

NUCLEAR MATTER PROPERTIES
WITH THE RE-EVALUATED COEFFICIENTS
OF LIQUID DROP MODEL

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The coefficients of the volume, surface, Coulomb, asymmetry and pairing energy terms of the semiempirical liquid drop model mass formula have been determined by furnishing best fit to the observed mass excesses. Slightly different sets of the weighting parameters for liquid drop model mass formula have been obtained from minimizations of χ^2 and mean square deviation. The most recent experimental and estimated mass excesses from Audi–Wapstra–Thibault atomic mass table have been used for the least square fitting procedure. Equation of state, nuclear incompressibility, nuclear mean free path and the most stable nuclei for corresponding atomic numbers, all are in good agreement with the experimental results.

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1. Introduction

A mass formula is not only useful to the evaluation of atomic masses and nuclear binding energies but provides theoretical predictions concerning a number of features of nuclei and their behavior. Mass models started out as empirical formulas and have now evolved into more or less full scale nuclear models, many capable of predicting a whole host of nuclear properties besides the mass. The atomic or nuclear mass is one of the most decisive factors governing nuclear stability. Enjoying reasonable success and relative ease of use are the class of mesoscopic models, where a gross macroscopic component is simply dependent on the number of nucleons and is finely sculpted using microscopic corrections for shell behavior, deformation, nuclear incompressibility and nucleon pairing. This approach finds its origin in the famous Bethe–Weizsäcker [1, 2] mass formula which is an empirically

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refined form of the liquid drop model for the binding energy of nuclei. Since Bethe–Weizsäcker mass formula is based on a liquid drop description of the nucleus, it is expected to reproduce the gross features of nuclear binding energies better for medium and heavy nuclei than for light nuclei.

Different approaches to the evaluation of the coefficients of the volume, surface, Coulomb, asymmetry and pairing energy terms of the semiempirical Bethe–Weizsäcker mass formula furnishing the best fit to the observed masses, have yielded different sets of results. Some of which are compared in Refs. [3,4]. The mass of an atom and especially of a nucleus is not merely the sum of the masses of its constituents but is accompanied by a binding energy. In nuclear physics accurate mass measurements have a great influence as the binding energy reveals a host of nuclear structure effects, magic configuration number effects known as shell and subshell closures, shape and deformation effects and also nucleon pairing. Finally, as the binding energy determines how much energy is available for a given nuclear reaction, the impact of masses in nuclear astrophysics is also far reaching. A host of quantities derived from the masses are in play, notably, neutron and proton separation energies. This atomic mass evaluation currently containing almost harmoniously adjusted values is published every few years and the recent one is available in Ref. [5]. In the present work, a five parameter least square fit to the Bethe–Weizsäcker mass formula has been achieved for new evaluations of energy coefficients by minimizing the χ^2 and the mean square deviation for atomic mass excesses using the latest compilation [5] and its consequences on nuclear properties such as incompressibility of nuclear matter, Coulomb radius constant and effect of asymmetry energy coefficient have been investigated.

2. The liquid drop model nuclear binding energy and the atomic mass excess

The Bethe–Weizsäcker formula [1,2] is an empirically refined form of the liquid drop model for the binding energy of nuclei. Expressed in terms of the mass number A and the atomic number Z for a nucleus, the binding energy $B(A, Z)$ is given by

$$B(A, Z) = a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - \frac{a_{\text{sym}}(A-2Z)^2}{A} + \delta, \quad (1)$$

where

$$\delta = \begin{cases} a_p A^{-1/2}, & \text{for even } N\text{--even } Z, \\ -a_p A^{-1/2}, & \text{for odd } N\text{--odd } Z, \\ 0, & \text{for odd } A, \end{cases}$$

and N is the neutron number of the nucleus.

