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Weak cosmic censorship conjecture with pressure and volume in the Gauss-Bonnet AdS black hole

Xiao-Xiong Zeng^{a,b}, Xin-Yun Hu^{c,*}, Ke-Jian He^d

^a State Key Laboratory of Mountain Bridge and Tunnel Engineering, Chongqing Jiaotong University, Chongqing 400074, China

^b Department of Mechanics, Chongqing Jiaotong University, Chongqing 400074, China
 ^c College of Economic and Management, Chongqing Jiaotong University, Chongqing 400074, China
 ^d Physics and Space College, China West Normal University, Nanchong 637000, China

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Abstract

With the Hamilton-Jacobi equation, we obtain the energy-momentum relation of a charged particle as it is absorbed by the Gauss-Bonnet AdS black hole. On the basis of the energy-momentum relation at the event horizon, we investigate the first law, second law, and weak cosmic censorship conjecture in both the normal phase space and extended phase space. Our results show that the first law, second law as well as the weak cosmic censorship conjecture are valid in the normal phase space for all the initial states are black holes. However, in the extended phase space, the second law is violated for the extremal and near-extremal black holes, and the weak cosmic censorship conjecture is violable for the near-extremal black hole, though the first law is still valid. In addition, in both the normal and extended phase spaces, we find the absorbed particle changes the configuration of the near-extremal black hole, while don't change that of the extremal black hole.

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⁶ Corresponding author. *E-mail address:* xxzengphysica@163.com (X.-Y. Hu).

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1. Introduction

According to the singularity theorems developed by Penrose and Hawking [1], we know that the formation of a singularity with infinite matter density is inevitable during the gravitational collapse. The existence of a singularity will destroy the deterministic nature of general relativity. To circumvent this problem, Penrose thus claimed that the singularity produced in the gravitational collapse must be hidden within a black hole so that a distant observer cannot perceive it [2], which is the so-called weak cosmic censorship conjecture. In this case, the expected predictability and deterministic nature of general relativity is reassured.

There is not a concrete proof of the weak cosmic censorship conjecture, we thus should check its validity in different spacetimes. Wald proposed firstly a gedanken experiment to check this conjecture by examining whether the black hole horizon can be destroyed by injecting a point particle [3]. For an extremal Kerr-Newman black hole, he found that a particle which violates the weak cosmic censorship conjecture will not be absorbed by the black hole. Until now, there are some debates on the test particle model. In the near-extremal Reissner-Nordstöm black hole and Kerr black hole, the cosmic censorship conjecture was found to be violated in [4] and [5] respectively. As the higher order terms in the energy, angular momentum, and charge of the test particle are taken into account, the weak cosmic censorship conjecture was found to be violated too even for an extremal Kerr-Newman black hole [6]. Later, it was claimed that in all of these situations, the test particle assumption was not perfect since they did not take into account the self force and back reaction effects [7]. As these effects were considered, the weak cosmic censorship conjecture was found to be valid for both the extremal and near-extremal black holes [8]. Especially, by applying the Wald formalism rather than matter, a new version of gedanken experiment has been designed recently [9,10]. The weak cosmic censorship conjecture was found to be valid for the non-extremal black holes [9-12]. In this framework, the second order variation of the mass of the black hole emerges, which somehow incorporates both the self force and back reaction effects.

With different methods, there have been some counter examples to the weak cosmic censorship conjecture, especially in spacetimes with more than four dimensions [13–16]. Even in the four dimensional AdS black holes, there are also some counter examples recently. In the Einstein-Maxwell theory [17], the weak cosmic censorship conjecture was found to be violated since the curvature grows without bound in the future to the infinite boundary observers. In the Einstein-Maxwell-dilaton theory, it was found that there was not a horizon covering the singularity and the singularity are connected by the traversable wormholes [18]. The weak cosmic censorship was found to be related with the weak gravity. It was suggested that the above counterexamples might be removed if the weak gravity conjecture holds [19–22].

Besides the weak cosmic censorship conjecture, the test particle model also can be used to discuss the first law and second law of thermodynamics of black holes. With the relation between the energy and momentum of the test particle, the first law and second law of a three dimensional black hole have been investigated [23]. The merit of the test particle model is that it also can be used to study the thermodynamics and weak cosmic censorship conjecture in the extended phase space, where the cosmological parameter and its conjugate quantity are regarded as the pressure and volume respectively [24,25]. In the extended phase space, the first law of thermodynamics and Van der Waals-like phase transition have been investigated extensively [24–31]. However, there is little work to discuss the second law as well as the weak cosmic censorship conjecture. The validity of the first law does not imply that the second law and the weak cosmic censorship conjecture are valid. Therefore, it is of great importance and necessity to study the second law

and the weak cosmic censorship in the extended phase space. The laws of thermodynamics and weak cosmic censorship conjecture with pressure and volume in the high dimensional Reissner Nordström-AdS black hole have been investigated recently [32]. It was found that the first law and the weak cosmic censorship conjecture were valid, while the second law was violated for the extremal and near-extremal black holes, which is different from the case without pressure and volume. Now, the idea in [32] has been extended to the Born-Infeld-anti-de Sitter black hole [33–35], torus-like AdS black hole [36], and three dimensional BTZ black holes [37,38]. Especially, thermodynamics and weak cosmic censorship conjecture in the Kerr-AdS black hole have also been investigated [39]. Different from the spherically symmetric black holes, the validity or violation of the second law in the extended phase space was found to be dependent of the spin parameter, radius of the AdS spacetime, and their variations.

In this paper, we intend to investigate the thermodynamics and weak cosmic censorship conjecture with pressure and volume in the Gauss-Bonnet AdS black hole. In the extended phase space, it has been found that the Gauss-Bonnet coefficient should be treated as a dynamical variable besides the cosmological parameter in order to satisfy the Smarr relation [40]. Thus in the Gauss-Bonnet gravity, the thermodynamic phase space is more extensive than that in [32]. In this paper, we want to explore how the Gauss-Bonnet coefficient affects the laws of thermodynamics and weak cosmic censorship conjecture besides the pressure and volume. In addition, in [32], the weak cosmic censorship conjecture was fond to be valid in the extended phase space for the near-extremal Reissner Nordström-AdS black hole. However, we found that there were some approximations. In our paper, we want to explore whether the weak cosmic censorship conjecture is valid in the extended phase space without approximations. As a result, we find the weak cosmic censorship conjecture for the near-extremal Gauss-Bonnet AdS black hole is violable, depending on the values of α , *l*, *r_{min}* and their variations.

The remainder of this article is organized as follows. In section 2, we will briefly review the thermodynamics of the Gauss-Bonnet AdS black hole. In section 3, we are going to get the relation between the energy and momentum of the absorbed particle near the horizon. The laws of thermodynamics and weak cosmic censorship conjecture will be discussed in the normal and extended phase space in section 4 and section 5, respectively. We employ the variation of entropy to check the second law of thermodynamics. We adopt the variation of the minimum value of the function which determine the locations of the horizons to check the weak cosmic censorship conjecture. Section 6 is devoted to our conclusions. Throughout this paper, we will set $G_d = \hbar = c = k = 1$.

