

# THE PION-HYPERON COUPLINGS, THE $\bar{K}N$ INTERACTIONS, AND THE $Y^*$ RESONANT STATES

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(invited paper presented by R. H. Dalitz)

## I. THE DETERMINATION OF THE $K\Sigma$ AND $KA$ PARITIES

First let us review briefly the evidence bearing most directly on the  $K\Sigma$  and  $KA$  parities which has been obtained over the past year. Although it is not appropriate for us to analyze the data in detail here, it will be important for us to keep this evidence in mind in the later discussion.

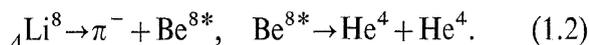
It now appears rather well-established that spin zero holds for the ( ${}_A\text{H}^4$ ,  ${}_A\text{He}^4$ ) doublet. This conclusion has been reached in at least three ways:

- i) from the branching ratios observed in the  $\pi^-$  decay of  ${}_A\text{H}^4$  by Levi-Setti *et al.*<sup>1)</sup> and by Block *et al.*<sup>2)</sup>, taken together with the theoretical calculations<sup>3)</sup> on these decay modes. For this comparison, the accurate determination of the form of the decay amplitude  $A \rightarrow p + \pi^-$ , carried out by Beall *et al.*<sup>4)</sup> and by Cronin *et al.*<sup>5)</sup>, has been essential.
- ii) from the short lifetime observed<sup>6)</sup> for  ${}_A\text{H}^4$  (and for  ${}_A\text{H}^3$ ), taken together with the theoretical calculations<sup>7)</sup> of hypernuclear decay rates.
- iii) from the isotropy of  ${}_A\text{H}^4$  decay observed by Block *et al.*<sup>2)</sup> in the reaction sequence

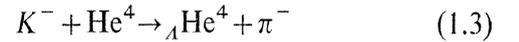


for  $K^-$  mesons coming to rest in helium.

This spin determination requires that the  $A$ -nucleon interaction is most strongly attractive in the  ${}^1S_0$  state. This conclusion has been supported also by the determination of the spin value  $J = 1$  for  ${}_A\text{Li}^8$  from the analysis<sup>8, 9)</sup> of the angular correlations in the sequence

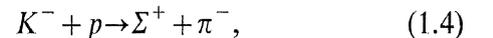


With  $J = 0$  for the ( ${}_A\text{He}^4$ ,  ${}_A\text{H}^4$ ) doublet, the observations of the reactions (1.1) and the related reaction



requires that the  $KA$  parity be odd. However, as is well known<sup>9)</sup>, this otherwise convincing argument would not be completely conclusive if there were established the existence of an excited bound state (with  $J = 1$ ) for the ( ${}_A\text{He}^4$ ,  ${}_A\text{H}^4$ ) doublet.

Evidence on the  $K\Sigma$  parity has been obtained recently by Tripp *et al.*<sup>10)</sup> in the course of their study of the 1520 MeV  $Y_0^{**}$  resonance. The angular and energy dependence of the  $K^-p$  scattering and charge-exchange processes shows clearly that the resonance is in the  $d_{3/2}$  wave for the  $\bar{K}N$  channel. The inclusion of the strongly-absorptive  $s$ -wave interactions together with the resonant amplitude suffices to give a good account of these data. The analysis of the angular distributions of the reactions  $K^- + p \rightarrow \Sigma + \pi$ , and of the angular and energy dependence of the  $\Sigma^+$  polarization produced in the reaction



deduced from the known asymmetry parameter for  $\Sigma^+ \rightarrow p + \pi^0$  decay, then leads rather directly to the conclusion the resonance is also in the  $d_{3/2}$  wave for the  $\Sigma\pi$  channel, which requires that the  $K\Sigma$  parity is odd. More accurate data on the processes occurring in the neighbourhood of this resonance, especially on the  $\Sigma^+$  polarization, would be very desirable in order to strengthen this conclusion, since it is not completely excluded that a much more complicated fit to the data could be obtained with the opposite parity assumption.

## II. HYPERON-NUCLEON FORCES

The  $\Lambda$ -nucleon potential is believed to be attractive in both  $^1S_0$  and  $^3S_1$  states. The stronger  $^1S_0$  potential may be estimated rather well from the analysis of the  ${}_{\Lambda}H^3$  system, since the mean potential (per nucleon) effective in this system is  $(3V_0 + V_1)/4$ , which is dominated by  $V_0$ . The strong binding ( $B_{\Lambda} = 3.1$  MeV) observed for the  $\Lambda$  particle in  ${}_{\Lambda}He^5$  requires that the  $^3S_1$  potential is also attractive, since the mean potential effective in  ${}_{\Lambda}He^5$  is  $(3V_1 + V_0)/4$ , dominated by  $V_1$ . The precise well-depth parameters obtained depend on the form of  $\Lambda$ - $N$  potential adopted, especially on the hard core radius. The calculations available on these systems for a realistic hard-core radius are still rather inadequate.

On theoretical grounds, the range parameter for these potentials is expected to be of order  $(2\mu)^{-1}$ . With this value, the potential strengths may then be characterized by specifying the zero-energy scattering amplitudes. After considering the calculations available de Swart and Dullemond<sup>11)</sup> have adopted the estimates

$$a_0 = 3.6_{-1.8}^{+3.6} \text{ F}, \quad a_1 = 0.53 \pm 0.1 \text{ F} \quad (2.1)$$

for the singlet and triplet amplitudes, respectively.

At Chicago, it seemed of interest to us to calculate these potentials meson-theoretically, in terms of the general pion-hyperon coupling

$$-if_{\Sigma\Sigma}\underline{\Sigma}^+ \times \underline{\Sigma} \cdot \underline{\pi} + f_{\Sigma\Lambda}(A^+\underline{\Sigma} + \underline{\Sigma}^+A) \cdot \underline{\pi}. \quad (2.2)$$

These calculations have been carried out by de Swart and Iddings<sup>12, 13)</sup>. The hyperon-nucleon system has been considered as a two-channel system,  $\Lambda N$  and  $\Sigma N$ , so that a matrix potential ( $V_{YY}$ ) was calculated, as first done by Lichtenberg and Ross<sup>14)</sup>, with elements

$$\begin{pmatrix} V_{\Lambda\Lambda} & V_{\Lambda\Sigma} \\ V_{\Sigma\Lambda} & V_{\Sigma\Sigma} \end{pmatrix} \quad (2.3)$$

For one-pion exchange, the potentials were calculated with the inclusion of the  $\Sigma$ - $\Lambda$  mass difference; there is, of course, no contribution to  $V_{\Lambda\Lambda}$  in this order. The two-pion exchange potentials were computed following the prescriptions of Brueckner and Watson<sup>15)</sup>; here the effects of the  $\Sigma$ - $\Lambda$  mass difference on the potentials were neglected. These potentials were then inserted in the Schrödinger equations for the hyperon-nucleon system

$$-\frac{1}{2\mu_{\Sigma N}}\nabla^2\psi_{\Sigma} + V_{\Sigma\Sigma}\psi_{\Sigma} + V_{\Sigma\Lambda}\psi_{\Lambda} = (E - (m_{\Sigma} - m_{\Lambda}))\psi_{\Sigma}, \quad (2.4a)$$

$$-\frac{1}{2\mu_{\Lambda N}}\nabla^2\psi_{\Lambda} + V_{\Lambda\Sigma}\psi_{\Sigma} + V_{\Lambda\Lambda}\psi_{\Lambda} = E\psi_{\Lambda}, \quad (2.4b)$$