The above formula for nuclear binding energy was prescribed by Bethe [1] and Weizsäcker [2] where the nucleus was considered as a droplet of incompressible matter and all nuclei have the same density. Scattering experiments suggest that nuclei have approximately constant density, so that the nuclear radius can be calculated by using that density as if the nucleus were a drop of a uniform liquid. Thus the first approximation to the binding energy was identified as due to the saturated exchange force and the term $a_v A$ was called the volume energy. The second term $-a_s A^{2/3}$, called the surface energy, was a correction to it since a deficit of binding energy for surface nucleons is expected, due to those nucleons which are visualized as being at the nuclear surface, to have fewer near neighbors than deep within the nuclear matter. Only well known infinite range force in a nucleus is the Coulomb repulsion among protons that gives rise to the repulsive Coulomb energy term $-a_c Z(Z-1)/A^{1/3}$. This term was derived by calculating the Coulomb energy assuming that the total nuclear charge Ze to be spread uniformly throughout the spherical nuclear volume of radius equal to $r_0 A^{1/3}$, r_0 being the Coulomb radius constant. The term $Z(Z-1)$ instead of Z^2 appears due to the fact that the nuclear charges are integral multiples of electronic charge e and the single proton should not have any contribution to Coulomb energy and hence Coulomb self energy contribution by each of Z protons should be removed. The Coulomb energy coefficient a_c is given by

$$a_c = \frac{3e^2}{5r_0}. \quad (2)$$

Another deficit of binding energy depends on the neutron excess $N - Z = A - 2Z$ since these excess neutrons have to go into previously unoccupied quantum states having larger kinetic energy and smaller potential energy than those already occupied. This asymmetry energy term $-a_{\text{sym}}(A-2Z)^2/A$ thus, arises due to purely quantum mechanical effect. The facts that there is a finite pairing energy between odd- A and even- A nuclei and the anomalously large binding energy for nuclei containing a magic number of neutrons and protons, fail to appear in liquid drop model since intrinsic spins of the nucleons have been omitted. To correct this omission the pairing energy term δ was empirically incorporated.

Using the definition of nuclear binding energy $B(A, Z)$ which is defined as the energy required to separate all the nucleons constituting a nucleus, the mass equation can be expressed as

$$M_{\text{nucleus}}(A, Z) = Zm_p + (A - Z)m_n - B(A, Z), \quad (3)$$

where m_p and m_n are the masses of proton and neutron, respectively, and $M_{\text{nucleus}}(A, Z)$ is the actual mass of the nucleus.

The reason that the atomic mass is considered rather than the nuclear mass is that historically, the former has been the actual experimentally measured quantity, whereas the latter is less accurate because its extraction requires a knowledge of binding energy of the Z atomic electrons. However, recent developments now allow the nuclear masses to be measured directly [6]. For those applications where it is necessary to know the actual mass of the nucleus $M_{\text{nucleus}}(A, Z)$ itself, its value (in MeV) can be found from Eq. (3) using the nuclear binding energy or from the atomic mass excess by use of the relationship

$$M_{\text{nucleus}}(A, Z) = Au + \Delta M_{A,Z} - Zm_e + a_{\text{el}}Z^{2.39} + b_{\text{el}}Z^{5.35}, \quad (4)$$

where m_e is the mass of an electron, the atomic mass unit u is 1/12 the mass of ^{12}C atom, $\Delta M_{A,Z}$ is the atomic mass excess of an atom of mass number A and atomic number Z and the electronic binding energy constants [7] $a_{\text{el}} = 1.44381 \times 10^{-5}$ MeV and $b_{\text{el}} = 1.55468 \times 10^{-12}$ MeV. Hence from Eq. (3) and Eq. (4) the atomic mass excess is given by

$$\Delta M_{A,Z} = Z\Delta m_{\text{H}} + (A - Z)\Delta m_n - a_{\text{el}}Z^{2.39} - b_{\text{el}}Z^{5.35} - B(A, Z), \quad (5)$$

where $\Delta m_{\text{H}} = m_p + m_e - u = 7.28897050 + a_{\text{el}} + b_{\text{el}}$ MeV and $\Delta m_n = m_n - u = 8.07131710$ MeV.

