

MULTIQUARK HADRONS I.
THE PHENOMENOLOGY OF $Q^2\bar{Q}^2$ MESONS †

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ABSTRACT

The spectra and dominant decay couplings of $Q^2\bar{Q}^2$ mesons are presented as calculated in the quark bag model. Certain known 0^+ -mesons ($\epsilon(700)$, S^* , δ , κ) are assigned to the lightest cryptoexotic $Q^2\bar{Q}^2$ nonet. The usual quark model 0^+ -nonet ($Q\bar{Q}$ L=1) must lie higher in mass. All other $Q^2\bar{Q}^2$ mesons are predicted to be broad, heavy and usually inelastic in formation processes. Other $Q^2\bar{Q}^2$ states which may be experimentally prominent are discussed.

(Submitted to Phys. Rev.)

†This work is supported in part through funds provided by the Energy Research and Development Administration under contracts AT(11-1)-3069 and E(04-3)-515.

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This is the first of two papers on multiquark states in the quark-bag model. This paper is concerned primarily with phenomenology—the spectrum of $Q^2\bar{Q}^2$ mesons, their important couplings and the possibility that certain known mesons are actually made of two quarks and two antiquarks. The second paper¹ (known hereafter as II) summarizes the calculational methods developed to handle multiquark hadron states with particular reference to $Q^2\bar{Q}^2$ mesons. Here we shall defer all detailed calculations and instead quote liberally from the results of II. A preliminary report of some of these results was given in collaboration with K. Johnson.²

I. INTRODUCTION

In his 1964 paper³ introducing the notion of quarks, Gell-Mann comments that, "It is amusing that the lowest baryon configuration (QQQ) gives just the representations 1, 8 and 10 that have been observed, while the lowest meson configuration (Q \bar{Q}) similarly gives just 1 and 8." In recent years attention has focused on developing a quark dynamics consistent with the absence of free quarks or other states of nonzero triality. The apparent spectroscopic absence of multiquark hadrons ($Q^2\bar{Q}^2$ mesons, $Q^4\bar{Q}$ baryons, etc.) has remained essentially where Gell-Mann left it.⁴ Indeed, it seems foolish to attempt an explanation of the absence of exotics without at least some understanding of the peculiar forces which confine quarks.

On the other hand, one hopes that any dynamical scheme which confines quarks would also shed some light on the problem of multiquark hadrons. The masses of S-wave baryons and mesons are approximately in the ratio 3 to 2 (the S-wave baryons are the $\frac{3}{2}^+$ decouplet and the $\frac{1}{2}^+$ octet; the S-wave mesons are the 0^- and 1^- nonets). This is to be expected if confined quarks interact relatively weakly (as motivated by Bjorken scaling): The mass of a hadron should

increase roughly linearly with the number of quarks. With (nonstrange) $Q\bar{Q}$ mesons at 700 MeV, and Q^3 baryons at 1100 MeV one expects $Q^2\bar{Q}^2$ mesons at around 1500 MeV. Given the splittings within SU(6)-multiplets, the lightest $Q^2\bar{Q}^2$ mesons might be expected at masses less than a GeV. Certainly there is no evidence of mesons with exotic quantum numbers in this range.

In this paper we examine the S-wave $Q^2\bar{Q}^2$ sector of a colored quark-gluon model⁵ based on a semiclassical approximation⁶ to the MIT Bag theory.⁷ Confinement is built into the model ab initio. It has been quite successful in describing the S-wave $Q\bar{Q}$ mesons and Q^3 baryons.⁵ Furthermore the model may be applied to the S-wave $Q^2\bar{Q}^2$ sector without introducing any further parameters or approximations.

Surprisingly we find that it is possible to accommodate $Q^2\bar{Q}^2$ mesons relatively comfortably within the restrictions imposed by experimental meson spectroscopy. We do not claim to resolve the problem by elevating unwanted multi-quark states to very high masses. Quite the contrary we will attempt to identify the lowest $Q^2\bar{Q}^2$ multiplet—a $J^P = 0^+$ -nonet—with some of the known 0^+ -mesons ($\epsilon(700)$, $S^*(993)$, $\delta(976)$, $\kappa(?)$). The masses and decay systematics of the observed 0^+ -mesons support the $Q^2\bar{Q}^2$ assignment. Other exotics (mesons not classifiable as flavor octets or singlets) and cryptoexotics (flavor singlets or octets nevertheless constructed from $Q^2\bar{Q}^2$) are broad, heavy and often couple weakly to formation channels. For one or more of these reasons, most of them are unlikely to have been seen at this time. The model nevertheless makes many striking predictions. For example, there should be another, entire nonet of 0^+ -mesons in the vicinity of the f. These are the ordinary P-wave $Q\bar{Q}$ states erroneously (we claim) identified with the ϵ , S^* , δ and κ in quark model compendia. A recent Notre Dame-Argonne experiment⁸ has found a

$J^P = 0^+$ isovector resonance in $K_S K_S$ at 1255 MeV which could be a member of this nonet. There should be resonances in exotic channels at high mass: We find a $\pi^+ \pi^+$ S-wave resonance somewhere above 1 GeV. The resonance will be inelastic and will most likely be associated with an inelastic background, making it difficult to see. It should be looked for. These predictions and others are discussed at length later in the paper.

The spectrum of $Q^2 \bar{Q}^2$ states depends crucially on residual gluon interactions among bound quarks which persist in color singlet states. These gluon exchanges are magnetic in character. In the model, they are also responsible for the Δ being heavier than the nucleon and the ρ being heavier than the π . The prediction of a light cryptoexotic 0^+ -nonet is a strong test of the character of the spectroscopically important forces among quarks. We have found it useful to introduce a new SU(6) group in order to discuss multiquark states. The group, which we shall call "colorspin" ($SU(6)_{CS}$), is the direct product of the SU(3) of color ($SU(3)_C$) and the SU(2) generated by the angular momentum of relativistic $j=1/2$ quarks (which we refer to loosely as spin). The role of this symmetry group in determining the spectra of multiquark states is discussed thoroughly in II.

This paper is divided roughly into two parts. The first part outlines the phenomenological quark-bag model (Section II) and its application to $Q^2 \bar{Q}^2$ mesons (Section III). The spectrum of $Q^2 \bar{Q}^2$ mesons is presented in Section IV. The remainder of the paper is devoted to the phenomenology of the $Q^2 \bar{Q}^2$ mesons, first to the 0^+ nonet (Section V) and then to the remaining channels (Section VI). Section VII contains a summary of our results and predictions.

II. REVIEW OF THE BAG QUARK MODEL

In our model⁵⁻⁷ colored quarks and massless colored gluons are confined to the interior of hadrons by the introduction of a constant energy density ($B \approx 50 \text{ MeV/fm}^3$) into the hadron Hamiltonian. The quarks are massless (u or d) or light (s) and are coupled to the gluons a la Yang-Mills. The field equations of motion and boundary conditions guarantee that only color singlet (triality zero) hadrons exist. A hadron is an extended region of space (a bag) containing quanta of quark and gluon fields.

In Ref. 5 we constructed a semiclassical quark model based on this picture. For a discussion of the assumptions and approximations involved, the reader is referred to Ref. 5. Here we wish only to review the ingredients in the phenomenological quark Hamiltonian. An S-wave hadron is made by populating the lowest Dirac eigenmode of a spherical cavity of radius R with quarks of the appropriate color, flavor and spin. The energy is a function of R and contains four terms:

$$E(R) = E_q + E_V + E_0 + E_g$$

The first term is the quark kinetic energy:

$$E_q = \frac{1}{R} \sum_{i=1}^N \left[x(m_i R)^2 + m_i^2 R^2 \right]^{1/2} \quad (2.1)$$

where N is the number of quarks in the state and $x(m_i R)$ is the (dimensionless) wave number of an S-wave quark in a cavity with bag boundary conditions. $x(mR)$ is plotted as a function of mR in Fig. 1. The quark kinetic energy dominates the phenomenological Hamiltonian — about 3/4 of the mass of a typical hadron arises from the motion of the quarks.

The next two terms, E_V and E_0 , are consequences of doing field theory in a finite domain. The first

$$E_V = BV \quad (2.2)$$

is the energy associated with the confining pressure, B ($V = \frac{4}{3} \pi R^3$). The second term

$$E_0 = -z_0/R \quad (2.3)$$

is a phenomenological estimate of the effects of zero point fluctuations of fields confined to a sphere of radius R .⁹

The final term is the energy associated with the gluon interactions of the quarks. The long range, strong, confining forces are provided by the bag term BV . It is therefore not necessary for the quark-gluon coupling to be strong (since the bag provides an infrared cutoff, the quark-gluon coupling need never become large). We treat the gluon interaction perturbatively in $\alpha_c = g^2/4\pi$ and find a relatively small value of α_c is consistent with the S-wave $Q\bar{Q}$ and Q^3 spectrum.^{5, 10}

To lowest order, only the Born graph of Fig. 2a and the self-energy graph of Fig. 2b contribute. Only the magnetic contribution is important:

$$E_g = -\frac{g^2}{2} \sum_{i \neq j} \sum_a \int d^3x \vec{B}_i^a(x) \cdot \vec{B}_j^a(x) \quad (2.4)$$

Other contributions—the electric energy and the self-energy graphs (Fig. 2b) are either small or included in the definitions of the observable (renormalized) parameters of the model.⁵ In Eq. (2.4) $\vec{B}_i^a(x)$ is the color ($a=1,2,\dots,8$) magnetic field produced by the i th quark. In Ref. 3, this integral was evaluated:

$$E_g = -\frac{\alpha_c}{R} \sum_{i>j} \sum_a \vec{\sigma}_i \cdot \vec{\sigma}_j \lambda_i^a \lambda_j^a M(m_i R, m_j R) \quad (2.5)$$

λ_i^a is the color of the i th quark just as $\vec{\sigma}_i$ is its spin.¹¹ $M(m_i R, m_j R)$ is the result of an integral over cavity wave functions which is graphed in Fig. 3.

