Extreme Physics and Informational/Computational Limits

Paolo Di Sia

Department of Computer Science, Faculty of Science, Verona University, Strada Le Grazie 15, I-37134 Verona, Italy
Faculty of Computer Science, Free University of Bozen, Piazza Domenicani 3, I-39100 Bozen-Bolzano, Italy

E-mail: `paolo.disia@univr.it`; `10alla33@virgilio.it`

Abstract. A sector of the current theoretical physics, even called "extreme physics", deals with topics concerning superstring theories, multiverse, quantum teleportation, negative energy, and more, that only few years ago were considered scientific imaginations or purely speculative physics. Present experimental lines of evidence and implications of cosmological observations seem on the contrary support such theories. These new physical developments lead to informational limits, as the quantity of information, that a physical system can record, and computational limits, resulting from considerations regarding black holes and space-time fluctuations. In this paper I consider important limits for information and computation resulting in particular from string theories and its foundations.

1. Introduction

Many of the fundamental limits on information processing derive from physics, in particular from thermodynamics, relativity, quantum mechanics and the following unified theories, like string theories. Recent physically motivated computing paradigms like reversible computing and quantum computing may help for same aspects, but in every case they remain subject to some basic limits. Computers are physical systems, therefore the physical laws establish their operativity. In particular, the speed with which a physical device can process information is limited by its energy and the amount of information which it can process by the number of degrees of freedom possessed. These physical limits of computation appear depending from important physical constants, the speed of light c, the quantum scale $\hbar$, and the gravitational constant G [1]. The so called "Extreme Physical Information", seen as an extension of the information theory, includes much theoretical physics and it has been used to derive some fundamental laws not only in the sectors of physics, but also for biology, chemistry, economics. The holographic principle is a property of string theories and quantum gravity declaring that the description of a volume of space can be thought as encoded on the region boundary, of preference a light-like boundary. Through this theory the entire universe can be seen as a 2-D information structure "painted" on the cosmological horizon, such that the 3 observed dimensions result to be a description at macroscopic scales and low energies. Therefore the 3-D universe could be totally equivalent to alternative quantum fields and physical laws "painted" on a big and distant surface [2]. The physics of black holes suggests credibility to this theory; studies about black holes have showed that the maximum entropy or information content of a space region is defined

by its surface area and not by its volume. In particular, a hole with a horizon area A has $A/4$ units of entropy, with the Planck area, approximately $10^{-66}\,cm^2$, the fundamental quantum unit of area. The holographic principle shows the complete equivalence between two universes with different dimensions and obeying different physical laws. From a theoretical viewpoint this property has been demonstrated mathematically for 5-D anti-de Sitter spacetimes and the corresponding 4-D boundary, considering the 5-D universe recorded like a hologram on the 4-D surface. As example, a black hole in the 5-D spacetime appears equivalent to hot radiation on the hologram; the hole and the radiation have the same entropy even if the physical origin of the entropy is completely different. Theoretical studies suggested also that a black hole with appropriate properties can serve as a basis for a description of string theory [3].

2. Informational limits

Entropy is measurable in bits, if considered as information. The total quantity of bits is related to the total degrees of freedom of matter/energy. Considering a given volume, there is an upper limit to the density of information in relation to all the particles costituting the matter in that volume. The fundamental difference between the thermodynamic entropy of physics and the Shannon's information entropy regards the units of measure; the former is expressed in units of energy divided by temperature, the latter in bits of information.

By information theory, the maximal information I_{MAX} of a system, obtainable knowing the details of its state, is given by:

$$I_{MAX} = S_{MAX}/K_B \ln 2 \tag{1}$$

with K_B the Boltzmann constant. If this information collapses into a black hole, the resulting black hole has an entropy S_{BH} related to S_{MAX} as:

$$I_{MAX} K_B \ln 2 = S_{MAX} < S_{BH} < \frac{K_B A}{4 L_{PL}^2} \tag{2}$$

with L_{PL} the Planck length. From relation (2) it follows:

$$I_{MAX} < \frac{A}{4 L_{PL}^2 \ln 2} \; . \tag{3}$$

The important bound (3) was deduced by 't Hooft (1993) and Susskind (1995). Previously, in 1981, Bekenstein had reasoned in this way: putting an information content of overall radius R and total mass-energy E into a black hole, it is possible to obtain the bound:

$$I_{MAX} < \frac{2 \pi R E}{\hbar c \ln 2} \; . \tag{4}$$

The bound (4) does not contain the gravitational constant G and for simple closed systems it can also be derived from quantum statistical considerations without considering black holes [4]. Numerically, it requires that $1\,cm^3$ of ordinary material holds no more than $\approx 10^{38}$ bits. The statistical mechanics allows the computation of the entropy of its molecules, apparently ignoring the atomic and subatomic deeper degrees of freedom of matter as contributions to the total entropy; but "de facto" all atomic and subatomic particles gravitate. This is one of the interesting peculiarities of black hole thermodynamics with respect to detailed aspects of physical systems. The thermodynamics of black holes allows therefore to deduce important limits on the density of entropy or information in a lot of circumstances.