3. The mass fitting and the extraction of energy coefficients

Different approaches to the evaluation of the weighting parameters a_v , a_s , a_c , a_{sym} and a_p furnishing the best fit to the observed masses when inserted in the mass formula have yielded different sets of results. Some of which are compared in Refs. [3, 4]. The two quantities of importance that can be used for least square fitting are the root mean square deviation σ and χ^2/N which are defined as

$$\sigma^2 = \frac{1}{N} \sum (\Delta M_{\text{Th}} - \Delta M_{\text{Ex}})^2 \quad (6)$$

and

$$\frac{\chi^2}{N} = \frac{1}{N} \sum \left[\frac{(\Delta M_{\text{Th}} - \Delta M_{\text{Ex}})}{\Delta M_{\text{Ex}}} \right]^2, \quad (7)$$

respectively, where the summations extend to N data points for which measured atomic mass excesses and corresponding binding energies are known. The quantity ΔM_{Th} is theoretically calculated atomic mass excess obtained using Eq. (5) while ΔM_{Ex} is the corresponding experimental atomic mass excess obtained from the Audi–Wapstra–Thibault atomic mass table [5].

For the calculations of χ^2/N and the mean square deviation σ^2 , masses [5] of 3179 nuclei including the 951 extrapolated values which are predicted according to systematics, have been used for least square fitting. These calculations have been repeated using only the 2228 experimental data. The χ^2/N and σ^2 minimizations have been performed separately since both cannot be minimized with identical sets of values for energy coefficients. In case of the χ^2/N minimization, the denominator has to be kept equal to unity only for ^{12}C in order to accommodate it in the calculation as the atomic mass excess of ^{12}C is, by definition, equal to zero. In Table I, values of the energy coefficients along with the minimized quantities with their values have been listed. Since Bethe–Weizsäcker mass formula is more appropriate for nuclear binding energies of medium and heavy nuclei than for light nuclei, the values obtained for σ^2 and χ^2/N are not very small. However, as can be seen from Table I, the errors associated with the energy coefficients a_v , a_s , a_c and a_{sym} are very small while that with a_p is reasonably small.

TABLE I

Energy coefficients for the Bethe–Weizsäcker mass formula.

a_v [MeV]	a_s [MeV]	a_c [MeV]	a_{sym} [MeV]	a_p [MeV]
(a) Minimized quantity $\sigma^2 = 15.15$				
15.260	16.267	0.689	22.209	10.076
± 0.020	± 0.062	± 0.001	± 0.048	± 0.854
(a) Minimized quantity $\chi^2/N = 3.336$				
15.777	18.340	0.710	23.210	11.998
± 0.053	± 0.174	± 0.003	± 0.103	± 1.912
(b) Minimized quantity $\sigma^2 = 11.74$				
15.409	16.873	0.695	22.435	11.155
± 0.026	± 0.080	± 0.002	± 0.065	± 0.864
(b) Minimized quantity $\chi^2/N = 2.484$				
15.777	18.341	0.710	23.211	11.996
± 0.037	± 0.133	± 0.002	± 0.060	± 1.536

- (a) Using both measured 2228 + extrapolated 951 atomic mass excesses.
- (b) Using only the experimentally measured 2228 atomic mass excesses.

As can be seen from Table I, both σ^2 , which are a measure of absolute error, and the χ^2/N , which are a measure of relative error, cannot be minimized simultaneously with the same set of parameters. The set of parameters obtained by minimizing χ^2/N are almost the same as those listed in Refs. [8,9]. However, in many cases, such as reaction Q value calculations, absolute errors involved in atomic mass excesses are the quantities of concern and σ^2 minimization plays more important role than χ^2/N minimization. On the contrary, χ^2/N minimization causes minimization of relative, that is, uniform percentage error involved in the mass predictions.