The recipe for calculating the masses of S-wave hadrons is as follows: First construct wave functions properly antisymmetrized in flavor, color, and spin.

Second, diagonalize the energy in this basis. Third, minimize the energy eigenvalues with respect to the cavity radius R for each eigenstate:

$$\left. \frac{\partial E(R)}{\partial R} \right|_{R=R_0} = 0 \quad (2.6)$$

The mass of a hadron is given by $E(R=R_0)$. Minimizing the energy with respect to R is equivalent (for S-wave hadrons) to balancing the field pressure locally against the confining pressure B , which is a requirement of the original bag boundary conditions.

The parameters of the model were fixed once and for all by a fit to the masses of the $Q\bar{Q}$ and Q^3 S-wave hadrons. The results were

$$B^{1/4} = 146 \text{ MeV}$$

$$m_s = 279 \text{ MeV}$$

$$\alpha_c = .55$$

$$z_0 = 1.84$$

The reader may wish to consult Ref. 5 to judge the success of this simple model in describing the masses and static parameters (axial vector charges, magnetic moments, etc.) of the lightest hadrons.

The phenomenological Hamiltonian of Eqs. (2.1-2.3) and (2.5) is the basis of the rest of this paper. The only modification we have made is to approximate the mass dependence of $M(m_i R, m_j R)$ in Eq. (2.5) so that E_g is diagonal in flavor for states with a definite number of S-quarks. Specifically in a state with n_s strange quarks, we set

$$M(m_i R, m_j R) \rightarrow M\left(\frac{n_s}{N} m_s R, \frac{n_s}{N} m_s R\right) \quad (2.7)$$

for all i and j . For $n_s=0$ or N , this agrees with the exact calculation, in between it interpolates linearly. In the Q^3 sector, the approximation amounts to ignoring the Λ - Σ splitting relative to the N - Δ splitting. While this ignores interesting physics, it does not affect the spectroscopy of $Q^2\bar{Q}^2$ states at the level at which we are working.

III. DYNAMICS OF $Q^2\bar{Q}^2$ MESONS

The spectrum of S-wave $Q^2\bar{Q}^2$ mesons is much richer than that of S-wave $Q\bar{Q}$ mesons or Q^3 baryons. The general features of the spectrum are related to the various terms in the quark-bag Hamiltonian written down in the previous section. The first part of this section will concern the diagonalization of the Hamiltonian and the hierarchy of states which emerges. Afterwards we discuss the dynamics of production and decay of $Q^2\bar{Q}^2$ states which is essential in understanding their place in meson phenomenology.

A. Diagonalizing the Quark-Bag Hamiltonian

In the absence of gluon interactions the masses of $Q^2\bar{Q}^2$ states would range from 1460 MeV (no strange quarks) to 2140 MeV (four strange quarks). The quark kinetic energy operator, E_q , is diagonal in states with a well-defined number, n_s , of strange quarks. This is a generalization of "magic mixing" observed in, say, the ω - ϕ system. There the octet and singlet mix so that (to a first approximation) the ϕ is pure $s\bar{s}$, the ω pure $1/\sqrt{2}(u\bar{u}+d\bar{d})$. The phenomenology of magic mixing is quite different in the $Q^2\bar{Q}^2$ sector than the $Q\bar{Q}$ sector. For example, in a $Q^2\bar{Q}^2$ nonet the isovector state is $(u\bar{d}s\bar{s}, 1/\sqrt{2}(u\bar{u}-d\bar{d})s\bar{s}, d\bar{u}s\bar{s})$. The isosinglet analogue of the ω -degenerate with the isotriplet—is $1/\sqrt{2}(u\bar{u}+d\bar{d})s\bar{s}$, while the analogue of the $\phi(u\bar{u}d\bar{d})$ contains no strange quarks. The mixing induced by the kinetic energy

operator is very important in the phenomenology of the $Q^2\bar{Q}^2 0^+$ -nonet which we identify with physical scalar mesons.

As the magnetic gluon interaction is turned on the degeneracies in color and spin are lifted. E_g as approximated by Eq. (2.7) is diagonal in a basis of states which are magically mixed. The splittings and mixing induced by the magnetic exchange interaction are the subject of II. Here we wish only to emphasize that these splittings are essential in ordering the spectrum in a way compatible with experiment. This is no different than the state of affairs among the S-wave $Q\bar{Q}$ and Q^3 hadrons. There the magnetic gluon exchange is responsible for the Δ being heavier than the nucleon and the ρ being heavier than the π .^{5, 12} Any vector exchange (treated to lowest order) between particle and antiparticle makes the 1^- state heavier than the 0^- state. However, the fact that the $\frac{3}{2}^+$ state is heavier than the $\frac{1}{2}^+$ state depends crucially on the λ matrices which appear at the quark-gluon vertices.¹³

The success or failure of our treatment of $Q^2\bar{Q}^2$ mesons must be attributed primarily to the magnetic gluon interaction of Fig. 2a. In this sense our model is more general than the bag framework in which our calculations are performed. Nevertheless, an attempt to study $Q^2\bar{Q}^2$ mesons based on symmetry considerations alone would be much less predictive because of the large number of reduced matrix elements which are left undetermined. It is amusing that a large chunk of hadron spectroscopy might be explained in terms of as simple a mechanism as that illustrated in Fig. 2a.

B. Dynamics of Production and Decay of $Q^2\bar{Q}^2$ Hadrons

If it is heavy enough an S-wave $Q^2\bar{Q}^2$ meson will be unstable against decay into two S-wave $Q\bar{Q}$ mesons. The $Q^2\bar{Q}^2$ state simply falls apart as illustrated in Fig. 4a. In contrast, decay of a $Q\bar{Q}$ meson into two $Q\bar{Q}$ mesons (for example

$\rho \rightarrow 2\pi$ or $f \rightarrow 2\pi$) requires creation of a $Q\bar{Q}$ pair (Fig. 4b). We interpret the Okubo-Zweig-Iizuka rule¹⁴ as an inhibition associated with the creation or annihilation of quark lines. If decays like $\rho \rightarrow 2\pi$ or $f \rightarrow 2\pi$ are "Zweig allowed" then decays of $Q^2\bar{Q}^2$ states as in Fig. 4a are "Zweig super allowed."

Of course a given $Q^2\bar{Q}^2$ meson cannot "fall apart" into any arbitrary $Q\bar{Q}$ mesons. The decay products must be S-wave mesons in relative S waves. Thus a $Q^2\bar{Q}^2$ 2^+ -state cannot fall apart into two π -mesons since they would have to be in a D-wave. It could however fall apart into two ρ -mesons. This 2^+ -state might couple to $\pi\pi$ but only by a mechanism like that shown in Fig. 4c. Two gluons are required because the final state mesons must be color singlets. We assume such couplings are suppressed.

The Zweig super-allowed decays of a $Q^2\bar{Q}^2$ state are calculated by a change of coupling transformation. Any $Q^2\bar{Q}^2$ state may be written as a linear superposition of $(Q\bar{Q})(Q\bar{Q})$ states coupled to the same total flavor, spin and color (singlet). The recoupling coefficients, which weight the terms in the sum, tell us the amplitude for the $Q^2\bar{Q}^2$ meson to fall apart into that particular channel. Thus, for example, a $Q^2\bar{Q}^2$ 0^+ -meson could fall apart into four channels: (1) two color-singlet, pseudoscalar $Q\bar{Q}$ mesons; (2) two color-singlet, vector $Q\bar{Q}$ mesons; (3) two color-octet, pseudoscalar $Q\bar{Q}$ mesons; and (4) two color-octet, vector $Q\bar{Q}$ mesons. The first two are physical, Zweig super-allowed decays. The latter two cannot occur—all physical mesons are color-singlets—without further exchange of at least one gluon (Fig. 4d), which we assume to be suppressed.

For the purposes of this paper we consider only Zweig super-allowed decays. All suppressed decays are assumed forbidden. This is allowable so long as all $Q^2\bar{Q}^2$ states have large recoupling coefficients to at least one super-allowed decay channel which is also allowed by energy momentum conservation. This

is true for all $Q^2\bar{Q}^2$ mesons except certain members of the lowest nonet which we discuss individually.

We are now in a position to enumerate a set of rules for determining the phenomenology of $Q^2\bar{Q}^2$ mesons.

- (1) If a given $Q^2\bar{Q}^2$ state is above threshold for decay into a "fall apart" channel, it is very broad into that channel.
- (2) All non-fall apart decays are to be ignored (to a first approximation).
- (3) An S-wave $Q^2\bar{Q}^2$ meson can only couple (in light of (2)) to two color singlet S-wave $Q\bar{Q}$ mesons in a relative S-wave.
- (4) The coupling of any $Q^2\bar{Q}^2$ meson to any $(Q\bar{Q})(Q\bar{Q})$ fall apart channel is determined by a change of coupling transformation up to one universal multiplicative factor, g_0 . g_0 represents the coupling of a $Q^2\bar{Q}^2$ meson to a $(Q\bar{Q})(Q\bar{Q})$ channel with which it has perfect overlap of quantum numbers.
- (5) For any $(Q\bar{Q})(Q\bar{Q})$ channel of definite spin and flavor the sums of the squares of the couplings to all $Q^2\bar{Q}^2$ states is unity. There is but one elastic $Q^2\bar{Q}^2$ state per channel though its strength may be spread over several inelastic resonances. This follows from the orthogonality of the recoupling matrices.

