Q^2)/\Delta\ln Q^2$ provided by Fox.[19]

Fig. 4:   Comparison of the Field-Feynman model and OGX with

(a) $\alpha = 2$, (b) $\alpha = 2$ and $F(x, Q^2)$ included, (c) $\alpha = 2$, $F(x, Q^2)$,

$D(z, Q^2)$ included, (d) $\bar{\alpha}(Q^2)$, $F(x, Q^2)$, $D(z, Q^2)$ included.

(a) Invariant cross section versus p_T.

(b) Invariant cross section times p_T^8 versus x_T and the

effective exponents F and N.

Fig. 5:   The angular dependence of OGX compared with the Field-

Feynman model.   The value $\alpha = 2$ is used unless $\alpha = \bar{\alpha}(Q^2)$.

Fig. 6:   OGX (with $\alpha = 2$) and the Field-Feynman model compared

to models CT I (Eq. (5)), CT II (Eq. (5')), and CT I ($\delta = 0.1$)

as described in the text.

Fig. 7:   (a) the effective exponents F and (b) N for models OGX

($\alpha = 2$), Field-Feynman, CT I, CT II, and CT I ($\delta = 0.1$).

For clarity, the value of F for model CT II has been displaced

upward two units.

Fig. 8:   The average value of transverse momentum of quarks in a proton as a function of the Q^2 used to observe the quarks (Eq. (7)).

Fig. 9:   Data from ref. 2 compared to the model CT I with $\delta = 0.1$ and $\langle k_T(Q^2) \rangle$ determined by Eq. (7). The curve and data for $\sqrt{s} = 23.4$ GeV^2 has been displaced down one decade for clarity.

Fig. 10:   Data from ref. 2 compared to (a) the $m^2(x)$ model (section IV. C(3)), (b) the $\delta = 0.1$ model, and (c) the $\bar{\alpha}(Q^2)$ model (section IV. C(2)). $\langle k_T \rangle$ effects are included in each curve. For clarity, the $\sqrt{s} = 23.4$ GeV^2 set has been displaced down one decade.

Fig. 11:   The effective exponents F and N for (a) the Field-Feynman model, (b) the $\delta = 0.1$ and $\bar{\alpha}(Q^2)$ models, which give indistinguishable results, and (c) the $m^2(x)$ model.

Fig. 12:   The quantity $-m^2$ of Landshoff and Polkinghorne[36] evaluated for different values of Q^2 and $\langle k_T^2 \rangle$.

Fig. 13:   Angular distributions of invariant cross sections for (a) model CT I with $\delta = 0.1$ and $\langle k_T \rangle$ effects included, (b) the Tyburski model with $\delta = 0.1$ and $\langle k_T \rangle$ effects included, (c) the Field-Feynman model, and (d), (e) same as (a), (b) but without $\langle k_T \rangle$ effects.

Fig. 14:   Pattern of scaling violations for the model CT I with δ = 0.1

and $<k_T>$ effects included for (a) fixed values of p_T, (——)

13.4 $\leq$ $\sqrt{s}$ $\leq$ 63 GeV^2, (---) otherwise, and (b) fixed values

of $\sqrt{s}$, (——) 2 $\leq$ p_T $\leq$ 8 GeV/c, (---) otherwise. The "data"

points are from the Field-Feynman model.
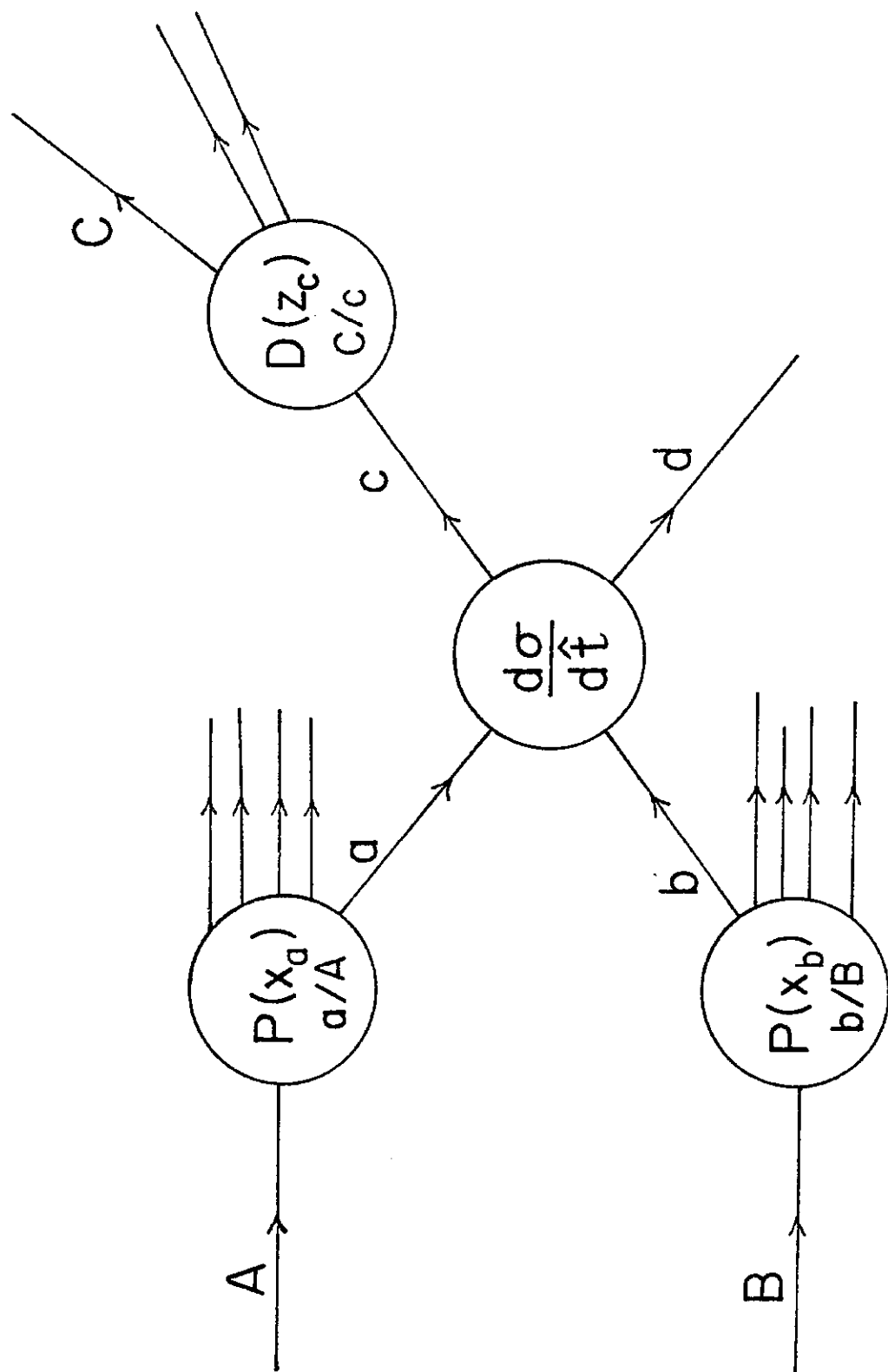
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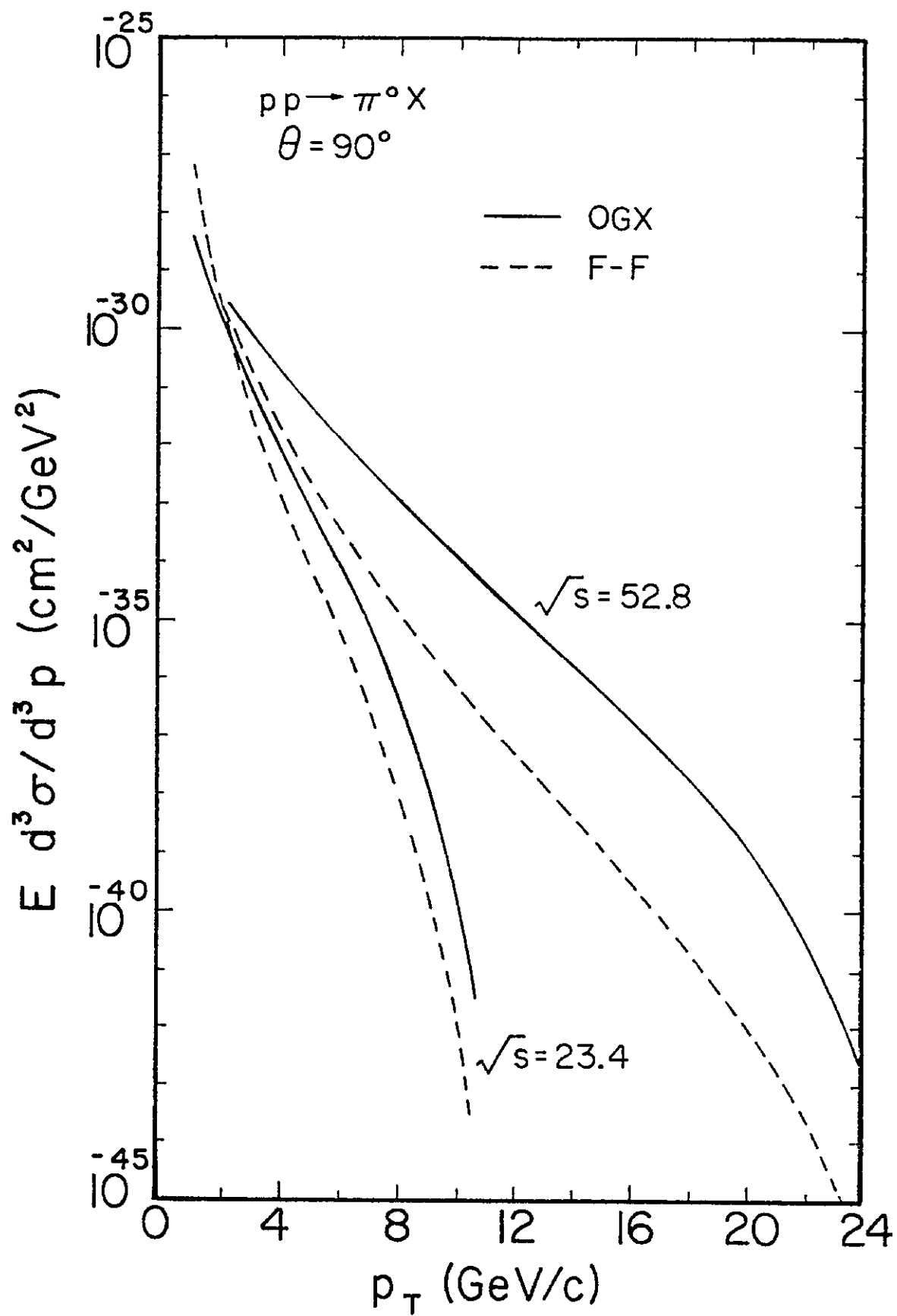
Fig. 2