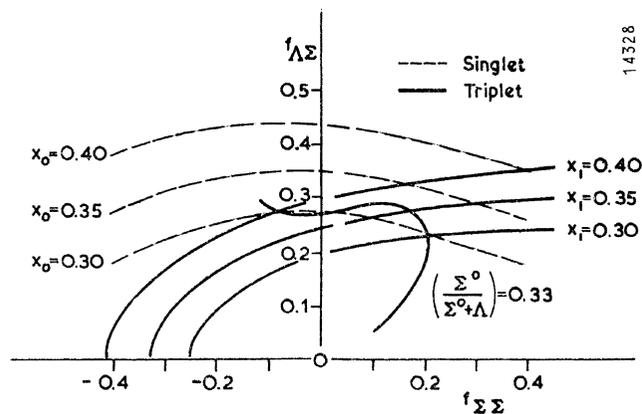
and these equations were then solved for the  $^1S_0$  and the  $^3S_1$  states, with the inclusion of the couplings to states of higher orbital angular momentum wherever appropriate.

In order to connect these potentials with our knowledge of the  $NN$  forces, the particular case of  $f_{\Sigma\Lambda} = f_{\Sigma\Sigma} = f_{NN}$  and even  $\Sigma\Lambda$  parity, known as "global symmetry", was first considered, since the potentials (2.3) are then directly related with the meson-theoretic  $NN$  potentials. With the known coupling parameter  $f_{NN}^2 = 0.081$ , de Swart and Iddings found that their calculated potentials gave an excellent fit to the scattering length and effective range for choice of the hard-core radius  $R_0 = 0.5$  F for the  $^1S_0 NN$  system,  $R_1 = 0.46$  F for the  $^3S_1 NN$  system.

It is not completely unambiguous what hard-core radii should be used in (2.3). If the repulsive core is due to the exchange of a neutral vector meson coupled universally with the baryon current, then it would be reasonable to expect the hard-core radii in  $V_{YY}$  to have essentially the same value,  $R \approx 0.5$  F, as that found for the  $^1S_0$  and  $^3S_1 NN$  systems.

Alternatively, this assumption for  $R$  is certainly the most economical assumption which can be made for  $V_{YY}$ , and this was the course followed, in general, the parameters  $f_{\Sigma\Lambda}$  and  $f_{\Sigma\Sigma}$  being regarded as free parameters available to fit the input data (2.1). The effects of  $K$ -exchange have not yet been included in these potentials, but these are not expected to give rise to major modifications of the results, since they have short range and are suppressed by the presence of the repulsive core.

*Even  $\Sigma\Lambda$  parity.* Here the couplings (2.2) were taken of the pseudovector form  $\underline{\sigma} \cdot \underline{q}_{\pi}/\mu$ . From the calculations for  $\Lambda N$  scattering at low energies, the fits to the scattering amplitudes (2.1) are shown on Fig. 1, for several choices of the hard-core radius  $R = x/\mu$ . It will be seen that  $f_{\Sigma\Lambda}$  is relatively well-determined by the  $^1S_0$  potential alone. The  $^3S_1$  amplitude has more dependence on  $f_{\Sigma\Sigma}$ , especially for negative  $f_{\Sigma\Sigma}$ . For  $R_1 = R_0 = 0.5$  F, the two



**Fig. 1** For even  $\Sigma\Lambda$  parity, the lines for which  $a_0 = -3.6$  F and  $a_1 = -0.53$  F are plotted on the  $(f_{\Sigma\Sigma}, f_{\Sigma\Lambda})$  plane for several values of the hard core radii  $R_0 = x_0/\mu$ ,  $R_1 = x_1/\mu$ . Also shown is the line along which the spin-average ratio  $(\Sigma^0/(\Sigma^0+\Lambda))$  of Eq. (2.7) has the experimental value 0.33,  $R_0$  and  $R_1$  both being chosen to have the value  $0.35/\mu$ .

curves cross at the point  $f_{\Sigma\Lambda} \approx f_{\Sigma\Sigma} \approx f_{NN} = 0.29$ , compatible with global symmetry. However, it is clear that  $f_{\Sigma\Sigma}$  is relatively poorly determined, since the two sets of curves intersect at a small angle; large negative values of  $f_{\Sigma\Sigma}$  do seem to be excluded, on the other hand. The sensitivity of these curves to the scattering amplitudes used is shown in Fig. 2, from which it will be apparent that the uncertainties in the hard-core radii are of far greater importance than the uncertainties in the experimental data at present.

De Swart and Iddings<sup>13)</sup> have also made calculations on the  $\Sigma^-p$  reactions

$$\Sigma^- + p \rightarrow \Sigma^0 + n, \quad (2.5a)$$

$$\rightarrow \Lambda + n, \quad (2.5b)$$

with these potentials. For zero energy, the effects of the  $(\Sigma^-, \Sigma^0)$  mass difference on these matrix elements have also been calculated and prove to be quite significant. The experimental data available refers to  $\Sigma^-$  capture from rest in hydrogen, giving<sup>16)</sup> the ratio

$$\Sigma^0/(\Sigma^0 + \Lambda) = 0.33 \pm 0.06. \quad (2.6)$$

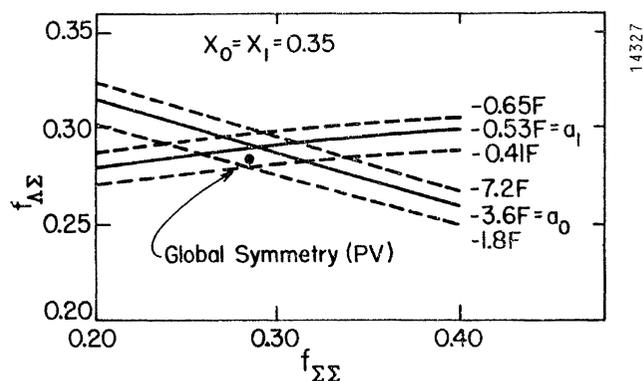
The interpretation to be given to this ratio is not completely clear. If the  $\Sigma^-p$  capture is  $s$ -wave (in high  $n$  orbitals, through the Stark-mixing mechanism of Day, Sucher and Snow) and if there is no mixing of singlet and triplet spin states for the  $\Sigma^-p$  system, then this ratio would be simply the weighted average of the singlet and triplet capture ratios

$$\left(\frac{\Sigma^0}{\Sigma^0 + \Lambda}\right)_{\text{expt.}} = \frac{1}{4} \left(\frac{\Sigma^0}{\Sigma^0 + \Lambda}\right)_{S=0} + \frac{3}{4} \left(\frac{\Sigma^0}{\Sigma^0 + \Lambda}\right)_{S=1}. \quad (2.7)$$

However, in the  $\Sigma^-p$  atomic states, the spin-orbit potential is not symmetric in  $\underline{\sigma}_\Sigma$  and  $\underline{\sigma}_p$  and can give rise to mixing of the  ${}^3(J)_J$  and  ${}^1(J)_J$  states; in this case it is conceivable that the relative magnitudes of the capture rates from singlet and triplet states should also enter into the expression for  $(\Sigma^0/(\Sigma^0+\Lambda))_{\text{expt.}}$ . This question has not yet been resolved and this uncertainty must be borne in mind, as the data (2.6) has been discussed in terms of expression (2.7).

