COLLIDING-BEAM EFFECT

PART ONE: PERTURBATION UNDERGONE
BY A PARTICLE CROSSING A BUNCH

by

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In an electron-positron storage ring the motion of a particle crossing a bunch of the opposed beam is modified by the electromagnetic force due to the charge of the bunch (Amman-Ritson effect\textsuperscript{1}).

The bunches of particles have a trigaussian density\textsuperscript{2,3}, and this property is used here to calculate exactly the angular perturbation undergone by the particle, which is obtained in particular in the form of a series expansion having as parameters the amplitudes of the particle's betatron oscillations.

The result will be applied to various effects pertaining to the space-charge compensation in a four-beam machine (Coppelia project\textsuperscript{4}), as well as
the resultant distribution of wave numbers in the beams.

1. **ANGULAR PERTURBATION IN A HEAD-ON COLLISION**

We shall consider the case of a head-on collision. The case of crossing at an angle, which will be discussed in the next section, leads to analogous formulae insofar as the drift of the particle wave-number is concerned.

We assume that:

a) the distribution of the particles in the bunch is not affected by the crossing;

b) this crossing occurs within a short enough time so that only the particle's momentum is affected.

The calculation consists in making a Lorentz transformation for reducing to the system in which the particle bunch is at rest. The space-charge field is then purely electric, and the resultant force very simple.

The coordinates in the laboratory system will be indicated by capital letters, those in the bunch's rest system by lower-case. The index 1 is assigned to a traveling point* of the bunch, the index 2 to the particle that crosses it.

* **Translator's Note:** The French says "point courant," which translates literally as "running point" or "current point."
In the laboratory, the density of particles in the bunch is

\[ dF(X_1, Y_1 + vT, Z_1) = \frac{N}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} e^{-\left[ \frac{X_1^2}{2\sigma_x^2} + \frac{(Y_1 + vT)^2}{2\sigma_y^2} + \frac{Z_1^2}{2\sigma_z^2} \right]} dX_1 dY_1 dZ_1, \]

where \( N \) is the number of particles in the bunch; \( \sigma_x, \sigma_y, \sigma_z \) the radial, longitudinal and vertical standard deviations of the distribution; \( v = \beta c \) the velocity of the bunch (which is equal and opposite to that of the particle); the equation for the motion of the particle being

\[
\begin{align*}
x_2 &= x_0 \\
y_2 &= y_0 + vT \\
z_2 &= z_0.
\end{align*}
\]

The equations of the transformation that reduce to the bunch's rest system are:

\[
\begin{align*}
x &= X \\
y &= \gamma(Y + \beta ct) \\
z &= Z \\
t &= \gamma(t - \frac{\beta}{c} y)
\end{align*}
\]

so that the density becomes

\[ dp = dF \left( x, \frac{y}{\gamma}, z \right), \]

and the particle motion

\[
\begin{align*}
x_2 &= x_0 \\
y_2 &= \gamma y_0 + \beta' ct \\
z_2 &= z_0
\end{align*}
\]

with \( \beta' = \frac{2\beta}{1+\beta^2} \approx 1 \) and \( \gamma' = (1-\beta'^2)^{-1/2} \approx 2\gamma^2 \).
The electric field on the particle is 
\[ \mathbf{E} = \frac{q_1}{4\pi\varepsilon_0} \frac{\mathbf{e}}{r^3} \]

with \( q_1 = \pm e \) being the charge of a particle of the bunch, \( \mathbf{e} \) the vector \( x_2 - x_1, y_2 - y_1, z_2 - z_1 \).

The momentum received by the particle of charge \( q_2 \) is:

\[ \delta \mathbf{p} = \int_{-\infty}^{+\infty} \left[ \frac{q_2}{c} \mathbf{E} \, dt \right] \mathbf{y}_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 c} \int_{-\infty}^{+\infty} \frac{\mathbf{r}}{r^3} \, dy_2. \]

Translator's Note: Intended meaning of the isolated traversée above unclear. It could be the past participle of the verb traverser, which means "traverse" or "cross" (though the author hitherto has been using croiser to mean "cross"). Or it could be the noun traversée, which means "crossing," "passage," etc. If the past participle, the meaning would be "crossed" or "traversed."