4. Energy coefficients and the nuclear matter properties

A density dependent $M3Y$ effective nucleon–nucleon (NN) interaction based on the G -matrix elements of the Reid–Elliott NN potential has been used to determine the nuclear matter equation of state. The equilibrium density of the nuclear matter has been determined by minimizing the energy per nucleon. The density dependence parameters have been chosen to reproduce the saturation energy per nucleon and the saturation density of spin and isospin symmetric cold infinite nuclear matter. The general expression for the density dependent effective NN interaction potential $v(s)$ is written as

$$v(s, \rho, \epsilon) = t^{M3Y}(s, \epsilon) g(\rho, \epsilon) \text{ MeV}, \quad (8)$$

where $M3Y$ effective interaction potential supplemented by a zero range pseudopotential t^{M3Y} is given by

$$t^{M3Y}(s, \epsilon) = 7999 [\text{MeV}] \frac{e^{-4s}}{4s} - 2134 [\text{MeV}] \frac{e^{-2.5s}}{2.5s} + J_{00}(\epsilon)\delta(s), \quad (9)$$

where the constants 4 and 2.5 have dimensions of fm^{-1} , $\delta(s)$ has the dimension of fm^{-3} and the zero-range pseudo-potential $J_{00}(\epsilon)$ representing the single-nucleon exchange term is given by

$$J_{00}(\epsilon) = -276 (1 - \alpha \epsilon) [\text{MeV fm}^3] \quad (10)$$

and the dimensionless density dependent part is given by

$$g(\rho, \epsilon) = C \left(1 - \beta(\epsilon)\rho^{2/3} \right). \quad (11)$$

The energy per nucleon ϵ obtained using the effective nucleon–nucleon interaction $v(s)$ for the spin and isospin symmetric cold infinite nuclear matter, which will henceforth be called the standard nuclear matter, is given by

$$\epsilon = \frac{3\hbar^2 k_F^2}{10m} + \frac{g(\rho, \epsilon)\rho J_v}{2} = \frac{3\hbar^2 k_F^2}{10m} + \rho J_v C \frac{(1 - \beta\rho^{2/3})}{2}, \quad (12)$$

where m is the nucleonic mass, $k_F = (1.5\pi^2\rho)^{1/3}$ is the Fermi momentum, ρ is the nucleonic density while ρ_0 being the saturation density for the standard nuclear matter and J_v represents the volume integral of t^{M3Y} , the $M3Y$ interaction supplemented by the zero-range pseudopotential. The Eq. (12) can be differentiated with respect to ρ to yield equation

$$\frac{\partial\epsilon}{\partial\rho} = \frac{\hbar^2 k_F^2}{5m\rho} + J_v C \frac{[1 - (5/3)\beta\rho^{2/3}]}{2}. \tag{13}$$

The equilibrium density of the nuclear matter is determined from the saturation condition $\partial\epsilon/\partial\rho = 0$. Then Eq. (12) and Eq. (13) with the saturation condition can be solved simultaneously for fixed values of the saturation energy per nucleon ϵ_0 and the saturation density ρ_0 of the standard nuclear matter, to obtain the values of the density dependence parameters β and C given by

$$\beta = \frac{[(1 - p)\rho_0^{-2/3}]}{[3 - (5/3)p]}, \tag{14}$$

$$p = \frac{10m\epsilon_0}{\hbar^2 k_{F_0}^2}, \tag{15}$$

$$k_{F_0} = (1.5\pi^2\rho_0)^{1/3}, \tag{16}$$

$$C = \frac{-[2\hbar^2 k_{F_0}^2]}{[5mJ_v\rho_0(1 - (5/3)\beta\rho_0^{2/3})]}, \tag{17}$$

respectively. It is quite obvious that the density dependence parameter β obtained by this method depends only on the saturation energy per nucleon ϵ_0 , the saturation density ρ_0 but not on the parameters of the $M3Y$ interaction while the other density dependence parameter C depends on the parameters of the $M3Y$ interaction also through the volume integral J_v . The energy per nucleon can be rewritten as

$$\epsilon = \frac{3\hbar^2 k_F^2}{10m} - \left(\frac{\rho}{\rho_0}\right) \frac{[\hbar^2 k_{F_0}^2(1 - \beta\rho^{2/3})]}{[5m(1 - (5/3)\beta\rho_0^{2/3})]} \tag{18}$$

and the incompressibility K_0 of the spin and isospin symmetric cold infinite nuclear matter is given by

$$K_0 = \frac{k_F^2 \partial^2 \epsilon}{\partial k_F^2} = 9\rho^2 \frac{\partial^2 \epsilon}{\partial \rho^2} \Bigg|_{\rho=\rho_0} = - \left(\frac{3\hbar^2 k_{F_0}^2}{5m}\right) - 5J_v C \beta \rho_0^{5/3}. \tag{19}$$