2. Review of the Gauss-Bonnet AdS black hole

The action admitting the d-dimensional Einstein-Maxwell theory with a Gauss-Bonnet term and a cosmological constant term is [41–44]

$$S = \frac{1}{16\pi} \int d^d x \sqrt{-g} \left[R - 2\Lambda + \alpha_{\rm GB} \left(R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \right) - 4\pi F_{\mu\nu} F^{\mu\nu} \right],$$
(1)

where α_{GB} is the Gauss-Bonnet coefficient, Λ is the cosmological constant that relates to the AdS radius *l* with the relation $\Lambda = -\frac{(d-1)(d-2)}{2l^2}$, *R* is the Ricci scalar, *g* is determinant of the metric tensor, and $F_{\mu\nu}$ is the Maxwell field strength with the definition $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, where A_{μ} is the vector potential. The Gauss-Bonnet coefficient is set to be positive for it is proportional to the inverse string tension with positive coefficient in the low energy effective

action of heterotic string theory [41,42]. The dimension of the spacetime is supposed to be larger than four since the Gauss-Bonnet term has no dynamics in four dimensions. From the action in Eq. (1), we can obtain the following solution [41–44]

$$dS^{2} = -f(r)dt^{2} + f^{-1}(r)dr^{2} + r^{2}h_{ij}dx^{i}dx^{j},$$
(2)

with

$$f(r) = \kappa + \frac{r^2}{2\alpha} - \frac{r^2}{2\alpha} \sqrt{1 + \frac{64\pi\alpha M}{(d-2)\Omega_{\kappa}r^{d-1}} - \frac{2\alpha Q^2}{(d-2)(d-3)r^{2d-4}} - \frac{4\alpha}{l^2}},$$
(3)

where $\alpha = (d-3)(d-4)\alpha_{GB}$ is the redefined Gauss-Bonnet coefficient, M and Q are the mass and charge of the black hole respectively, and $r^2h_{ij}dx^i dx^j$ is the line element of a (d-2)-dimensional Einstein space with constant curvature $(d-2)(d-3)\kappa$ and volume Ω_{κ} . The value of κ can be 1, 0, -1, corresponding to spherical, flat and hyperbolic topology of black hole horizon. In this paper, we are interested in the case $\kappa = 1$, and the corresponded volume of the (d-2)-dimensional space is labeled as Ω .

The non-vanishing component of the vector potential is

$$A_t = -\frac{Qr^{3-d}\Omega}{16(d-3)\pi}.$$
 (4)

From the equation f(r) = 0, we can obtain two solutions, which correspond to the inner horizon and outer horizon. The outer horizon is the event horizon, labeled as r_h thereafter. At the event horizon, the mass M can be calculated as

$$M = \frac{\Omega r_h^{-d-5} \left(l^2 \left(2 \left(d^2 - 5d + 6 \right) r_h^{2d} \left(a + r_h^2 \right) + Q^2 r_h^8 \right) + 2 \left(d^2 - 5d + 6 \right) r_h^{2d+4} \right)}{32\pi (d-3) l^2}.$$
 (5)

According to the definition of the surface gravity, the Hawking temperature can be written as

$$T = \frac{-l^2 \pi Q^2 r^8 + 2(d-2)\pi r_h^{2d} \left(\alpha (d-5)^2 l^2 + (d-3) l^2 r_h^2 + (d-1) r_h^4\right)}{8r_h^{2d+1} (d-2) l^2 \left(2\alpha + r_h^2\right)}.$$
 (6)

The Bekenstein-Hawking entropy, and electric potential can be obtained by [42]

$$S_{h} = \int_{0}^{r_{h}} T^{-1} \left(\frac{\partial M}{\partial r_{h}}\right)_{Q} dr_{h} = \frac{r_{h}^{d-4} \left(2\alpha (d-2) + (d-4)r_{h}^{2}\right)\Omega}{4(d-4)\pi^{2}},$$
(7)

$$\Phi = \left(\frac{\partial M}{\partial Q}\right)_{S_h} = A_t(\infty) - A_t(r_h) = \frac{Qr_h^{3-d}\Omega}{16(d-3)\pi},$$
(8)

in which we have employed the first law of black hole thermodynamics in the normal phase space, namely the cosmological parameter is fixed. Recent investigations have shown that the cosmological parameter can be a dynamical quantity, the thermodynamic phase space thus is extended. In the extended phase space, we also can construct the first law of thermodynamics by treating the cosmological constant as a dynamical pressure and its conjugate quantity as the thermodynamic volume. In this case, the black hole mass M is explained as enthalpy H rather than internal energy U of the system. In addition, as the cosmological constant is regarded as thermodynamic pressure in the first law, the Smarr relation for black hole thermodynamics can be obtained by scaling argument. In the Gauss-Bonnet gravity, to satisfy the Smarr relation, the

$$dH = TdS_h + \Phi dQ + VdP + \mathcal{A}d\alpha, \tag{9}$$

in which P is the pressure, V is its conjugate quantity interpreted as volume, and A is the conjugate quantity of Gauss-Bonnet coefficient α , which are defined respectively as

$$P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2},\tag{10}$$

$$V = \left(\frac{\partial H}{\partial P}\right)_{S_h, Q, \alpha} = \frac{r_h^{d-1}\Omega}{d-1},\tag{11}$$

$$\mathcal{A} = \left(\frac{\partial H}{\partial a}\right)_{S_h, Q, P} = \frac{(d-2)^2 r_h^{d-5} \Omega}{16\pi}.$$
(12)

One can check that the following Smarr relation is also satisfied

$$(d-3)H = (d-2)TS_h - 2PV + 2A\alpha + (d-3)Q\Phi.$$
(13)

In the Born-Infeld AdS black hole, the conjugate quantity of Born-Infeld parameter is interpreted as Born-Infeld vacuum polarization [46]. While in the Gauss-Bonnet AdS black hole, the physical meanings of the conjugate quantity \mathcal{A} is still not known, we only know it has the dimension $[length]^{-3}$ [40].

3. Energy-momentum relation of the absorbed particle

In this section, we intend to obtain the energy-momentum relation of a charged particle near the event horizon as it is absorbed by the Gauss-Bonnet AdS black hole. We are interested in the scalar particle, so we will employ the following Hamilton-Jacobi equation to study the dynamical of the absorbed particle

$$g^{\mu\nu} \left(p_{\mu} - eA_{\mu} \right) \left(p_{\nu} - eA_{\nu} \right) + \mu^{2} = 0,$$
(14)

where $p_{\mu} = \partial_{\mu}I$ is the momentum, *e* is the charge, μ is the mass, and *I* is the Hamilton action of the particle. In the spherically symmetric spacetimes, the Hamilton action of the moving particle can be separated into

$$I = -Et + W(r) + \bigcap_{i=1}^{d-3} I_{\theta_i}(\theta_i) + L\psi,$$
(15)

in which E and L are the energy and angular momentum of the particle respectively, and the (d-2)-dimensional sphere has been expressed as

$$h_{ij}dx^{i}dx^{j} = \sum_{i=1}^{d-2} \left(\prod_{j=1}^{i} \sin^{2}\theta_{j-1}\right) d\theta_{i}^{2}, \theta_{d-2} \equiv \psi.$$
(16)

