Symmetry Breaking through Higgs Mechanism in $SU(6)$ GUT

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2007

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a thesis
submitted in partial fulfillment
of the requirements for the degree

Master of Science

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2007

Letter of Approval

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Salemba, 1 February 2007
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Preface

Elementary particle physics has made remarkable progress in the past ten years. We now have, for the first time, a comprehensive theory of particle interactions. One can argue that it gives a complete and correct description of all non-gravitational physics. This theory is based on the principle of gauge symmetry. Strong, weak, and electromagnetic interactions are all gauge interactions. The importance of a knowledge of gauge theory to anyone interested in modern high energy physics can scarcely be overstated. Regardless of the ultimate correctness of every detail of this theory, it is the framework within which new theoretical and experimental advances will be interpreted in the foreseeabie future.

Quantum field theory is a set of ideas and tools that combines three of the major themes of modern physics: the quantum theory, the field concept, and the principal of relativity. Today most working physicists need to know some quantum field theory, and many others are curious about it. The theory underlies modern elementary particle physics, and supplies essential tools to nuclear physics, atomic physics, condensed matter physics, and astrophysics. In addition, quantum field theory has led to new bridges between physics and mathematics.

It is a pleasure to acknowledge the aid I have received from my family, my colleagues and friends in my group. I am very grateful to Mr. Handoko and Mr. Andreas for fruitful discussion, and also to Mr. Terry, Mr. Anto, Mr. Agus, and Mr. Imam for having encouraged and for helping me to finish my thesis. I also would like to thank to Nowo, Bayu, and many other friends in my group that I could not tell one by one. Finally, I also gratefully acknowledge the encouragement and help given by my family in PALABS.

Depok, February 2007

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Abstract

We impose the Higgs mechanism to break the Grand Unified Theory based on $SU(6)$ symmetry, $SU(6)$ GUT. We investigate and search for the Higgs multiplets which are approriate to generate masses for both fermions and gauge bosons. We have found that the most minimal Higgs multiplets are $< \Phi^{15} >, < \Phi^{20} >$ and $< \Phi^{21} >$ to realize three steps of symmetry breaking in $SU(6)$ GUT down to the standar $SU(2) \times U(1)$ model. However within this minimal $SU(6)$ GUT, we can reproduce only some fermions spectra as $N_{4e}, \nu_e, N_{6e}, N_e$ and d quarks, although we have deployed the most general forms of Higgs multiplets. On the other hand, the present approach have succeeded in generating the reguler gauge bosons, $W^{\pm}, Z^0$ and A_μ.

Contents

Chapter 1

Introduction

Nowadays, all phenomenons in the high energy physics have been explained within the standard model (SM) which is a gauge theory based on $SU(3)_{\rm C} \otimes SU(2)_{\rm L} \otimes U(1)_{\rm Y}$ symmetry [1]. This set of symmetry represent strong, weak and electromagnetic interactions in a universal framework. In contrast to the weak and electromagnetic interactions which have been successfully unified in the electroweak theory based on $SU(2)_{\rm L} \otimes U(1)_{\rm Y}$ symmetry, the strong interaction with $SU(3)_{\rm C}$ symmetry remains independent from the others.

So far, the electroweak is in impressive agreement with the most of experimental observables [2]. However, there are recently several experimental results which disagree with the SM's predictions, as the oscillation in the neutrino sector [3] and the discrepancy in the NuTeV measurement [4]. There are also undergoing or forthcoming experiments to measure the double β decay [5], to search the Higgs particle(s) needed to break the symmetry [6], to relate the high energy phenomenon with the cosmology one and so on. All of them have been expected to be able to distinguish some physics beyond the SM. As mentioned above, the SM is lacking of explaining the unification of three gauge couplings at a particular scale, especially under an assumption that our nature should be explained by a single unified theory, the so called grand unified theory (GUT).

In order to realize GUT at some scale, most of works in the last decades have dealed with gauge theory inspired by the successfull electroweak theory. Those theories assumed the gauge invariance under particular symmetries larger than the SM's one, but contain $SU(3)_{\rm C} \otimes SU(2)_{\rm L} \otimes U(1)_{\rm Y}$ as a part of its subgroups at electroweak scale. One of these models is $SU(6)$ GUT [7].