The pressure P and the energy density ε of nuclear matter can be given by

$$P = \rho^2 \frac{\partial \epsilon}{\partial \rho} = \frac{\rho \hbar^2 k_F^2}{5m} + \rho^2 J_v C \frac{[1 - (5/3)\beta \rho^{2/3}]}{2}, \quad (20)$$

$$\varepsilon = \rho(\epsilon + mc^2) = \rho \left[\left(\frac{3\hbar^2 k_F^2}{10m} \right) + \rho J_v C \frac{(1 - \beta \rho^{2/3})}{2} + mc^2 \right], \quad (21)$$

respectively, and thus the velocity of sound v_s in standard nuclear matter is given by

$$\frac{v_s}{c} = \sqrt{\frac{\partial P}{\partial \varepsilon}} = \sqrt{\left[2\rho \frac{\partial \epsilon}{\partial \rho} - \frac{\hbar^2 k_F^2}{15m} - \frac{5}{9} J_v C \beta \rho^{5/3} \right] / \left[\epsilon + mc^2 + \rho \frac{\partial \epsilon}{\partial \rho} \right]}. \quad (22)$$

One of the most directly derivable information provided by the mass formula is the magnitude of the Coulomb radius constant contained within the expression for the Coulomb energy constant $a_c = (3e^2/5r_0)$ assuming a uniform volume distribution of charge. Using the values of Coulomb energy coefficients a_c , the values obtained for the Coulomb radius constant r_0 have been listed in Table II. The present value for the Coulomb radius constant is $r_0 = 1.22$ – 1.25 fm which like previous such mass fittings still overpredicts. The crucial property of nuclear matter is the saturation density. It has been recognized right from the beginning that Bethe–Weizsäcker mass formula or its improved versions cannot give this property through its Coulomb radius

TABLE II

Coulomb radius constant and the compression modulus using the energy dependence parameter $\alpha = 0.005 \text{ MeV}^{-1}$ and values of the saturation energy per nucleon ϵ_0 determined by a_v and using the saturation density $\rho_0 = 0.1533 \text{ fm}^{-3}$ for the standard nuclear matter. Rows (a) and (b) correspond to a_v and a_c values obtained from minimizing σ^2 and χ^2/N , respectively, using both measured 2228 + extrapolated 951 atomic mass excesses whereas rows (c) and (d) correspond to a_v and a_c values obtained from minimizing σ^2 and χ^2/N , respectively, using only the experimentally measured 2228 atomic mass excesses.

	ϵ_0 [MeV]	a_c [MeV]	r_0 [fm]	β [fm ²]	C	K_0 [MeV]
(a)	15.260	0.689	1.254	1.668	2.07	293.4
(b)	15.777	0.710	1.217	1.676	2.11	301.1
(c)	15.409	0.695	1.244	1.671	2.08	295.6
(d)	15.777	0.710	1.217	1.676	2.11	301.1

constant using the simple relation $\rho_0 = 3/4\pi r_0^3$. This density is measured through electron scattering experiments on heavy nuclei. This gives a value corresponding to $r_0 = 1.12\text{--}1.13$ fm. Identifying a_v as the saturation energy per nucleon for the spin and the isospin symmetric cold infinite nuclear matter and hence using the saturation energy per nucleon equal to -15.260 MeV along with the density dependent $M3Y$ effective interaction and the commonly used value for saturation density equal to 0.1533 fm^{-3} [10], as one can see in Table II, the nuclear incompressibility is found to be 293.4 MeV which is in close agreement with experimental data [11, 12]. In Fig. 1 the energy per nucleon $E/A = \epsilon$ of standard nuclear matter is plotted as a function of nucleonic density ρ . The minima of the energies per nucleon equaling saturation energies of -15.260 MeV and -15.777 MeV for the present calculations occur correctly at the saturation density $\rho_0 = 0.1533\text{ fm}^{-3}$ while that for the A18 (VCS) model occurs around $\rho = 0.28\text{ fm}^{-3}$ with a saturation energy of about -17.3 MeV.