To solve the Hamilton-Jacobi equation, we will use the inverse metric of the black hole in Eq. (2)

$$g^{\mu\nu}\partial_{\mu}\partial_{\nu} = -f(r)^{-1}(\partial_{t})^{2} + f(r)(\partial_{r})^{2} + r^{-2}\sum_{i=1}^{d-2} {i \choose j=1} \sin^{-2}\theta_{j-1} \partial_{\theta_{i}}^{2} \partial_{\theta_{i}}^{2} d_{\theta_{i}}^{2} d_{\theta_{i$$

Substituting Eqs. (15) and (17) into Eq. (14), we can obtain

$$-\frac{1}{f(r)}\left(-E - eA_{t}\right)^{2} + f(r)\left(\partial_{r}W(r)\right)^{2} + r^{-2}\sum_{i=1}^{d-3} \left(\prod_{j=1}^{i} \sin^{-2}\theta_{j-1}\right) \left(\partial_{\theta_{i}}I(\theta_{i})\right)^{2} + r^{-2} \left(\prod_{j=1}^{d-2} \sin^{-2}\theta_{j-1}\right) L^{2} + u^{2} = 0.$$
(18)

With a variable \mathcal{K} to separate this equation, we can get the radial equation and angular equation

$$-\frac{r^2}{f(r)}\left(-E - eA_t\right)^2 + r^2 f(r) \left(\partial_r W(r)\right)^2 + r^2 \mu^2 = -\mathcal{K},$$
(19)

$$\sum_{i=1}^{d-3} \left(\prod_{j=1}^{i} \sin^{-2} \theta_{j-1} \right) \left(\partial_{\theta_i} I(\theta_i) \right)^2 + \left(\prod_{j=1}^{d-2} \sin^{-2} \theta_{j-1} \right) L^2 = \mathcal{K}.$$

$$(20)$$

Lastly, we obtain the radial momentum

$$p^{r} \equiv g^{rr} \partial_{r} W(r) = f(r) \sqrt{\frac{-\mu^{2} r^{2} + \mathcal{K}}{r^{2} f(r)} + \frac{1}{f(r)^{2}} (-E - eA_{t})^{2}}.$$
(21)

As a particle drops into the black hole, we will pay attention to the thermodynamics of the black hole. Therefore, we are interested only the near horizon behavior of the particle, where $f(r_h) = 0$. Near the event horizon, Eq. (21) will be simplified as

$$E = -A(r_h)e + \left| p_h^r \right|, \tag{22}$$

in which $p_h^r = p^r(r_h)$. For the $|p_h^r|$ term, we will choose the positive sign thereafter as done in [47–49] in order to assure the particle drops into the black hole in a positive flow of time direction.

4. Thermodynamics and weak cosmic censorship conjecture in the normal phase space

In this section, we will investigate the thermodynamics and weak cosmic censorship conjecture in the normal phase space on the basis of Eq. (22). We want to explore whether the energy-momentum relation can check the validity of the second law of thermodynamics and weak cosmic censorship conjecture besides produce the first law of thermodynamics.

4.1. The first law of thermodynamics in the normal phase space

In the normal phase space, the cosmological parameter is fixed, and the black hole is characterized by the mass M and charge Q. The mass M is interpreted as the internal energy of the thermodynamic system. As a particle is absorbed by the black hole, we suppose the energy and charge are conserved, that is, the change of the internal energy and charge of the black hole satisfy

$$E = dM, e = dQ. \tag{23}$$

In this case, Eq. (22) changes into

$$dM = \frac{Qr_h^{3-d}\Omega}{16(d-3)\pi} dQ + p_h^r.$$
(24)

The absorbed charged particle will also change the location of the event horizon. We label the final state of the event horizon as $r_h + dr_h$, which satisfies $f(r_h + dr_h) = 0$ too. According to $f(r_h + dr_h) = f(r_h) = 0$, there is always a relation

$$df_h = \frac{\partial f_h}{\partial M} dM + \frac{\partial f_h}{\partial Q} dQ + \frac{\partial f_h}{\partial r_h} dr_h = 0.$$
⁽²⁵⁾

Inserting Eq. (24) into Eq. (25), we find both dM and dQ are eliminated. The solution of dr_h thus can be expressed as

$$dr_h = \frac{32l^2\pi\Omega^{-1}p_h^r r_h^{6+d}}{2\left(2-3d+d^2\right)r_h^{4+2d} + l^2\left(-Q^2r_h^8 + 2(d-2)r_h^{2d}\left(\alpha(d-5) + (d-3)r_h^2\right)\right)}.$$
 (26)

In addition, based on Eq. (7), the variation of the entropy can be expressed as

$$dS_h = \frac{(d-2)r_h^{d-5} \left(2\alpha + r_h^2\right) \Omega}{4\pi^2} dr_h.$$
 (27)

Substituting Eq. (26) into Eq. (27), we have

$$dS_h = \frac{8\pi^{-1}(d-2)l^2 p_h^r r_h^{1+2d} \left(2\alpha + r_h^2\right)}{2\left(2-3d+d^2\right)r_h^{4+2d} + l^2\left(-Q^2 r_h^8 + 2(d-2)r_h^{2d} \left(\alpha(d-5) + (d-3)r_h^2\right)\right)}.$$
 (28)

From Eqs. (6) and (28), we get

$$TdS_h = p_+^r. (29)$$

Combining Eqs. (24) and (29), we find

$$dM = T dS_h + \Phi dQ, \tag{30}$$

which is nothing but the first law of thermodynamics in the normal phase space. That is, the first law is valid in the normal phase space as a charged particle is absorbed by the Gauss-Bonnet AdS black hole.

4.2. The second law of thermodynamics in the normal phase space

The second law of black hole thermodynamics states that the entropy of the black holes never decrease in the clockwise direction. As a particle is absorbed by the Gauss-Bonnet AdS black hole, the entropy of the black hole also should increase if the second law is valid, namely the variation of the entropy should satisfy $dS_h > 0$. In this section, we will employ Eq. (28) to check whether this is true.

For an extremal black hole, its temperature vanishes. From Eq. (29), we know that the variation of entropy is divergent, which is meaningless. Thus we mainly concentrate on the near-extremal black hole thereafter. We will obtain dS_h by numeric method. During the numeric calculation, we focus on studying how α and d affect the value of dS_h by fixing Q = 7, $l = p^r = \Omega = 1$. For a given values of α and d, we can obtain the mass of the extremal black hole by solving equation $f(r_h) = 0$. For the case that the two roots are the same, the corresponded mass is that of the extremal black hole. For example, when d = 5, the masses of the extremal black hole is shown in Table 1. The mass of the non-extremal black holes should be larger than that of the extremal black hole. For different d and α , the values of dS_h are given in Table 1, Table 2, and Table 3. From these tables, we can see clearly that the variation of the entropy is positive always for the extremal and non-extremal black holes. For the extremal black holes, though the values seem finite, there are much larger than the near extremal black hole, which can be regarded as

Table 1 The relation between dS_h , M and r_h for d = 5 in the normal phase space.