$$\frac{\Delta \ln F_2(x,Q^2)}{\Delta \ln Q^2}$$
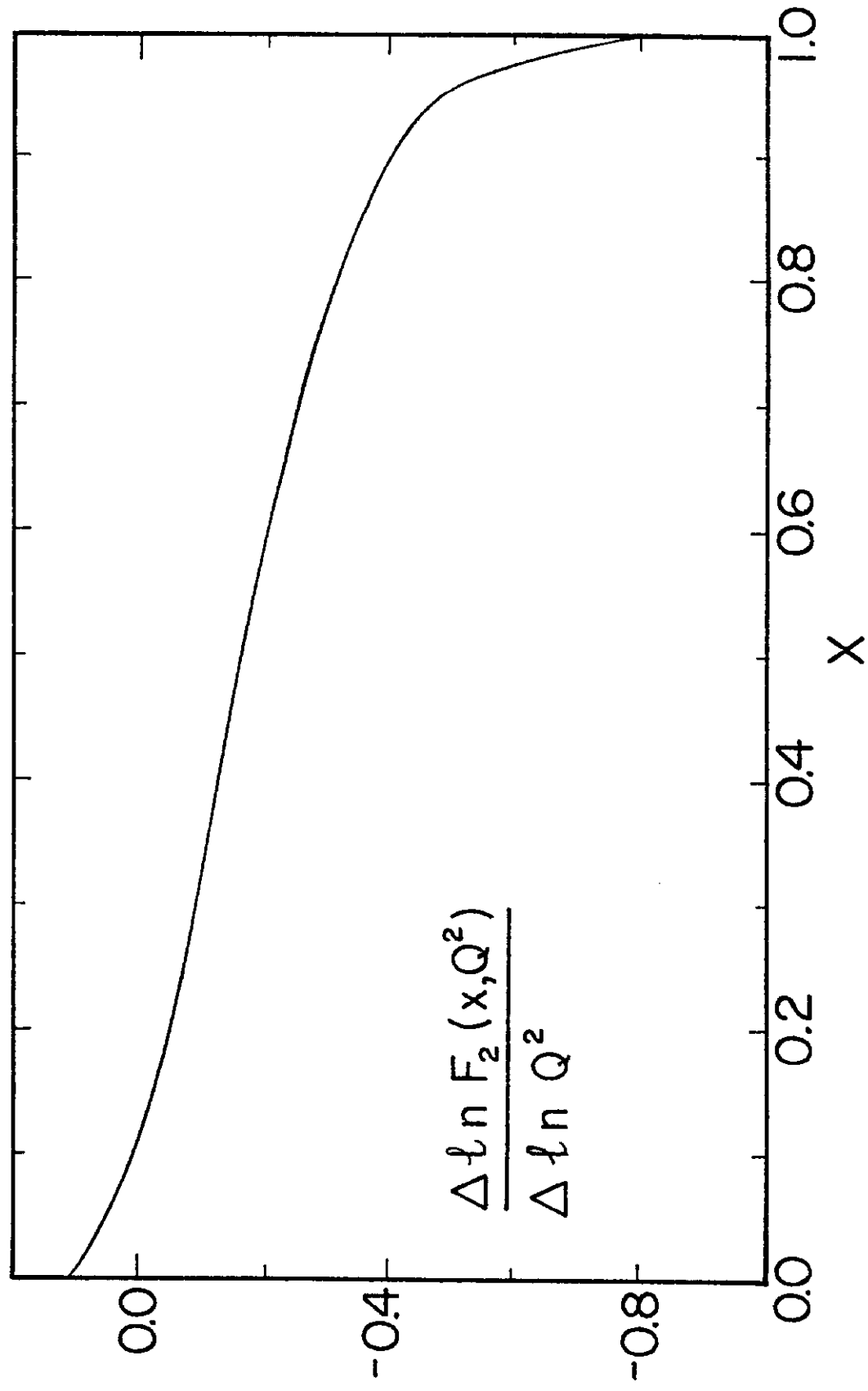
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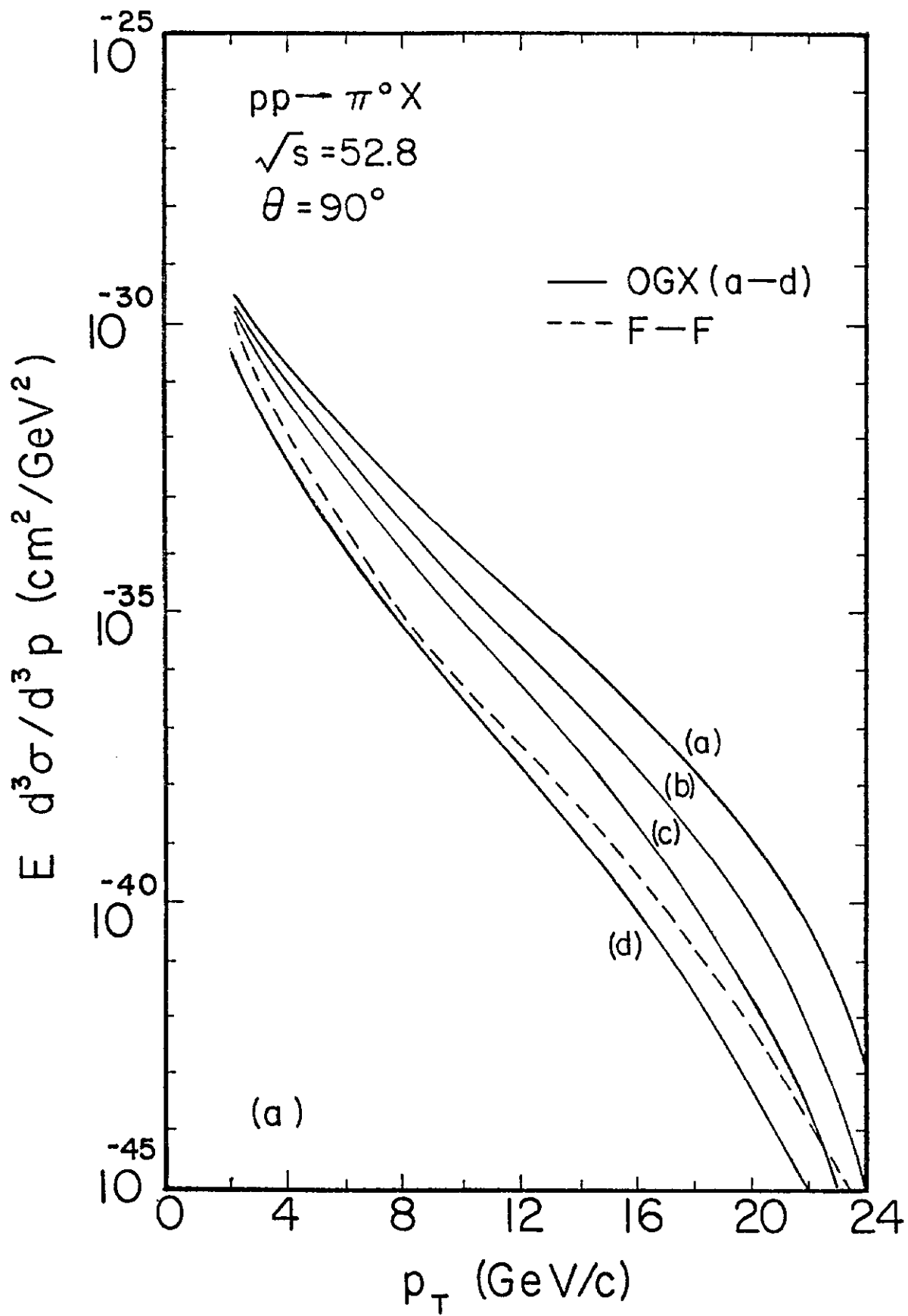