The curve of points  $(f_{\Sigma\Lambda}, f_{\Sigma\Sigma})$  giving the value (2.6) has been plotted on Fig. 1. The three curves shown do not have a common intersection. For the intersection  $f_{\Sigma\Lambda} = f_{\Sigma\Sigma} = f_{NN}$  mentioned above, the value calculated for  $\Sigma^0/(\Sigma^0+\Lambda)$  is 0.45, two standard deviations too high. However, an acceptable fit to all three data can easily be obtained by relaxing slightly the condition  $R_0 = R_1$ ; the values  $R_0 \approx 0.48$  F,  $R_1 \approx 0.51$  F,  $f_{\Sigma\Sigma} \approx 0.2$ ,  $f_{\Sigma\Lambda} \approx 0.29$  achieve this purpose within the uncertainties. It is apparent that, if further reduction is permitted for  $R_0$ , values of  $f_{\Sigma\Sigma}$  down to  $f_{\Sigma\Sigma} = 0$  can be permitted by the data, since the curve for  $(\Sigma^0/(\Sigma^0+\Lambda))_{\text{expt.}}$  runs roughly parallel with the curve for  $a_1$  in the range  $f_{\Sigma\Sigma} \approx 0.2$  to zero.

With global symmetry, the  ${}^1S_0$   $I = 3/2$  potential is known to be equal to that for the  ${}^1S_0$   $NN$  system. With  $f_{\Sigma\Sigma} = 0.29$  (and  $f_{\Sigma\Lambda} = 0.29$ ,  $R = 0.5$  F.), the potential calculated actually predicts a bound state for the  $\Sigma^-n$  system. It is of interest to note that the  ${}^1S_0$  zero-energy scattering amplitude predicted falls



**Fig. 2** The intersections of the curves of constant  $a_0$  and  $a_1$  calculated for the case of even  $\Sigma\Lambda$  parity are shown in detail in the  $(f_{\Sigma\Sigma}, f_{\Sigma\Lambda})$  plane for  $R_0 = R_1 = 0.35/\mu$ , in order to illustrate the uncertainties arising from the present uncertainties in  $a_0$  and  $a_1$ .

rapidly with decreasing  $f_{\Sigma\Sigma}$ . At  $f_{\Sigma\Sigma} = 0.2$ , this amplitude has fallen to 1.3 F; at the value  $f_{\Sigma\Sigma} = 0$ , this interaction has become weakly repulsive, with scattering length  $-0.13$  F.

The possibility  $f_{\Sigma\Sigma} = 0$  is of special interest, since the hypothesis of invariance with respect to hypercharge reflection discussed by several workers<sup>17)</sup> excludes the interaction  $\Sigma \rightarrow \Sigma + \pi$ . In this case, a fit to the  $^1S_0$  amplitude with  $R_0 = 0.5$  F, requires  $f_{\Sigma\Lambda}^2 = 0.12$ ; for the  $^3S_1$  state, such a large value of  $f_{\Sigma\Lambda}$  (with  $R_1 = 0.5$  F) leads to a strongly attractive potential, sufficient to bind the  $\Lambda N$  system, contrary to observation. However, if the value of  $R_0$  is dropped as far as 0.42 F, then these three data can be fitted with  $R_1 = 0.53$  F and  $f_{\Sigma\Lambda}^2 = 0.073$ . This remark illustrates the strong sensitivity of the calculated results to the assumptions concerning the hard core radii for  $V_{YY}$ . Clear-cut conclusions can therefore not be obtained from these comparisons until there is some understanding of the origin of the hard-core repulsion in the baryon-baryon interactions.

*Odd  $\Sigma\Lambda$  parity.* Here the couplings were taken of the pseudovector form  $\underline{\sigma} \cdot \underline{q}_\pi / \mu$  for  $f_{\Sigma\Sigma}$ , but of the scalar form (1) for  $f_{\Sigma\Lambda}$ . We note that, with  $f_{\Sigma\Sigma} = 0$ , the couplings do not depend on the hyperon spin, so that no spin-dependence could then be obtained for the  $\Lambda$ - $N$  interactions. As shown on Fig. 3, the triplet

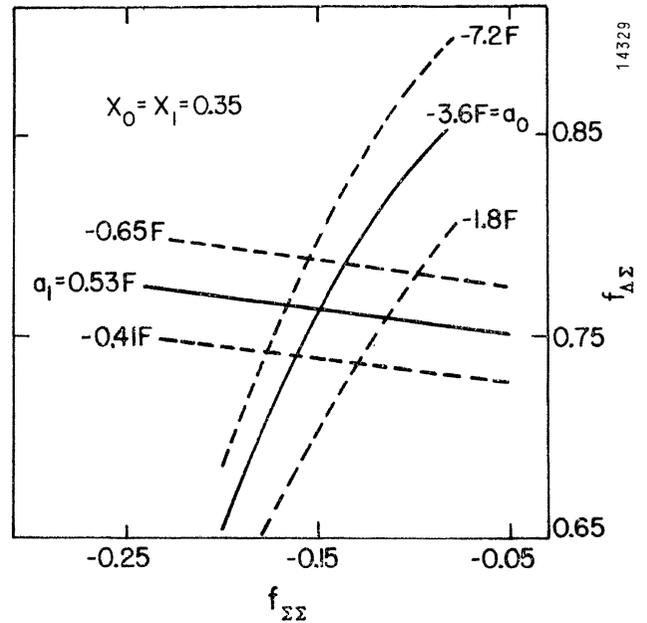


Fig. 4 The intersections of the curves of constant  $a_0$  and  $a_1$  calculated for the case of odd  $\Sigma\Lambda$  parity are shown in detail in the  $(f_{\Sigma\Sigma}, f_{\Sigma\Lambda})$  plane for  $R_0 = R_1 = 0.35/\mu$ , to illustrate the uncertainties which result from the present uncertainties in  $a_0$  and  $a_1$ .

potential calculated is insensitive to  $f_{\Sigma\Sigma}$  and the amplitude  $a_1$  determines a value  $f_{\Sigma\Lambda}^2 \approx 0.6$ . This value is appreciably smaller than the estimate  $f_{\Sigma\Lambda}^2 \approx 1.5$ , which has been given in the literature, based on the  $\Sigma$ - $\Lambda$  mass relationship and odd parity for the  $\Sigma\Lambda$  system. The singlet potential depends quite strongly on  $f_{\Sigma\Sigma}$ ; with  $R = 0.5$  F, the intersection of the curves for  $a_0$  and  $a_1$  leads to  $f_{\Sigma\Sigma} \approx -0.15$ . Again, Fig. 4 shows that the location of this intersection is far less sensitive to the uncertainties in  $a_0$  and  $a_1$  than to the uncertainties associated with the hard-core radii to be used.