As a new model, there is still a lot of work to be done. One of these work is how this model can reproduce masses fermions and gauge bosons as in SM and also can have very heavy masses for some new exotic fermions that introduce in this model. With some new exotic fermion, this model is also expected to explain some experiments that can not be explained with SM . To construct such masses we use Higgs Mechanism.

1.1 An example

Now we are going to give a brief history about the arising of Higgs mechanism. For the sake of simplicity, let us consider the case of the Abelian gauge theory.

Consider a complex scalar field coupled both to itself and to an electromagnetic field:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\phi|^2 - V(\phi), \tag{1.1}$$

with $D_\mu = \partial_m u + ieA_\mu$. This Lagrangian is invariant under the local $U(1)$ transformation

$$\phi(x) \to e^{i\alpha(x)}\phi(x), \quad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x). \tag{1.2}$$

If we choose the potential in $\mathcal{L}$ to be of the form

$$V(\phi) = -\mu^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2, \tag{1.3}$$

with $\mu^2 > 0$, the field ϕ will acquire a vacuum expectation value and the $U(1)$ global symmetry will be spontaneously broken. The minimum of this potential occurs at

$$\langle\phi\rangle = \phi_0 = \left(\frac{\mu^2}{\lambda}\right)^{1/2}, \tag{1.4}$$

or at any other value related by the $U(1)$ symmetry (1.2).

Let us expand the Lagrangian (1.1) about the vacuum state (1.4). Decompose the complex field $\phi(x)$ as

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x)). \tag{1.5}$$

The potential (1.3) is rewritten

$$V(\phi) = -\frac{1}{2\lambda}\mu^4 + \frac{1}{2}\cdot 2\mu^2\phi_1^2 + \mathcal{O}(\phi_i^3), \tag{1.6}$$

so that the field ϕ_1 acquires the mass $m\sqrt{2}\mu$ and ϕ_2 is the massless Goldstone boson.

Now consider how the kinetic energy term of ϕ is transformed. Inserting the expansion (1.5), we rewrite

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \sqrt{2}e\phi_0 \cdot A_\mu\partial^\mu\phi_2 + e^2\phi_0^2 A_\mu A^\mu + \cdots, \tag{1.7}$$

where we have omitted terms cubic and quartic in the fields A_μ, ϕ_1, and ϕ_2. The last term written explicitly in (1.7) is a photon mass term

$$\Delta\mathcal{L} = \frac{1}{2}m_A^2 A_\mu A^\mu, \tag{1.8}$$

where the mass

$$m_A^2 = 2e^2\phi_0^2 \tag{1.9}$$

arises from the nonvanishing vacuum expectation value of ϕ. Notice that the sign of this mass term is correct; the physical spacelike components of A_μ appear in (1.8) as

$$\Delta\mathcal{L} = -\frac{1}{2}m_A^2(A^i)^2, \tag{1.10}$$

with the correct sign for a potential energy term.

A model with a spontaneously broken continuous symmetry must have massless Goldstone bosons. These scalar particles have the quantum numbers of the symmetry currents, and therefore have just the right quantum numbers to appear as intermediate states in the vacuum polarization. In the model we are now discussing, we can see this pole arise explicitly in the following way: The third term in Eq. (1.7) couples the gauge boson directly to the Goldstone boson ϕ_2; this gives a vertex of the form

$$i\sqrt{2}e\phi_0(-ik^\mu) = m_A k^\mu. \tag{1.11}$$

If we also treat the mass term (1.8) as a vertex in perturbation theory, then the leading-order contributions to the vacuum polarization amplitude give the expression

$$\begin{aligned} &= im_A^2 g^{\mu\nu} + (m_A k^\mu)\frac{i}{k^2}(-m_A k^\nu) \qquad (1.12)\\ &= im_A^2(g^{\mu\nu}) - \frac{k^\mu k^\nu}{k^2} \end{aligned}$$

The Goldstone boson supplies exactly the right pole to make the vacuum polarization amplitude properly transverse.