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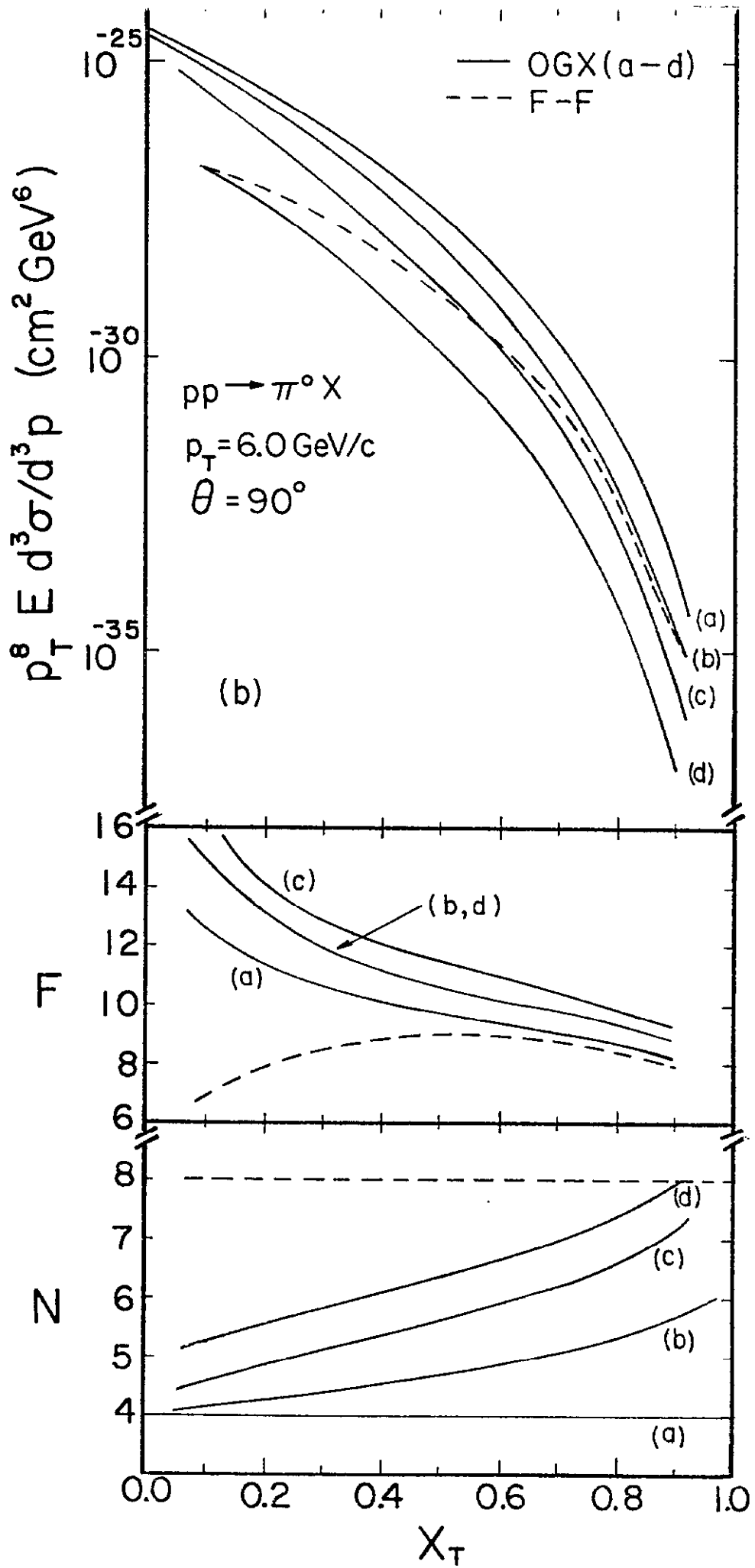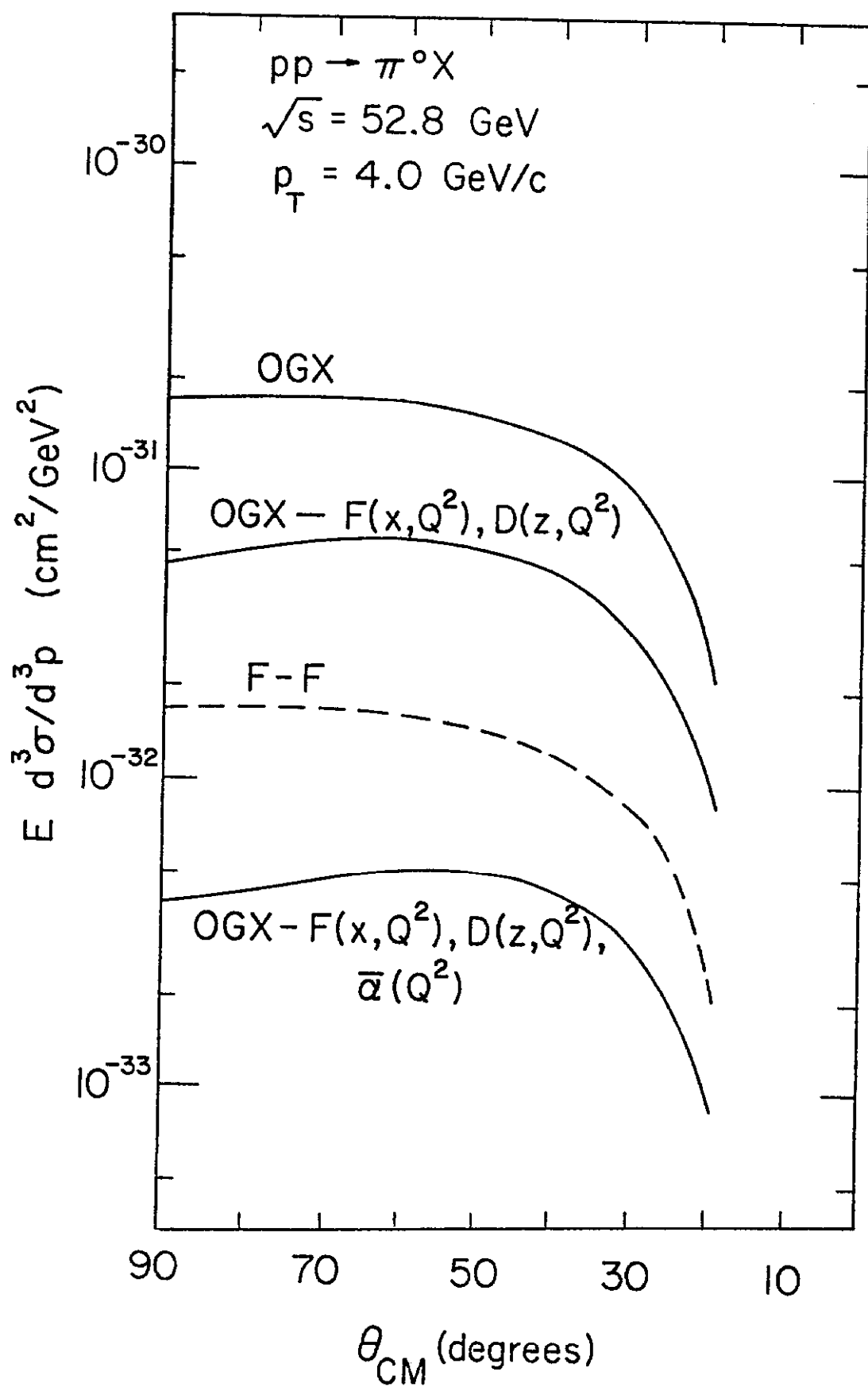