Calculations of the  $\Sigma^- - p$  capture ratio (2.7) have also been made for odd  $\Sigma\Lambda$  parity, as shown on Fig. 3. At the intersection of the curves  $a_0$  and  $a_1$  for  $R = 0.5$  F, the value (2.7) calculated is 0.21, too low by more than two standard deviations. It turns out to be quite difficult to improve this disagreement by independent variation of the radii  $R_0$  and  $R_1$ , unless very widely different values are used for  $R_0$  and  $R_1$  with values  $f_{\Sigma\Sigma}$  and  $f_{\Sigma\Lambda}$  quite far from those mentioned above for  $R_0 = R_1 = 0.5$  F. However, if the hard-core radii are to be allowed to differ, it is no longer clear what are reasonable assumptions about these radii; this is especially the case since the  $^3S_1$  (or  $^1S_0$ )  $\Sigma N$  states are coupled with the  $^3P_1$  and  $^1P_1$  (or  $^3P_0$ )  $\Lambda N$  states, whereas the  $^3S_1$  (or  $^1S_0$ )  $\Lambda N$  states are coupled

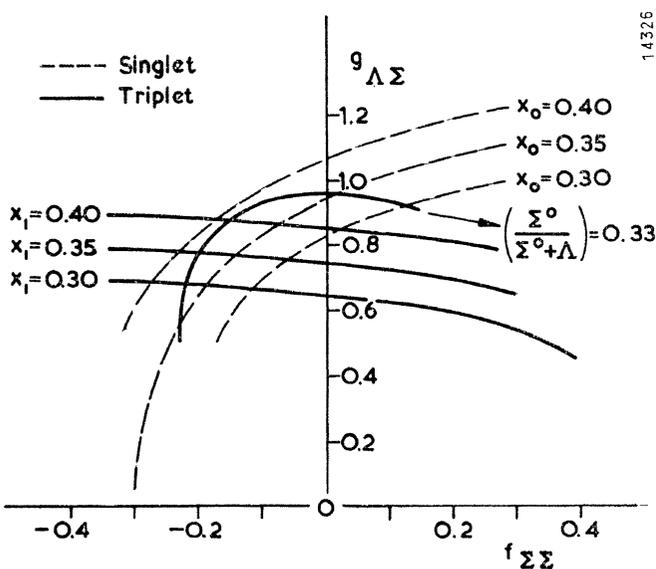


Fig. 3 For odd  $\Sigma\Lambda$  parity, the lines for which  $a_0 = -3.6$  F and  $a_1 = -0.53$  F are plotted on the  $(f_{\Sigma\Sigma}, f_{\Sigma\Lambda})$  plane for several values of the hard core radii  $R_0 = x_0/\mu$ ,  $R_1 = x_1/\mu$ . Also shown is the line along which the spin-average ratio  $(\Sigma^0 / (\Sigma^0 + \Lambda))$  of Eq. (2.7) has the experimental value 0.33,  $R_0$  and  $R_1$  both being chosen to have the value  $0.35/\mu$ .

with the  ${}^3P_1$  and  ${}^1P_1$  (or  ${}^3P_0$ )  $\Sigma N$  states, so that the  $\Lambda N$  and  $\Sigma N$   $S$ -interactions actually refer to two different sets of states. We confine ourselves to the remark that for  $R_0 = R_1 = 0.5 F$ , it does not appear possible to fit these three data within their experimental uncertainties for one choice of  $(f_{\Sigma A}, f_{\Sigma\Sigma})$ .

### III. PION-HYPERON RESONANCES

Next, it is of interest to consider the pion-hyperon resonances in the light of the above indications concerning the pion-hyperon interaction (2.2).

For even  $\Sigma\Lambda$  parity, the possibility of  $j = 3/2$   $\pi$ - $Y$  resonances analogous to the  $(3,3)$   $N^*$  resonance is very well-known, especially for the case of global symmetry. The existence of these resonances is governed by the baryon pole term, whereas their location depends also on the effective range of the interaction, which depends more on the distant singularities (i.e. on the short range interactions). For the  $p_{3/2}$  state, the pole terms of the scattering amplitude are as follows<sup>18)</sup>:

$$I = 2 : \frac{2}{3} \left( \frac{k_\Sigma}{\mu} \right)^2 \left( \frac{f_{\Sigma A}^2}{\omega - 2\Delta} + \frac{f_{\Sigma\Sigma}^2}{\omega - \Delta} \right), \quad (3.1)$$

$$I = 1 : \left\{ \begin{array}{ll} \frac{2}{3} \left( \frac{k_A}{\mu} \right)^2 \frac{f_{\Sigma A}^2}{\omega + \Delta}, & \frac{2\sqrt{2}}{3} \frac{k_A k_\Sigma}{\mu^2} \frac{f_{\Sigma A} f_{\Sigma\Sigma}}{\omega} \\ \frac{2\sqrt{2}}{3} \frac{k_A k_\Sigma}{\mu^2} \frac{f_{\Sigma A} f_{\Sigma\Sigma}}{\omega}, & \frac{2}{3} \left( \frac{k_\Sigma}{\mu} \right)^2 \left( \frac{-f_{\Sigma A}^2}{\omega - 2\Delta} + \frac{f_{\Sigma\Sigma}^2}{\omega - \Delta} \right) \end{array} \right\} \quad (3.2)$$

$$I = 0 : \frac{2}{3} \left( \frac{k_\Sigma}{\mu} \right)^2 \left( \frac{f_{\Sigma A}^2}{\omega - 2\Delta} - \frac{2f_{\Sigma\Sigma}^2}{\omega - \Delta} \right), \quad (3.3)$$

where  $\omega$  denotes the c.m. energy relative to  $m_A + m_\pi$ , and  $\Delta = m_\Sigma - m_A$ . Diagonalization of the matrix (3.2) with  $k_\Sigma = k_A = k$  and with neglect of  $\Delta$  in the denominators gives an attractive pole term with residue  $\frac{2}{3}(k/\mu)^2(f_{\Sigma A}^2 + f_{\Sigma\Sigma}^2)$ .

We note that the existence of  $Y_1^*$  and  $Y_2^*$  resonances does not depend sensitively on global symmetry. Such resonances appear naturally if there is a sufficiently strong  $\Sigma\Lambda\pi$  coupling. Further, the attractive interaction is comparable in the  $I = 1$  and  $I = 2$  configurations, irrespective of the value of  $f_{\Sigma\Sigma}$ . The experimental evidence on the  $Y_1^*$  resonance does not yet allow the determination of its spin value. Some

evidence has been published by Ely *et al.*<sup>19)</sup> that the decay angular distribution of  $Y_1^*$  produced in 1150 MeV/c  $K^-p$  collisions has a strong anisotropy (of form  $1 + (1.5 \pm 0.4) \cos^2\psi$ ) relative to the normal to their production plane, which was suggestive that the  $Y_1^*$  spin is not less than  $3/2$ ; however, in a similar experiment recently carried out at 1220 MeV/c by Alston *et al.*<sup>20)</sup>, the anisotropy observed is quite small. If  $j = 3/2$  does hold for the  $Y_1^*$  resonance, then it is natural to expect the existence of an  $I = 2$   $Y_2^*$  resonance at a mass value in the region 1500-1600 MeV.

