SU_4-SYMMETRY

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Strong-interaction symmetry in a number of cases agrees well with the symmetry of the unitary group SU_3 [1 – 6]. In the present paper we consider the most important consequences of the hypothesis of the extension of the properties of strong-interaction symmetry to the framework of the SU_4 -group. For the boson supermultiplet 1^- this assumption was considered in [7]. The properties of the group of unitary and unimodular SU_4-transformations are determined by the commutation relations for 15 generating operators. The simplest form is obtained by introducing 16 operators a^i_k $(i, k = 1, 2, 3, 4)$, interconnected by the condition $a^i_i = 0$

$$\left[a^i_k a^j_l\right] = \delta^i_l a^j_k - \delta^j_k a^i_l \,. \qquad (1)$$

The operators a^1_1, a^2_2, a^3_3, a^4_4 commute with one another. If we introduce three independent commuting operators

$$I_3 = \frac{1}{2}(a^1_1 - a^2_2), \quad K_3 = \frac{1}{2}(a^3_3 - a^4_4),$$
$$X = \frac{1}{2}(a^1_1 + a^2_2 - a^3_3 - a^4_4), \qquad (2)$$

then their eigenvalues form the simplest system of additive quantum numbers, characterizing the states. The quantum number I_3 may be interpreted as the projection of the isotopic spin, and X can be related to the hypercharge. The charge definition

$$Q = I_3 + \frac{1}{2}X - \frac{1}{4}B \qquad (3)$$

(B being the baryon number) gives the simplest interpretation of the quantum numbers of SU_4, agreeing with the experimental data. We assume that for the third additive quantum number K_3 there is no exact conservation law. Setting

$$I_1 = \frac{1}{2}(a^1_2 + a^2_1),\ I_2 = \frac{i}{2}(a^1_2 - a^2_1),\ K_1 =$$
$$= \frac{1}{2}(a^3_4 + a^4_3),\ K_2 = \frac{i}{2}(a^3_4 - a^4_3),$$

we easily see that the operator triplets I_1, I_2, I_3 and K_1, K_2, K_3 form two independent subgroups of rotations in space. Since all the three operators K_1, K_2, K_3 commute with the operators of the isotopic subgroup and the charge operator, they can all be included among the allowed symmetry-violating operators in the derivation of the mass formula [2, 3, 8]; the fourth allowed operator is X. Four similar allowed operators can be constructed from the bilinear in terms of the a^i_k tensor operator

$$b^i_k = \frac{1}{2}\left(a^j_i a^i_k + a^j_k a^i_j\right) . \qquad (4)$$

Introducing the R-transformation $a^i_k \to -a^k_i$ which preserves equation (1), it is possible to classify the eight allowed operators with respect to R-parity.

The tensors of the representations D (p, q) containing q covariant and p contravariant indices, are reduced with the aid of the unit tensor δ^i_k and the fully antisymmetric pseudoscalar tensors $\epsilon^{ijkl} = \pm 1$ and $\epsilon_{ijkl} = \pm 1$ [4]. Under these operations the parity of $p - q$ does not change. This makes it possible to relate $p - q$ to the baryon number

$$B = q - p \tag{5}$$

and interpret the representation of SU_4 in terms of the composite model: the base representation $D\ (0;\ 1)$ is associated with fermions, the representation $D\ (1;1)$ with bosons. The statistics are not altered upon reduction of the tensors. Relating the more complicated representations* $D\ (2,1;\ 1^4) = D\ (1;\ 1^2)$ and $D\ (3;\ 1^4) = D\ (2;\ 1^3)$ allowed for baryons to the known supermultiplets, the octet and the decuplet, one can obtain a unique selection of the charges for the four states of the base representation: -1, 0, 0, 0. This corresponds to the presence in the base of a doublet with the quantum numbers of the Ξ-hyperon and two singlets of the Λ type.