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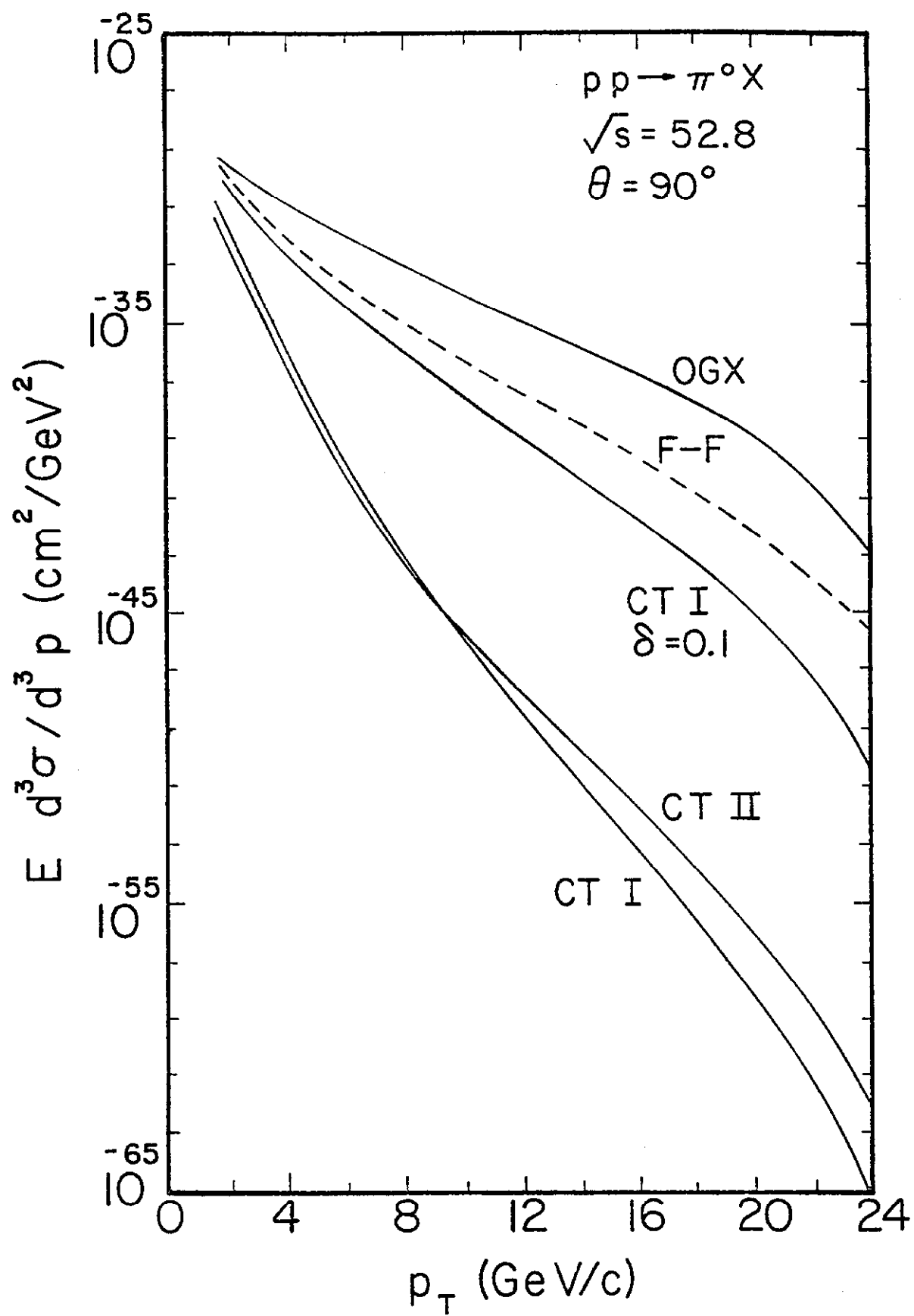
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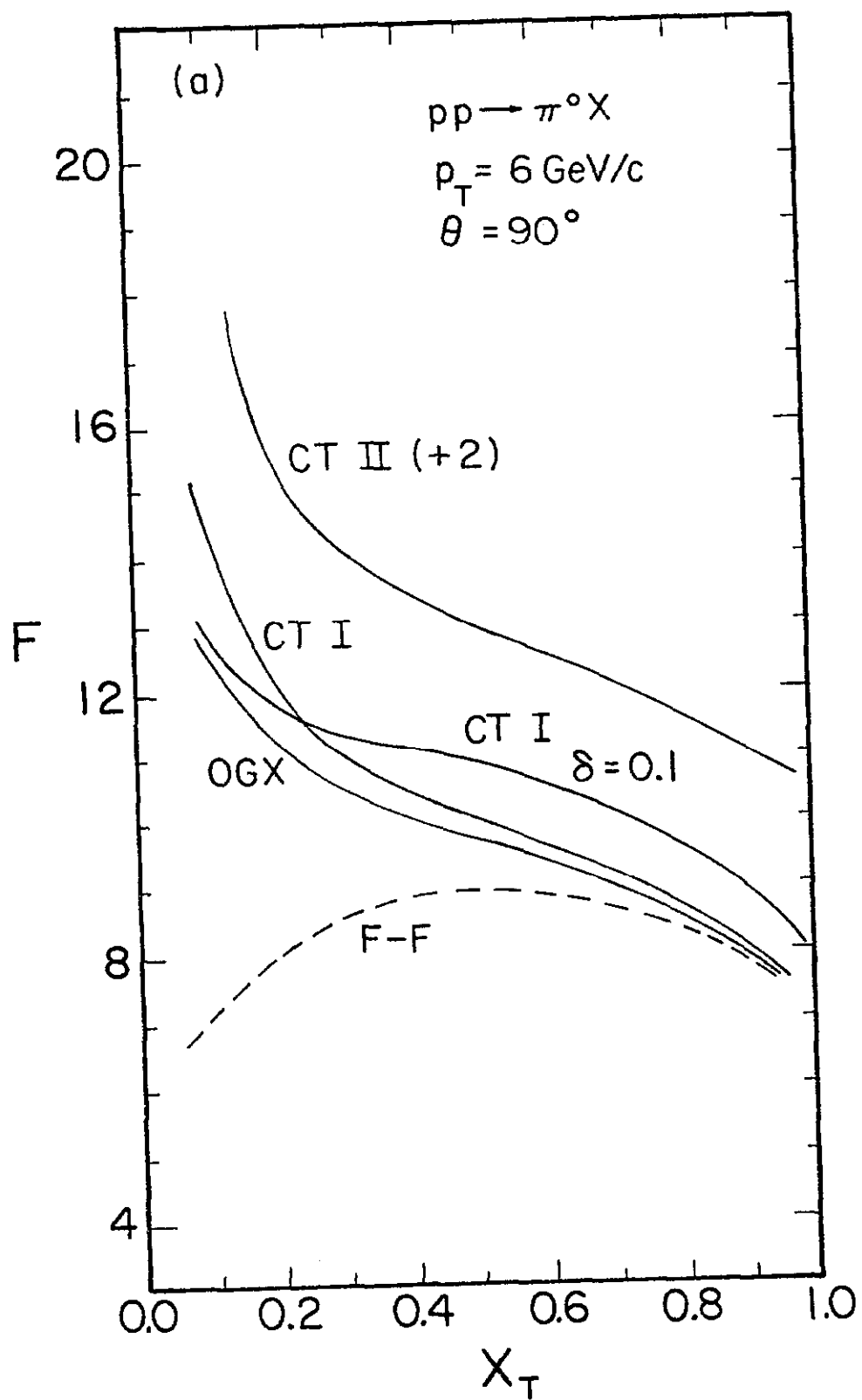
Fig. 7a