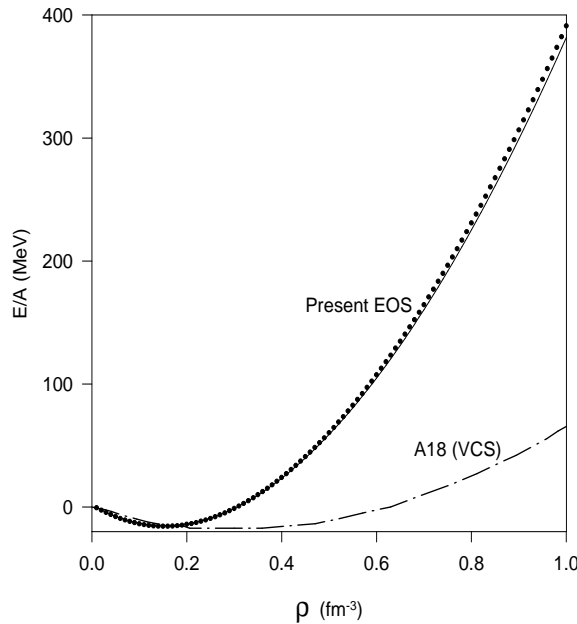


Fig. 1. The energy per nucleon ϵ of nuclear matter as a function of ρ . The continuous line represents the curve for the present calculations using saturation energy per nucleon of -15.26 MeV and the dots represent the same using saturation energy per nucleon of -15.78 MeV whereas the dash-dotted line represents the same for the A18 model using variational chain summation (VCS) [13] for the spin and isospin symmetric infinite nuclear matter.

In Table III the theoretical estimates of the pressure P and velocity of sound v_s of standard nuclear matter have been listed as functions of nucleonic density ρ and energy density ε using the usual value of 0.005/MeV for the parameter α of energy dependence, given in Eq. (10), of the zero range pseudo-potential. As for any other non-relativistic EOS, present EOS also suffers from super-luminosity at very high densities. According to present calculations the velocity of sound becomes imaginary for $\rho \leq 0.1 \text{ fm}^{-3}$ and exceeds the velocity of light c at $\rho \geq 5.3\rho_0$ and the EOS obtained using $v_{14} + TNI$ [14] also resulted in sound velocity becoming imaginary at the same nuclear density and super-luminous at about the same nuclear density. But in contrast, the incompressibility K_0 of infinite nuclear matter for the $v_{14} + TNI$ was chosen to be 240 MeV while that obtained by the present theoretical estimate is 290–300 MeV which is in excellent agreement with the experimental value of $K_0 = 300 \pm 25$ MeV obtained from the giant monopole resonance [11] and with the recent experimental determination of K_0 based upon the production of hard photons in heavy ion collisions which led to the experimental estimate of $K_0 = 290 \pm 50$ MeV [12].

In Fig. 2, the plots of the velocity of sound v_s in nuclear matter, the pressure P and the energy density ε of nuclear matter as functions of nucleonic density ρ have been shown.

TABLE III

Energy per nucleon ϵ , pressure P , energy density ε and velocity of sound v_s as functions of nuclear density ρ using saturation energy per nucleon equal to -15.260 MeV and saturation density $= 0.1533 \text{ fm}^{-3}$ for standard nuclear matter.