Dividing up the states among the supermultiplets of the subgroup SU_3 and the multiplets of the isotopic subgroup, we obtain for the representation $D\ (3;\ 1^4)$ with multiplicity 20

$$\left.\begin{array}{llll} 20 = 10 + & 6 + & 3 + & 1 \\ \Delta & & & \\ \Sigma & \Sigma' & & \\ \Xi & \Xi' & \Xi'' & \\ \Omega & \Omega' & \Omega'' & \Omega''' \end{array}\right\} \tag{6}$$

In what follows symbols of known particles are used for denoting states having a definite baryon number, isotopic spin, and hypercharge, irrespective of the spatial spin and the parity. Investigation of symmetry violations predicts for this representation equidistant positions of the particle masses over the rows and columns. The first column can be associated with the known decuplet $\frac{3}{2}^+$. If the resonance Y^*_1 with mass 1660 [9, 10], whose spin is apparently $\frac{3}{2}^+$, is associated with Σ^1, then the nearest observable member of the 20-dimensional family of Ξ should have a mass of about 1810. Assuming a spin $\frac{1}{2}$ for base baryons and writing this representation with a minimum number of indices, $D\ (2;\ 1^3)$, it can be easily seen that the assumption of an S-wave for all the components results in a total spin of $\frac{3}{2}$, in agreement with experiment.

A similar subdivision procedure for another 20-dimensional representation $D\ (2;\ 1;\ 1^4)$ gives

$$\left.\begin{array}{llll} 20 = 8 + & 6 + & \bar{3} + & 3 \\ N & & & \\ \Sigma\Lambda & \Sigma' & \Lambda' & \\ \Xi & \Xi' & \Xi'' & \Xi''' \\ & \Omega & & \Omega' \end{array}\right\} \tag{7}$$

Investigation of symmetry violations leads to the Okubo – Gell-Mann relation $2N + 2\Xi = \Sigma + 3\Lambda$ for the octet and a similar relation for the states N, Σ', Λ', Σ'''. This makes it possible to associate the first column with the baryon octet $\frac{1}{2}^+$. Definite candidates for the 12 remaining states cannot be indicated yet [11]. The states Ξ' and Ξ'' are degenerate with respect to the quantum numbers K_3 and X. Upon perturbation they form a superposition.

Of the other representations for baryons, the 36-dimensional representation $D\ (1^4;\ 3;\ 1^2) = D\ (1;\ 2)$ should be mentioned. It contains the octet SU_3, but the Λ-state of this octet is degenerate and may mix with the Λ-state of the unitary singlet. As a result, Okubo's formula may be violated, and the octet contained in $D\ (1;\ 2)$ probably does not correspond to the observed baryon octet.

* In the notation used a semicolon separates the number of contravariant and covariant indices; the numbers determining permutational symmetry (the length of the rows in Yang's schemes) are separated by commas and the superscript denotes repetition of rows of equal length.

Passing to the description of representations for bosons, we note that the requirement of constant boson masses upon charge conjugation excludes all R-odd operators from the number of possible perturbation operators. The excluded operators include K_3 and X. Subdivision of the unitary multiplets over the mass is determined only by the tensor (4), which does not contribute to some representations. In these cases the subdivision of the representations of SU_4 over the representations of the subgroup SU_3 may be only arbitrary and may serve for the classification of the particles with respect to the quantum numbers but not to the mass.

The regular representation $D\,(1;1)$ contains the following set of states:

$$\begin{array}{ccccccc} 15 = & 8 & + & 1 & + & 3 & + & 3 \\ & \pi\eta & & \eta' & & \lambda & & \overline{\lambda} \\ & KK & & & & K' & & \overline{K'}. \end{array} \qquad (8)$$

The symbols λ and $\overline{\lambda}$, according to the assumption of Tarjanne and Teplitz [7], denote singlets with strangeness $S = 0$, which are not truly neutral particles. The combinations ${}_{+}\lambda = \lambda + \overline{\lambda}$ and ${}_{-}\lambda = \lambda - \overline{\lambda}$ possess a definite charge parity. The state ${}_{+}\lambda$, whose charge parity is equal to the parity of η, η', forms a superposition with them.