Fig. 7b

Fig. 8

Fig. 9
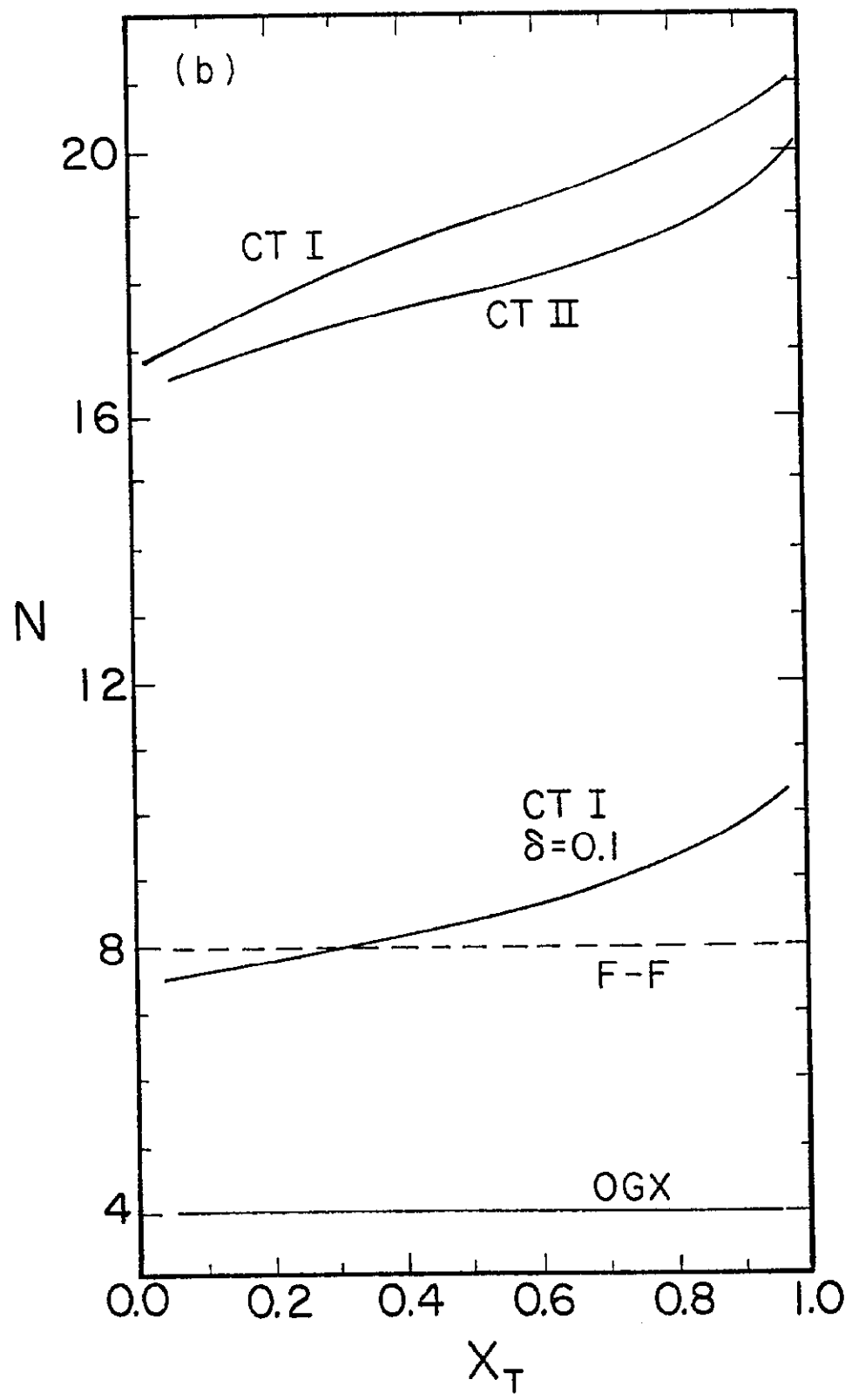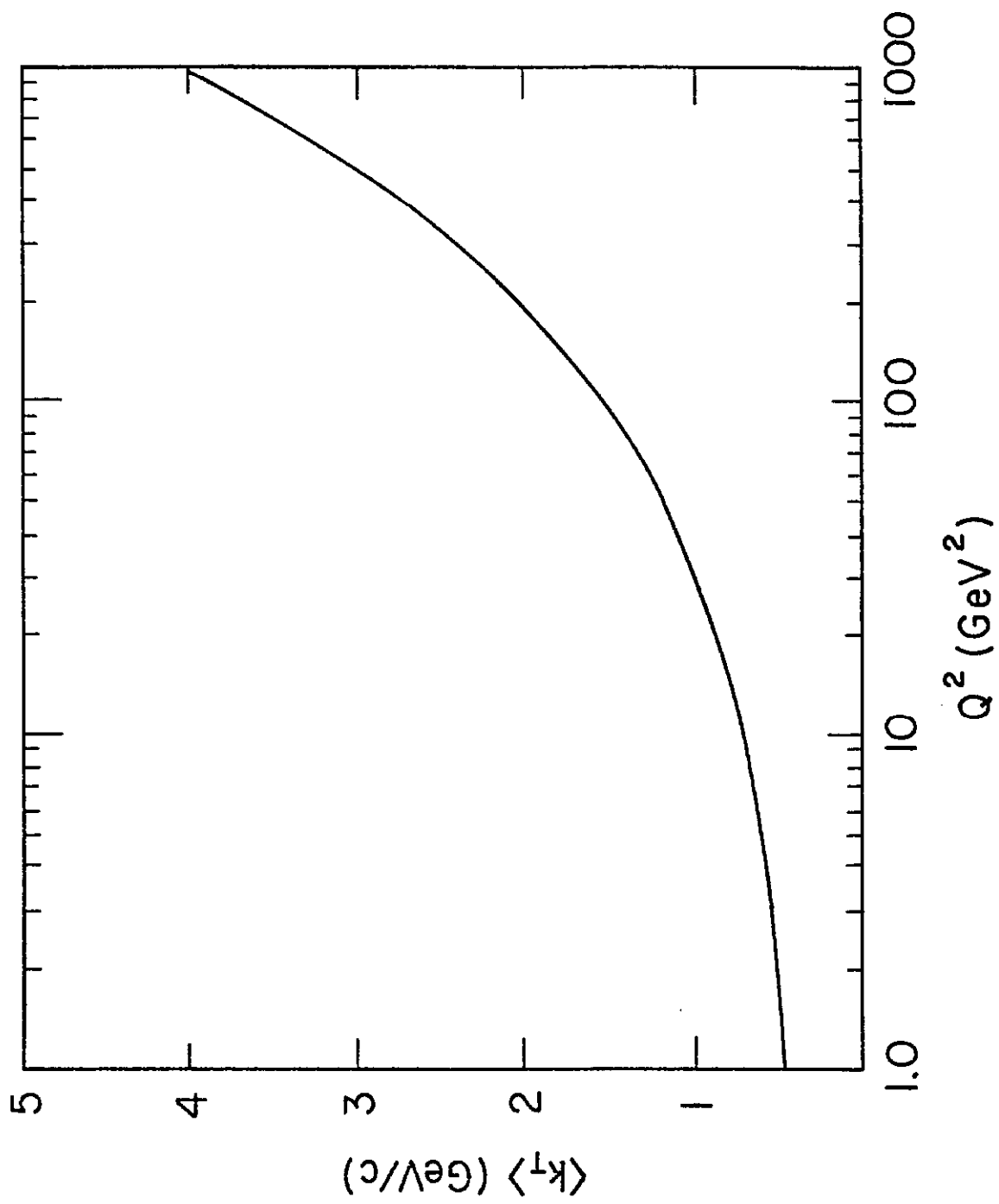