As expected, \( \delta p_y \) is zero. The momentum received by the particle is purely transverse in the system in which the bunch is at rest. The collision having been head-on, \( \delta \mathbf{p} \) is also perpendicular to the direction of the Lorentz transformation, and is conserved on return to the laboratory.

Hence the angular kick will be 
\[ \delta \theta = \frac{\delta \mathbf{p}}{p}, \text{ i.e.,} \]

\[ \delta \mathbf{r} = \frac{\varepsilon r_e}{\gamma} \int \mathcal{F} \left( \frac{x_1}{\gamma}, \frac{y_1}{\gamma}, \frac{z_1}{r^3} \right) \mathbf{r} \, dy_2 \]

with \( \varepsilon = \pm 1 \), depending on whether the particle has the same charge (+) or one opposite (-) to that of the bunch, and \( r_e = e^2/4\pi\varepsilon_0 m_e c^2 \) is the classical electron radius.
We let
\[ \delta \theta_x(x_2, z_2) = \int_\mathbb{R} e^{\frac{eN\rho e}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z\gamma^2}} \frac{\left[ \frac{x^2_1}{2\sigma_x} + \frac{y^2_1}{\gamma^2\sigma_y} + \frac{z^2_1}{2\sigma_z} \right]}{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}^{3/2} dx_1 dy_1 dz_1 \]

and
\[ \delta \theta_z = \delta \theta_x [x_1 \leftrightarrow z_1, x_2 \leftrightarrow z_2, \sigma_x \leftrightarrow \sigma_z]. \]

The integration is given in the supplement. It can be summarized with the formula
\[ \delta \theta_x + i\delta \theta_z = K(x+iz) \int_0^1 dt \, e^{-(1-t)\left[ x^2 + z^2 + \psi t(x^2 - z^2 + 2ixz) \right]}, \]

where
\[ K = e \frac{r\gamma}{2\gamma^2} \frac{2\gamma^2}{\gamma\sigma_x\sigma_z}, \quad x = \frac{X_0}{\sqrt{2}\sigma_x}, \quad z = \frac{Z_0}{\sqrt{2}\sigma_z}, \quad \psi = \frac{\sigma_x - \sigma_z}{\sigma_x + \sigma_z}, \]

\( N \) being the number of particles per bunch of the beam crossed, with \( \epsilon = \pm 1 \), depending on whether the particle has the same charge as or one opposite to that of the bunch.

With \( x = \rho \cos \phi, y = \rho \sin \phi \), we also write:
\[ \delta \theta_x + i\delta \theta_z = K \rho e^{i\phi} \int_0^1 dt \exp \left[ -\rho^2(1-t)(1+t\psi e^{2i\phi}) \right]. \]
1.1 Round Beam: \( \psi = 0 : \sigma_x = \sigma_z \)

\[
\delta\theta_x + i \delta\theta_z = K (\rho \cos\phi + i\rho \sin\phi) \int_0^1 dt \ e^{-\rho^2(1-t)}
\]

\[
\delta\theta_x = \frac{r_N X}{\gamma \sigma_x^2} \frac{1-e^{-\rho^2}}{\rho^2}
\]

\[
\delta\theta_z = \frac{r_N Z}{\gamma \sigma_z^2} \frac{1-e^{-\rho^2}}{\rho^2}.
\]

1.2 Flat Beam: \( \psi = 1 : \sigma_x >> \sigma_z \)

\[
\delta\theta_x + i \delta\theta_z = K e^{-\rho^2 \cos^2 \phi} \int_0^1 dt \ e^{-\rho^2} (e^{i\phi} t - i \sin\phi)^2
\]

We let

\[
\delta\theta_x = \frac{2r_N X}{\gamma \sigma_x (\sigma_x + \sigma_z)} \int_0^1 e^{-x^2(1-v^2)} \ dv
\]

\[
\delta\theta_z = \frac{2r_N Z}{\gamma \sigma_z (\sigma_x + \sigma_z)} e^{-z^2} \int_0^1 e^{-z^2v^2} \ dv
\]

with \( x = \frac{X}{\sqrt{2} \sigma_x} \) and \( z = \frac{Z}{\sqrt{2} \sigma_z} \).
1.3 General Case