The great width of most $Q^2\bar{Q}^2$ mesons will account for their experimental elusiveness. It gives us two headaches in return: first we are confronted with mesons whose width is a substantial fraction of their mass. A calculation of their masses which ignores decay processes (as does ours) must not be taken too literally. We should not expect the accuracy we demanded in our treatment of $Q\bar{Q}$ mesons and Q^3 baryons.⁵ Second, the great width of these states may make it difficult to establish their resonant character at all. The $\epsilon(700)$ (which

we claim to be a $Q^2\bar{Q}^2$ state) provides a clear example of this. Many higher mass $Q^2\bar{Q}^2$ states not only may be as broad and confusing as the ϵ , but also will probably occur in channels with substantial inelastic background obscuring their resonant behavior. This must be kept in mind when we discuss exotic resonances in Section VI.

C. Zweig Rule Violation Among $Q^2\bar{Q}^2$ Mesons

Finally we turn to the modification of magic mixing by Zweig rule violating processes. Annihilation of quarks into some number of gluons provides an SU(3)-flavor singlet force which can alter the magic mixing dictated by E_q . It has been conjectured¹⁵ that the success of magic mixing in the ω - ϕ system follows from the smallness of Fig. 5a. Figure 5a is of order α_c^3 and this is the lowest order in which ω - ϕ mixing can occur. Since α_c is small this may account for the suppression.

Curiously, $Q^2\bar{Q}^2$ states may mix to order α_c via the diagram in Fig. 5b. Some fraction of the $Q^2\bar{Q}^2$ wavefunction has at least one $Q\bar{Q}$ pair in a color-octet, vector state. This $Q\bar{Q}$ pair may annihilate virtually into a single vector gluon. Since this diagram is of order α_c one might expect large violations of Zweig's rule, i. e., physical states which are not magically mixed. This is not always the case. The reason lies in the recoupling coefficient which determines the fraction of the $Q^2\bar{Q}^2$ state which can annihilate. In the case of immediate interest to us, the $Q^2\bar{Q}^2$ 0^+ -nonet, the projection onto color-octet vector mesons introduces an additional factor of ~ 0.15 . Thus a large underlying matrix element ($0(\alpha_c)$) is multiplied by a small coefficient and induces only a small mixing.

The problem only arises for us in the 0^+ -nonet which we discuss in Section V. We mention it here because the mechanism is somewhat unconventional and might be of wider interest.

IV. THE SPECTRUM OF $Q^2\bar{Q}^2$ MESONS

Two quarks may reside in a $\underline{\bar{3}}$ or $\underline{6}$ of flavor $SU(3)$. When coupled to two antiquarks the following flavor multiplets arise

$$\underline{\bar{3}} \otimes \underline{3} = \underline{1} \oplus \underline{8} \equiv \underline{9} \quad (4.1)$$

$$\underline{6} \otimes \underline{\bar{6}} = \underline{1} \oplus \underline{8} \oplus \underline{27} \equiv \underline{36} \quad (4.2)$$

$$\begin{aligned} \underline{6} \otimes \underline{3} + \underline{\bar{3}} \otimes \underline{\bar{6}} &= \underline{8}_f \oplus \underline{8}_d \oplus \underline{10} \oplus \underline{\bar{10}} \\ &\equiv \underline{18} \oplus \underline{\bar{18}} \end{aligned} \quad (4.3)$$

These multiplets mix magically so that the $SU(3)_f$ labels $\underline{1}$, $\underline{8}$, etc. apply only to states which do not mix.

Which flavor multiplets occur with a given total spin is determined by the exclusion principle. In Π we find

$$\begin{aligned} J^P = 2^+ &: \underline{9}, \underline{36} \\ J^P = 1^+ &: \underline{9}, \underline{36}, \underline{18}, \underline{18}^*, \underline{\bar{18}}, \underline{\bar{18}}^* \\ J^P = 0^+ &: \underline{9}, \underline{9}^*, \underline{36}, \underline{36}^* \end{aligned} \quad (4.4)$$

Two multiplets with identical spin and flavor content are distinguished by an asterix (applied to the heavier).

States are labelled as follows:

1. Exotics (E) carry a subscript denoting the (pseudoscalar) flavor channel to which they couple. They are also labelled by their spin-parity and the $SU(3)_f$ "multiplet" ($\underline{36}$ or $\underline{18}$ or $\underline{\bar{18}}$) in which they reside.
2. Cryptoexotics (C) carry as a subscript the name of the corresponding pseudoscalar with the same flavor quantum numbers. The number of $s\bar{s}$ -pairs in the state is denoted

by a superscript s (one pair) or ss (two pairs). States at the center of the $SU(3)$ weight diagram carry no subscript.

Occasionally another superscript is necessary to distinguish the G -parity (or more generally $SU(3)$ -parity) of otherwise degenerate states.

The weight diagrams for the three multiplets of interest, Eqs. (4.1-4.3), are given in Figs. 6-8. Also in Figs. 6-8, we give the recoupling to $Q\bar{Q}$ octet and singlet decay channels. Thus, for example, according to Fig. 7, $C_K^S(\underline{36})$ is a $Q^2\bar{Q}^2$ meson with the flavor quantum numbers of a kaon but containing a hidden $s\bar{s}$ -pair. It has the same flavor content as $K\eta_s$ (η_s is shorthand for $s\bar{s}$, η_0 denotes $1/\sqrt{2}(u\bar{u}+d\bar{d})$). The actual fall apart decay channels of a given meson are determined by its spin and color content in conjunction with the flavor recoupling given in Figs. 6-8. For example, a $C_K^S(\underline{36}, 2^+)$ could decay to $K^*\phi$ (we take ϕ to be pure $s\bar{s}$), while a $C_K^S(\underline{36}, 0^+)$ decays to $K^*\phi$, $K\eta$ or $K\eta'$ (the physical η and η' are linear combinations of η_0 and η_s). Figures 6-8 refer only to the flavor recoupling calculation.

The masses of $Q^2\bar{Q}^2$ states are listed in Tables I-III. The spin and color recouplings of the $Q^2\bar{Q}^2$ multiplets are listed in Tables IV-VI. To calculate the coupling of a particular $Q^2\bar{Q}^2$ meson to decay channels multiply the flavor recoupling coefficients of Figs. 6-8 by the color and spin coefficients of Tables IV-VI.¹⁶ Thus, for example, the $C_K^S(\underline{36}, 0^+)$ with mass 1750 MeV couples as follows to decay channels:

$$C_K^S(\underline{36}, 0^+) = -.644 K\eta_s + .269 K^*\phi - .322 \underline{K}\cdot\underline{\eta_s} - .639 \underline{K^*}\cdot\underline{\phi}$$

(the boldface $\underline{\quad}$ denotes a color-octet meson, the dot product $\underline{K^*}\cdot\underline{\phi}$ denotes the coupling of the two color octets to a color singlet). Thus the $C_K^S(\underline{36}, 0^+)$ appears primarily in the ηK channel.

The masses and recoupling coefficients collected in Figs. 6-8 and Tables I-VI enable one to study any channel of interest. We turn to specific channels for phenomenological support for our model.

V. THE 0^+ -CRYPTOEXOTIC NONET

The most striking feature of the spectra of the previous section is the low mass nonet of 0^+ -mesons. This multiplet is light for the same reason that the pseudoscalar $Q\bar{Q}$ mesons and spin-1/2 Q^3 -baryons are light: attractive gluon-magnetic interactions. Since these states are predicted to lie well within the mass range covered by good phase shift analyses of $\pi\pi$, $K\bar{K}$ and πK scattering they should already be known. In this section we construct a case that these $Q^2\bar{Q}^2$ states are none other than the best known scalar mesons. We first review the qualitative features of the 0^+ -mesons and simple attempts to classify them as $Q\bar{Q}$ P-wave states. Then we turn to our $Q^2\bar{Q}^2$ nonet and compare its features with the physical states.

A. The 0^+ -Mesons in the $Q\bar{Q}$ Quark Model

Not much is known about the 0^+ -mesons. It has taken a long time to establish their resonant parameters¹⁷ and quite a bit of controversy remains.¹⁸⁻²⁰ We will allow for some controversy by constructing alternative quark models. We note the following properties:

- a) The $S^*(993)$ is a narrow resonance which couples more strongly to $K\bar{K}$ than to $\pi\pi$ ¹⁹:

$$M(S^*) = 993.2 \pm 4.4 \text{ MeV}$$

$$\Gamma(S^*) = 40.0 \pm 7.4 \text{ MeV}$$

$$1.3 \lesssim \frac{g_{S^* K\bar{K}}}{g_{S^* \pi\pi}} \lesssim 2.5$$

- b) The $\epsilon(700)$ is a very broad resonance which couples weakly if at all to $K\bar{K}$ ²¹:

$$M(\epsilon) = 660 \pm 100 \text{ MeV}$$

$$\Gamma(\epsilon) = 640 \pm 140 \text{ MeV}$$

- c) The $\delta(976)$ is a narrow isospin-1 resonance coupling to both $\eta\pi$ and $K\bar{K}$:

$$M(\delta) = 976.4 \pm 5.4 \text{ MeV}$$

$$\Gamma(\delta) = 46.9 \pm 11.2 \text{ MeV}$$

- d) The κ is a very broad and poorly known resonance in $K\pi$ scattering

$$M(\kappa) \sim 1300 \text{ MeV}$$

Conventional quark models¹⁸ assign these mesons to a nonet with $L=1$.

This raises severe problems. Since the S^* and δ are nearly degenerate one might assume (in analogy to the ρ - ω - ϕ system) that the 0^+ -mesons are magically mixed:

$$S^* = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d})$$

$$\epsilon = s\bar{s}$$

This fails because (in decreasing order of severity)

1. ϵ couples to $\pi\pi$ not $K\bar{K}$.
2. S^* couples strongly to $K\bar{K}$ and only weakly to $\pi\pi$.
3. $M(\epsilon) < M(S^*)$ in contrast to our expectation in a quark model.
4. $M(\kappa)$ (though poorly known) is too large.
5. Very large spin orbit forces are necessary to split the 0^+ $L=1$ states so far from the 2^+ $L=1$ states.