Although the Goldstone boson plays an important formal role in this theory, it does not appear as an independent physical particle. The easiest way to see this is to make a particular choice of gauge, called the *unitarity gauge*. Using the local $U(1)$ gauge symmetry (1.2), we can choose $\alpha(x)$ in such a way that $\phi(x)$ becomes real-valued at every point x. With this choice, the field ϕ_2 is removed from the theory. The Lagrangian (1.1) becomes

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (\partial_\mu\phi)^2 + e^2\phi^2 A_\mu A^\mu - V(\phi). \tag{1.13}$$

If the potential $V(\phi)$ favors a nonzero vacuum expectation value of ϕ, the gauge field acquires a mass; it also retains a coupling to the remaining, physical field ϕ_1.

This mechanism, by which spontaneously symmetry breaking generates a mass for a gauge boson, was explored and generalized to the non-Abelian case by Higgs, Kibble, Guralnik, Hagen, Brout, and Englert, and is now known as the *Higgs mechanism*.

Chapter 2

$SU(6)$ GUT

In this chapter we would like to give some brief to the new model, $SU(6)$ GUT [7]. As an introduction to this model, first, we present the pattern of symmetry breaking that is used in $SU(6)$ GUT [7]. Based on these patterns, the basic of $SU(6)$ group and its generators are then given in section 2.2. Before presenting the extended Gell-Mann Okubo relation, we perform a detail study of the quantum numbers contained in the model in section 2.3. Finally, in the last section we present a new particle assignment in the $SU(6)$ multiplets.

2.1 Pattern of Symmetry Breaking

First of all, determining the pattern of symmetry breaking in a GUT model is a crucial step. In the case of $SU(6)$ group, concerning only the sub-matrices of its generators, intuitively there are several possibilities to break the symmetry, for example

$$SU(6) \to \begin{cases} SU(5) \otimes U(1) \\ SU(2) \otimes SU(2) \otimes SU(2) \otimes U(1) \otimes U(1) \\ SU(3) \otimes SU(3) \otimes U(1) \end{cases} \tag{2.1}$$

The first choice is clearly similar to the known $SU(5)$ GUT where it is followed by the breaking pattern of $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$ to obtain the SM. This breaking pattern has been introduced by [10], however this is not much preferred due to too fast proton decay. On the other hand, the second example can be excluded since it is not able to accomodate the SM. Then the last one is the only pattern we should choice and it has actually not been studied so far.

At this present stage, we can straightforward put the first $SU(3)$ as $SU(3)_{\mathrm{C}}$ representing the strong interaction, while the second one should break further to $SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}}$ to reproduce the electroweak theory. So, there are two stages to break $SU(6)$ down to the electroweak scale,

$$\begin{aligned} SU(6) &\rightarrow SU(3)_{\rm C} \otimes SU(3)_{\rm H} \otimes U(1)_{\rm C} \\ &\rightarrow SU(3)_{\rm C} \otimes SU(2)_{\rm L} \otimes U(1)_{\rm B} \otimes U(1)_{\rm C} \ , \end{aligned} \tag{2.2}$$

where H denotes a new quantum number which we later on call as hyper-isospin. The combination of quantum numbers induced by $U(1)_{\rm B}$ and $U(1)_{\rm C}$ will reproduce the familiar hypercharge associated with $U(1)_{\rm Y}$ in the electroweak theory.

Next, we should consider the fundamental representation and the minimal multiplets to accomodate the particle contents. The fundamental representations of $SU(6)$ group is represented as $\{6\}$ and its anti-symmetric $\{\overline{6}\}$. A tensor product of two fundamental representations gives, $\{6\} \otimes \{6\} = \{21\} \oplus \{\overline{15}\}$. Following the general requirement for the anomaly free combination of representations of fermions in any particular $SU(N)$ group [12], one should choose the combination of $2\{6\} \oplus \{\overline{15}\}$ in the case of $SU(6)$. The second $\{6\}$–dimensional representation comes up from the decomposition of $\{21\}$ in the above tensor product. Therefore we can conclude here that the fermions must be assigned in these multiplets, namely sextet ($\{6\}$) and decapentuplet ($\{15\}$).