Of interest for the contest is important to remember two important bound: the holographic bound and the universal entropy bound. The holographic bound explains how much information can be contained in a specific space region. Let's consider an approximately spherical matter

distribution contained within a surface of area A; the matter is induced to collapse to form a black hole. The black hole's area must be smaller than A, so its entropy must be less than $A/4$. For the fact that entropy cannot decrease, it follows that the original distribution of matter also must carry less than $A/4$ units of entropy or information.

The universal entropy bound establishes how much information can be carried by a mass m of diameter D. For its derivation we can imagine that a capsule of matter falls in a black hole of about the same dimension. The size of the black hole increases and this gives a limit on how much entropy is contained in the capsule. This limit is tighter than the holographic bound, except the case in which the density of the capsule is about the same with respect to the black hole (in this case the two bounds become equivalent).

The information content of a big quantity of computer chips increases in proportion with the occuped volume. Continuing to add chips, we must stop when the information exceeds the holographic bound, which depends on the surface area. The "breakdown" will occur when the huge quantity of chips collapses to form a black hole [5].

3. Computational limits and quantum fluctuations of space-time

The laws of quantum mechanics and gravitation lead to physical bounds on computation and on precision of clocks; the same laws control also the quantum fluctuations of space-time. The limits regard the speed of computation v and the memory space I of a simple computer. These evaluations hold also for the so-called "*simple* clocks", intending for "simple" that a separation of components of a clock is not involved (and itself in relation to "simple computer"). These bounds are saturated for black holes, which, for some aspects, may be regarded as the ultimate simple computers and ultimate simple clocks [6,7]. It is possible to use quantum mechanics to find fundamental limits on any system with mass m that serves as a time-registering device. For a given T and t, with T the total running time over which a simple clock can remain accurate and t the smallest time interval with the clock capable of resolving, the lower bound on m will be [8]:

$$m \geq \frac{\hbar}{c^2 t} \left(\frac{T}{t} \right) . \tag{5}$$

It is possible to obtain also an upper bound considering the general relativity. Suppose a simple light-clock consisting of two parallel mirrors, everyone with mass $m/2$, and with a beam of light bouncing between them. The separation d between the mirrors must satisfy the relation $d \leq c\,t$ for resolving a time interval of the order of t, and d is necessarily larger than the Schwarzschild radius $G\,m/c^2$. These two requirements permit to find the upper bound on m:

$$t \geq \frac{G\,m}{c^3} . \tag{6}$$

A bound on the speed of computation v of any information processor [9] is obtainable considering the mean input power $P = m\,c^2/T$. Eq. (5) and the fastest possible processing frequency $v = 1/t$ imply:

$$v^2 \leq \frac{m\,c^2}{\hbar\,T} = \frac{P}{\hbar} . \tag{7}$$

Eqs. (5) and (6) lead to a relation between T and t:

$$T \leq t \left(\frac{t}{t_{PL}} \right)^2 \tag{8}$$

with t_{PL} the Planck time. If we consider now that the ratio T/t (maximum step number of information processing), is (except constant factors) the amount of information I recordable by the computer, it is possible to write a limit for the memory space I of a simple computer:

$$I \approx \frac{T}{t} \leq \frac{1}{(v\,t_{PL})^2} \approx \frac{\hbar}{P\,t_{PL}^2} \; . \tag{9}$$

The combination of Eqs. (7) and (9) is very interesting:

$$I\,v^2 \leq \frac{1}{t_{PL}^2} = \frac{c^5}{\hbar\,G} \approx 10^{86}\Big/ s^2 \; , \tag{10}$$

because of its independence from mass, size, and details of the simple computer. This expression (valid for simple computers) unites together the concepts of information, gravity, and quantum uncertainty. This bound is realized in particular for black holes [10]. Considering that at short distance scales the space-time is foamy, the same physics underlies the foaminess of space-time, the limits to computation, and clock precision. The bound given from Eq. (10) is saturated for black holes, with both v and I bounds saturated, confirming the conceptual importance of black holes as the simplest and most fundamental constructs of space-time.

The experimental research, in particular in the gravitational-wave interferometer field, can sure help these theoretical aspects providing confirmations or denials.