Alternatively one may abandon the assumption that the S^* and ϵ mix magically, and adjust their mixing to obtain the observed decay couplings. But then:

- 1'. The S^* (mostly $s\bar{s}$)- δ degeneracy is accidental.
- 2'. The ϵ (mostly $(u\bar{u} + d\bar{d})$)- δ splitting is unaccounted for.
- 3'. The large violation of Zweig's rule (magic mixing) is unaccounted for.

and 4 and 5 remain.

Finally one can ignore the $\epsilon(700)$ entirely and find instead a broad $0^+ \pi\pi$ -resonance near 1200 MeV, which is not the $\epsilon'(1240)$ that goes along with the $\epsilon(700)$.¹⁷ If magic mixing is still maintained this cures problems 3, 4 and perhaps 5 but does not address problems 1 and 2. If magic mixing is abandoned in this scheme the mass splittings and degeneracies again become accidental. Furthermore the $\epsilon(700)$ effect is left out in the cold.

In all the 0^+ -nonet is not very attractive from the viewpoint of the $Q\bar{Q}$ $L=1$ assignment. The elegance of the quark model has always been its simplicity—consider for example the 1^- or 2^+ mesons. We turn now to a simple explanation of the 0^+ -phenomenology.

B. The 0^+ -Mesons as $Q^2\bar{Q}^2$ States

The lowest nonet of $Q^2\bar{Q}^2$ states are natural candidates for the observed 0^+ -mesons. The quark content of the nonet is shown in Fig. 9. The object with no strange quarks, $C^0(\underline{9}, 0^+)$, is predicted to have a mass of 650 MeV. The degenerate isosinglet, $C^S(\underline{9}, 0^+)$, and isotriplet, $C^S_\pi(\underline{9}, 0^+)$, are predicted to have a mass of 1100 MeV. We propose the following identifications:

$$\epsilon(700) = C^0(\underline{9}, 0^+) = u\bar{u}d\bar{d}$$

$$S^*(993) = C^S(\underline{9}, 0^+) = \frac{1}{\sqrt{2}} s\bar{s}(u\bar{u} + d\bar{d})$$

$$\delta(976) = C_{\pi}^S(\underline{9}, 0^+) = u\bar{d}s\bar{s}, \text{ etc.}$$

$$\kappa(\ ?) = C_K(\underline{9}, 0^+) = u\bar{s}d\bar{d}, \text{ etc.}$$

The solution to the $S^*-\epsilon$ puzzle is immediate. The $\epsilon(700)$ falls apart into $\pi\pi$. Since the state is well above threshold the ϵ is very broad. The $S^*(993)$ is naturally degenerate with the $\delta(976)$ and falls apart into $K\bar{K}$. The $S^*(993)$ would be very broad into $K\bar{K}$ were it not so close to threshold. The decay $S^* \rightarrow \pi\pi$ requires Zweig rule violation of the type discussed in Section III. Thus the $S^*(993)$ is narrow and couples predominantly to $K\bar{K}$.

The couplings of $\delta(976)$ to $K\bar{K}$ and $\eta\pi$ are determined by SU(3) and the $\eta-\eta'$ mixing angle. If the η is pure octet we predict

$$\frac{g_{\pi}^{\delta}}{g_{K\bar{K}}^{\delta}} = \sqrt{\frac{2}{3}}$$

The δ coupling to $\pi\eta$ is reduced as the $s\bar{s}$ content of the η is reduced. If η is pure octet we predict the width of δ into $\pi\eta$ to be larger than observed. Of course the η is not pure octet. We shall parametrize the $\delta \rightarrow \pi\eta$ width in terms of an $\eta-\eta'$ mixing angle θ_{η} ($\eta = \eta_{\underline{8}} \cos\theta_{\eta} + \eta_{\underline{1}} \sin\theta_{\eta}$) which may be compared with $\eta-\eta'$ mixing determined from other processes. We require a larger value of θ_{η} than obtained from a quadratic Gell-Mann Okubo relation.²²

We predict the κ -state to have mass 900 MeV. It should be extremely broad into πK . The present high mass of the κ is in apparent conflict with this assignment.

The recoupling tables introduced in the last section allow us to predict the decay couplings of the 0^+ -mesons in terms of two parameters: one overall constant, \bar{g}_0 ($\bar{g}_0 = .74 g_0$, where g_0 is the fundamental coupling introduced in

Section III); and the η - η' mixing angle θ_η . For the present we ignore Zweig rule violation. The decay couplings are given in Table VII. The actual widths are strongly influenced by phase space which is not included in Table VII.

The $Q^2\bar{Q}^2$ assignment is in qualitative agreement with all features of the 0^+ -nonet except for the κ -mass.

C. Another 0^+ -Nonet

Another obvious feature of our assignment is the need for a second 0^+ -nonet to incorporate the conventional $Q\bar{Q}$ 0^+ -states. These will presumably lie near the 2^+ -states (naive bag model estimates predict $M(0^+) > M(2^+)$) and should form a rather ordinary multiplet, i. e., magically mixed with a ϕ -like state lying higher. Tentatively we associate the $\epsilon'(1240)$ ¹⁷ and the $\delta'(1255)$ of Ref. 8 with this nonet. A ϕ -analogue remains to be discovered. It should be approximately as massive as the f' and couple predominantly to $K\bar{K}$.

Finally we return to the isospin-1/2 πK S-wave. We are led to predict a broad resonance in the region of about 900 MeV followed by a narrower state ($Q\bar{Q}$) at about 1300 MeV. We suggest this as a possibility in future phase shift analyses.

VI. OTHER EXOTIC AND CRYPTOEXOTIC MESONS

Meson states may be observed in phase shifts obtained from Chew-Low type analyses of meson-nucleon scattering or as bumps in mass spectra. Here we discuss the contribution of our $Q^2\bar{Q}^2$ mesons to both sorts of searches. After noting some general principles we catalog the effects of $Q^2\bar{Q}^2$ mesons on experimentally accessible channels.

A. General Remarks

S-wave $Q^2\bar{Q}^2$ mesons are either 0^+ , 1^+ or 2^+ states. Only 0^+ or 2^+ states could be observed in formation experiments (we assume only $\pi\pi$, πK and KK phase shifts are accessible). For reasons discussed in Section III the $Q^2\bar{Q}^2$ 2^+ -mesons couple weakly to two pseudoscalars. On the other hand it is apparent from the spectrum of Table III and the recoupling coefficients of Table VI that $Q^2\bar{Q}^2$ 2^+ -states couple strongly to two vector mesons and are massive enough to decay into them. Thus all $Q^2\bar{Q}^2$ 2^+ -states would be weakly excited, broad and very inelastic in formation channels. Consequently only $Q^2\bar{Q}^2$ 0^+ -states can be seen in phase shift analyses.

According to Table IV the lighter $Q^2\bar{Q}^2$ 0^+ -state with a given flavor content couples strongly to two pseudoscalars. The heavier couples strongly to two vectors. Consequently, the heavier 0^+ -multiplets (9* and 36*) will not be seen in phase shift analyses. The lighter will be seen but generally will be broad because the (dominant) pseudoscalar decay channels are open.

We are left with the 0^+ 9 and 36 as serious candidates for phase shift analyses. The 9 was the subject of the previous section, the 36 will be discussed below.

$Q^2\bar{Q}^2$ resonances are generally so broad that they would not show up in bump hunts. An exception is the δ which (in our assignment) is narrower because the $K\bar{K}$ channel is closed and because the η is deficient in $s\bar{s}$ content inhibiting the $\pi\eta$ decay. We expect to see bumps only if the $Q^2\bar{Q}^2$ resonance is forced to be narrow because of some phase space or recoupling inhibition. This will be true especially for 2^+ -states which decay preferentially into two (relative massive) vector mesons.

B. Phase Shift Analyses

Only the lighter 0^+ 36-plet might be observed. The states in this multiplet are predicted to range in mass from 1150 to 1950 MeV. It is convenient to enumerate the states by their flavor quantum numbers.

$(\pi\pi)^{I=2}$

The $E_{\pi\pi}(\underline{36}, 0^+)$ state occurs at a mass of 1150 MeV. Its coupling to $\pi\pi$ is identical to the coupling of $\epsilon(700)$ to $\pi\pi$ (.64 g_0 , see Tables IV and VII and Figs. 6 and 7). Recent $\pi\pi$ phase shifts show no sign of resonance structure up to about 1300 MeV assuming no inelasticity.⁴ If our picture is correct, the estimate of 1150 MeV must be too low. If the $E_{\pi\pi}(\underline{36}, 0^+)$ is heavier it is increasing inelastic into $\rho\rho$ and will be more difficult to see above an inelastic background.

Given the enormous width of the $E_{\pi\pi}(\underline{36}, 0^+)$ ($\Gamma(\epsilon)$ is 640 MeV) it is not surprising that our estimate of its mass should be in error. Nevertheless a very broad, somewhat inelastic $\pi^+\pi^+$ resonance should occur somewhere below, say, 2 GeV.

$(\pi K)^{I=3/2}$ and $(KK)^{I=1}$

The same remarks apply to these channels as apply to $(\pi\pi)^{I=2}$. Broad resonances at masses 200 MeV (for πK) and 400 MeV (for KK) heavier than the $(\pi\pi)^{I=2}$ resonance are expected. Current phase shift analyses are not sufficiently accurate to look for these states.