2.2 $SU(6)$ Group

In this section, we construct the generators for $SU(6)$ group. In general, the generators for $SU(N)$ group can be determined using the existing generators of $SU(N-1)$ group and expanding its $(N-1) \times (N-1)$ matrices [13]. Then there are three considerable types of matrices which could form an $SU(N)$ group,

$$\lambda_i = \begin{cases} \left(\begin{array}{ccc|c} & & & 0 \\ & \widetilde{\lambda}_i & & \vdots \\ & & & 0 \\ \hline 0 & \cdots & 0 & 0 \end{array} \right) & , \ \text{for } i = 1, 2, \cdots, (N-1)^2 - 1 \\ \left(\begin{array}{c|c} & 0 \\ & \vdots \\ (0)_{(N-1)\times(N-1)} & a_{jN} \\ & \vdots \\ & 0 \\ \hline 0 \cdots a_{jN} \cdots 0 & 0 \end{array} \right) & , \ \text{for } (N-1)^2 - 1 < i < N^2 - 1 \\ \lambda_{N^2-1} & , \ \text{for } i = N^2 - 1 \end{cases} , \tag{2.3}$$

where $\widetilde{\lambda}_i$ is the i-th generator of $SU(N-1)$ group and $a_{jN} = a^*_{Nj} = 1$ or $-i$ with $j = 1, 2, \cdots, N-1$. This confirms that the total number of generators in an $SU(N)$ group equals to $[(N-1)^2 - 1] + [2 \times (N-1)] + 1 = N^2 - 1$. Note that special (physical) consideration must be taken to determine the last generator, $i, e.$ λ_{N^2-1} beside the basic mathematical requirement $\mathrm{tr}(\lambda_i \lambda_j) = 2\delta_{ij}$. Of course one should remark that the order of numbering the generators can be changed for the sake of convenience due to some physical considerations as discussed soon. Now we are ready to move forward to the case of $SU(6)$ group.

Throughout the thesis we use the notation $\overline{\lambda}_i$ to indicate the generators of $SU(3)$ (Gell-Mann matrices [15]), $\widetilde{\lambda}_i$ for $SU(5)$, λ_i for the $SU(6)$ and $\sigma_{1,2,3}$ for the Pauli matrices. We start from the well-known generators of $SU(5)$ [14]. It is considerable to bring the $\widetilde{\lambda}_{1,\cdots,20}$ as they are and extend them to be $\lambda_{1,\cdots,20}$ by adding the 6-th rows and columns with null elements. This implies that the color quantum number is preserved as the conventional quantum chromodynamics (QCD), *i.e.* the upper left 3×3 block still represents the $SU(3)_C$ symmetry. Since the last generator should form the Cartan sub-algebra which determines the (physically meaningfull) eigenvalues, that is having non-zero diagonal elements, we eliminated the $\widetilde{\lambda}_{24}$. Instead of that we put $\lambda_{21,\cdots,26}$ as the type of extended σ_1 and σ_2 matrices filling in the upper-right and lower-left 3×3 blocks.

Further, $\widetilde{\lambda}_{21,22,23}$ are kept and extended to be $\lambda_{27,28,29}$ to represent the $SU(3)_H$ group after the first step of symmetry breaking in Eq.(2). The extended σ_1 and σ_2 types with its non-zero elements filling the last rows and columns in the lower-right 3×3 block form $\lambda_{30,31,32,33}$.

Since $SU(6)$ group is a rank 5 group, it should have five generators form its Cartan sub-algebra. In a more technical term, there must be five generators with non-zero diagonal elements. Since we already have three of them ($\lambda_{3,8,29}$), therefore we should define the remaining two diagonal generators. From the fact that $\lambda_{27,\cdots,33}$ have the same form as the extended $\overline{\lambda}_{1,\cdots,7}$ of $SU(3)$, it is then appropriate to choose λ_{34} as the extended form of $\overline{\lambda}_8$.