4. Computational capacity of the universe

It is possible to quantify the amount of information processing performed by the Universe, considered as a whole, since the big bang. The Universe has the capacity to perform a maximum of $I_{UNIV} \leq 10^{122}$ (10^{123} considering all the constant factors like 2π, etc...) elementary quantum logic operations on $\approx 10^{90}$ bits registered in quantum fields, considering $t \approx 10^{10} yr$ the age of the Universe. If we consider a closed Universe, the previous numbers represent the amount of elementary logic operations (ops) and bits available in the entire Universe; if on the contrary the Universe is open, and of infinite extension, then the numbers give the amount of computation performed within the event horizon [2].

The previous information limit can be derived looking at the link between physics and information, discovered by Bekenstein (1973) and Hawking (1975), when they applied quantum mechanics to black holes. They have found that an uncharged and not rotating black hole has entropy S given by:

$$S = \frac{4\,\pi\,K_B\,G\,M^2}{\hbar\,c} = \frac{K_B}{4L_{PL}^2}\,A \tag{11}$$

with M and A respectively mass and area of the black hole horizon. Seeing the entropy as a measure of the information I (or lost information) through the relation $S = K_B \ln 2\,I$, Eq. (11) binds the total information content of a space region to the surface area including such volume. The idea to associate entropy and information with the area of the horizon has been soon enlarged to include all the event horizons, not only those that surround a black hole.

5. Black hole computation

Supposing to perform a high speed serial computation, a way can be to compress the computer dimensions to reduce the time of sending signals. "Smallering" the computer, for fixed values of energy, the density to its inside increases, requesting the "exploration" of different regimes of the high energy physics. At first we find the weak unification scale, therefore that of great unification; when the linear dimensions of the computer are near to its Schwarzschild radius, the Planck scale is reached. To such scale both gravitational and quantum effects become

important; the Compton wavelength of a particle of mass m, $\lambda_C = 2\pi\hbar/mc$, is of order of its Schwarzschild radius $R_S = 2mG/c^2$. A theory of quantum gravity unification or a superstring theory is required.

Although it is not possible to know the exact bit number recordable from a computer of the mass of 1 Kg confined in the volume of 1 litre, it is possible to know the exact bit number recordable from a computer of 1 Kg which has been compressed to the dimension of a black hole [11]. Using Page's results in relation to the mechanism of black holes evaporation [12], the time needed for a black hole to evaporate is given by:

$$t_{EVAP} = \frac{m^3 G^2}{3\hbar c^4 C} \ , \tag{12}$$

where C is a constant depending on the number of particles kinds with mass smaller than $K_B T$, with T the black hole temperature. In the interval $10 - 100$ of such kinds, C is of order of $10^{-3} - 10^{-2}$ and this implies a lifetime around $\approx 10^{-19} s$ for a black hole of 1 Kg of, during which the black hole can treat around 10^{32} operations on its 10^{16} bits. The effective massless particle number at Hawking temperature for a black hole of 1 Kg is greater than 100. The Schwarzschild radius of a computer of 1 Kg results to be $R_S = 1.485 \cdot 10^{-27} m$. Therefore, it is possible to evaluate the information quantity stored in a black hole through the relation:

$$I = \frac{4\pi m^2 G}{\hbar c \ln 2} \ . \tag{13}$$

For a computer of 1 Kg in the black hole limit, relation (13) gives $I = 3.827 \cdot 10^{16} \, bits$, from which follows $N = 5.426 \cdot 10^{50} \, ops/s$.

6. Velocity, memory space, relativistic effects

An interesting evaluation concerns the calculation of the computational capacity of a computer that we could call "extreme" or "ultimate". The actual netbooks have a mass of order of 1 Kg. We consider therefore a mass of 1 Kg and look for a limit regarding the speed of such a device. An elementary logic operation in a time Δt requires an average energy given by:

$$E \geq \frac{\pi\hbar}{2\Delta t} \ \Rightarrow \ N = \frac{1}{\Delta t} \approx \frac{2E}{\pi\hbar} \ . \tag{14}$$

For a mass of 1 Kg, $E = mc^2 \cong 8.987 \cdot 10^{16} \, J$ and consequently $N = 5.426 \cdot 10^{50} \, ops/s$.

A real computer operates much slower, because a lot of energy constitutes the mass of particles of which it is composed, leaving only a very small fraction for logical execution; mereover a conventional computer employs many degrees of freedom (billions of electrons) to record a single bit.