ρ [fm^{-3}]	ρ/ρ_0	ϵ [MeV]	P [MeV fm^{-3}]	ε [MeV fm^{-3}]	v_s in units of c
.01	$.6523 \times 10^{-1}$	$-.7537 \times 10^0$	$-.1677 \times 10^{-1}$	$.9382 \times 10^1$	$.0000 \times 10^0$
.10	$.6523 \times 10^0$	$-.1325 \times 10^2$	$-.7633 \times 10^0$	$.9257 \times 10^2$	$.0000 \times 10^0$
.20	$.1305 \times 10^1$	$-.1378 \times 10^2$	$.2520 \times 10^1$	$.1850 \times 10^3$	$.2879 \times 10^0$
.30	$.1957 \times 10^1$	$-.1138 \times 10^1$	$.1689 \times 10^2$	$.2813 \times 10^3$	$.4700 \times 10^0$
.40	$.2609 \times 10^1$	$.2341 \times 10^2$	$.4829 \times 10^2$	$.3849 \times 10^3$	$.6207 \times 10^0$
.50	$.3262 \times 10^1$	$.5896 \times 10^2$	$.1020 \times 10^3$	$.4989 \times 10^3$	$.7442 \times 10^0$
.60	$.3914 \times 10^1$	$.1048 \times 10^3$	$.1830 \times 10^3$	$.6262 \times 10^3$	$.8443 \times 10^0$
.70	$.4566 \times 10^1$	$.1605 \times 10^3$	$.2958 \times 10^3$	$.7696 \times 10^3$	$.9248 \times 10^0$
.80	$.5219 \times 10^1$	$.2254 \times 10^3$	$.4447 \times 10^3$	$.9315 \times 10^3$	$.9895 \times 10^0$
.90	$.5871 \times 10^1$	$.2993 \times 10^3$	$.6340 \times 10^3$	$.1114 \times 10^4$	$.1042 \times 10^1$
1.00	$.6523 \times 10^1$	$.3819 \times 10^3$	$.8675 \times 10^3$	$.1321 \times 10^4$	$.1084 \times 10^1$

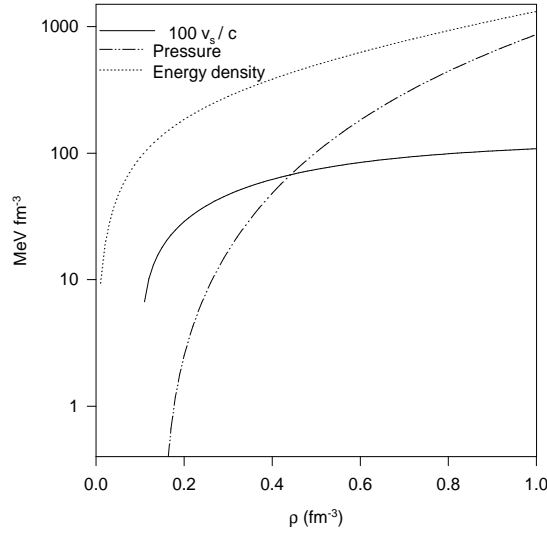


Fig. 2. The velocity of sound v_s in nuclear matter, the pressure P and the energy density ε of nuclear matter as a function of nucleonic density ρ . The continuous line represents the velocity of sound in units of $10^{-2}c$ for the standard nuclear matter, the dash-dotted line represents pressure in MeV fm^{-3} while the dotted line represents energy density in MeV fm^{-3} for the standard nuclear matter.

Keeping A constant while differentiating Eq. (3) and using Eq. (1) and setting the term $\partial M_{\text{nucleus}}(A, Z)/\partial Z|_A$ equal to zero, one obtains

$$Z_{\text{stable}} = \frac{[A + (a_c A^{2/3}/2x)]}{[(4a_{\text{sym}}/x) + (a_c A^{2/3}/x)]}, \quad (23)$$

where $x = 2a_{\text{sym}} + [(m_n - m_p)/2]$. The second term in the numerator of the above equation is small compared to the atomic mass number A . The above relation which connects Z with A for the most stable nuclei can be written in a closed form as

$$Z_{\text{stable}} = \frac{[A + 0.5a_1 A^{2/3}]}{[a_2 + a_1 A^{2/3}]}, \quad (24)$$

where $a_1 = a_c/x$ and $a_2 = 4a_{\text{sym}}/x$. Table IV provides values of a_1 , a_2 for different sets of coefficients a_c , a_{sym} . Since the values of a_1 and a_2 do not differ significantly for the different sets of coefficient values a_c and a_{sym} , in Fig. 3 the plot of Z versus $N = A - Z$ for the most stable nuclei is shown for the first set of values only. The continuous line represents the theoretical curve following the exact expression given by Eq. (24) while the dots represent the observed stable nuclei from the recent nuclide chart [15].