Considering mass perturbation by the four even operators

$$K_2,\ \frac{1}{2}(b_1^1 + b_2^2 - b_3^3 - b_4^4),\ \frac{1}{2}(b_3^3 - b_4^4),$$
$$\frac{1}{2}(b_4^3 + b_3^4)$$

with the coefficients a_2, $\beta\chi$, β_3, β_1, we obtain the mass additions $\pi \to \to 2\beta x$, K, $K' \to \pm\sqrt{\frac{1}{4}a_2^2 + \beta_3^2 + \beta_1^2}$, ${}_{-}\lambda \to \to - 2\beta x$. For the three combinations of the singlets η, η', ${}_{+}\lambda$ the mass correction is determined by the characteristic equation

$$\begin{vmatrix} -2\beta_x - \mu & -i\alpha_2 & \sqrt{2}\beta_1 \\ i\alpha_2 & -2\beta_x - \mu & \sqrt{2}\beta_3 \\ \sqrt{2}\beta_1 & \sqrt{2}\beta_3 & -\mu \end{vmatrix} = 0 \qquad (9)$$

For the unitary multiplet 0^- (π, K, η) there are yet no clear candidates for the role of K', η', ${}_{+}\lambda$, ${}_{-}\lambda$. Probably, it would be advisable to investigate the possible type of electromagnetic decay of the hypothetic strange particle $K' \to K + 2\gamma$ with spin 0^-.

If the nine bosons ρ, ω, K^*, φ with spin 1^- are complemented by the resonance κ [12, 13] with a mass of 725, then the symmetry scheme with the representation $D\,(1;1)$ predicts the appearance of two additional resonances ${}_{+}\lambda$ ($J^{PG} = 1^{--}$) and ${}_{-}\lambda$ ($J^{PG} = 1^{-+}$), with a mass of about 930. We note that the resonances ρ, ω, K^*, κ, φ fit into the mass formula only if the possible perturbation operators are complemented by the simplest operator with $I = K = 0$ of the 84-dimensional representation $D\,(2;2)$ (the analog of the 27-dimensional one in SU_3). The question of whether the resonance in the system ($\pi^+\pi^-\eta$) with a mass of 959 belongs to this family is still difficult to solve [14]. There are indications [15, 16] that a $\pi^+\pi^-$ resonance exists in the region of the expected ${}_{-}\lambda$.

After the regular representation, the simplest representation for bosons is the 20-dimensional one $D\,(1^2;1^2) = D\,(2^2;1^4)$. Subdividing it into multiplets, we obtain

$$\begin{array}{cccccc} 20 = & 8 & + & 6 & + & \overline{6} \\ & \pi\eta & & \pi' & & \pi'' \\ & KK & & \overline{K'} & & K' \\ & & & \xi & & \overline{\xi}, \end{array}$$

where the symbol ξ is introduced for bosons with strangeness ± 2 and isotopic spin 0. The triplets π' and π'' form the combinations $\pi =$

$= \pi' + \pi''$ and $_{-}\pi = \pi' - \pi''$ with different charge parity, and $_{+}\pi$ forms a superposition with π. Of the four allowed perturbation operators, only K_2 gives a nonzero result. The triplets π and $_{+}\pi$ form superpositions with masses $m_0 \pm a_2$, the doublets K, K' give $m_0 \pm \frac{1}{2}\, a_2$; the masses of the remaining particles are equal to m_0, if there are no more complicated perturbations.

The two $\pi\rho$ resonances A_1 and A_2 with masses 1080 and 1320 [17] can be formed as a superposition of π and $_{+}\pi$; the resonance B [18] in the $\pi\omega$-system with a mass of 1220 is a good candidate for the role of $_{-}\pi$. In this case the charge parity of the η-type singlet is positive and, if $J^P = 2^+$ for the resonances A_1, A_2, and B, then the f_0 resonance with a mass of 1250 may be a singlet. The two strange particles with a mass difference of about 120 and an average mass of 1200 – 1250 may be resonances of the $K\pi$ and $K\pi\pi$ type. The observed $K\pi\pi$ resonances [19] – [21] are possibly connected with these particles, but in this case the spin should be 1^+ or 2^-, instead of 2^+, and the inclusion of f_0 in the family is forbidden. However, SU_4-symmetry allows such a choice of symmetry-violating perturbations, for which one of the strange particles of this representation has mainly the decays (K) and ($K\pi\pi$), and the other ($K\pi$) even in the case of spin 2^+. Existing measurements of the spin of the B resonance [22, 23] do not yet exclude the value 2^+. The strange particle should show up as a resonance in the system K^+K^0 or $KK\pi$.

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