As mentioned earlier, we must take physical considerations to determine the remaining λ_{35}. Concerning the first step of symmetry breaking in Eq.(2), λ_{35} should reflect the quantum number of $U(1)_C$ and be independent from both $SU(3)_C$ and $SU(3)_H$. It yields that,

$$\lambda_{35} = \frac{1}{\sqrt{3}} \left(\begin{array}{c|c} (-1)_{3\times 3} & (0)_{3\times 3} \\ \hline (0)_{3\times 3} & (1)_{3\times 3} \end{array} \right) . \tag{2.4}$$

Finally, the generators for $SU(6)$ group can be defined in a common way using these matrices as follows,

$$F_i = \frac{1}{2}\lambda_i, \quad i = 1, \cdots, 35 \; , \tag{2.5}$$

which satisfies the relation $[F_i, F_j] = i f_{ijk} F_k$ with f_{ijk} is the structure constant respectively. Complete expressions for all matrices are given in the appendix.

Before going on to the next section, we would like to make several remarks here,

- The Gell-Mann like matrices $\lambda_{27,\cdots,34}$ with non-zero elements in the lower-right 3×3 block represents the $SU(3)_H$ at the first symmetry breaking. This generates a new quantum number namely hyper-isospin.
- Since $SU(6)$ contains $SU(5)$ as its sub-group, $\widetilde{\lambda}_{24}$ should be able to be contained in a 6×6 matrix which is the linear combination of λ_{34} and λ_{35}, *i.e.*,

$$c_{34}\lambda_{34} + c_{35}\lambda_{35} = \frac{2}{\sqrt{15}} \left(\begin{array}{ccccc|c} 1 & & & & & 0 \\ & 1 & & & & \\ & & 1 & & & \vdots \\ & & & -\frac{3}{2} & & \\ & & & & -\frac{3}{2} & 0 \\ \hline 0 & & \cdots & & 0 & 0 \end{array} \right) = \left(\begin{array}{c|c} \widetilde{\lambda}_{24} & 0 \\ \hline 0 & 0 \end{array} \right) \ , \quad (2.6)$$

  where the multiplication factors are choosed to be $c_{34} = -1/\sqrt{5}$ and $c_{35} = -2/\sqrt{5}$.
- λ_{35} represents the hypercharges exist in the strong and weak forces with opposite signs. This reflects the property of its short and long range interactions. We label this kind of hypercharge as $C-$hypercharge.
- On the other hand, the hypercharge induced by λ_{34} exists only in the weak sector. We label it as $B-$hypercharge.

2.3 Quantum Number

Since the $SU(3)_C$ symmetry is kept till the low energy scale, in the sense of quantum number there is no new physical consequence on it. Then, let us focus on the generators form $SU(3)_H$ relevant for the electroweak interaction. We should reconsider the Gell-Mann Okubo relation which has been well established within the SM. This relation constitutes that the isospin and hypercharge are the constituents of charge, *i.e.* $Q = I_3 + \frac{1}{2}Y$. In the present framework we have several new hypercharges as mentioned in the preceeding section. This motivates us to consider the extended Gell-Mann Okubo relation.

The $B-$hypercharge induced by λ_{34} has non-identical hypercharges, *i.e.* (1 1 -2), in contrast with the identical C-hypercharge, (1 1 1). If the total hypercharge Y is defined as,

$$Y \equiv Y_B + Y_C \ , \quad (2.7)$$

we obtain non-identical hypercharge and isospin configurations for the doublets which are possibly built from $SU(3)_H$ triplet. This strange behaviour can be

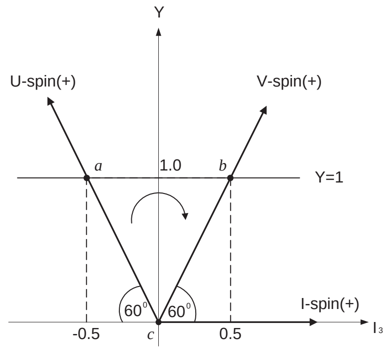

Figure 2.1: $I-$, $U-$ and $V-$ spin on the $I_3 - Y$ plane for a, b and c in the $SU(3)_H$ triplet.