$\alpha = 2$			$\alpha = 1$			$\alpha = 0.1$		
М	r_h	dS_h	М	r_h	dS_h	М	r_h	dS_h
0.478672011	1.059899	275.84	0.41898843	1.05925	4004.7	0.36527364	1.05928	825.02
0.4787	1.064454	35.889	0.4190	1.06256	34.250	0.3653	1.06426	9.6753
0.48	1.095201	5.3898	0.42	1.09059	3.7818	0.370	1.12751	0.8177
0.5	1.206054	1.4548	0.5	1.34925	0.5288	0.5	1.43458	0.2413
0.6	1.415228	0.6771	0.6	1.49469	0.3878	0.6	1.55497	0.2086
0.7	1.540696	0.5243	0.7	1.60113	0.3296	0.7	1.64970	0.1914
0.8	1.638024	0.4479	0.8	1.68814	0.2951	0.8	1.72953	0.1799
0.9	1.719498	0.3996	0.9	1.76288	0.2715	0.9	1.79928	0.1717

Table 2 The relation between dS_h , M and r_h for d = 6 in the normal phase space.

$\alpha = 2$			$\alpha = 1$			$\alpha = 0.1$		
М	r_h	dS_h	М	r _h	dS_h	М	r _h	dS_h
0.469357085	0.927431	12640.4	0.39485042	0.94529	1587.9	0.32655982	0.961894	1084.7
0.4694	0.931772	23.3241	0.3949	0.95002	13.440	0.3266	0.96623	6.0210
0.47	0.944334	6.19426	0.40	0.99447	1.4585	0.33	1.00257	0.7310
0.5	1.047712	1.09073	0.5	1.17392	0.4427	0.5	1.25735	0.1915
0.6	1.180504	0.63647	0.6	1.26516	0.3576	0.6	1.33130	0.1744
0.7	1.264667	0.52439	0.7	1.33420	0.3171	0.7	1.39099	0.1645
0.8	1.330526	0.46491	0.8	1.39119	0.2917	0.8	1.44180	0.1577
0.9	1.385745	0.42571	0.9	1.44030	0.2735	0.9	1.48639	0.1526

Table 3 The relation between dS_h , M and r_h for d = 7 in the normal phase space.

$\alpha = 2$			$\alpha = 1$			$\alpha = 0.1$		
М	r _h	dS_h	М	r_h	dS_h	М	r_h	dS_h
0.463091855	0.878523	1642.9	0.38442131	0.90054	1129.2	0.31006521	0.92257	294.88
0.4631	0.879883	40.076	0.3845	0.90504	8.4265	0.3101	0.92564	5.1496
0.47	0.919834	1.5810	0.40	0.96557	0.7360	0.32	0.97599	0.3784
0.5	0.975948	0.8116	0.5	1.08097	0.3697	0.5	1.15811	0.1586
0.6	1.069715	0.5323	0.6	1.14703	0.3125	0.6	1.21152	0.1475
0.7	1.131048	0.4581	0.7	1.19760	0.2843	0.7	1.25492	0.1409
0.8	1.179453	0.4183	0.8	1.23951	0.2663	0.8	1.29194	0.1364
0.9	1.220202	0.3917	0.9	1.27567	0.2533	0.9	1.32443	0.1330

divergent. So, the second law of thermodynamics is valid for all the black holes in the normal phase space.

In addition, from Table 1, Table 2, and Table 3, we also can observe how d and α affect the values of dS_h . For fixed values of d and α , we find the values of dS_h decrease as the black holes move far away from the extremal black hole. As the value of d is fixed, the value of the mass of the extremal black hole decreases as the value of α reduces. As the mass of a black hole is fixed,

the value of dS_h decreases while the value of r_h increases as α reduces. When the value of α is fixed, we also can see how d affect the value of dS_h and r_h . As the value of d increases, the value of the mass of extremal black hole decreases, while dS_h and r_h increases.

4.3. Weak cosmic censorship conjecture in the normal phase space

The weak cosmic censorship conjecture states that an observer located at future null infinity can not observe the singularity of a spacetime for the singularity is hidden by the horizon. In this section, we want to check whether this is true as a particle drops into the Gauss-Bonnet black hole. We will explore whether there is a horizon after the particle is absorbed.

The event horizon of the black hole is determined by the function f(r), so we concentrate on studying how f(r) changes. The function f(r) has a minimum value at radial coordinate r_{\min} . For the case $f(r_{\min}) < 0$, there are two roots, for the case $f(r_{\min}) = 0$, the two roots coincide, and the black hole becomes into an extremal black hole, for the case $f(r_{\min}) > 0$, the function has no real root so that there is not an event horizon. Our motivation is to explore how $f(r_{\min})$ moves as a charged particle is absorbed by the Gauss-Bonnet AdS black hole. For the initial state is a black hole, $f(r_{\min})$ satisfies the following conditions [50–55]

$$f(M, Q, \alpha, l, r)|_{r=r_{\min}} \equiv f_{\min} = \delta \le 0,$$
(31)

$$\partial_r f(M, Q, \alpha, l, r)|_{r=r_{\min}} \equiv f'_{\min} = 0, \tag{32}$$

$$(\partial_r)^2 f(M, Q, \alpha, l, r)|_{r=r_{\min}} \equiv f_{\min}^{\prime\prime} > 0.$$
(33)

For the extremal black hole, $\delta = 0$, r_h and r_{\min} are coincident. For the near-extremal black hole, δ is a small quantity, r_{\min} locates at the middle of the two horizons.

In the normal phase space, the state parameters of the black hole are the mass M and charge Q. As a charged particle is absorbed, the mass and charge of the black hole will change into M + dM and Q + dQ. Correspondingly, the horizon r_h and radial coordinate r_{\min} will change into $r_h + dr_h$ and $r_{\min} + dr_{\min}$. Note that Eq. (32) is satisfied at both r_{\min} and $r_{\min} + dr_{\min}$, which implies

$$df'_{\min} = \frac{\partial f'_{\min}}{\partial M} dM + \frac{\partial f'_{\min}}{\partial Q} dQ + \frac{\partial f'_{\min}}{\partial r_{\min}} dr_{\min} = 0.$$
(34)

In addition, at $r_{\min} + dr_{\min}$, the function $f(r_{\min} + dr_{\min})$ can be expressed as

$$f(r_{\min} + dr_{\min}) = f_{\min} + df_{\min} = \delta + \frac{\partial f_{\min}}{\partial M} dM + \frac{\partial f_{\min}}{\partial Q} dQ.$$
(35)

Firstly, we are interested in the extremal black hole, where $f_{\min} = \delta = 0$ and Eq. (24) is applicable. Inserting Eq. (24) into Eq. (35), we can get finally

$$df_{\min} = -\frac{16\pi p_h^r r_{\min}^{-(d-5)}}{(d-2) \left(2\alpha + r_{\min}^2\right) \Omega},$$
(36)

which seems to be negative always. However, note that the black hole is an extremal black hole here, from Eq. (29), we know that $p_h^r = 0$, so $df_{\min} = 0$, implying that the extremal Gauss-Bonnet AdS black hole is stable. In other words, as a particle drops into the extremal Gauss-Bonnet AdS black hole, the final state of the black hole is still an extremal black hole. The weak cosmic censorship conjecture thus is valid in this case.



Fig. 1. The value of df_{\min} for different dQ and r_h for $p^r = \alpha = \Omega = 1$, $\epsilon = 0.0001$, d = 5.