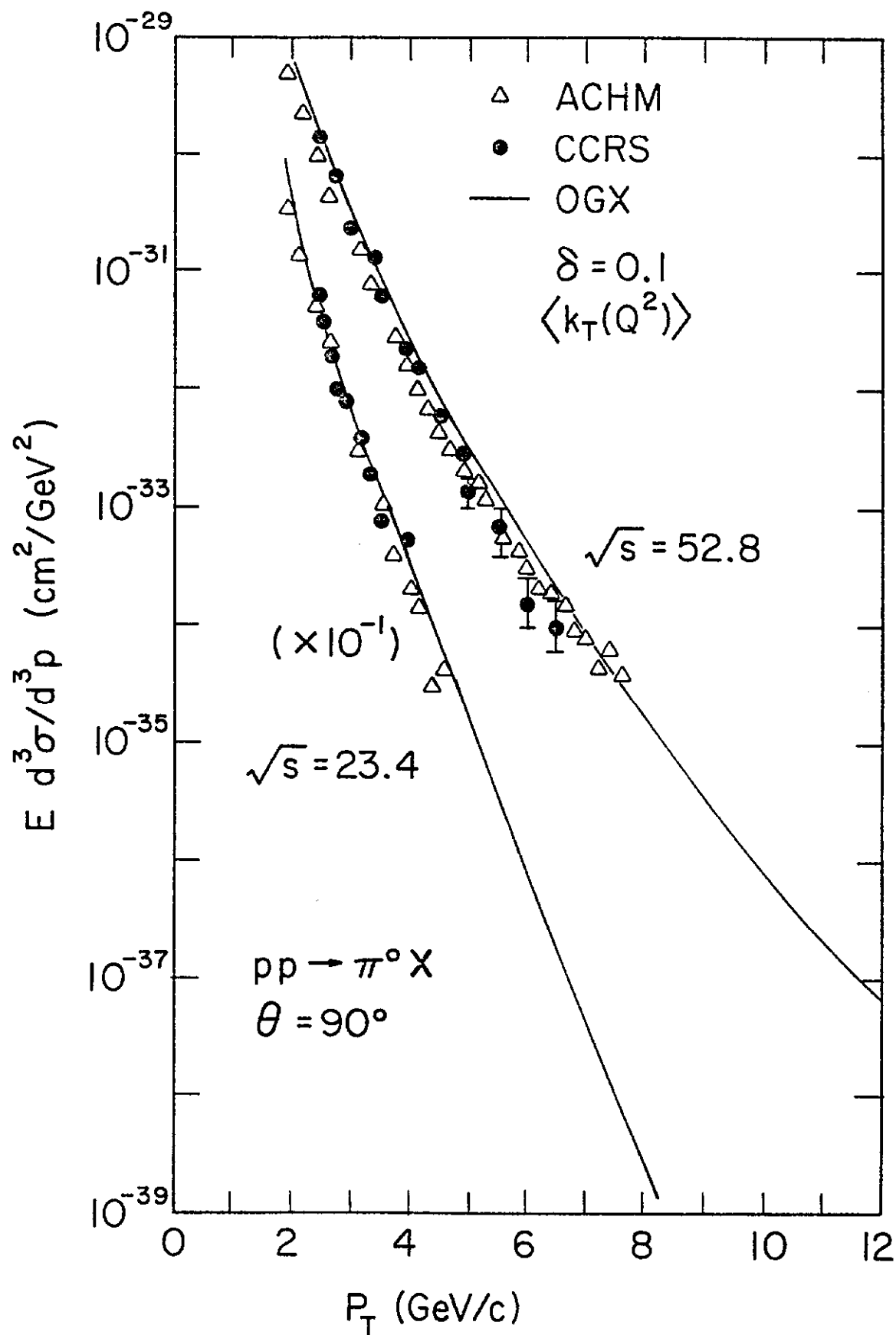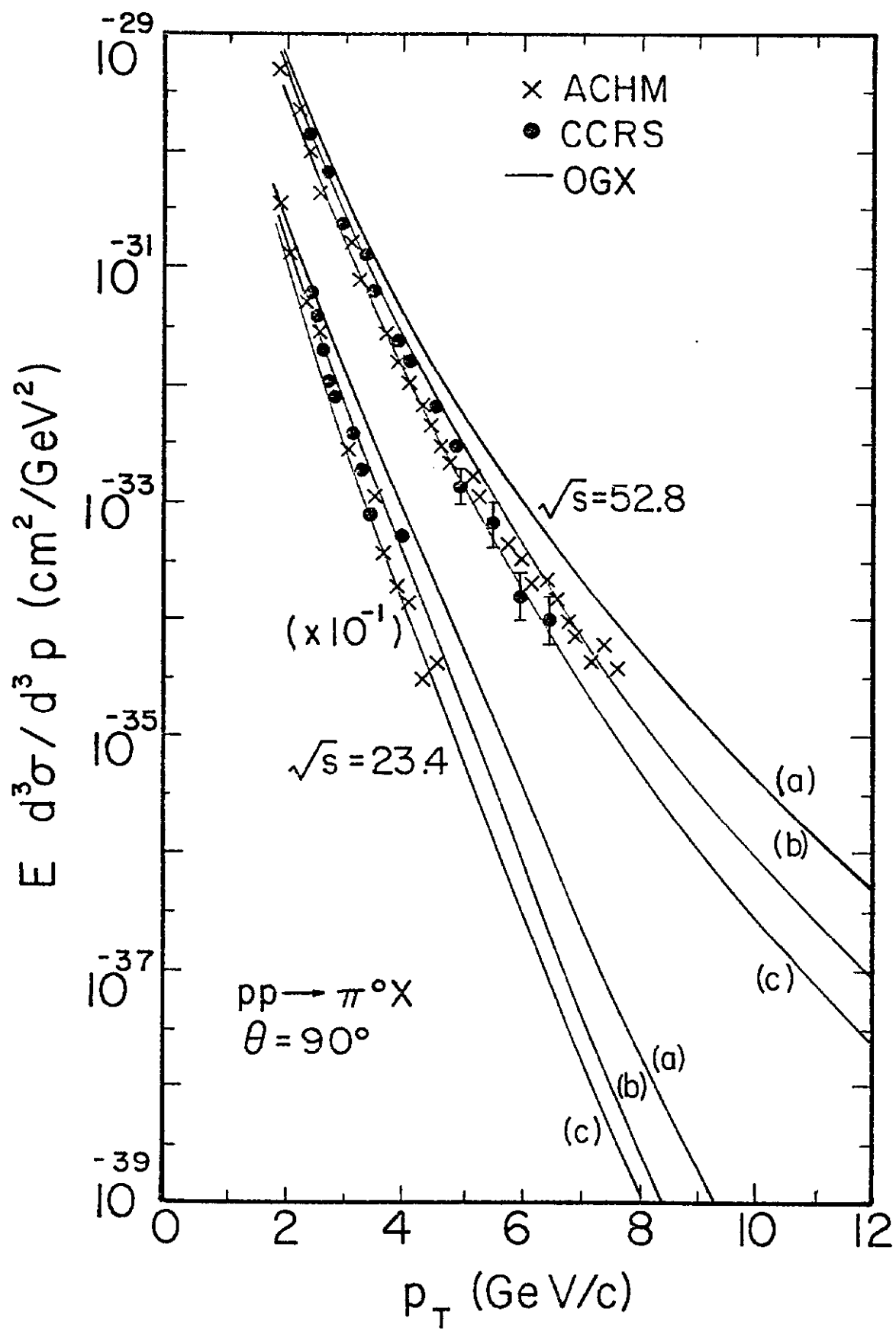
Fig. 10