If we set \( \psi = e^{-2\lambda} \), i.e., \( \lambda = \log \sqrt{\frac{z}{x \cdot z}} \), and \( y = -\lambda + i\phi \)

with \( \rho \ch y = x \ch \lambda - i\pi \sh \lambda \)

\( \rho \sh y = -x \sh \lambda + iz \ch \lambda \),

we have

\[
\delta \theta_x + i \delta \theta_z = K e^\lambda \cdot e^{-\rho^2 \ch^2 y} \int_{-\rho \sh y}^{\rho \ch y} e^{t^2 \ho^2 \sh^2 y} dt
\]

or

\[
\delta \theta_x + i \delta \theta_z = K e^\lambda \cdot e^{-\rho^2 \ch^2 y} \left\{ \rho \ch y \int_0^1 e^{t^2 \rho^2 \ch^2 y} dt + \rho \sh y \int_0^1 e^{t^2 \rho^2 \sh^2 y} dt \right\}.
\]

The first formula allows simple derivations. Numerical calculation of the second, which introduces an oscillating term of frequency \( xz \sh 2\lambda \), is easier, in the case of a flat beam, than numerical calculation of formula (1), where the frequency is of the order of \( xz\psi \).

1.4 Series Expansion

Starting with formula (1), we get

\[
\delta \theta_x = K x \sum_{n=0}^{\infty} (-)^n n! \sum_{p=0}^{n} \frac{\psi^p}{(x^2 + z^2)^{\frac{n-p}{2}}} \frac{P}{(n-p)! (n+p+1)!} (-)^q \frac{2^q}{C} x^2 (p-q) z^2q,
\]

which, if we separate the powers of \( x \) and \( z \), can be written
\[ \delta \theta_i = k x \sum_{n=0}^{\infty} (-)^n \frac{n!}{(2n+1)!} \sum_{s=0}^{n} (x^2)^{n-s} (z^2)^s \psi_{n,s}(x) \]

\[ P_{n,s}(\psi), \text{ an } n\text{-th-degree polynomial in } \psi, \text{ is given by} \]
\[ P_{n,s}(\psi) = \sum_{p=0}^{n} \psi^p c_{2n+1}^{n-p} \sum_{x+q=s, r+q=n-p \neq 0}^{} \]

whence
\[ \delta \phi = \epsilon \frac{2r e N}{(\sigma_x + \sigma_z) \sigma_x} X_0 \left( 1 - \frac{X_0^2}{2 \sigma_x^2} \left( \frac{3+\psi}{6} \right) - \frac{Z_0^2}{2 \sigma_z^2} \left( \frac{1-\psi}{2} \right) + \ldots \right) \]
\[ \delta \phi = \epsilon \frac{2r e N}{(\sigma_x + \sigma_z) \sigma_z} Z_0 \left( 1 - \frac{X_0^2}{2 \sigma_x^2} \left( \frac{1+\psi}{2} \right) - \frac{Z_0^2}{2 \sigma_z^2} \left( \frac{3-\psi}{6} \right) + \ldots \right) \]

2. **CASE OF CROSSING AT AN ANGLE**

The main differences between crossing at an angle and a head-on collision stem from two facts:

- The synchronism affects the result, which depends on the particle's longitudinal coordinate \( Y_0 \).

- There exists a longitudinal component of the momentum variation undergone by the particle, which is either accelerated or decelerated by the opposed bunch.
The treatment is similar to that of a head-on collision. Through a Lorentz transformation, we achieve reduction to the system in which the bunch is at rest. In that system the particle has a velocity forming an angle $\varepsilon$ with the bunch's longitudinal axis, so that \( \tan \varepsilon = \frac{1}{\gamma} \frac{\sin 2 \alpha}{1 + \cos 2 \alpha} = \frac{1}{\gamma} \tan \alpha \) and, during the crossing, it has over its trajectory a velocity \( B = \frac{2 \beta \cos \varepsilon}{1 + \beta^2 \cos^2 \varepsilon} \gamma \cdot 1 \) for $\alpha \neq 2\pi$.