For a near-extremal black hole, Eq. (24) is not applicable since r_m and r_h are not coincident. With the condition $r_h = r_{\min} + \epsilon$, we can expand Eq. (24) at r_{\min} , which leads to

$$d\mathbf{M} = \left(p_h^r + \frac{Qr_{\min}^{3-d}\Omega dQ}{16(d-3)\pi}\right) - \frac{\left(Qr_{\min}^{2-d}\Omega dQ\right)\epsilon}{16\pi} + O(\epsilon)^2.$$
(37)

Combining Eq. (35) and Eq. (37), we have

$$df_{\min} = -\frac{16\left(\pi p_h^r r_{\min}^{5-d}\right)}{(d-2)\left(2\alpha + r_{\min}^2\right)\Omega} + \frac{Qr_{\min}^{7-2d}\epsilon dQ}{(d-2)\left(2\alpha + r_{\min}^2\right)} + O(\epsilon)^2.$$
(38)

In the Eq. (38), ϵ is a very small quantity, so the third terms can be neglected approximately. In addition, comparing with the first term, the second term is smaller than it always. In this case, Eq. (38) is negative too, please refer to Fig. 1. Therefore, for the near-extremal black hole, the weak cosmic censorship conjecture is also valid under a charged particle absorption in the normal phase space.

In fact, the high order corrections are important to discuss the weak cosmic censorship conjecture [9]. However, in our paper, we find it has little effect, which is shown in Fig. 1. From this figure, we know that for different dQ and r_h , the value of df_{\min} is negative always. As the values of p^r , α , Ω , ϵ , d change, it is still negative too though the values of df_{\min} changes, which is not shown here.

5. Thermodynamics and weak cosmic censorship conjecture in the extended phase space

In the normal phase space, we have derived the first law of thermodynamics and found that the second law as well as the weak cosmic censorship conjecture are valid. In this section, we intend

to discuss the thermodynamics and weak cosmic censorship conjecture in the extended phase space, where the pressure and volume emerge. We will explore whether the first law, second law, as well as the weak cosmic censorship conjecture are valid in this framework.

5.1. The first law of thermodynamics in the extended phase space

In the extended phase space, the mass M is not the internal energy U but the enthalpy H of the thermodynamic system. The relation between the internal energy and enthalpy is [40]

$$M = U + PV + \alpha \mathcal{A}.$$
(39)

In this case, the variation of the energy and charge takes the form as

$$E = dU = d(M - PV - \alpha \mathcal{A}), e = dQ.$$
(40)

Correspondingly, the relation between the energy and momentum in Eq. (22) should be rewritten as

$$dU = d(M - PV - \alpha A) = \frac{Qr_h^{3-d}\Omega}{16(d-3)\pi} dQ + p_h^r.$$
 (41)

In addition, in order to obtain the first law of thermodynamics in the extended phase space, besides dS, we also should find dV and dA. From Eq. (11) and Eq. (12), we have

$$dV = \Omega r_h^{d-2} dr_h,$$

$$dA = \frac{(d-5)(d-2)\Omega r_h^{d-6}}{dr_h} dr_h.$$
(42)
(43)

To get the final results of dV and dA, we should find dr_h .

In the extended phase space, the state parameters of the Gauss-Bonnet AdS black hole are M, Q, l, α , as a particle drops into the black hole, the state parameters change into M + dM, Q + dQ, l + dl, $\alpha + d\alpha$. Correspondingly, the event horizon will change into $r_h + dr_h$. Based on the fact that $f(r_h + dr_h) = f(r_h) = 0$, we find

$$df_h = \frac{\partial f_h}{\partial M} dM + \frac{\partial f_h}{\partial Q} dQ + \frac{\partial f_h}{\partial l} dl + \frac{\partial f_h}{\partial r_h} dr_h + \frac{\partial f_h}{\partial \alpha} d\alpha = 0.$$
(44)

Combining Eqs (44) and (41), we find all the variables are eliminated except for dr_h and p_h^r , so we get

$$dr_h = -\frac{32\pi p_h^r r_h^{4+d}}{\left(Q^2 r_h^6 - 12r_h^{2d} + 10dr_h^{2d} - 2d^2 r_h^{2d}\right)\Omega}.$$
(45)

Inserting Eq. (45) into Eqs. (27), (42), and (43), we get

$$dS_h = \frac{8(d-2)p_h^r \left(2\alpha + r_h^2\right)}{\pi r_h \left(2\left(6 - 5d + d^2\right) - Q^2 r_h^{6-2d}\right)},\tag{46}$$

$$dV = \frac{32\pi p_h^r r_h^{2+2d}}{-Q^2 r_h^6 + 2\left(6 - 5d + d^2\right) r_h^{2d}},\tag{47}$$

$$d\mathcal{A} = \frac{2(d-5)(d-2)p'_h}{r_h^2 \left(2\left(d^2 - 5d + 6\right) - Q^2 r_h^{6-2d}\right)}.$$
(48)

Combining Eqs (6), (10), (46), (47), (48), we find

$$TdS_h - PdV - \alpha d\mathcal{A} = p_h^r.$$
⁽⁴⁹⁾

Substituting Eq (49) into Eq (41), we get

$$dU = \Phi dQ + T dS_h - P dV - \alpha d\mathcal{A}.$$
(50)

The relation between the internal energy and enthalpy in Eq (39) also can be written as

$$dM = dU + PdV + VdP + \alpha d\mathcal{A} + \mathcal{A}d\alpha.$$
⁽⁵¹⁾

Substituting Eq (51) into Eq (50), we can obtain lastly

$$dM = T dS_h + \Phi dQ + V dP + \mathcal{A} d\alpha, \tag{52}$$

which is the first law of thermodynamics in the extended phase space. That is, the first law of the Gauss-Bonnet AdS black hole in the extended phase space can be obtained under a charged particle absorption.

5.2. The second law of thermodynamics in the extended phase space

In the extended phase space, we have proved that the first law of thermodynamics is valid. However, the validity of the first law does not mean the second law is valid [32]. So we should check the second law in the extended phase space. The second law of thermodynamics states that the entropy of the black hole never decreases. As the particle is absorbed, the entropy of the final state thus should be larger than the initial state according to the second law of thermodynamics. Next, we will check whether this is true with Eq. (46).

We first study the case of the extremal black hole, for which the temperature vanishes. On the basis of Eq. (6), we can obtain a critical charge

$$Q_c = \frac{\sqrt{2}\sqrt{d-2}r_h^{d-4}\sqrt{\alpha dl^2 - 5\alpha l^2 + dl^2 r_h^2 + dr_h^4 - 3l^2 r_h^2 - r_h^4}}{l}.$$
(53)

Substituting Eq. (53) into Eq. (46), we get finally

$$dS_h = -\frac{4l^2 p_h^r r_h \left(2\alpha + r_h^2\right)}{\pi \left(\alpha (d-5)^2 l^2 + (d-1)r_h^4\right)},$$
(54)

which is negative, implying that the second law is invalid for the extremal Gauss-Bonnet AdS black hole.