If this is the interpretation of the  $Y_1^*$  resonance, we note that the  $(\Sigma\pi)/(\Lambda\pi)$  ratio of  $Y_1^*$  decay will be sensitive to the coupling constant  $f_{\Sigma\Sigma}$ . With  $f_{\Sigma\Sigma} \approx f_{\Sigma A}$ , the ratio predicted is about 11%, whereas the experimental upper limit on this ratio is about 5%. It is clear that a value for  $f_{\Sigma\Sigma}$  in the range 0-0.2 allowed in the last section will lead to an acceptable prediction for this ratio. Trueman<sup>21)</sup> has considered the effect of the  $\bar{K}N$  channel on these simple predictions. He has found that the coupling to the  $\bar{K}N$  channel tends to lower the  $Y_1^*$  resonance energy, and to reduce the  $(\Sigma\pi)/(\Lambda\pi)$  ratio; for reasonable coupling strengths, (say,  $f_{\Sigma K}^2$  and  $f_{\Lambda K}^2$  smaller than  $f_{NN}^2$  by a factor 4), this reduction is only from 11% to about 8%. This evidence does appear to require that  $f_{\Sigma\Sigma}^2/f_{\Sigma A}^2$  be no more than about 0.5.

From (3.3), the residue of the pole term for the  $I = 0$  state is  $(f_{\Sigma A}^2 - 2f_{\Sigma\Sigma}^2)$ . When  $f_{\Sigma\Sigma} = 0$ , this pole term is as attractive as for the  $I = 1$  and 2 states, and it is well known that an  $I = 0$  resonance is then predicted at the same energy as the  $I = 2$  resonance. However, it is clear that the location predicted for this resonance does depend very sensitively on the precise value of  $f_{\Sigma\Sigma}$ .

For odd  $\Sigma\Lambda$  parity, the only  $I = 1$   $\pi$ - $\Lambda$  resonance predicted in this way is in the  $p_{1/2}$  state. In this resonance, the  $p_{1/2}$   $\pi$ - $\Lambda$  channel is coupled with the  $s_{1/2}$   $\pi$ - $\Sigma$  channel; since the  $\pi\Lambda\Sigma$  coupling involves only  $s$ -wave pions, it is apparent that the existence of this  $\pi$ - $\Lambda$  resonance requires the presence of a strong  $\pi\Sigma\Sigma$  coupling as well as of a strong  $\pi\Lambda\Sigma$  coupling. Under these circumstances, the prediction of a large  $\Sigma/\Lambda$  ratio follows naturally, especially as the  $\pi$ - $\Sigma$  channel is  $s$ -wave and the  $\pi$ - $\Lambda$  channel  $p$ -wave, so that there is no natural prediction for a  $Y_1^*$  resonance with the observed properties, without an essential appeal to the coupling with the  $\bar{K}N$  channels.

We conclude that the assumption of even  $\Sigma\Lambda$  parity, with  $f_{\Sigma\Lambda} \approx f_{NN} = 0.29$  and  $0 \leq f_{\Sigma\Sigma} < 0.2$ , can account naturally for the spin-dependence of  $\Lambda$ -nucleon forces and for the  $\Sigma^-p$  capture data. In this case, there is a natural interpretation of  $Y_1^*$  as a  $p_{3/2}$  analogue to the (3,3)  $N^*$  resonance, and a strong prediction of the existence of a  $p_{3/2}$   $Y_2^*$  resonance. On the other hand, with odd  $\Sigma\Lambda$  parity, there is no such natural interpretation for the  $Y_1^*$  resonance with the parameters appropriate to the hyperon-nucleon interactions.

#### IV. THE $\bar{K}N$ s-WAVE INTERACTION AND THE $Y_0^*$ RESONANCE

There has been much progress recently concerning our knowledge of the  $\bar{K}N$  s-wave scattering parameters from an unexpected quarter, namely the analysis of the  $K^-p$  scattering data in the neighbourhood of the 1520 MeV  $Y_0^{**}$  resonance.

Following the model discussed in Section I, the amplitudes for the  $\Sigma^\pm$  production reactions for  $K^-p$  interactions near this resonance take the form

$$T(K^-p \rightarrow \Sigma^\pm \pi^\mp) = \frac{m_0}{\sqrt{6}} \mp \frac{m_1}{2} + \frac{1}{\sqrt{6}} \frac{(\Gamma_{\bar{K}N} \Gamma_{\Sigma\pi})^{\frac{1}{2}}}{E^* - E - i\Gamma/2} \times \\ \times (3 \cos^2 \theta - 1 - 3i \underline{\sigma} \cdot \underline{n} \cos \theta \sin \theta). \quad (4.1)$$

Since the phase of the resonant amplitude is known, the absolute phases of  $m_0$  and  $m_1$  can be determined from the energy and angular dependence of the  $(\Sigma^\pm + \pi^\mp)$  cross-sections in the resonance region (irrespective of the  $K\Sigma$  parity). From the diagrammatic analysis given by Tripp *et al.*<sup>10)</sup>, this phase  $(\psi_0 - \psi_1)$  may be read off as about  $-110^\circ$ . The magnitude of  $(\psi_0 - \psi_1)$  can be obtained from the s-wave cross-sections for the three  $\Sigma + \pi$  production reactions in this region; the determination of the sign requires a knowledge of the interference of the s-wave amplitudes with an amplitude of known phase.

Humphrey and Ross<sup>22)</sup> have carried out a detailed least-square analysis of all the data available on  $K^-p$  reactions from zero momentum up to 250 MeV/c in terms of the two complex scattering amplitudes,  $A_0$  and  $A_1$ , of the zero range theory. They obtained two sets of amplitudes, listed in Table I under the headings "Solution I" and "Solution II". We note

that the phase difference  $(\psi_0 - \psi_1)$  they obtained (actually the phase difference between  $\psi_0$  and  $\psi_1$  at the  $\bar{K}^0n$  threshold) has the value  $+100^\circ$  for solution I, and  $-50^\circ$  for solution II. The  $\Sigma^-/\Sigma^+$  ratio is a sensitive indicator of the phase difference  $(\psi_0 - \psi_1)$ , since

$$\frac{\Sigma^-}{\Sigma^+} = \frac{\frac{1}{6}|M_0|^2 + \sqrt{\frac{1}{6}}|M_0^*M_1| \cos(\psi_0 - \psi_1) + \frac{1}{4}|M_1|^2}{\frac{1}{6}|M_0|^2 - \sqrt{\frac{1}{6}}|M_0^*M_1| \cos(\psi_0 - \psi_1) + \frac{1}{4}|M_1|^2}.$$

This ratio falls from 2.15 at zero energy, through a value of unity somewhere in the region 100-150 MeV/c, and then smoothly to a value less than unity at 400 MeV/c. As pointed out by Akiba and Capps<sup>23)</sup>, the continuity of this phase difference  $(\psi_0 - \psi_1)$  as function of energy and its negative value at 400 MeV/c jointly require that solution II holds for the low energy scattering amplitudes.

With solution II, there is no possibility for the interpretation of the  $Y_1^*$  resonance as an s-wave  $\bar{K}N$  virtual bound state, since  $a_1$  has the value  $+1.2(\pm 0.6)F$ , which corresponds to an attractive  $\bar{K}N$  potential but insufficiently attractive for the formation of a bound state.