explained by introducing the $U-$, $V-$ spins beside the conventional $I-$spin. If a, b, c denote the first, second and third elements in the $SU(3)_H$ triplet, there are three combinations of doublet respectively,

$$I - \text{spin} \ : \begin{pmatrix} b \\ a \end{pmatrix} \ , \ U - \text{spin} \ : \begin{pmatrix} a \\ c \end{pmatrix} \ , \ V - \text{spin} \ : \begin{pmatrix} c \\ b \end{pmatrix} \ . \tag{2.8}$$

These combinations can be illustrated on the $I_3 - Y$ plane as shown in Fig. 1. note that the conventional isospin I_3 is determined by λ_{29}.

From these results, we have found that the third component of hyper-isospin I_H is related with isospin and hypercharge as follows,

$$I_{H_3} = I_3 + (\Delta I_3 \cdot \Delta Y) \ , \tag{2.9}$$

where the delta means the difference between the upper and lower elements in each doublet (see Eq.(8)). Secondly, the charge for each element can be derived using this hyper-isospin and the total hypercharge,

$$\begin{aligned} Q &= I_{H_3} + \frac{1}{2} \qquad (2.10) \\ &= I_3 + [\Delta I_3 \cdot (\Delta Y_B + \Delta Y_C)] + \frac{1}{2}(Y_B + Y_C) \ , \end{aligned}$$

using Eqs.(7) and (9). This is the extended Gell-Mann Okubo relation in the framework of $SU(6)$ GUT under consideration.

2.4 Particle Assignment

With the extended Gell-Mann Okubo relation at hand, we are now ready to go on assigning the particle contents appropriately. As mentioned earlier we must fill all fermions in the combining of the $\{6\}-$ and $\{\overline{15}\}-$plets.

Taking into account the quantum numbers (charge, isospin and hypercharge) defined above, we should take,

$$(\Psi^6_1)^i_R = \begin{pmatrix} d^i_1 \\ d^i_2 \\ d^i_3 \\ (\nu_{l^i})^C \\ (l^i)^+ \\ N_{l^i} \end{pmatrix}_R \quad \text{and} \quad (\Psi^6_2)^i_R = \begin{pmatrix} D_{1d^i} \\ D_{2d^i} \\ D_{3d^i} \\ (N_{4l^i})^C \\ (E_{5l^i})^+ \\ N_{6l^i} \end{pmatrix}_R . \tag{2.11}$$

for the sextet, while the $\{\overline{15}\}$−plet should consist of,

$$(\Psi^{15})^i_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & (u^i_3)^C & -(u^i_2)^C & -u^i_1 & -d^i_1 & -D_{7d^i} \\ -(u^i_3)^C & 0 & (u^i_1)^C & -u^i_2 & -d^i_2 & -D_{8d^i} \\ (u^i_2)^C & -(u^i_1)^C & 0 & -u^i_3 & -d^i_3 & -D_{9d^i} \\ u^i_1 & u^i_2 & u^i_3 & 0 & (l^i)^+ & -E_{10l^i} \\ d^i_1 & d^i_2 & d^i_3 & -(l^i)^+ & 0 & N_{11l^i} \\ D_{7d^i} & D_{8d^i} & D_{9d^i} & E_{10l^i} & -N_{11l^i} & 0 \end{pmatrix}_L . \tag{2.12}$$

where $u^i : u, c, t$; $d^i : d, s, b$; $l^i : e, \mu, \tau$; $N_{l^i} : N_e, N_\mu, N_\tau$; $D_{d^i} : D_d, D_s, D_b$ and $1, 2, 3$ denote the colors respectively. N_l's and D_d's are newly introduced fermions with neutral charges and $-1/3$. L and R are the projection operators, $L \equiv \frac{1}{2}(1-\gamma_5)$ and $R \equiv \frac{1}{2}(1+\gamma_5)$.

