PROTON-PROTON ELASTIC SCATTERING AT 8.35 BeV

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A more accurate value is obtained for the differential cross section for pp scattering at 8.35 BeV by measurement in photographic emulsions. It is found that the cross section is greater than cited in previous communications [3-5] in a large range of angles. The data are analyzed on the basis of the Regge pole technique and compared with those of other experiments. The total pp elastic scattering cross section is found to be 10.8 ± 0.8 mb; the mean square interaction radius is (1.07 ± 0.08) F.

INTRODUCTION

A theoretical interpretation of the experimental data within the framework of the Regge pole theory [5] predicts a slow decrease in the differential cross section with increasing primary-particle energy at a fixed momentum transfer.

A comparison of the data on the differential cross section of elastic pp scattering, obtained by various methods, has raised the suspicion that the results obtained with emulsions are subject in the region of large scattering angle to systematic errors, which can lead to the false deduction that the interaction radius increases rapidly. This was pointed out, for example, by Marquit [2].

In the present investigation we used a water-soaked emulsion stack and a scanning method that yields reliable data at large scattering angles. The investigation is a continuation of earlier experiments [3-4] aimed at improving the statistical accuracy in the region of small scattering angles (< 8.5° in the c.m.s.) and at obtaining more reliable data in the large angle region (> 8.5° in the c.m.s.).

1. EXPERIMENTAL SETUP

A stack of 30 water-soaked NIKFI-BR emulsions was irradiated by the internal beam of the proton synchrotron of the Joint Institute for Nuclear Research with protons of 8.22 ± 0.01 BeV energy. The pellicles measured 10 by 10 cm and had initial thickness of 435 μ. The stack was irradiated perpendicular to the emulsion plane, and the average flux density was 1.8 × 10² particles/cm². The beam was inclined 89° to the plane of the emulsion. Each cm² of irradiated emulsion contained (5.38 ± 0.13) × 10²² hydrogen nuclei. The sensitivity of the emulsions, as determined by the positrons from the π-μ-e decays, was 14 ± 1 grains per 100.

In area scanning for elastic-scattering events, the registration efficiency decreases with increasing scattering angle, since the ionization of the recoil proton decreases; the fact that the events are not of the same type can therefore play an important role. If this is so, then a systematic error, connected with the increased scanning efficiency, arises in the differential cross section. In order to avoid systematic errors, the emulsions which were previously area-scanned twice were scanned for a third time “along the group of tracks” entering perpendicularly into the emulsion, and the deviations of the track from the beam direction were sought. Owing to the small angular half-width of the beam (10°), the group configuration was retained. The presence of a large base (the average emulsion thickness during the irradiation time was 1100 μ) permitted easy registration of the deviation of the track from the beam direction, provided the scattering angle was > 0.5°. The method has the following advantages:

1) The efficiency of registration of the events does not depend on the recoil-proton ionization.

2) The scanning efficiency is higher than in area scanning, particularly for large scattering angles.

3) In scanning “along the group of tracks” in a soaked emulsion, the rate of finding the events on hydrogen is approximately double that of finding them in standard emulsions.

It is much more difficult to apply this scanning method to unsoaked emulsions, since their thickness is 2-2.5 times smaller. An analogous scan-
ning method was used by Bull and Garbutt [4]. The methodological problems connected with the use of soaked emulsions are treated elsewhere [7,8].

2. MEASUREMENT AND IDENTIFICATION OF THE EVENTS

The selection criteria, the measurements, and the identification of events of elastic scattering on hydrogen are similar to those previously described [3].

The measurement of the angle \( \psi \) of the scattered proton was greatly simplified and accelerated because of the small half-width of the beam and the large thickness of the soaked pellicle (the base for the measurement of the scattered-proton angle was 1100 \( \mu \)). This made it possible to measure the angle \( \psi \) relative to the mean beam direction with accuracy \( 6'-7' \) (method 1 in [3]). The second method, which was more laborious, was used only to measure events with range \( R \approx 200 \mu \) when the scattering angles \( \psi \) were smaller than 0.5°, and in doubtful cases, when the error in the measurement of the angle \( \psi \) or of the non-coplanarity angle \( \gamma \) exceeded somewhat or was close to the triple error. At the given accuracy of measurement of the angle \( \psi \), the contribution of "quasi-events" as estimated by the method described in [3] amounted to 1–1.3 per cent.

The accuracy with which the proton recoil angle \( \psi \) was measured was \( \approx 1.5' \). This angle is defined as \( \varphi = 90° - \alpha \), where \( \alpha \) is the dip angle of the recoil track in the emulsion. In this case the 1° deviation of the beam from perpendicular was neglected.