$(\pi\pi)^{I=0}$

The 36-plet contains only one state which couples to this channel: $C^0(\underline{36})$. This state couples strongly to $\eta\eta$ and $\eta\eta'$. Most of the $\pi\pi$ isoscalar strength was used up in the $\epsilon(700)$.²³ $C^0(\underline{36})$ is therefore a broad state coupling primarily

to $\eta\eta$:

$$\frac{g_{\eta\eta}^2}{g_{\pi\pi}^2} = 3 \cos^2(\theta_\eta - \theta_0)$$

which will be difficult to see in $\pi\pi$ -scattering.

States like $C^S(\underline{36})$ and $C^{SS}(\underline{36})$ have the quantum numbers of $(\pi\pi)^{I=0}$ but do not couple to $\pi\pi$ without Zweig rule violation. Like the $S^*(993)$ they manifest themselves as a drop in $\pi\pi$ elasticity. Unlike the $S^*(993)$ they are very broad (way above $K\bar{K}$ or $\eta\eta$ threshold) and will be difficult to see in $\pi\pi$ scattering.

So it goes with the rest of the $\underline{36}$ -plet. The states are heavy, broad and usually inelastic. The reader may construct a scenario for the channel of his choice by combining the flavor recoupling of Fig. 7, the color-spin recoupling of Table IV, and the mass spectra of Table I. We turn now to production experiments.

C. Bump Hunts

There are broad S-wave exotic and cryptoexotic enhancements in all 0^+ , 1^+ and 2^+ channels. We assume very broad states cannot be distinguished from background in mass spectra. Here we discuss only those states which by virtue of their light mass might be inhibited in their fall apart decays and be narrower than otherwise expected. Sometimes a state may recouple with small amplitude to an open channel but with large amplitude to a closed or nearly closed channel. With our present poor understanding of decay mechanisms we cannot judge how narrow such states may be, nevertheless we enumerate them below.

0^+ -States

The excited nonet $\underline{9}^*$ lies close to vector-vector decay thresholds, furthermore these states do not recouple strongly to two pseudoscalars. If their mass

is not underestimated and their coupling to pseudoscalars is indeed suppressed we are led to expect relatively narrow states: an isosinglet at about 1450 MeV coupling to $\rho\rho$; an isovector at about 1800 MeV coupling to $\rho\phi$ and $K^*\bar{K}^*$, an isosinglet also at 1800 MeV coupling to $K^*\bar{K}^*$, and a quartet of K-like states at about 1600 MeV coupling to ρK^* and ωK^* . All other 0^+ -states (the 36 and 36*) are massive enough to be broad.

1⁺-States

The lightest 1^+ -multiplet is a nonet with the same flavor structure as the 0^+ -nonet discussed in the previous section. The lightest state is an isosinglet 1^{+-} with mass about 1200 MeV coupling strongly to $\rho\pi$. This broad enhancement has the quantum numbers of the long sought H (the isoscalar brother of the B) and is in a mass region in which it might obscure the hunt for the H meson. The remaining members of this multiplet are presently of little interest. The object with the quantum numbers of the B couples to $\phi\pi$ rather than $\omega\pi$ (a consequence of the hidden $s\bar{s}$ pair) and would not obscure the B.

The lightest 18 and 18 multiplets couple strongly to vector-pseudoscalar channels—all of which are open. Hence they are broad. These multiplets contain states with the quantum numbers of the A_1 and B mesons with masses estimated to be about 1250 MeV.

The 36 of 1^+ -mesons is also sufficiently heavy to be very broad into vector-pseudoscalar channels, which are favored by the recoupling calculation. In contrast the 18* and 18* multiplets while heavy, decay preferentially into two vector mesons. The lightest such states are estimated to lie at about 1650 MeV, somewhat above $\rho\rho$ threshold. If our estimates are slightly too high, these states may be narrow enough to observe.

The $\underline{18}^*$ and $\overline{18}^*$ occur at the same mass as the $Q^2\overline{Q}^2$ 2^+ -states, and like the 2^+ -states are characterized by decay into two vectors. The 2^+ -states are a clearer case since they can fall apart only into two vectors (the 1^+ $\underline{18}^*$ and $\overline{18}^*$ couple to vector-pseudoscalar and may be very broad into those channels) so we confine our (illustrative) remarks to those channels.

2^+ -States

The $Q^2\overline{Q}^2$ 2^+ -states all fall apart exclusively into two vectors. Although we estimate the states to be at least about a hundred MeV above the appropriate thresholds it is tantalizing to speculate on the possibility that we have over-estimated their masses.

Supposing that these states are light enough to be narrow, we would expect to observe them as threshold enhancements in vector-vector channels. Good signatures are in the K^*K^* and $K^*\overline{K}^*$ channels at an estimated mass of 1950 MeV ($2m_{K^*} = 1880$ MeV); and in the $\phi\phi$ channel at an estimate mass of 2250 MeV. $\omega\omega$ and $\rho\rho$ enhancements are harder to pick up because of the difficulty of detecting neutrals.

Clearly we have only scratched the surface of this spectroscopy. Without a guide from experiment it is too early to enumerate the couplings of the many heavy, broad $Q^2\overline{Q}^2$ states we have found. At any event, the reader may reconstruct the model's predictions for the channel of his choice by juggling the tabulated recoupling coefficients.

VII. CONCLUSIONS AND DISCUSSION

We have proposed a rather radical solution to the $Q^2\overline{Q}^2$ problem. The solution was not tailored to the problem, rather it was forced on us by the structure of the effective Hamiltonian which successfully describes $Q\overline{Q}$ and Q^3 states. Fortunately this picture of the $Q^2\overline{Q}^2$ meson has the virtue of making

rather concrete predictions, which may be tested experimentally. In closing we wish first to summarize the predictions and then to mention some theoretical and phenomenological avenues which might be explored in the future.

First the predictions:

I. The ϵ , δ , S^* , κ are $Q^2\bar{Q}^2$ mesons.

1. The couplings of Table VII should roughly describe the decay couplings of these states. S-wave phase space must be included before comparing with actual decay processes. A more sophisticated approach might be to use the couplings to Table VII as input to some dynamical unitarization scheme.
2. The $\epsilon(700)$ coupling to $K\bar{K}$ and the $S^*(993)$ coupling to $\pi\pi$ are small and consequences of Zweig rule violation. They should be described by a single mixing angle.
3. Either the δ coupling of the $\pi\eta$ is larger than naively extracted from the $\pi\eta$ -mass plot, or the $s\bar{s}$ content of the η is less than Gell-Mann-Okubo implies (see footnote 22).

II. There is another entire nonet of 0^+ -mesons nearly degenerate with the f , A_2 , $K^*(1420)$, f' .

These are the $Q\bar{Q}$ $L=1$ states. In particular,

1. The $\epsilon'(1240)$ and $\delta'(1255)$ of Cason et al.⁸ are the 0^+ -analogues of the ω - ρ or f - A_2 . They will be conventional quark model states.
2. Another 0^+ -state coupling to $K\bar{K}$ will complete the hypercharge zero sector of the nonet. It should lie approximately beneath the $f'(1520)$.

III. The $K\pi$ $I=1/2$ S-wave is not correctly interpreted at present.

There should be a very broad $K\pi$ enhancement (analogous to the $\epsilon(700)$) at roughly 900 MeV. This is the lowest $Q^2\bar{Q}^2$ state. At approximately the mass of the $K^*(1420)$ there should be a (conventional) relatively narrow $K\pi$ S-wave resonance. This is the strange member of the $Q\bar{Q}$ P-wave nonet. The situation may look similar to the $\pi\pi$ -system where the phase grows slowly through the ϵ -region until the narrow S^* makes its appearance.

IV. There should be a very broad, probably inelastic resonance in the isospin 2, S-wave of $\pi\pi$ -scattering, somewhere in the region below 2 GeV.

Since present analyses⁴ show a negative phase in the region less than 1300 MeV there is presumably an inelastic background in this channel. Finding a broad inelastic resonance over an inelastic background will not be easy.

V. There may be less broad enhancements, particularly with spin 1^+ or 2^+ near vector-vector thresholds.

Some of these states have exotic quantum numbers ($\rho^+\rho^+$, K^*K^* , etc.). We calculate their masses to be sufficiently high to allow them to be broad into vector-vector. If the masses are overestimated the vector-vector decays may be inhibited by phase space making the states less broad. A particularly good candidate is the exotic K^*K^* $I=1$ resonance calculated to be at 1950 MeV.

VI. The predictions for $Q^2\bar{Q}^2$ resonances in every channel of meson-meson scattering can be read off from the recoupling coefficients of Figs. 6-8 and Tables IV-VI and from the spectra of Tables I-III.

Probably the most pressing phenomenological problem is to obtain a firmer grasp on the manner in which a $Q^2\bar{Q}^2$ resonance appears in a $(Q\bar{Q})(Q\bar{Q})$ channel. If the $\epsilon(700)$ and its brethren are any indication, the effects may be quite subtle.^{19, 20}

The zero width approximation masses and couplings we have calculated would be the input to such a calculation.

It is interesting to apply these quark-bag model ideas to the $Q^4\bar{Q}$ baryons. Baryon resonances are better known than meson resonances in general and there is less room to accommodate funny states at low masses. Preliminary calculations give some cause for optimism: the lowest multiplet is not exotic (a nonet) and like the 9 of $Q^2\bar{Q}^2$ mesons, many of the nonet members contain $s\bar{s}$ pairs making them heavy and coupled to relatively obscure channels. Progress in quark models is slow. It is perhaps then not surprising that the $Q^4\bar{Q}$ sector may remain refractory for some time to come.

Acknowledgements

Ken Johnson collaborated on the early phases of this work. I wish to thank him for many ideas and suggestions which were essential. I also wish to thank him and Jeff Mandula for allowing me to delve into their bags of group theory tricks and find several gems which made these calculations both more tractable and more enjoyable.