When in this system we integrate over the duration of the crossing, the particle's momentum variation $\delta p$ is still purely perpendicular to its motion. But it is no longer perpendicular to the axis of the Lorentz transformation. For a crossing in the vertical plane we have:

\[
\begin{align*}
\delta p_x & \quad \text{transverse} \\
\delta p_z \cos \varepsilon & \\
\delta p_z \sin \varepsilon & \quad \text{longitudinal}
\end{align*}
\]

Coming back to the laboratory system, we have $\delta p_z \cos \varepsilon$ and $\gamma \delta p_z \sin \varepsilon$ as the last two components, or $\delta p_z \cos \varepsilon$ and $\delta p_z \cos \varepsilon \tan \alpha$.

The component situated in the plane of the crossing is directed along the
bisector of the angle of the velocities:

In addition to a transverse component, we always have a longitudinal component, which is a variation $\Delta E$ of the particle's energy.

$\delta p_x$ and $\delta p_z$ are calculated, as in the case of a head-on collision, by integrating the force to which the particle is subjected over its trajectory in the system in which the bunch is at rest.

In the laboratory, the motion of the particle was:

$$
X_2 = X_0 \\
y_2 = Y_0 + vT \\
z_2 = Z_0
$$

which, if the crossing occurs in the vertical plane, becomes:

$$
x_{20} = X_0 \\
y_{20} = Bct + (Z_0 \sin \epsilon + Y_0 \cos \epsilon) \sqrt{1-B^2} \\
z_{20} = Z_0 \cos \epsilon - Y_0 \sin \epsilon.
$$

If $x_1$, $y_1$, $z_1$ are the coordinates along the axes of the bunch, we have:

$$
X_2 = X_1 \\
y_2 = y_1 \cos \epsilon - z_1 \sin \epsilon \\
z_2 = y_1 \sin \epsilon + z_1 \cos \epsilon
$$
and

\[ \delta \rho = \frac{q_1 q_2}{4 \pi \varepsilon_0 B} \int \frac{d\rho}{|x|^3} \frac{r}{4} dy_2. \]

with

\[ d\rho = \frac{N}{(2\pi)^{3/2}} \exp \left[ \frac{x_1^2}{2 \sigma_x^2} \frac{y_1^2}{2 \sigma_y^2} \frac{z_1^2}{2 \sigma_z^2} \right] dx_1 dy_1 dz_1. \]

After integrating with respect to \( y_2 \), we exchange the variables \( x_1 y_1 z_1 \) and \( x_2 y_2 z_2 \) through the change of variable \( A \). After integrating with respect to \( y_2 \), we get:

\[ \delta \rho = \frac{q_1 q_2}{4 \pi \varepsilon_0 B} \frac{N}{\pi \sigma_x \sigma_y \sigma_z} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \exp \left[ \frac{x^2}{2 \sigma_x^2} \frac{z^2}{2 \sigma_z^2} \right] \frac{(x-x_0) dx dz}{(x-x_0)^2 + (z-z_0')^2}. \]

with

\[ \sigma_x' = \cos \varepsilon \sqrt{\sigma_x^2 + t \sigma_y^2} \quad \text{and} \quad Z_0' = \cos \varepsilon \left( Z_0 - \frac{Y_0 \tan \alpha}{\sigma_x} \right). \tag{B} \]

This formula is identical to that for the head-on collision (A-1).

We have \( B = 1 \) and \( \cos \varepsilon = 1 \) so long as \( \alpha < \frac{\pi}{2} \).

So we see that substitution (B) permits calculation of the angular perturbation undergone by the particle.

The longitudinal effect due to the crossing of the beams is examined elsewhere.\(^5\)

In part two of this report these formulae will be applied to calculation of the wave-number distribution due to the space-charge effect.
in the crossing of two beams, and to calculation of the space-charge compensation in a four-beam machine.

SUPPLEMENT

We wish to calculate:

\[ \delta \theta_x = \frac{\varepsilon r_e}{\gamma} \frac{N}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(x_2 - x_1) dy_2}{\left[ (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right]^{3/2}} \]

\[ \delta \theta_z = \delta \theta_x \left[ x_1 \leftrightarrow z_1, x_2 \leftrightarrow z_2, \sigma_x \leftrightarrow \sigma_z \right] \]

with \( x_2 = x_0 \); \( z_2 = z_0 \).