Next, we focus on investigating the non-extremal black hole. We will adopt the same numerical method as that in the normal phase space. The difference is that we will employ Eq. (46) to obtain the variance of entropy. We also set $\Omega = l = p^r = 1$, and Q = 7. For given values of d and α , we can obtain the masses of the extremal black holes. For example, for d = 6, the masses of extremal black holes are 0.469357085, 0.39485042, 0.32655982 for $\alpha = 2, 1, 0.1$ respectively. For any non-extremal black holes with different d and α , we can calculate the values of r_h and dS_h , which are listed in Table 4, Table 5, and Table 6. From these tables, we find that as the mass of the black hole increases, the event horizon of the black hole increases. While for dS_h , there is a divergent point, which divides the variation of entropy into positive region and negative region. The variation of entropy is negative for the near-extremal black holes while positive for the

Table 4 The relation between dS_h , M and r_h for d = 5 in the extended phase space.

$\alpha = 2$			$\alpha = 1$			$\alpha = 0.1$		
М	r_h	dS_h	М	r_h	dS_h	М	r_h	dS_h
0.478672011	1.059899	-1.377	0.41898843	1.05925	-0.8363	0.36527364	1.05928	-0.3542
0.4787	1.064454	-1.408	0.4190	1.06256	-0.8509	0.3653	1.06426	-0.3652
0.48	1.095201	-1.644	0.42	1.09059	-0.9869	0.37	1.12751	-0.5442
0.50	1.206054	-3.096	0.5	1.34925	-7.7667	0.45	1.35594	-4.6024
0.60	1.415228	-150.8	0.5315	1.40189	-31.478	0.4865	1.41508	-54.04
0.60125	1.417078	-214.2	0.5485	1.42717	114.439	0.4925	1.42388	150.74
0.605	1.422573	913.54	0.55	1.42932	83.1982	0.5	1.43458	27.901
0.65	1.482854	17.119	0.6	1.49470	9.9134	0.6	1.55497	3.5542
0.70	1.540696	9.5658	0.7	1.60113	4.79158	0.7	1.64970	2.5127
0.80	1.638024	6.0014	0.8	1.68814	3.67834	0.8	1.72953	2.1607
0.90	1.719498	4.8332	0.9	1.76288	3.19564	0.9	1.79928	1.9925

Table 5

The relation between dS_h , M and r_h for d = 6 in the extended phase space.

$\alpha = 2$			$\alpha = 1$			$\alpha = 0.1$		
М	r_h	dS_h	М	r_h	dS_h	М	r_h	dS_h
0.469357085	0.927431	-1.007	0.39485042	0.945294	-0.698	0.32655982	0.96189	-0.315
0.4694	0.931772	-1.046	0.3949	0.950017	-0.729	0.3266	0.96623	-0.331
0.47	0.944334	-1.170	0.40	0.994473	-1.149	0.35	1.06945	-1.463
0.50	1.047712	-3.799	0.450	1.10992	-13.43	0.38	1.12536	-107.1
0.5475	1.122003	-85.22	0.4515	1.12219	-55.23	0.381	1.12691	178.29
0.5525	1.128293	190.46	0.4615	1.12662	787.30	0.395	1.14719	5.3786
0.56	1.137380	34.707	0.5	1.17392	5.5541	0.50	1.25735	1.2439
0.60	1.180504	7.8941	0.60	1.26516	2.4055	0.60	1.33130	0.9929
0.70	1.264667	3.7510	0.70	1.33420	1.8846	0.70	1.39099	0.9070
0.80	1.330526	2.9123	0.80	1.39119	1.6712	0.80	1.44180	0.8681
0.90	1.385746	2.5478	0.90	1.44030	1.5566	0.90	1.48639	0.8486

far-extremal black holes. That is, the second law of thermodynamics is violated for the extremal Gauss-Bonnet AdS black hole in extended phase space. This conclusion is independent of the values of d and α .

We also can investigate how d and α affect the values of dS_h . From Table 4, Table 5, and Table 6, we find that as the values of α decrease, the values of the critical horizon where dS_h is divergent become smaller. And as the values of d decrease, the values of the divergent point become smaller too.

In fact, the relation between dS_h and r_h also can be plotted on the basis of Eq. (46). In Fig. 2, Fig. 3, and Fig. 4, we fix the dimension d to investigate the effect of α on dS_h . Obviously, there is a phase transition point which divides dS_h into two branches. Interestedly, the phase transition point is independent of the values of α . Taking the case d = 6 as an example, which is shown in Table 5. From Table 5, we know that the radius of the extremal black holes are 0.927431, 0.945294, 0.96189. While from Fig. 3, we know that the phase transition point is about 1.12. Thus, for the near-extremal black holes, dS_h is negative always, which is independent of α . From

Table 6 The relation between dS_h , M and r_h for d = 7 in the extended phase space.

$\alpha = 2$			$\alpha = 1$			$\alpha = 0.1$		
М	r _h	dS_h	М	r_h	dS_h	М	r_h	dS_h
0.463091855	0.878523	-0.705	0.38442131	0.900537	-0.542	0.31006521	0.922567	-0.272
0.4631	0.879883	-0.717	0.3845	0.905037	-0.576	0.3101	0.925639	-0.286
0.47	0.919834	-1.206	0.39	0.939131	-0.953	0.34	1.01595	-4.866
0.50	0.975948	-3.307	0.40	0.965567	-1.556	0.3515	1.03265	7.4142
0.545	1.025403	-694.5	0.435	1.01910	-17.93	0.3715	1.05687	1.8621
0.548	1.028150	82.552	0.44	1.02494	-160.9	0.4	1.08520	1.1128
0.55	1.029951	47.969	0.441	1.02607	321.84	0.5	1.15810	0.6817
0.60	1.069715	5.3613	0.6	1.14703	1.5564	0.6	1.21152	0.5953
0.70	1.131048	2.7381	0.7	1.19760	1.2847	0.7	1.25492	0.5621
0.80	1.1794532	2.1622	0.8	1.23951	1.1641	0.8	1.29194	0.5468
0.90	1.220203	1.9073	0.9	1.27567	1.0967	0.9	1.32443	0.5395



Fig. 2. The relation between dS and r_h for different α with d = 5.

Fig. 3, we can conclude that the second law is violated for the near-extremal black holes, which is consistent with that obtained in Table 4. For the case d = 7, the phase transition point is about 1.02, which is larger than the radius of the extremal black holes listed in Table 6. Therefore, we also can conclude that the second law is violated for the near-extremal black holes. This conclusion will not be changed for d = 5, which is shown in Fig. 2. The invalidity of the second law for the near-extremal Gauss-Bonnet AdS black holes thus is universal, independent of the values of α and d.

5.3. Weak cosmic censorship conjecture in the extended phase space

In the extended phase space, the second law of thermodynamics was found to be invalid for the extremal and near-extremal black holes, which is different from that in the normal phase space.



Fig. 3. The relation between dS and r_h for different α with d = 6.



Fig. 4. The relation between dS and r_h for different α with d = 7.

Therefore, it is necessary to check whether the weak cosmic censorship conjecture is valid in these cases.