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10. The characteristic wavelengths of the gluon fields are governed by the size of the hadron, which for all the states we consider is approximately 1 fm. In the language of the renormalization group α_c is the running coupling constant at a q^2 of approximately 1 fm^{-2} .
11. We normalize $\text{Tr } \lambda^2 = 2$.
12. To my knowledge, this was first observed by A. deRujula, H. Georji and S. L. Glashow, Phys. Rev. D 12, 147 (1975) in the context of a nonrelativistic quark model.
13. The dependence of the Δ -N mass difference on color is as follows. If only spin were involved higher spin states would be lower. The ρ is more massive than the π because the mesons are a particle-antiparticle system which introduces an additional minus sign. To make a color singlet baryon, however, two quarks must be in a $\bar{\mathbf{3}}$ representation (rather than a $\mathbf{6}$) to couple with the final quark to make a singlet. Thus, together, any pair of quarks behaves like an antiquark, an additional minus sign appears and the Δ is heavier than the nucleon.
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15. A. deRujula and S. L. Glashow, Phys. Rev. Letters 34, 46, 1252 (1975); J. Pasupathy and G. Rajasekaran, ibid., 1250. See also Ref. 12.
16. It's not always quite this simple. The 18 and $\bar{18}$ multiplets are mixed by final state interactions. Consider, for example, the states $C_\pi(\underline{18})$ and $C_\pi(\overline{18})$ both of which couple to $\rho\pi$ and $\omega\pi$. They are obviously not G-parity

eigenstates. The linear combinations $C_{\pi}^{\pm} = 1/\sqrt{2} (C_{\pi}(\underline{18}) \mp C_{\pi}(\overline{18}))$ are G -parity eigenstates and recouple as follows:

$$C_{\pi}^{+} = \frac{\sqrt{2}}{3} (\pi\omega + \eta_0\rho) + \frac{1}{\sqrt{2}} (\underline{\rho} \cdot \underline{\rho}) + \frac{1}{3\sqrt{2}} \underline{\rho} \cdot \underline{\pi}$$

$$C_{\pi}^{-} = \frac{2}{3} \rho\pi - \frac{1}{\sqrt{2}} \underline{\omega} \cdot \underline{\rho} - \frac{1}{3\sqrt{2}} (\underline{\omega} \cdot \underline{\pi} - \eta_0 \cdot \underline{\rho})$$

(boldface denotes color octet states). Since they couple to distinct decay channels C_{π}^{\pm} will not be mixed further. All of the states in 18, $\overline{18}$, 18* and $\overline{18}^*$ except the exotics, $C_K^S(\underline{18})$ and $C_K^S(\overline{18})$ mix in this manner.

17. After much of this work was completed the $\epsilon(700)$ was declared dead by the Particle Data Group (T. G. Trippe et al., Rev. Mod. Phys. 48, 51 (1976)), and replaced with a very broad $\epsilon(1200)$. Resonance fits to phase shifts are compatible with either, the choice of a state at 1200 MeV seems prejudiced by a desire to make the 0^+ -nonet into a "conventional" $Q\bar{Q}$ multiplet following Morgan (Ref. 19). Without this theoretical prejudice the question seems to remain open. We shall retain the $\epsilon(700)$ of Protopopescu et al. (Phys. Rev. D 7, 1279 (1973)).

The $\epsilon'(1240)$ to which we refer is the state quoted by J. T. Carroll et al., Phys. Rev. 28, 318 (1972) and by P. Estabrooks et al., in Proc. of the International Conference on $\pi\pi$ Scattering and Associated Topics at Talahassee, ed. by P. K. Williams and V. Hagopian (AIP, New York, 1973).

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22. The Gell-Mann-Okubo relation quadratic in meson masses predicts

$|\theta_\eta| \approx 10^\circ$, linear it predicts $|\theta_\eta| \approx 23^\circ$. $\Gamma(\delta) = 50$ MeV requires $\theta_\eta = 36^\circ$. Mixing schemes based on Zweig rule violation (see, for example, the Appendix to ref. 5) imply a larger octet singlet mixing. In fact the calculation in ref. 5 gives $\theta_\eta \approx 40^\circ$. Clearly the η - η' system is not well understood but it is intriguing that a value of $\theta_\eta \approx 36^\circ$ is not excluded.

23. The sum of the squares of the couplings of all $Q^2\bar{Q}^2$ states to isoscalar $\pi\pi$ is unity—a special case of the result quoted in Section III.

TABLE I

The predicted masses of $Q^2\bar{Q}^2 0^+$ -mesons listed according to their SU(3) multiplet (see Eqs. (4.1) - (4.3)) and name (see Figs. 6-8). Masses are quoted to the nearest 50 MeV.

SU(3) Multiplet	State	Mass (MeV)
<u>9</u>	C^0	650
	C_K	900
	C^S, C_π^S	1100
<u>36</u>	$E_{\pi\pi}, C_\pi, C^0$	1150
	$E_{\pi K}, C_K$	1350
	$E_{(KK)^1}, C_\pi^S, C^S$	1550
	C_K^S	1750
	C^{SS}	1950
<u>9*</u>	C^0	1450
	C_K	1600
	C^S, C_π^S	1800
<u>36*</u>	$E_{\pi\pi}, C_\pi, C^0$	1800
	$E_{\pi K}, C_K$	1950
	$E_{(KK)^1}, C_\pi^S, C^0$	2100
	C_K^S	2200
	C^{SS}	2350

TABLE II

Predicted masses of $Q^2\bar{Q}^2 1^+$ -mesons listed according to their SU(3) multiplet (see Eqs. (4.1) - (4.3)) and name (see Figs. 6-8). Masses are quoted to the nearest 50 MeV.

SU(3) Multiplet	State	Mass (MeV)
<u>9</u>	C^0	1200
	C_K	1400
	C^S, C_π^S	1600
<u>18</u>	C_π	1250
	$E_{\pi K}, C_K$	1450
	$E_{(\bar{K}\bar{K})^0}, C_\pi^S, C^S$	1650
	$C_{\bar{K}}^S$	1850
<u>36</u>	$E_{\pi\pi}, C_\pi, C^0$	1450
	$E_{\pi K}, C_K$	1600
	$E_{(KK)^1}, C_\pi^S, C^S$	1800
	C_K^S	1950
	C^{SS}	2150
<u>18*</u>	C_π	1650
	$E_{\pi K}, C_K$	1800
	$E_{(\bar{K}\bar{K})^0}, C_\pi^S, C^S$	1950
	$C_{\bar{K}}^S$	2100

TABLE III

Predicted masses of $Q^2\bar{Q}^2$ 2^+ -mesons listed according to their SU(3) multiplet (see Eqs. (4.1) - (4.3)) and name (see Figs. 6-8). Masses are quoted to the nearest 50 MeV.

SU(3) Multiplet	State	Mass (MeV)
<u>9</u>	C^0	1650
	C_K	1800
	C^S, C_π^S	1950
<u>36</u>	$E_{\pi\pi}, C_\pi, C^0$	1650
	$E_{\pi K}, C_K$	1800
	$E_{(KK)^1}, C_\pi^S, C^S$	1950
	C_K^S	2100
	C^{SS}	2250

TABLE IV

Recoupling coefficients for $Q^2\bar{Q}^2 0^+$ -mesons into two $Q\bar{Q}$ mesons. P and V are color singlet pseudoscalar and vector $Q\bar{Q}$ -mesons; \underline{P} and \underline{V} are color octets of the same.

	PP	VV	$\underline{P}\cdot\underline{P}$	$\underline{V}\cdot\underline{V}$
$ \underline{9}, 0^+\rangle$	0.743	0.328	-0.432	-0.393
$ \underline{36}, 0^+\rangle$	-0.644	0.269	-0.322	-0.639
$ \underline{9}^*, 0^+\rangle$	0.178	-0.556	0.479	-0.655
$ \underline{36}^*, 0^+\rangle$	0.041	0.715	0.692	-0.089

TABLE V

Recoupling coefficients for $Q^2 \bar{Q}^2 1^+$ mesons into two $Q\bar{Q}$ mesons.
P and V are color singlet pseudoscalar and vector $Q\bar{Q}$ -mesons; \underline{P} and \underline{V} are color octet of the same.

	VV	$\underline{V}\cdot\underline{V}$	$\frac{1}{\sqrt{2}}(\underline{V}\cdot\underline{P}+\underline{P}\cdot\underline{V})$	$\frac{1}{\sqrt{2}}(\underline{V}\cdot\underline{P}-\underline{P}\cdot\underline{V})$	$\frac{1}{\sqrt{2}}(\underline{V}\underline{P}+\underline{P}\underline{V})$	$\frac{1}{\sqrt{2}}(\underline{V}\underline{P}-\underline{P}\underline{V})$
$ \underline{9}, 1^+ \rangle$	0	0	$-\frac{1}{\sqrt{3}}$	0	$\sqrt{\frac{2}{3}}$	0
$ \underline{36}, 1^+ \rangle$	0	0	$\sqrt{\frac{2}{3}}$	0	$\frac{1}{\sqrt{3}}$	0
$ \underline{18}, 1^+ \rangle$	0	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{3\sqrt{2}}$	0	$-\frac{2}{3}$
$ \underline{18}, 1^+ \rangle$	0	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{3\sqrt{2}}$	0	$\frac{2}{3}$
$ \underline{18}^*, 1^+ \rangle$	$\frac{1}{\sqrt{2}}$	0	0	$\frac{2}{3}$	0	$-\frac{1}{3\sqrt{2}}$
$ \underline{18}^*, 1^+ \rangle$	$\frac{1}{\sqrt{2}}$	0	0	$-\frac{2}{3}$	0	$\frac{1}{3\sqrt{2}}$

TABLE VI

Recoupling coefficients for $Q^2\bar{Q}^2$ 2^+ -mesons into two $Q\bar{Q}$ mesons.
 \underline{V} and \underline{V} are color singlet and octet $Q\bar{Q}$ vector mesons respectively.