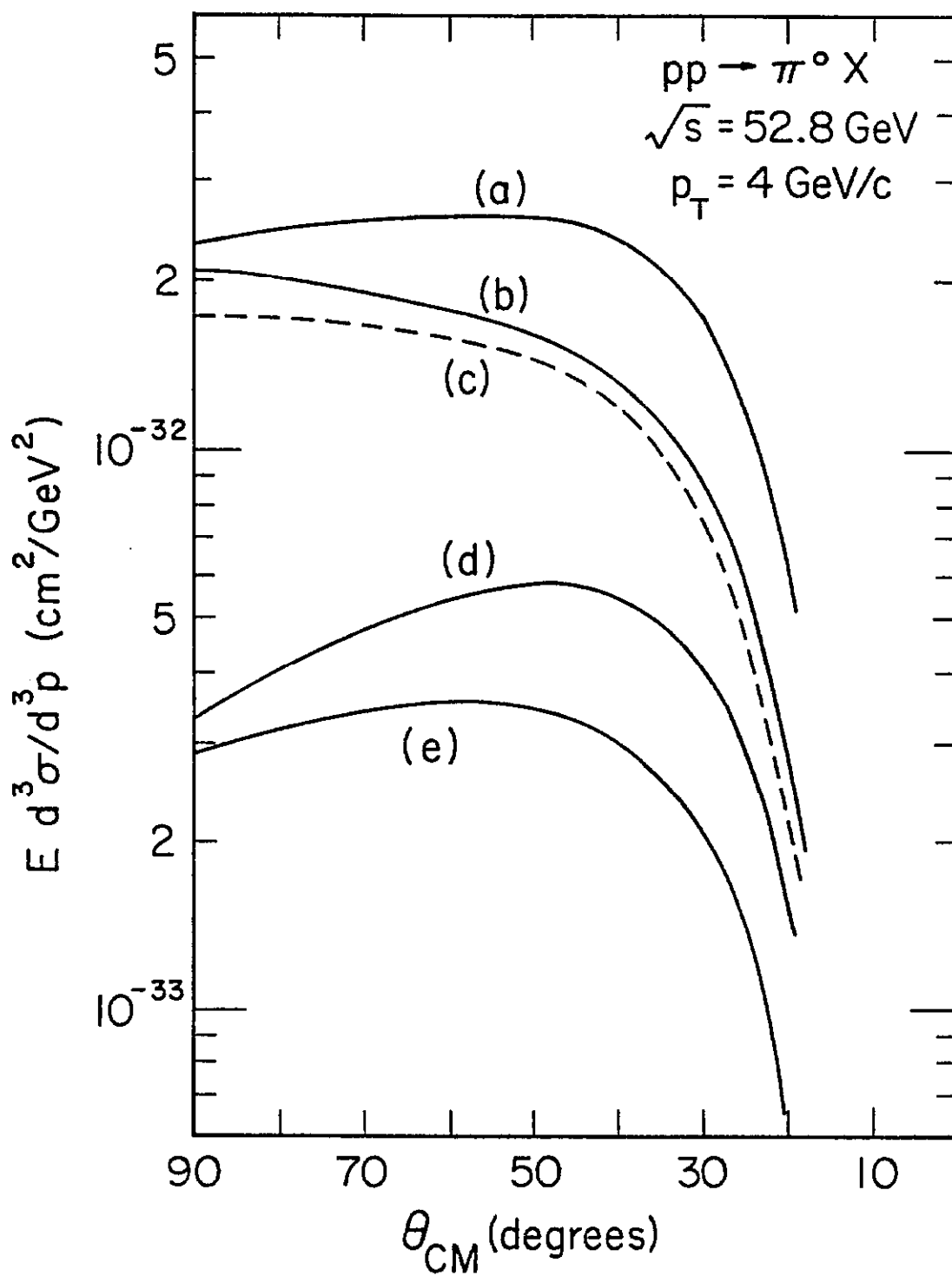$$pp \rightarrow \pi^\circ X$$
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$$\theta = 90^\circ$$