A preliminary least-squares analysis of all the  $K^-p$  interaction data in the range 350-450 MeV/c has been made by Watson<sup>24)</sup>, with the inclusion of  $I = 0$  and 1 amplitudes for the  $p_{1/2}$  and  $p_{3/2}$  initial states (but not including terms beyond  $\cos^2 \theta$  in the angular distribution). Two similar sets of parameters, A and B, have been obtained to fit the data. The s-wave parameters for these solutions have been listed in Table I. For  $I = 1$ , these amplitudes differ strongly from those obtained from the low energy data, the strongest energy dependence being that of  $a_1$ . It is most appropriate to compare the values of  $1/A_1$  at these energies, as follows

$$k^2 \approx 0, \quad A_1^{-1} = (0.7 - 0.32i)F, \quad (4.2)$$

$$k^2 = 1.61F^{-2}, \quad A_1^{-1} = (-0.2 - 2.4i)F, \quad (4.2a)$$

$$= (-0.7 - 2.2i)F, \quad (4.2b)$$

so that the effective range must be large ( $R \approx 2iF$ ) and dominantly imaginary. The  $I = 0$  amplitude has only a moderate energy dependence. It is of interest to note here that these energy dependences parallel those estimated by Dalitz<sup>25)</sup> (correcting and simplifying earlier results of Ferrari *et al.*<sup>26)</sup>) for  $\bar{K}N$

TABLE I

The  $s$ -wave scattering amplitudes  $A_0 = a_0 + ib_0$ ,  $A_1 = a_1 + ib_1$  (in unit  $F$ ) for the  $\bar{K}N$   $s$ -wave interaction as obtained (a) by Humphrey and Ross<sup>22)</sup> from the low-energy  $K^-p$  interaction data, and (b) by Watson<sup>24)</sup> from the analysis of  $K^-p$  interaction data in the region of the 1520 MeV  $Y_0^{*}$  resonance.

	Humphrey-Ross Solutions		Watson parameters at 400 MeV/c	
	Solution I	Solution II	Solution A	Solution B
$a_0$	$-0.22 \pm 1.07$	$-0.59 \pm 0.46$	$-0.90 \pm 0.25$	$-0.96 \pm 0.24$
$b_0$	$2.74 \pm 0.31$	$0.96 \pm 0.17$	$2.50 \pm 0.20$	$1.71 \pm 0.17$
$a_1$	$0.02 \pm 0.33$	$1.20 \pm 0.06$	$-0.03 \pm 0.06$	$0.14 \pm 0.05$
$b_1$	$0.38 \pm 0.08$	$0.56 \pm 0.15$	$0.41 \pm 0.03$	$0.42 \pm 0.03$
$\epsilon$	$0.40 \pm 0.03$	$0.39 \pm 0.02$	$0.31 \pm 0.03$	$0.31 \pm 0.03$
$\psi_0 - \psi_1$	$(+100^\circ)$	$(-50^\circ)$	$-119^\circ \pm 3$	$-109^\circ \pm 2$

potentials generated by  $\rho$  and  $\omega$  exchange, together with zero-range absorptive processes. In the work just cited, a strong energy dependence was found for the scattering amplitude  $A$  in the case of an attractive  $\bar{K}N$  potential insufficient to generate a bound state, whereas relatively little energy dependence of  $A$  was found for an attractive  $\bar{K}N$  potential sufficiently strong for the formation of a bound state (as may be the case for the  $I = 0$  channel, see below).

Earlier, data obtained by Luers *et al.*<sup>27)</sup> on the  $(K_1^0 p)/(Y\pi)$  ratio in the reactions

$$K_2^0 + p \rightarrow K_1^0 + p, \quad (4.3a)$$

$$\rightarrow \Sigma^+ + \pi^0, \quad \Sigma^0 + \pi^+, \quad \text{and} \quad \Lambda + \pi^+, \quad (4.3b)$$

for  $K_2^0 p$  collisions in the momentum range 250-500 MeV/c had been claimed as evidence favouring the solution I parameters. We now see that the comparison made with the low-energy amplitudes  $A_0, A_1$  is invalid because of the strong energy-dependence of  $A_1$ . In fact, we may note that these  $K_2^0 p$  processes depend only on the  $\bar{K}^0 p$  amplitude, i.e., on the  $I = 1$   $\bar{K}N$  amplitude  $A_1$ , and on the  $K^0 p$  elastic scattering amplitudes. In the region of 400 MeV/c, the  $A_1$  of Watson's amplitudes are quite similar to the value of  $A_1$  of the old solution I. In other words, the agreement found previously for the old solution I now holds just as well for the new energy-dependent solution II.

Similarly, the relation of the  $K^-d$  cross-sections to the  $K^-p$  cross-sections depends primarily on the

magnitude of the additional  $K^-n$  interactions, which correspond to the  $I = 1$  amplitude alone. The old solution II disagreed with the data on  $K^-d$  cross-sections in the range 200-300 MeV/c because the additional  $I = 1$  scattering predicted was too strong. Although the calculations have not yet been made, it seems probable that the new energy-dependent solution II will give  $K^-d$  cross-sections in adequate accord with the data.

Finally, the data on the branching ratios for  $K^-d$  capture from rest has provided a puzzle for several years. Since the final pion spectra show no indications of secondary scattering and the  $\pi N$  cross-sections in the energy range appropriate are known to be small, we may calculate the total rates for reactions leading to  $\pi^\pm$  mesons with momenta in the peak ( $q_\Sigma$ ) corresponding to the primary capture reaction  $K^- + p \rightarrow \Sigma + \pi$ . Neglecting the effects of initial state scattering, these expressions are as follows:

$$\pi^+ \Sigma^- n; \quad \left| \frac{M_0}{\sqrt{6}} + \frac{M_1}{2} \right|^2, \quad (4.4a)$$

$$\pi^- \Sigma^+ n, \pi^- \Sigma^0 p, \text{ and } (\pi^- \Lambda p)_c; \quad \left| \frac{M_0}{\sqrt{6}} - \frac{M_1}{2} \right|^2 + \frac{1}{2} |M_1|^2, \quad (4.4b)$$

where  $M_0, M_1$  denote the reaction amplitudes appropriate to the  $\bar{K}N$  capture at zero energy. The rate (4.4b) represents the total of all reactions giving a negative pion of momentum  $q_\Sigma$ , including those

$\pi^-Ap$  events which have resulted from  $\Sigma$ - $A$  conversion. The puzzle consists of the contrast between the  $\pi^+/\pi^-$  ratios observed with excellent statistics for  $K^-d$  and  $K^-p$  capture,

$$\left(\frac{\pi^+}{\pi^-}\right)_p = \left(\frac{\Sigma^-}{\Sigma^+}\right)_p = 2.15, \quad \left(\frac{\pi^+}{\pi^-}\right)_d = 0.7, \quad (4.5)$$

Expression (4.4a) and the first term of (4.4b) represent the intensity of  $K^-p$  capture, the second term of (4.4b) represents the intensity of  $K^-n$  capture. This additional term in (4.4b) is in the right direction for the change indicated in (4.5) but is far too small to account for this change, for it is well established that the  $I = 0$  absorptive rate is much stronger than the  $I = 1$  absorptive rate, i.e., that  $|M_1/M_0|^2 \ll 1$ .