The range-energy curve for the given chamber, which we obtained previously [8], is described by the formula

\[
E = (0.201 \pm 0.008) R^{0.472 \pm 0.008},
\]

where \( E \) is the proton energy in MeV and \( R \) its range in microns. This dependence holds true at least for the energy region \( 4 < E < 70 \) (MeV); it has been used to obtain the angle intervals for the determination of the differential cross section. Events located more than 5 per cent of the total layer thickness from the surface of the emulsion were not included in the analysis.

3. EXPERIMENTAL RESULTS

A total of 191 events of elastic scattering were found in the water-soaked emulsions. The table lists the values of the differential cross sections and the corresponding scanning efficiencies \( \epsilon_{12} \) obtained with two stacks: I—with standard emulsion [5] and II—with water-soaked emulsion (present work). The present results and those of the earlier investigation [5] agree in the c.m.s. angle region 4.5°–8.5°, and the data have been combined.

As a result of the scanning "along the group of tracks" many new events were observed in the region of large angles, and the values of the cross sections obtained in the present work are systematically higher than those given in earlier papers [3–5]. It was observed that the efficiency of single area scanning is greatly overvalued and the use of the well known formula for the efficiency of double area scanning, \( \epsilon_{12} = (1 - \epsilon_1)(1 - \epsilon_2) \) also gives too high values for the efficiency in the region of large scattering angles (\( > 8.5° \) in the c.m.s.). In this angle region, the efficiency of area registration of events is insufficient and the fact that the events are not of the same type plays an important role. The overvaluation of the efficiency due to non-standardization of the events at insufficiently high registration efficiency was obtained by Marquit with a model. It is probable that a similar effect explains the lower values of the cross sections obtained in [5] as compared with the present work. The same circumstances could occur, for example, in investigations of pp scattering at other energies [10,11], where the events were area-scanned. Consequently

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\theta_{\text{c.m.s., deg}} & E = 8.5 \text{ BeV} & E = 8.2 \text{ BeV} & E = 8.5 \text{ BeV} & E = 8.2 \text{ BeV} & E = 8.5 \text{ BeV} & E = 8.2 \text{ BeV} \\
\hline
& \text{mb/sr} & & \text{mb/sr} & & \text{mb/sr} & \text{mb/sr} \\
\text{present work} & \text{combined data} & \text{combined data} & \text{combined data} & \text{combined data} & \text{combined data} & \text{combined data} \\
\hline
1.5–2.5 & 0.0048 & 91.6±5.0 & 154±33 & 99.3±0.7 & 150±25 & 154±33 \\
2.5–4.5 & 0.0146 & 97.0±0.7 & 124±15 & 88.2±1.1 & 86.2±15.2 & 130±13 \\
4.5–6.5 & 0.0301 & 98.3±0.9 & 93±11 & 97.9±1.4 & 98.0±12.5 & 90.7±8.9 \\
6.5–8.5 & 0.0071 & 94.5±1.3 & 63.3±7.7 & 98.0±14.3 & 98.9±12.7 & 75.8±5.3 \\
8.5–10.5 & 0.1278 & 84.5±3.1 & 35.9±5.5 & 97.9±4.5 & 53.4±10.7 & 53.4±1.7 \\
10.5–12.5 & 0.1577 & 89.0±3.5 & 13.3±2.9 & 81.8±5.0 & 36.2±8.9 & 36.2±8.9 \\
12.5–14.5 & 0.2174 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 \\
14.5–16.5 & & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 \\
16.5–18.5 & & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 \\
18.5–20.5 & & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 & 4.5±1.5 \\
\hline
\end{array}
\]
only the new data are used for the analysis in the angle interval 8.5-14.5°.

The final values of the cross sections are listed in the last column of the table. The total cross section for elastic pp scattering turned out to be somewhat higher than in \([5]\), namely \(\sigma_{el} = 10.8 \pm 0.8 \text{ mb}\).

4. ANALYSIS OF THE EXPERIMENTAL DATA

The analysis was made under the assumption that the principal role in the scattering is played by the vacuum pole. In this case the following relation holds true (see, e.g., \([13]\))

\[
\frac{16\pi^2}{k^2} \frac{d^2 \sigma}{d \Omega} = F(t) \left( \frac{s}{2M^2} \right)^{L(t)-1},
\]

where \(\sigma_t\) — total elastic pp scattering cross section, \(k\) — c.m.s. wave number, \(s\) — square of the total c.m.s. energy, \(M\) — proton rest mass, \(F(t)\) — residue of the vacuum pole, \(L(t)\) — a universal function describing the behavior of the vacuum pole, and \(t\) — square of the c.m.s. four-momentum transfer.