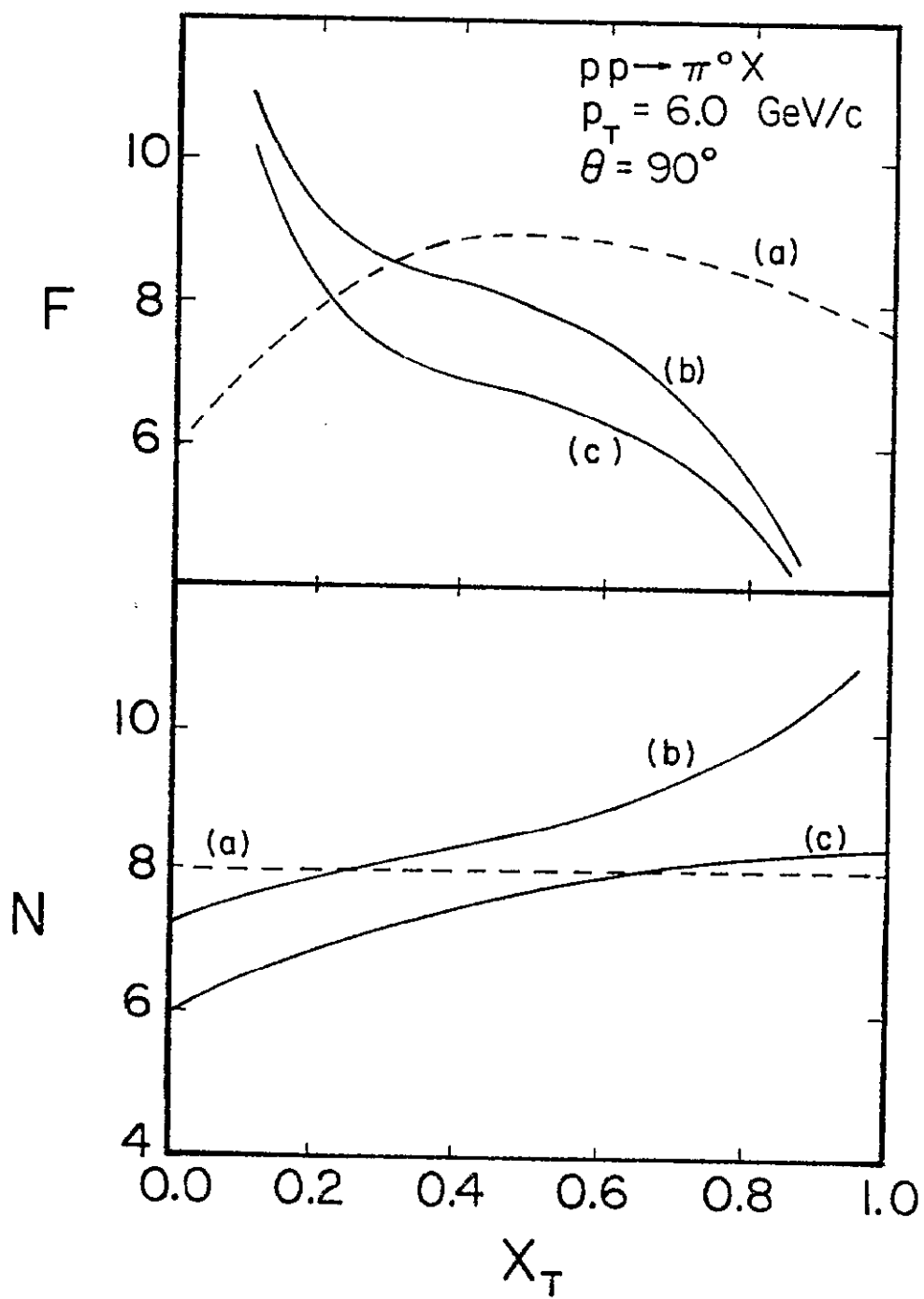
Fig. 11
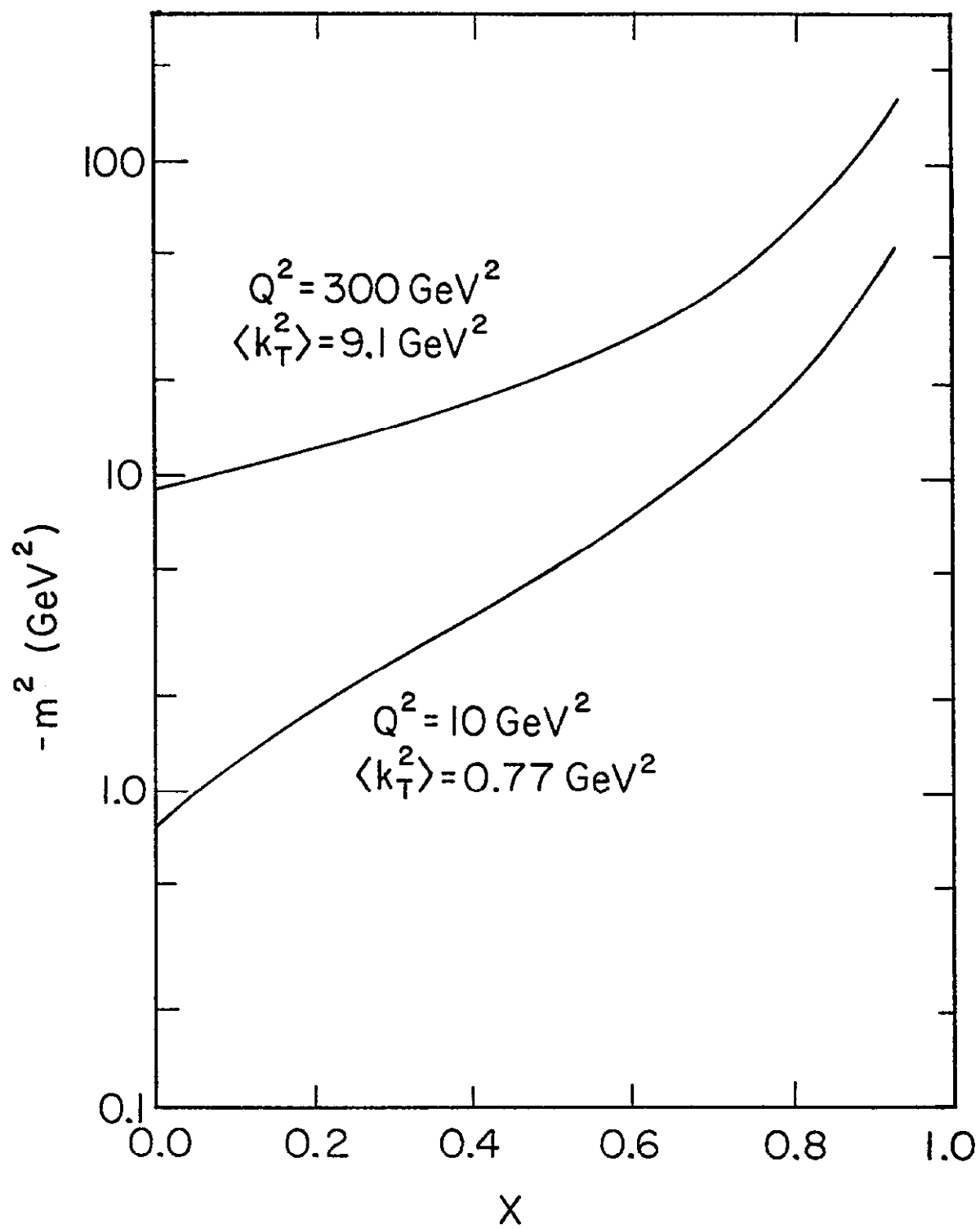
Fig. 12

Fig. 13

Fig. 14a

Fig. 14b