	$\underline{V}\underline{V}$	$\underline{V}\cdot\underline{V}$
$ \underline{9}, 2^+ \rangle$	$\sqrt{\frac{2}{3}}$	$-\frac{1}{\sqrt{3}}$
$ \underline{36}, 2^+ \rangle$	$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$

TABLE VII

Couplings of 0^+ -mesons in a $Q^2\bar{Q}^2$ assignment.
 $\bar{g}_0 = 0.74 g_0$ is the overall strength of fall apart decays for this multiplet.
 θ_η is the η - η' mixing angle ($\theta = 0$ implies η is pure octet).

Channel	Coupling
$\epsilon \rightarrow \pi\pi$	$\frac{\sqrt{3}}{2} \bar{g}_0$
$\epsilon \rightarrow K\bar{K}$	0 *
$\epsilon \rightarrow \eta\eta$	$\frac{1}{2} \bar{g}_0 \cos^2(\theta_\eta - \theta_0)^\dagger$
$S^* \rightarrow K\bar{K}$	$\frac{1}{\sqrt{2}} \bar{g}_0$
$S^* \rightarrow \pi\pi$	0 *
$S^* \rightarrow \eta\eta$	$\frac{1}{2\sqrt{2}} \bar{g}_0 \sin 2(\theta_\eta - \theta_0)$
$\delta \rightarrow K\bar{K}$	$-\frac{1}{\sqrt{2}} \bar{g}_0$
$\delta \rightarrow \pi\eta$	$-\frac{1}{\sqrt{2}} \bar{g}_0 \sin(\theta_\eta - \theta_0)$
$\kappa \rightarrow K\pi$	$\frac{\sqrt{3}}{2} \bar{g}_0$
$\kappa \rightarrow K\eta$	$-\frac{1}{2} \bar{g}_0 \cos(\theta_\eta - \theta_0)$

* These decays proceed by Zweig rule violation which is ignored in this compendium.

† $\theta_0 = \arcsin(\sqrt{2/3}) = 54.7^\circ$

FIGURE CAPTIONS

1. Eigenfrequency $x(mR)$ of the lowest S-wave quark mode with mass m in a spherical cavity of radius R .
2. Lowest order gluon exchange graphs; (a) exchange and (b) self-energy.
3. Magnetic gluon exchange energy of two quarks as a function of mR . M is the quantity referred to in Eq. (2.5). The solid line gives the interaction energy between equal-mass quarks, the dashed line gives the interaction energy between a massless quark and a quark of mass m .
4. Meson decay diagrams. (a) Fall apart decay of an S-wave $Q^2\bar{Q}^2$ into two S-wave $Q\bar{Q}$ mesons; (b) Conventional decay of a $Q\bar{Q}$ meson into two $Q\bar{Q}$ mesons; (c) Decay of an S-wave $J^P = 2^+ Q^2\bar{Q}^2$ meson into two $0^+ - Q\bar{Q}$ S-wave mesons in a relative D-wave; (d) An S-wave $Q^2\bar{Q}^2$ meson attempting to fall apart into two color octets and succeeding only after a gluon exchange neutralizes the color.
5. Zweig-Iizuka violation inducing mixing (a) in the 1^- channel of the $Q\bar{Q}$ sector; (b) in the $Q^2\bar{Q}^2$ sector.
6. The cryptoexotic nonet ($\bar{3} \otimes 3$) and its recoupling to flavor decay channels. $Q\bar{Q}$ mesons are labelled by the names of a magically mixed pseudoscalar nonet—no prejudice regarding spin for the $Q\bar{Q}$ -mesons is implied.
7. The exotic and cryptoexotic 36-plet ($6 \otimes \bar{6}$) and its recouplings to flavor decay channels. For clarity the weight diagram is divided in three according to the number of hidden $s\bar{s}$ pairs ((a) none; (b) one; (c) two). As in Fig. 6 no prejudice regarding the spin of the $Q\bar{Q}$ mesons is implied by the notation.
8. The exotic and cryptoexotic 18 and $\bar{18}$ -plets ($6 \otimes 3$ and $\bar{3} \otimes \bar{6}$) and their recoupling to flavor decay channels. Again the weight diagrams are divided in two according to the number of hidden $s\bar{s}$ pairs. As in Figs. 6

and 7, no prejudice regarding the spin of the $Q\bar{Q}$ mesons should be inferred from the notation. As described in Footnote 16 most states in these multiplets mix to give eigenstates of G-parity or SU(3) parity. The states which do not mix are the exotics and $C_K^S(\underline{18})$ and $C_K^S(\overline{18})$.

9. The quark content of the cryptoexotic nonet. (a) The $\bar{\underline{3}}$ formed from two quarks; (b) the $\underline{3}$ formed from two antiquarks; (c) the (magically mixed) nonet formed from the direct product of (a) and (b).

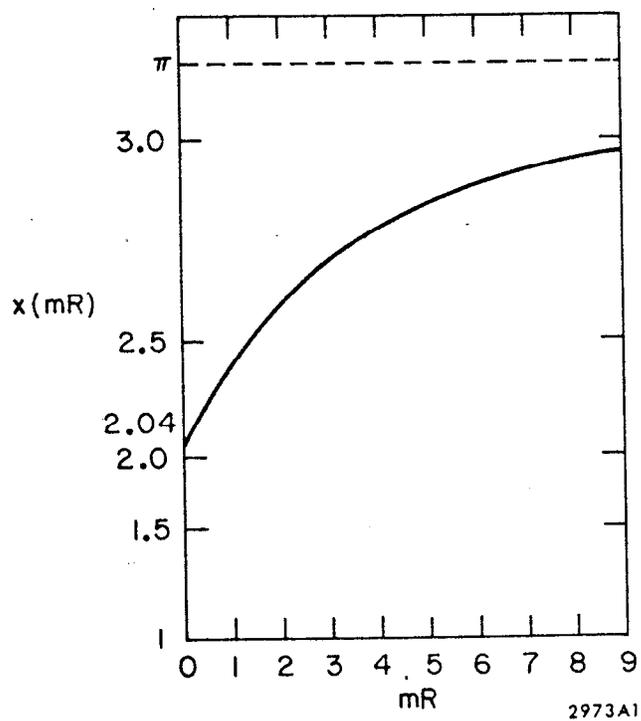


Fig. 1

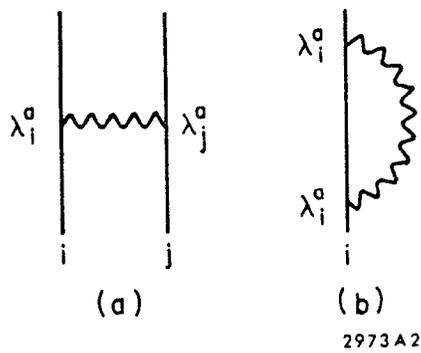


Fig. 2

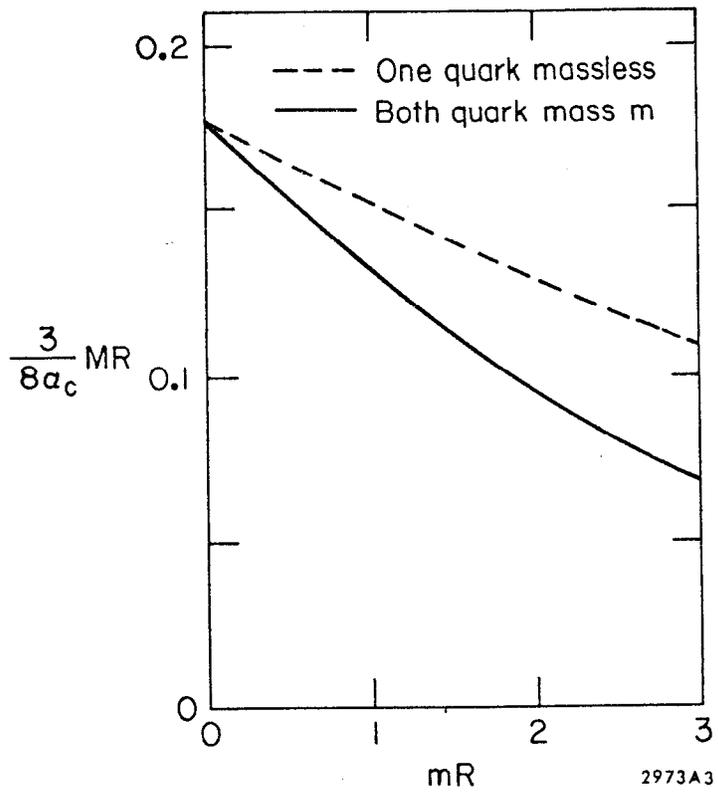


Fig. 3

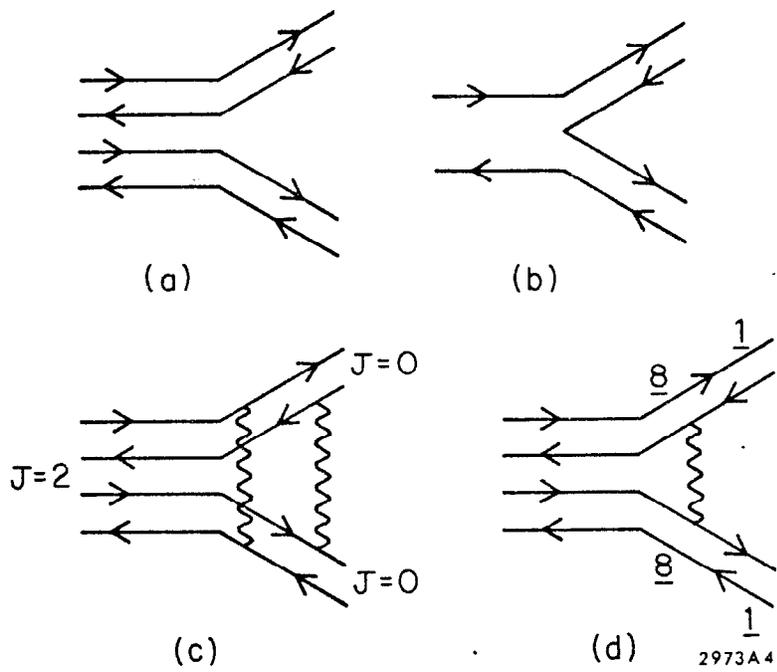
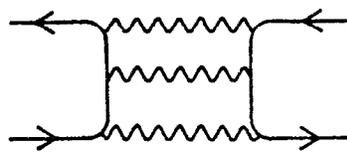
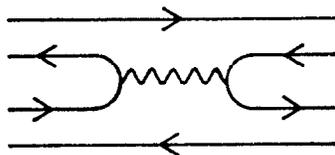