TABLE IV

The values of a_1 and a_2 .

a_c [MeV]	a_{sym} [MeV]	a_1	a_2
0.689	22.209	0.0153	1.971
0.710	23.210	0.0151	1.973
0.695	22.435	0.0153	1.972
0.710	23.211	0.0151	1.973

The density dependence parameter β has the dimension of cross section. The density dependent term $(1 - \beta\rho^{2/3})$ reduces the strength of the interaction which changes sign at high densities making it repulsive. It is a direct consequence of the Pauli blocking effect. Thus $(1 - \beta\rho^{2/3})$ can be interpreted as the probability of non-interaction arising due to the collision probability $\beta\rho^{2/3}$ of a nucleon in nuclear medium of density ρ . The density dependence parameter β can be identified as the in medium effective nucleon-nucleon interaction cross-section σ_0 . Density dependence parameter β along with nucleonic density of infinite nuclear matter ρ_0 can, therefore, provide the

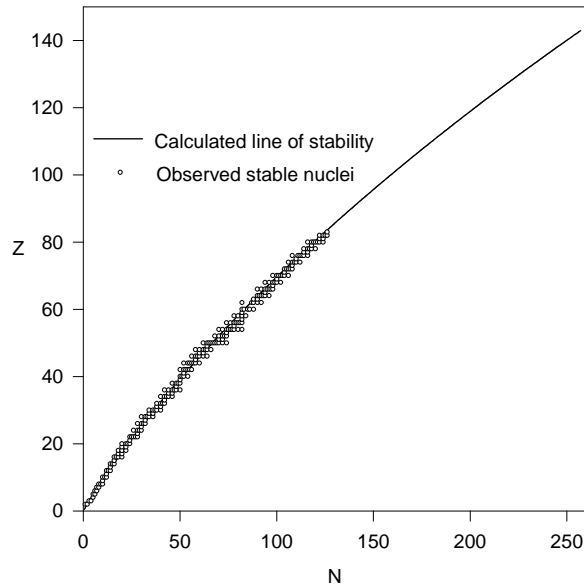


Fig. 3. The plot of Z versus N for the most stable nuclei. The continuous line represents the theoretical curve while the circles represent observed stable nuclei.

nuclear mean free path $\lambda = 1/(\rho_0\sigma_0)$. Using value of the density dependence parameter $\beta = 1.668 \text{ fm}^2$ along with the nucleonic density of 0.1533 fm^{-3} , the value obtained for the nuclear mean free path λ is about 3.9 fm.

5. Summary and conclusion

In summary, we conclude that five parameter least square fits to the Bethe–Weizsäcker mass formula by minimizing σ^2 and χ^2/N yield slightly different sets of energy coefficients a_v , a_s , a_c , a_{sym} and a_p . Both σ^2 and χ^2/N cannot be minimized simultaneously by the identical sets of parameters. The σ^2 minimization is more appropriate since it reduces absolute errors involved in mass predictions. Identifying a_v as the saturation energy per nucleon for the spin and the isospin symmetric cold infinite nuclear matter along with the value for saturation density equal to 0.1533 fm^{-3} , the nuclear incompressibility is found to be 290–300 MeV which is in excellent agreement with the experimental estimates from GMR [11] as well as determination based upon the production of hard photons in heavy ion collisions [12]. The present theoretical estimate of nuclear incompressibility is in reasonably close agreement with other theoretical estimates obtained by INM [16] model, using the Seyler–Blanchard interaction [10] or the relativistic Brueckner–Hartree–Fock theory [17]. The value of β provide nuclear mean free path $\lambda = 3.89\text{--}3.91 \text{ fm}$ which is in excellent agreement with the values derived by other methods [18].

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