After integrating with respect to \( y_2 \), then \( y_1 \), we get:

\[ \delta \theta_x(X_0, Z_0) = - \frac{\varepsilon N r_e}{\gamma \pi \sigma_x \sigma_z} \int_{-\infty}^{+\infty} \frac{dx dz}{2\sigma_x^2 + 2\sigma_z^2} \frac{(x - x_0) dx dz}{(x - x_0)^2 + (z - z_0)^2} \quad (A-1) \]
By setting \( x - X_0 = \sigma_x \sqrt{2} R \cos \alpha \) and \( z - Z_0 = \sigma_z \sqrt{2} R \sin \alpha \)

with \( 0 < \alpha < \pi \) and \( 0 < R < \infty \), we get:

\[
\delta \theta_x (X_0, Z_0) = -\frac{e \sqrt{2} \sigma_x}{\gamma \sqrt{2\pi}} \int_0^{2\pi} \frac{\cos \alpha \, d\alpha}{\sigma_x^2 \cos^2 \alpha + \sigma_z^2 \sin^2 \alpha} \left[ e^{-b^2} \int_0^{(R+a)^2} e^{-\frac{r^2}{2}} \left( 1 - \text{Erf} (a) \right) \right]
\]

with

\[
a = \frac{X_0}{\sigma_x} \cos \alpha + \frac{Z_0}{\sigma_z} \sin \alpha \quad \text{et} \quad b = -\frac{X_0}{\sigma_x} \sin \alpha + \frac{Z_0}{\sigma_z} \cos \alpha
\]

We have

\[
\int_0^{(R+a)^2} e^{-\frac{r^2}{2}} \left( 1 - \text{Erf} (a) \right) \, dr = \frac{\sqrt{\pi}}{2} \left( 1 - \text{Erf} (a) \right)
\]

with

\[
\text{Erf} (a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-t^2} \, dt
\]

The term \( \frac{\sqrt{2}}{\pi} \int_0^{2\pi} \frac{\cos \alpha \, d\alpha}{\sigma_x^2 \cos^2 \alpha + \sigma_z^2 \sin^2 \alpha} \left[ e^{-b^2} \int_0^{(R+a)^2} e^{-\frac{r^2}{2}} \left( 1 - \text{Erf} (a) \right) \right] \equiv 0 \), for the integrand is odd in \( \alpha + \pi + \alpha \), and finally (by removing the index zero from \( X \) and \( Z \))

we get:

\[
\delta \theta_x (X, Z) = \frac{e \sqrt{2} \sigma_x}{\gamma \sqrt{2\pi}} \int_0^{2\pi} \frac{\cos \alpha \, d\alpha}{\sigma_x^2 \cos^2 \alpha + \sigma_z^2 \sin^2 \alpha} \left[ e^{-b^2} \text{Erf} (a) \right]
\]

We shall write

\[
\text{Erf} (a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-t^2} \, dt = a \int_0^\frac{a^2}{2} e^{-t^2} \, dt \quad \text{et} \quad \int_0^\frac{\pi}{2} e^x \cos x \, dx
\]

or

\[
e^{-b^2} \text{Erf} (a) = \frac{2}{\sqrt{\pi}} e^{-\left( \frac{X^2}{2\sigma_x^2} + \frac{Z^2}{2\sigma_z^2} \right)} \int_0^\infty \sum_{n=0}^\infty \frac{a^{2n+1}}{n!} \cos^{2n+1} x \, dx
\]
By making use of

\[ \cos^{2n+1} x \, dx = \frac{2n \, (n!)^2}{(2n+1)!} \]

we get

\[ \delta \theta_\infty (X, Z) = \frac{\int_{-\pi}^{\pi} e^{-i B(x^2 + \frac{z^2}{2\sigma^2})} dx}{\gamma} \]