Similar to that in the normal phase space, we will also investigate how f(r) changes as a charged particle is absorbed. The difference is that in this case, the state parameters are M, Q, α, l . The final state of the black hole thus should be the function of M + dM, Q + dQ, $\alpha + d\alpha$, l + dl. Correspondingly, the horizon and minimum point of the final state are $r_h + dr_h$, $r_{\min} + dr_{\min}$. At $r_{\min} + dr_{\min}$, we find there is always a relation

$$\partial_r f(r)|_{r=r_{\min}+dr_{\min}} = f'_{\min} + df'_{\min} = 0.$$
(55)

Using the condition $f'_{\min} = 0$ in Eq. (32), we obtain $df'_{\min} = 0$. Expanding it further, we get

$$df'_{\min} = \frac{\partial f'_{\min}}{\partial M} dM + \frac{\partial f'_{\min}}{\partial Q} dQ + \frac{\partial f'_{\min}}{\partial l} dl + \frac{\partial f'_{\min}}{\partial r_{\min}} dr_{\min} + \frac{\partial f'_{\min}}{\partial \alpha} d\alpha = 0.$$
(56)

At $r_{\min} + dr_{\min}$, the function f(r) takes the form as

$$f|_{r=r_{\min}+dr_{\min}} = f_{\min} + df_{\min}$$
$$= \delta + \left(\frac{\partial f_{\min}}{\partial M} dM + \frac{\partial f_{\min}}{\partial Q} dQ + \frac{\partial f_{\min}}{\partial l} dl + \frac{\partial f_{\min}}{\partial \alpha} d\alpha\right).$$
(57)

For the extremal black holes, we know $f'_{\min} = 0$ and $f_{\min} = \delta = 0$. Substituting dM in (41) into Eq. (57), we get

$$df_{\min} = -\frac{r_{\min}^{3-d} \left(16\alpha d\mathcal{A}l^2 \pi r_{\min}^2 + 16l^2 \pi p_h^r r_{\min}^2 + \left(2 - 3d + d^2\right) dr_{\min} r_{\min}^d \Omega\right)}{(-2+d)l^2 \left(2\alpha + r_{\min}^2\right)\Omega}.$$
 (58)

Inserting p_h^r in Eq. (49) into Eq. (58), we find

$$df_{\min} = -\frac{l^2 \left(2(d-2)r_{\min}^{2d} \left(\alpha(d-5) + (d-3)r_{\min}^2\right) - Q^2 r_{\min}^8\right) + 2 \left(d^2 - 3d + 2\right) r_{\min}^{2d+4}}{2(d-2)l^2 \left(2\alpha + r_{\min}^2\right) r_{\min}^{2d+1}} dr_{\min}.$$
(59)

For the extremal black holes, the extremal charge in Eq. (53) is also applicable. Substituting Eq. (53) into Eq. (59), we gat lastly

$$df_{\min} = 0, \tag{60}$$

which means that $f(r_{\min})$ does not change as a charged particle drops into the extremal Gauss-Bonnet AdS black hole. The extremal black hole thus is still an extremal black hole. The weak cosmic censorship conjecture thus is valid for there is always a horizon to hidden the singularity.

For the near-extremal black hole, Eq. (41) is not applicable. But we can expand it near the minimum point with the relation $r_h = r_{\min} + \epsilon$. To the first order, we find

$$dM = \frac{r_{\min} \left(l^3 Q r_{\min}^8 dQ + (6 - 5d + d^2) r_{\min}^{2d} (d\alpha^2 l^3 - 2dl r_{\min}^4)\right) \Omega}{16(d - 3)l^3 \pi r_{\min}^{6+d}} \\ + \frac{(d - 3) \left(2 - 3d + d^2\right) r_{\min}^{4+2d} dr_{\min} \Omega}{16(d - 3)l\pi} \\ + \frac{(d - 3) \left(-Q^2 r_{\min}^8 + 2(d - 2)r_{\min}^{2d} (\alpha(d - 5) + (d - 3)r_{\min}^2)\right) \Omega}{32(d - 3) \pi} \\ - \frac{r_{\min} \left(l^3 Q r_{\min}^8 dQ - (d - 2)r_{\min}^{2d} ((d - 5)d\alpha^2 l^3 - 2(d - 1)dl r_{\min}^4)\right) \Omega \epsilon}{16l^3 \pi r_{\min}^{d+7}} \\ + \frac{(d - 2) \left(2 \left(2 - 3d + d^2\right) r_{\min}^{4+2d} dr_{\min}\right) \Omega \epsilon}{32l^2 \pi r_{\min}^{d+7}} \\ + \frac{(d - 2) \left(Q^2 r_{\min}^8 + 2r_{\min}^{2d} \left((30 - 11d + d^2)a + (12 - 7d + d^2)r_{\min}^2\right)\right) \Omega \epsilon}{32\pi r_{\min}^{d+7}} \\ + O(\epsilon)^2.$$

(61)

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Substituting Eq. (61) into Eq. (57), we have

$$df_{\min} = -\frac{2\left(2 - 3d + d^{2}\right)r_{\min}^{4+2d}dr_{\min}}{2\left((d - 2)l^{2}\left(2a + r_{\min}^{2}\right)\right)r_{\min}^{2d+1}} \\ - \frac{l^{2}\left(-Q^{2}r_{\min}^{8} + 2(d - 2)r_{\min}^{2d}\left(\alpha(d - 5) + (d - 3)r_{\min}^{2}\right)\right)dr_{\min}}{2\left((d - 2)l^{2}\left(2\alpha + r_{\min}^{2}\right)\right)r_{\min}^{2d+1}} \\ - \frac{-2r_{\min}\left(l^{3}Qr_{\min}^{8}dQ - (d - 2)r_{\min}^{2d}\left((d - 5)d\alpha^{2}l^{3} - 2(d - 1)dlr_{\min}^{4}\right)\right)\epsilon}{2\left((d - 2)l^{3}\left(2a + r_{\min}^{2}\right)\right)r_{\min}^{2d+2}} \\ - \frac{(d - 2)l\left(2 - 3d + d^{2}\right)r_{\min}^{4+2d}dr_{\min}\epsilon}{((d - 2)l^{3}\left(2\alpha + r_{\min}^{2}\right)\right)r_{\min}^{2d+1}} \\ - \frac{l^{2}\left(Q^{2}r_{\min}^{8} + 2r_{\min}^{2d}\left(\alpha\left(30 - 11d + d^{2}\right) + (12 - 7d + d^{2})r_{\min}^{2}\right)\right)\epsilon}{2\left((d - 2)l^{3}\left(2\alpha + r_{\min}^{2}\right)\right)r_{\min}^{2d+1}} \\ + O(\epsilon)^{2}.$$
(62)

For the equation $f(r_h) = 0$, we also can expand and solve it, which leads to lastly

$$l = \frac{\sqrt{2\sqrt{2-3d+d^2}r_{\min}^{(d+2)}}}{\sqrt{Q^2r_{\min}^8 - \alpha^2\left(20r_{\min}^{2d} - 14dr_{\min}^{2d} + 2d^2r_{\min}^{2d}\right) - r_{\min}^{2+2d}\left(12 - 10d + 2d^2\right)}}.$$
(63)