When \(t < 0.5 \text{ (BeV/c)}^2\) the function \(F(t)\) can be represented in the form \(F(t) = \exp(\lambda_1 t)\). Putting \(L(t) = 1 + \lambda_2 t\) we get

\[
\ln \left[ \frac{16\pi^2}{k^2} \frac{d^2 \sigma}{d \Omega} \right] = B(s) t,
\]

where

\[
B(s) = \lambda_1 + 2\lambda_2 \ln \left( \frac{s}{2M^2} \right).
\]

There are seven experimental points, with one of them (for the 1.5-2.5° interval) lying in the region of possible interference with the Coulomb scattering amplitude. Assuming that these data satisfy the linear dependence

\[
y = a + B(s) t
\]

(if the optical theorem is satisfied for \(t = 0\), then \(a = 0\) ), we obtained the coefficients \(a\) and \(B\) by the method of least squares. Two variants were calculated:

1) The straight line was drawn through the six experimental points lying outside the region of possible interference.

2) The straight line was drawn through all seven experimental points.

The following results were obtained:

<table>
<thead>
<tr>
<th>Variant 1</th>
<th>Variant 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d\sigma/d\Omega)</td>
<td>29±13</td>
</tr>
<tr>
<td>(B(s)) (c/BeV)^2</td>
<td>9.8±1.2</td>
</tr>
<tr>
<td>(\lambda_2) (c/BeV)^2</td>
<td>1.2±0.28</td>
</tr>
<tr>
<td>(\chi^2)</td>
<td>2.7</td>
</tr>
</tbody>
</table>

In accord with the literature data\([12]\), it has been assumed here that \(\sigma_t = 41 ± 1 \text{ mb}\), and in the calculation of \(\lambda_2\) the value of \(\lambda_1\) was taken from the paper of Diddens et al\([14]\). It is seen that both variants lead at \(t = 0\) to some overvaluation of the cross section compared with the prediction of the optical theorem in the spinless case.

Figure 1 shows the values of \(B\) as functions of the quantity \(2 \ln \left( \frac{s}{2M^2} \right)\), obtained from an analysis of the experimental data of several authors \([16-18]\) for \(t \leq 0.5 \text{ (BeV/c)}^2\) and from the present work. Disregarding the data of Fujii et al\([18]\) we have

\[
\lambda_1 = 2.7 ± 1.0 \text{ (c/BeV)}^2, \quad \lambda_2 = 1.2 ± 0.2 \text{ (c/BeV)}^2.
\]

A \(\chi^2\) test has shown that the line \(B = \lambda_1 + \lambda_2 \times 2 \ln \left( \frac{s}{2M^2} \right)\) passes well through the experimental points in this case \((\chi^2 = 2.5)\). If the data of Fujii et al are included in the analysis we obtain

\[
\lambda_1 = 4.0 ± 0.6 \text{ (c/BeV)}^2, \quad \lambda_2 = 0.9 ± 0.2 \text{ (c/BeV)}^2.
\]

In this case \(\chi^2 = 16.8\), i.e., the straight line agrees less with experiment. However, judging from the available experimental data there is apparently no reason for assuming that \(\lambda_2\) changes in the energy region 2-15 BeV.

Since we obtained in the present work somewhat larger values for the cross section than predicted by the optical theorem for \(t = 0\), the data of the other authors\([13-15,18]\) for \(t \leq 0.5 \text{ (BeV/c)}^2\) were also extrapolated, as in the present work, to the point \(t = 0\) and the value of the parameter \(a\) in \((4)\) was evaluated for them \((a = y|_{t=0})\). The obtained distribution of the values of \(a\) is shown in Fig. 2, from which it follows that \(a = 0.1 ± 0.1\).
Within the framework of the present theory, the mean-square radius of interaction depends on $s$:

$$(\langle r^2 \rangle^2 = n V^3 B(s).$$

If we use the values we obtained for $\lambda_1$ and $\lambda_2$, then the dependence of $r$ on $s$ is much weaker than obtained by Tsyganov\cite{11}. For 8.35 BeV it follows from our data that

$$(\langle r^2 \rangle^2 = 1.07 \pm 0.08 \text{ F}.$$}

In connection with the systematically lower values of the differential cross section observed by us at large scattering angles, owing to the over-valuation of the scanning efficiency, the experimental data obtained with emulsions should be used for analysis with great caution.

In conclusion, the authors are grateful to the laboratory staff for emulsion scanning and measurements.

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\end{itemize}