By setting \( X = \rho \cos \phi \) and \( Z = \rho \sin \phi \), we have \( a = \rho \cos (\phi - \alpha) \) and

\[ a^{2n+1} = \rho^{2n+1} \frac{1}{\sigma^{2n+1}} \sum_{p=0}^{n} \frac{n!}{(2n+1)!} \frac{(2n+1)\cos(2n + 1 - 2p)(\phi - \alpha)}{2n \, (n!)^2} \]

\[ \cos (2n+1-2p)(\phi - \alpha) = \cos(2n+1-2p)\phi \cos(2n+1-2p)\alpha + \sin(2n+1-2p)\phi \sin (2n+1-2p)\alpha \]

The sine part does not contribute to the integration with respect to \( a \), for it is odd in \( a \rightarrow -a \).

Whence \( \cos (2n+1-2p) a \cos a = \frac{1}{2} \left[ \cos 2(n+1-p)a + \cos 2(n-p)a \right] \).

We make use of

\[ \int_{0}^{2\pi} \frac{\cos 2n \alpha \, da}{\cos^2 \alpha + \sigma_z^2 \sin^2 \alpha} = \frac{2\pi (-1)^n}{\sigma_x \sigma_z} \psi^n \]

with \( \psi = \frac{\sigma_x - \sigma_z}{\sigma_x + \sigma_z} \)

and

\[ \int_{0}^{2\pi} \frac{a^{2n+1} \cos a \, da}{\cos^2 \alpha + \sigma_z^2 \sin^2 \alpha} = \frac{2\pi (-1)^n}{\sigma_x \sigma_z} \frac{\psi^{n-1}}{\sigma_z^2} \sum_{p=0}^{n-1} (-1)^{n-p} \frac{n!}{2n+1} \frac{(2n+1)\cos[(2n+1-2p)\phi]}{\sigma_x \sigma_z} \psi^{n-p} \]
We let:

\[ \delta_{\theta_x}(N^{1/2}(1-\psi) = e^{-\sum_{n=0}^{\infty} \frac{n!}{(2n+1)!} \psi^n \cos((2n+1-2p)\phi)} \] .

We obtain \( \delta_{\theta_z} \) by changing \( \psi \) to \(-\psi\) and \( \cos((2n+1)\phi) \) to 

\[ (-)^p \sin((2n+1)\phi) \), i.e., \( \psi^n \cos((2n+1-2p)\phi) \rightarrow (-)^p \psi^n \sin((2n+1-2p)\phi) . \]

By inverting the sums and making use of 

\[ t^\lambda (1-t)^\mu \frac{dt}{0} = \frac{\lambda ! \mu !}{(\lambda+\mu+1)!} , \]

we find them equal to 

\[ dt \rho \sum_{p=0}^{\infty} \frac{2^p p^p \psi^n \cos((2n+1)\phi)}{p!} \cdot (-)^p \psi^n \cos((2n+1)\phi) . \]

With \( \cos((2n+1)\phi) = \text{Re} \left[ e^{i(2n+1)\phi} \right] \), we get 

\[ \rho \text{Re} \left\{ \int_{0}^{1} dt \exp \left[ \rho^2 \psi \cos((2n+1)\phi) + i \phi \right] \right. \]

We let 

\[ \delta_{\theta_x}(X,Z) = e^{-\sum_{n=0}^{\infty} \frac{n!}{(2n+1)!} \psi^n \cos((2n+1-2p)\phi)} \] .

and 

\[ \delta_{\theta_z}(X,Z) = e^{-\sum_{n=0}^{\infty} \frac{n!}{(2n+1)!} \psi^n \cos((2n+1-2p)\phi)} \] .
or
\[
\delta x + i \delta z = \frac{2eN\sqrt{2}}{\gamma(x + i z)} \int_0^1 dt e^{-\rho^2(1-t)[1+\psi e^{2i\phi}]} e^{i\phi}
\]

or
\[
\delta x + i \delta z = K(x + i z) \int_0^1 dt e^{-(1-t)[x^2+z^2+t\psi(x^2-z^2+2ixz)]}
\]

with \( x = \frac{X_0}{\sqrt{2}\sigma_x} \) and \( z = \frac{Z_0}{\sqrt{2}\sigma_z} \).

References

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