Since the  $\bar{K}N$  interactions in the  $K^-d$  system are essentially at zero energy, the most plausible way in which the ratio of (4.4a) and (4.4b) may be changed from the value expected from the  $K^-p$  capture data is by a change in the relative phase of  $M_0$  and  $M_1$ , to carry this from the value  $-60^\circ$  appropriate to the  $K^-p$  situation to a value beyond  $-90^\circ$  for  $K^-d$  capture. Two mechanisms which have this effect have been discussed:

i) Capps and Schult<sup>28)</sup> have suggested that the presence of a  $Y_0^*$  resonance not far below the  $\bar{K}N$  threshold and with the same quantum numbers as the  $I = 0$   $\bar{K}N$   $s$ -wave system would give rise to a rapid fall in phase for the reaction amplitude  $M_0$  effective for the  $K^-p$  capture in the  $K^-d$  system as the energy of the recoil nucleon increases (i.e., as the energy of the final  $\pi$ - $\Sigma$  system decreases). This possibility is qualitatively in accord with solution II, since the sign of  $a_0$  is consistent with the existence of a virtual  $\bar{K}N$  bound state resonance, so that  $\delta_{\Sigma\pi}^0$  may be positive and greater than  $90^\circ$  at the  $\bar{K}N$  threshold. Since the phase of the  $K^-d \rightarrow \pi\Sigma N$  amplitude appropriate to  $I = 0$   $s$ -wave  $\bar{K}N$  absorption will be  $\delta_{\Sigma\pi}^0$  in the approximation

discussed here<sup>(\*)</sup>, the fall in  $\delta_{\Sigma\pi}^0$  through the resonance value with decreasing  $\pi\Sigma$  c.m. energy leads directly to the required decrease in the phase of  $(M_0/M_1)$  in (4.4). It was pointed out by Capps and Schult that, with the use of the zero-range  $\bar{K}N$   $s$ -wave amplitudes, a suitable phase change would require that  $a_0$  be at least as negative as  $-1.3$  F, and emphasized that at present the value of  $a_0 = (-0.6 \pm 0.5$  F) is not at all well known. The data presented at this conference<sup>30)</sup> has given very strong confirmation of the early indications<sup>31)</sup> of an  $I = 0$   $Y_0^*$  resonance at 1405 MeV, which may well be the resonance required by Capps and Schult. However, the  $Y_0^*$  spin value has not yet been directly established. This picture of the  $K^-d$  reaction processes suggests that the  $\pi^+/\pi^-$  ratio for the reactions  $\pi^\pm\Sigma^\mp n$  should vary rapidly with  $Q(\pi\Sigma)$ . A detailed study of the  $\pi^\pm\Sigma^\mp n$  spectra in  $K^-d$  capture at rest by Alexander *et al.*<sup>32)</sup> shows no evidence of this effect. However, this expectation ignores the distortive effect of the strong  $\Sigma N$  final state interactions in the  $\pi^-\Sigma^+n$  system, which might conceivably act to mask this effect. In fact, no direct indications of the  $Y_0^*$  resonance have been found in the at-rest or in-flight  $K^-d$  reaction data.

ii) Multiple scattering effects in the initial  $\bar{K}NN$  state, including the effects of virtual charge-exchange scattering, may also produce a phase change in  $(M_0/M_1)$ . Calculations of this effect<sup>33)</sup> show that, for capture at  $NN$  separation  $R$ , the capture amplitudes are changed as follows:

$$M_{0 \rightarrow D}(R) \left(1 + \frac{A_1}{R}\right) M_0, \quad M_{1 \rightarrow D}(R) \left(1 + \frac{A_0}{R}\right) M_1, \quad (4.6)$$

where  $D(R)$  is common to both amplitudes. Since the major contributions to the capture rate result from the region between  $R = 1$  F and  $R = 2$  F, the large values of  $A_0, A_1$  lead to appreciable

(\*) Note that the reaction amplitude used by Capps and Schult for negative  $\bar{K}N$  kinetic energy may be written in terms of the phase  $\delta_{\Sigma\pi}^0$ , given by

$$\tan \delta_{\Sigma\pi}^0 = qB = q(\gamma + i\beta^2 k / (1 - i\alpha k)) \quad (i)$$

in terms of the reaction matrix elements  $\alpha, \beta, \gamma$ , as follows<sup>29)</sup>:

$$(1 - ikA)^{-1} \beta (1 - iq\gamma)^{-1} = (1 - i\alpha k)^{-1} \beta (1 - iqB)^{-1} = (\beta \cos \delta_{\Sigma\pi}^0 / (1 - i\alpha k)) \exp(i\delta_{\Sigma\pi}^0) \quad (ii)$$

where (as pointed out to me by D. Miller) a convenient expression for the first factor is  $(k = +i|k|)$

$$\beta \cos \delta_{\Sigma\pi}^0 / (1 - i\alpha k) = (-1/\beta |k|) (\sin \delta_{\Sigma\pi}^0 / q - \gamma \cos \delta_{\Sigma\pi}^0). \quad (iii)$$

phase change for  $(M_0/M_1)_{\text{eff}}$ . This phase change is in the desired direction only for solution II. Some results of calculation<sup>33, 34)</sup> are given in Table II, as function of the poorly known amplitude  $A_0$ , the phase angle  $(\psi_0 - \psi_1)$  at zero energy being always adjusted to give agreement with the  $\pi^+/\pi^-$  ratio in  $K^-p$  capture at rest. As  $a_0$  becomes more negative, both of these ratios fall. For  $A_0 = (-1.4 + 0.9i)$  F, for example, the  $(\pi^+/\pi^-)_d$  ratio is 0.95, not yet in agreement with the experimental ratio but shifted far in this direction, whereas the fraction of direct  $\Lambda\pi^-p$  events (i.e., for which the pion momentum is in the peak  $q_\Lambda$ ) is 7.0%, in agreement with the data.

Each of these mechanisms requires that  $a_0$  be large and negative, and therefore the second mechanism probably also requires the existence of a  $I = 0$   $\bar{K}N$  virtual bound state, which may be the observed  $Y_0^*$  resonance. Hence it appears that the two mechanisms cannot really be discussed independently. These two effects are distinct and need both to be taken into account at the same time. This will not be an easy task, for it will require a complete three-body treatment of the  $K^-d$  interaction, including the effects of nucleon recoil.

Finally we comment that the  $\pi\Sigma$  resonance resulting from a  $\bar{K}N$  virtual bound state actually occurs for the  $\bar{K}N$  momentum  $|k| = -\alpha_0^{-1}$ , since

$$q_\Sigma \cot \delta_{\Sigma\pi}^0 = (\gamma_0 - |k|\beta_0^2/(1 + \alpha_0|k|))^{-1} \quad (4.7)$$

in terms of the  $I = 0$   $K$ -matrix elements  $\alpha, \beta, \gamma$  (cf. Ref. 29)). However the scattering amplitude is given by

$$\begin{aligned} A_0 &= \alpha_0 + iq_\Sigma\beta_0^2/(1 - iq_\Sigma\gamma_0) \\ &= (\alpha_0 - \gamma_0q_\Sigma b_0) + ib_0, \end{aligned} \quad (4.8)$$

where  $b_0 = q_\Sigma\beta_0^2/(1 + q_\Sigma^2\gamma_0^2)$ . Hence the sign of  $\gamma_0$  is of importance for the location of the peak of the  $\pi\Sigma$  resonance (i.e., the energy for which  $\delta_{\Sigma\pi}^0 = 90^\circ$ ). It is of interest to note that the phase of  $M_0$  at 400 MeV/c can be deduced directly from the diagram given by Tripp *et al.*<sup>10)</sup>, with the value  $+140^\circ$ . Since

$$M_0 = (1 + kb_0 - ika_0)^{-1}\beta_0(1 - iq_\Sigma\gamma_0)^{-1}, \quad (4.9)$$

we can deduce the value  $q_\Sigma\gamma_0 = -0.5$ , and therefore  $\gamma_0 = -0.37$  F, at this energy. If we assume that  $\gamma_0$  is constant, we then obtain  $b_0q_\Sigma\gamma_0 = -0.78$  F at the  $\bar{K}N$  threshold, quite an appreciable value. However, it is the sign of this term which is important, since

$$\alpha_0 = a_0 + b_0q_\Sigma\gamma_0. \quad (4.10)$$

Even with  $a_0 = -0.6$  F, the value obtained for  $\alpha_0$  is quite large,  $\alpha_0 = -1.38$  F. With neglect of effective range terms (a dangerous assumption here, as we have seen), the value of  $\alpha_0$  necessary to put the  $\pi\Sigma$  resonance at 1405 MeV is  $-1.5$  F. A more complete analysis of the  $I = 0$   $\bar{K}N$  interactions, using the  $K$ -matrix formalism with an effective range term<sup>35)</sup> would now seem rather desirable.