In addition, based on Eq. (56), we can get

$$dl = \frac{-\sqrt{2}\sqrt{2-3d+d^2}r_{\min}^{2+d}\left(Qr_{\min}^8dQ - (10-7d+d^2)d\alpha^2r_{\min}^{2d}\right)}{\left(Q^2r_{\min}^8 - 2(d-2)r_{\min}^{2d}\left(\alpha(d-5) + (d-3)r_{\min}^2\right)\right)^{3/2}} - \frac{\sqrt{2}\sqrt{2-3d+d^2}\left((d-2)dr_{\min}\left(-Q^2r_{\min}^8 + 2r_{\min}^{2d}\left(2\alpha(d-5) + (d-3)r_{\min}^2\right)\right)\right)}{r_{\min}^{-1-d}\left(Q^2r_{\min}^8 - 2(d-2)r_{\min}^{2d}\left(a(d-5) + (d-3)r_{\min}^2\right)\right)^{3/2}}.$$
(64)

Substituting Eq. (63) and Eq. (64) into Eq. (62), we find

$$df_{\min} = O(\epsilon)^2. \tag{65}$$

In [23], it was claimed that df_{\min} can be neglected for it is the high order terms of ϵ . In fact, δ is a small quantity, we can not neglect the contribution of $O(\epsilon)^2$ to $f(r_{\min} + dr_{\min})$ for both of them are small. We can find that δ is also a function of ϵ . As we expand $f(r_h)$ at r_{\min} to the second order, we find

$$f(r_h) = f(r_{\min}) + f'(r_{\min})\epsilon + \frac{1}{2}f''(r_{\min})\epsilon^2 + O(\epsilon)^3 = 0.$$
 (66)

For $f''(r_{\min}) \neq 0$, $O(\epsilon)^3$ thus can be omitted since the dominant term is that of ϵ^2 . In this case, we have

$$\delta = -\frac{1}{2}f''(r_{\min})\epsilon^{2}$$

$$= -\frac{\alpha \left(d^{2} - 9d + 20\right)l^{2} + r_{min}^{2}\left(\left(d^{2} - 3d + 2\right)r_{min}^{2} + (d - 3)^{2}l^{2}\right)}{l^{2}r_{min}^{2}\left(2\alpha + r_{min}^{2}\right)}\epsilon^{2}.$$
(67)



Fig. 5. The relation between $F(r_{\min} + dr_{\min})$ and r_{\min} for $Q = l = \alpha = 1, d = 5, da = dl = dr = 0.08$.

In addition, to the second order, df_{\min} can be simplified as

$$df_{\min} = \frac{X}{l^3 r_{\min}^3 \left(2\alpha + r_{\min}^2\right)} \epsilon^2 + O(\epsilon)^3,$$
(68)

in which

$$X = \alpha \left(16d^{2} - d^{3} - 83d + 140 \right) dr_{min}l^{3} - \left(d^{2} - 9d + 20 \right) d\alpha l^{3}r_{min} + 2 \left(d^{2} - 3d + 2 \right) dlr_{min}^{5} + (3 - d) \left(d^{2} - 8d + 15 \right) dr_{min}l^{3}r_{min}^{2} + (3 - d) \left(d^{2} - 3d + 2 \right) dr_{min}lr_{min}^{4}.$$
(69)

In this case, we can obtain the value of the final state of function f. For simplicity, we will discuss $F(r_{\min} + dr_{\min}) \equiv f(r_{\min} + dr_{\min})/\epsilon^2$, that is

$$F(r_{\min} + dr_{\min}) = \frac{x}{l^3 r_{\min}^3 \left(2\alpha + r_{\min}^2\right)} - \frac{\alpha \left(d^2 - 9d + 20\right) l^2 + r_{\min}^2 \left(\left(d^2 - 3d + 2\right) r_{\min}^2 + (d - 3)^2 l^2\right)}{l^2 r_{\min}^2 \left(2\alpha + r_{\min}^2\right)}.$$
 (70)

For different values of the parameters α , l, r_{min} , d, $d\alpha$, dl, dr_{min} , the configurations of $F(r_{\min} + dr_{\min})$ are different. In this paper, we find $d\alpha$, dl, dr_{min} affect the configuration drastically. For the case $d\alpha = dl = dr_{min} = 0.08$, the configuration of $F(r_{\min} + dr_{\min})$ is plotted in Fig. 5. We can see that $F(r_{\min} + dr_{\min})$ is negative, implying that there are horizons always. And for the case $d\alpha = dl = dr_{min} = 0.8$, the configuration of $F(r_{\min} + dr_{\min})$ is plotted in Fig. 6. We can see that $F(r_{\min} + dr_{\min})$ may be positive. In this case, there is not a horizon to cover the singularity and the weak cosmic censorship conjecture is violated. As we change the values of the parameters α , l, r_{min} , d, $d\alpha$, dl, dr_{min} , the configurations of $F(r_{\min} + dr_{\min})$ will change too. However, we can find that $F(r_{\min} + dr_{\min})$ is positive always for some values of the parameters. So, in the extended phase space, the weak cosmic censorship conjecture is violable, depending on the values of α , l, r_{min} and their variations.



Fig. 6. The relation between $F(r_{\min} + dr_{\min})$ and r_{\min} for $Q = l = \alpha = 1, d = 5, da = dl = dr = 0.8$.

6. Discussion and conclusions

As a charged particle drops into the Gauss-Bonnet AdS black hole, we obtained the energymomentum relation near the horizon via the Hamilton-Jacobi equation. We found that there was a relation between the energy, momentum, and chemical potential, which is conjectured to be the first law of black hole thermodynamics. To confirm this conjecture, we investigated the variation of the event horizon with the help of energy conservation as well as charge conservation and found that the energy-momentum relation was noting but the first law of thermodynamics in both the normal phase space and extended phase space.

With the variation of the event horizon, we also checked the second law of thermodynamics by investigating the variation of entropy. In the normal phase space, we found that the second law was valid for the variation of entropy was positive always. This conclusion is independent of the Gauss-Bonnet coefficient. In the extended phase space, the variation of the entropy is more sophisticated. We found that there was always a phase transition point, which divides the variation of entropy into positive and negative region. The variation of entropy is negative for the extremal and near-extremal black holes, while positive for the far-extremal black holes. In addition, we found that the phase transition point is independent of the Gauss-Bonnet coefficient though the value of the variation of entropy depends. Therefore, we concluded that in the extended phase space, the second law was violated for the extremal and near-extremal black holes.

In the normal and extended phase space, we also investigated the weak cosmic censorship conjecture. We mainly concentrated on studying how the minimum value of the function that determine the locations of the horizons move. In the normal phase space, we found that the function are stable and move downward respectively for the extremal and near-extremal black holes as a charged particle is absorbed, which implies that there are horizons always so that the weak cosmic censorship conjecture is valid. In the extended phase space, the validity or violation of the weak cosmic censorship conjecture is more subtle. We found that for the extremal Gauss-Bonnet AdS black hole, the weak cosmic censorship conjecture was found to be violable, depending on the values of α , *l*, *r_{min}* and their variations. Our result is different from that in [23], where the weak cosmic censorship conjecture for the near extremal black hole was found to be valid. The reason arises

from that they neglected the contribution of the second order term of ϵ to df_{\min} . As we shown, the second order term of ϵ can not be neglected for the initial state is also a function of ϵ^2 . Our result is thus more reasonable.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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