Thermodynamics and statistical mechanics provide also a fundamental limit to the quantity of performing information bits through the use of an energy quantity confined in a given volume. A system with N accessible states can record $\log_2 N$ bits of information. The accessible state number W of a physical system is also related to its thermodynamic entropy $S = K_B \ln W$. The quantity of information recordable from a physical system, i.e the total number of available memory space bits is given by:

$$I = \frac{S(E, V)}{K_B \ln 2} \ , \tag{15}$$

with $S(E, V)$ the thermodynamic entropy of a system with energy E confined in a volume V. It is possible to estimate this entropy considering the volume occupied by the computer as set of elementary particles modes with average total energy E. Let's assume as conserved quantities the angular momentum and the electric charge, considered equal to zero. Encapsulating the

number of kinds of particles/antiparticles, the polarization number and the particle statistics into a single quantity r, it is possible to write the entropy as function of energy:

$$S = \frac{4}{3} K_B \left(\frac{\pi^2 \, r \, V}{30 \, \hbar^3 \, c^3} \right)^{1/4} E^{3/4} \; . \tag{16}$$

Assuming entropy and energy dominated by black hole radiation consisting of photons, we have $r = 2$. For a computer of 1 Kg mass confined in a 1 litre volume, we find: $S \cong 2.04 \cdot 10^8 \, J/K$; $I \cong 2.13 \cdot 10^{31} \, bits$ [13].

With the increase of clock speed, also relativistic effects appear. The combination of relativistic and quantum effects in the logical circuits needs the use of relativistic quantum mechanics and quantum field theory. An important relativistic effect in quantum computation concerns the impossibility to locate an electron in a volume less than that characterized by the Compton wavelength of the electron. This implies that the time required by the light to cross such small distance, $\approx 10^{-21} s$, represents a minimum time for a logic process involving electrons, and therefore a working maximum frequency of order of $10^{21} \, Hz$. At such frequency, the energy-time Heisenberg relation involves voltage values going beyond the threshold of production of $e^+ - e^-$ pairs.

Estimating the power consumption of a quantum relativistic computer, we consider a device working on a clock cycle of $10^{-21} \, s$ and representing every bit through a single electron. From Heisenberg relation we deduce:

$$\Delta E \cong f \, \hbar = 1.06 \cdot 10^{-13} \, J \; . \tag{17}$$

Supposing that the elementary circuit process involves 10^3 electrons, at the uncertainty limit the power is given by the product of the number of electrons operating in parallel, times the energy of every electron, times the frequency. The required power in this case results in:

$$P = N_{e^-} \cdot \Delta E \cdot f = 1.06 \cdot 10^{11} \, W \; . \tag{18}$$

Since pratically the total of such power would go out of the computer in the form of gamma rays, a lot of problems would sure rise. Among the fastest fundamental processes known today in physics there is the creation or disintegration of the Z^0 boson. Its average timelife is of order of $10^{-26} \, s$, corresponding to a frequency of $10^{26} \, Hz$. Therefore if we assume that a circuit cannot operate in faster way, an ultimate limit to the computational speed is found. For the relativistic effects, the electrons cannot be measured in times less than $10^{-21} \, s$. If electrons are used, the maximum clock speed for an electronic circuit will be found in the range of $10^{21} \, Hz$ [14].

7. Closing remarks

The dimensions of the elementary circuit constituents are becoming smaller and smaller. The clock speed of computers is notably grown in the last years and it can well fit to an exponential growth. A huge progress has been done in the simulation of the material properties on computers and these applications accelerate manufacturing new, better devices. Quantum mechanics, together with the elementary properties of matter and radiation, has become fundamental for the future of computers and computation. This implies that the "ultimate computer" might necessarily incorporate the principles of relativistic quantum physics. A quantum computer allows a best understanding not only of subatomic world, but it also deeply penetrates the meaning of computation. Perhaps a final theory might be concerned not with fields and spacetime, but rather with information exchanged among physical processes. If so, the vision of information as stuff of the world would find a complete embodiment.

8. References

[1] Lloyd S 2000 *Nature* **406** 1047
[2] Lloyd S 2002 *Phys. Rev. Lett.* **88** 23
[3] Bekenstein J D 2003 *Sci. Am.* **17** 59
[4] Bekenstein J D 2001 *Stud. Hist. Phil. Mod. Phys.* **32** 511
[5] Bekenstein J D 1999 *Proceedings of the Eighth Marcel Grossman Meeting on General Relativity*, ed T Piran and R Ruffini (Singapore: WorldScientific) p 92
[6] Ng Y J and van Dam H 1994 *Mod. Phys. Lett.* A **9** 335
[7] Sasakura N 1999 *Prog. Theor. Phys.* **102** 169
[8] Salecker H and Wigner E P 1958 *Phys. Rev.* **109** 571
[9] Barrow J D 1996 *Phys. Rev.* D **54** 6563
[10] Ng Y J 2001 *Phys. Rev. Lett.* **86** 14
[11] Novikov I D and Frolov V P 1986 *Black Holes* (Berlin: Springer-Verlag)
[12] Page D N 1976 *Phys. Rev.* D **13** 198
[13] Lloyd S 2000 Ultimate physical limits to computation *Preprint* quant-ph/9908043v3
[14] Pakvasa S, Simmons W and Tata X 1999 Relativistic Quantum Computing *Preprint* quant-ph/9911091v1