Fig. 4



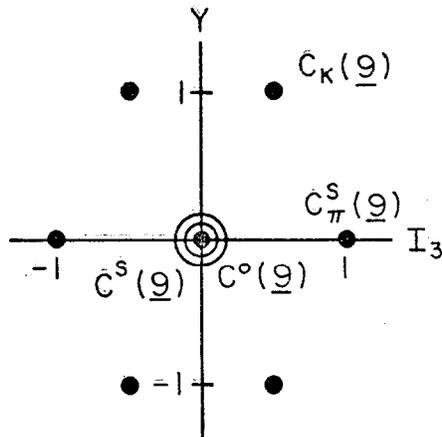
a



b

2973A5

Fig. 5



$$C_{\pi}^s(\underline{g}) = -\frac{1}{\sqrt{2}} K \bar{K} - \frac{1}{\sqrt{2}} \eta_s \pi$$

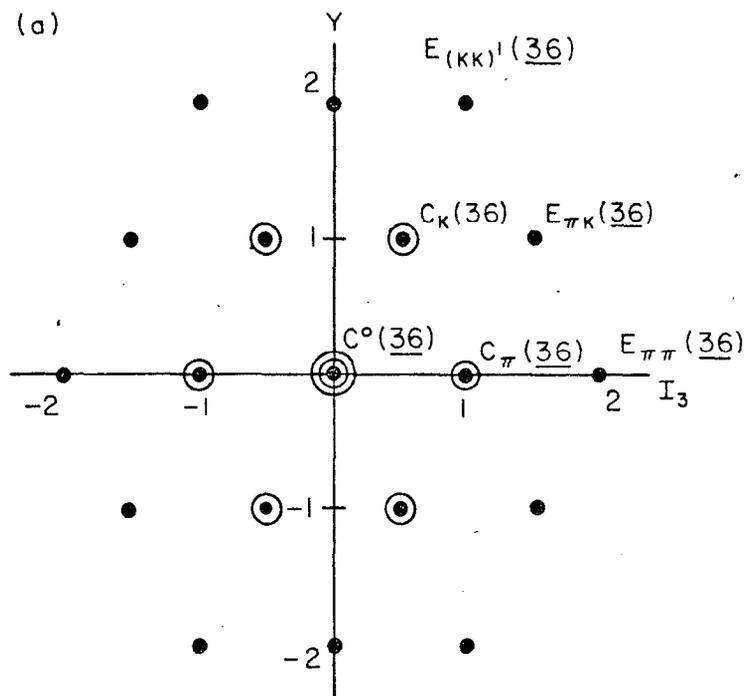
$$C_K(\underline{g}) = \frac{\sqrt{3}}{2} K \pi - \frac{1}{2} \eta_0 K$$

$$C^s(\underline{g}) = \frac{1}{\sqrt{2}} K \bar{K} + \frac{1}{\sqrt{2}} \eta_0 \eta_s$$

$$C^o(\underline{g}) = \frac{\sqrt{3}}{2} \pi \pi + \frac{1}{2} \eta_0 \eta_0$$

2973A6

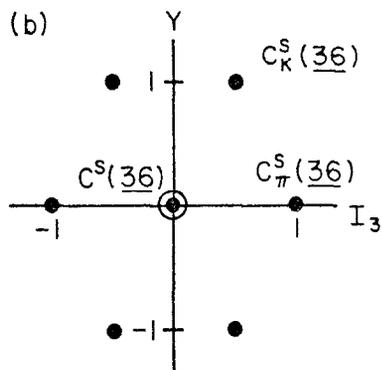
Fig. 6



$$E_{\pi\pi}(36) = \pi\pi \quad C_K(36) = \frac{1}{2} K\pi + \frac{\sqrt{3}}{2} K\eta_0$$

$$E_{\pi K}(36) = \pi K \quad C_\pi(36) = \pi\eta_0$$

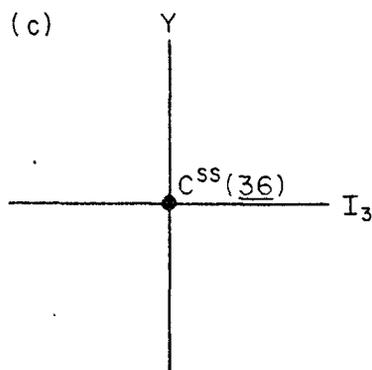
$$E_{(KK)'}(36) = (KK)^{I=1} \quad C^\circ(36) = -\frac{1}{2} \pi\pi + \frac{\sqrt{3}}{2} \eta_0\eta_0$$



$$C_\pi^S(36) = \frac{1}{\sqrt{2}} K\bar{K} - \frac{1}{\sqrt{2}} \pi\eta_s \quad C^{SS}(36) = \eta_s\eta_s$$

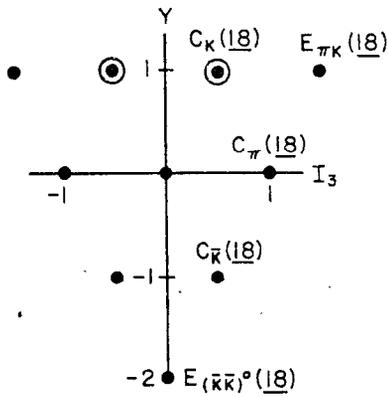
$$C_K^S(36) = K\eta_s$$

$$C^S(36) = \frac{1}{\sqrt{2}} K\bar{K} - \frac{1}{\sqrt{2}} \eta_0\eta_s$$



297387

Fig. 7



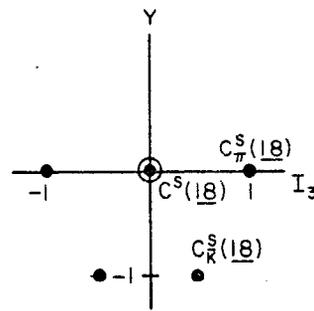
$$E_{\pi K}(18) = \pi K$$

$$E_{(\bar{K}\bar{K})^0}(18) = (\bar{K}\bar{K})^{I=0}$$

$$C_K(18) = -\frac{1}{2} K \pi + \frac{\sqrt{3}}{2} \eta_0 K$$

$$C_{\pi}(18) = \frac{1}{\sqrt{2}} \eta_0 \pi - \frac{1}{\sqrt{2}} \pi \pi$$

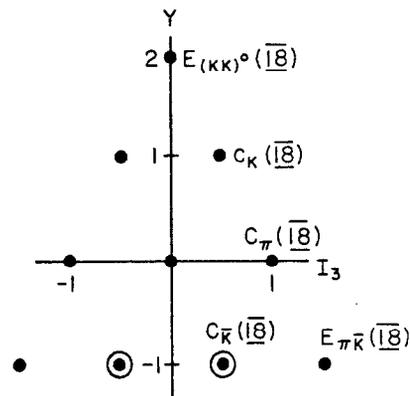
$$C_{\bar{K}}(18) = \frac{1}{2} \eta_0 \bar{K} - \frac{\sqrt{3}}{2} \bar{K} \pi$$



$$C_{\pi}^S(18) = -\frac{1}{\sqrt{2}} K \bar{K} + \frac{1}{\sqrt{2}} \eta_s \pi$$

$$C^S(18) = \frac{1}{\sqrt{2}} \eta_s \eta_0 - \frac{1}{\sqrt{2}} K \bar{K}$$

$$C_{\bar{K}}^S(18) = +\eta_s \bar{K}$$



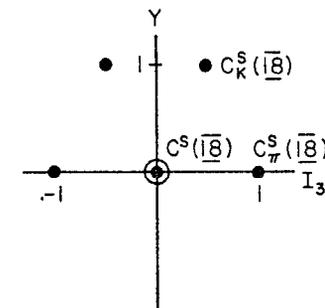
$$E_{\pi \bar{K}}(\bar{18}) = \pi \bar{K}$$

$$E_{(\bar{K}\bar{K})^0}(\bar{18}) = (\bar{K}\bar{K})^{I=0}$$

$$C_{\bar{K}}(\bar{18}) = \frac{\sqrt{3}}{2} \eta_0 \bar{K} + \frac{1}{2} \bar{K} \pi$$

$$C_{\pi}(\bar{18}) = \frac{1}{\sqrt{2}} \eta_0 \pi + \frac{1}{\sqrt{2}} \pi \pi$$

$$C_K(\bar{18}) = \frac{\sqrt{3}}{2} K \pi + \frac{1}{2} \eta_0 K$$



$$C_{\bar{K}}^S(\bar{18}) = \frac{1}{\sqrt{2}} K \bar{K} + \frac{1}{\sqrt{2}} \eta_s \pi$$

$$C^S(\bar{18}) = \frac{1}{\sqrt{2}} \eta_s \eta_0 + \frac{1}{\sqrt{2}} K \bar{K}$$

$$C_K^S(\bar{18}) = +\eta_s K$$

297388

Fig. 8