TABLE II

The calculated  $(\pi^+/\pi^-)_d$  and  $(\pi^- \Lambda p / \text{Total capture rate})_d$  ratios are given for several values of  $A_0 = a_0 + ib_0$  (F),  $A_1$  being held at the value  $(1.2 + 0.56i)$  F.

$(a_0, b_0)$	$(-0.2, 1.13)$	$(-1.0, 1.13)$	$(-1.8, 1.13)$	$(-0.2, 0.9)$	$(-1.0, 0.9)$	$(-1.8, 0.9)$
$(\pi^+/\pi^-)_d$	1.05	1.03	0.98	1.03	0.99	0.90
$(\pi^- \Lambda p)$ direct						
Total rate	10.8%	7.8%	5.7%	12.2%	8.1%	5.9%

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## DISCUSSION

BLOCK: I would like to comment that later in the meeting there will be evidence presented from the helium bubble chamber for the spin 3/2 assignment of the  $Y_1^*$ , which is shocking to me as well as to Dalitz, and furthermore a very preliminary indication on the branching ratio into the  $\Sigma\pi$  mode indicates a number not incompatible with the order of 10%.

CUTKOSKY: I would like to comment on some things that Tarjanne, Kalckar and I have been studying in connection with the hyperon resonances. Now we feel that it is necessary to take into account the  $\bar{K}$  nucleon channel on the same footing as the other channels for the reason that in the  $N/D$  method the main contribution to the  $D$  integral comes from energies at which the  $\bar{K}N$  phase space is equal to that for  $\Sigma\pi$ . If you do this, you get a smaller branching ratio, something like 7 or 8%, but you also get a different prediction with respect to the other resonances. In fact we find that any  $I = 2$  resonance should lie at a much higher energy, but there are indications that a  $I = 0$  resonance with  $J = 3/2$  might be lower.

DALITZ: Trueman at Chicago has also made calculations on the effects of the coupling to the  $\bar{K}N$  channel on these resonances. His conclusion was that the qualitative predictions were relatively insensitive to these couplings, except perhaps for the  $I = 0$   $P_{3/2}$  resonance, and I think that his results are essentially in accord with what you have just said.

CUTKOSKY: Well, I think it makes them a little better.

DALITZ: You probably have some freedom in choice of signs for the coupling parameters.

CUTKOSKY: We just took those from the unitary symmetry model. There is then no arbitrariness.

CAPPS: One question about the energy dependence. As you say, clearly there must be some energy dependence. Does it have to be in the  $I = 1$  channel? I mean, since the low-energy data is partly near threshold and partly up to 200 MeV/c, if you put in the energy dependence I had the impression this

might change the low-energy parameters so that one might get by with an  $I = 1$  amplitude that is fairly small everywhere.

DALITZ: Well, I think this has still to be tried quantitatively. You will note, of course, that the error on the amplitude  $a_1$  obtained from the Humphrey-Ross analysis of the low-energy region is, in fact, rather small. This amplitude appears relatively well determined and in fact its value has remained rather stable through all the changes in the data as the  $K^-p$  experiment progressed. This may not be a very strong argument

against your suggestion. On the other hand, I should remark again here that the calculations made by Ferrari, Frye and Pusterla, and by myself, found that, for the case where the  $\bar{K}$  nucleon interaction was attractive but not sufficient to give rise to a bound state, there followed a rather rapid energy dependence of the scattering amplitude. It has been difficult to understand intuitively why this rapid energy dependence should occur, and we are still looking into this question, but at least, the rapid energy dependence observed may not be in disagreement with those results.

## THE EVIDENCE OF ODD $K\Sigma N$ PARITY

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(presented by R. H. Capps)

Recently Tripp, Watson, and Ferro-Luzzi have presented evidence that the intrinsic  $K\Sigma N$  parity is odd<sup>1,2)</sup>, using a general method suggested by the author<sup>3)</sup>. The particular method of analysis used by TWF consisted of assuming specific forms for the  $K^- + N \rightarrow K^- + N$  and  $K^- + N \rightarrow \pi + \Sigma$  amplitudes, fitting them to the angular distribution data, and then showing that these assumed amplitudes are consistent with the  $\Sigma$ -polarization data only if the  $K\Sigma N$  parity is odd. In such a curve-fitting procedure it is not clear to what extent the parity conclusion depends on the specific assumptions regarding the forms of the amplitudes. In fact, Adair has recently questioned the validity of the parity conclusion<sup>4)</sup>. The purpose of this note is to present an analysis that is less subjective than the method of fitting curves to the data.

We shall consider only the  $K^- + p \rightarrow \pi^- + \Sigma^+$  data, since reliable polarization measurements have been made only for these events. We first list four key assumptions used in the parity determination; the detailed justification of the first three assumptions is given in references<sup>1-3)</sup>.

1) The 395 MeV/c resonance occurs in the  $D_{3/2}$ ,  $K^- + N$  state (amplitude denoted by  $f_3$ ).

2) The only non-resonant amplitude that is large in the resonance region is produced from the  $S_{1/2}$ ,  $K^- + N$  state (amplitude denoted by  $f_1$ ).

3) In the energy region of the resonance, the relative phase of  $f_3$  and  $f_1$  increases with energy. (This assumption comes from the well-known Wigner theorem.)

4) The proton helicity in the  $\Sigma^+ \rightarrow p + \pi^0$  decay is negative<sup>5)</sup>.

The simplest way to obtain the conclusion of odd  $K\Sigma N$  parity from the above assumptions and the data of TWF is as follows. The polar-equatorial ratio in the angular distribution  $\{R_{pe} = (P-E)/(P+E)\}$  is very large in the resonance region, i.e.,  $R_{pe} = 0.36 \pm 0.08$ ,  $0.50 \pm 0.8$  and  $0.36 \pm 0.08$  at 370, 390, and 410 MeV/c. Since the background cross-section from  $f_1$  is larger than the resonance bump for the  $\Sigma^+ - \pi^-$  events, and since  $R_{pe} = 0.375$  for a pure  $J = 3/2$  amplitude, a value of  $R_{pe}$  approximately equal to 0.15 would be expected in the resonance peak if there were no interference between  $f_1$  and  $f_3$ . Clearly, this interference term must be such as to increase  $R_{pe}$ . Since this term has the form  $(6 \cos^2 \theta - 2) |f_3 f_1| \cos \eta$ , where  $\eta$  is the relative phase, we conclude that  $\cos \eta$  is positive in the region 370-410 MeV/c.