We should make few remarks here. First, we assign different fermions for two sextets required to avoid the anomaly. Secondly, it is clear that this model on its own implies the existence of a new neutral fermion, N_l, to complete its multiplets. This exotic fermion then could be interpreted as the heavy Majorana neutrino to enable the seesaw mechanism naturally. Lastly, this is clearly the minimal particle assignment in the present model, *i.e.* the minimal $SU(6)$ GUT. One could also take other possibilities by introducing more exotic fermions as done in [10].

Chapter 3

Symmetry Breaking

In this chapter we review briefly the Higgs Mechanism to break the local symmetry in the theor. As thee results of symmetry breaking, the fermion and also the gauge boson masses are generated. In the elementary particle physics, this kind of mechanism has suceeded in realizing the spontaneous symmetry breaking in the SM of electromagnetic and weak interactions. Now, we follow the The same procedure and mechanism to break the $SU(6)$ GUT.

3.1 Higgs Mechanism in SM

Spontaneous breaking of gauge symmetries was the crucial new ingredient in the model of unified weak and electromagnetic interactions constructed independently by Weinberg and Salam. The general idea was that weak interactions should be mediated by gauge bosons ($W^{\pm}$), which are, 'to begin with', massless. The Lagrangian for the theory also contains terms for massless electrons, muons, and neutrinos, and is invariant under an internal symmetry group, which is a gauge symmetry. A scalar field (the Higgs field) is then introduced with a non-vanishing vacuum-expectation-value. The resulting spontaneous breakdown of symmetry gives masses to e, μ, and τ and to the gauge bosons, but not to the photon and neutrino. It is therefore indeed met with a good degree of success in describing weak interactions.

We begin with a theory with $SU(2)$ gauge symmetry. To break the symmetry spontaneously, we introduce a scalar field in the spinor representation of $SU(2)$. However, we know that this theory leads to a system with no massless gauge bosons. We therefore introduce an additional $U(1)$ gauge symmetry. We assign the scalar field a charge $+1/2$ under this $U(1)$ symmetry, so that its complete gauge transformation is

$$\phi \to e^{i\alpha^a \tau^a} e^{i\beta/2} \phi. \tag{3.1}$$

(Here $\tau^a = \sigma^a/2$.) If the field ϕ acquires a vacuum expectation value of the form

$$< \phi >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{3.2}$$

then a gauge transformation with

$$\alpha^1 = \alpha^2 = 0, \quad \alpha^3 = \beta \tag{3.3}$$

leaves $< \phi >$ invariant. Thus the theory will contain one massless gauge boson, corresponding to this particular combination of generators. The remaining three gauge bosons will acquire masses from the Higgs mechanism.

3.1.1 Gauge Boson Masses

In order to generate the mass spectrums using the Higgs mechanism, we consider the covariant derivative of ϕ appears in the kinetic term of Higgs particle, that is

$$D_\mu \phi = (\partial_\mu - igA^a_\mu \tau^a - i\frac{1}{2}g' B_\mu)\phi, \tag{3.4}$$

where A^a_μ and B_μ are, respectively, the $SU(2)$ and $U(1)$ gauge bosons. Since the factors associated with the $SU(2)$ and $U(1)$ gauge groups commute each other, they may generally have different coupling constants, namely g and g'.

The gauge boson mass terms come from the square of Eq.(3.4), evaluated at the scalar field vacuum expectation value (3.2). The relevant terms are

$$\Delta\mathcal{L} = \frac{1}{2}(0 \quad v) \left(gA^a_\mu \tau^a + \frac{1}{2}g' B_\mu \right) \left(gA^{b\mu} \tau^b + \frac{1}{2}g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{3.5}$$

If we evaluate the matrix product explicitly, using $\tau^a = \sigma^a/2$, we find

$$\Delta\mathcal{L} = \frac{1}{2}\frac{v^2}{4}[g^2(A^1_\mu)^2 + g^2(A^2_\mu)^2 + (-gA^3_\mu + g' B_\mu)^2]. \tag{3.6}$$

There are three massive vector bosons, which we notate as follows:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp A^2_\mu) \quad \text{with mass} \quad m_W = g\frac{v}{2};$$
$$Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(gA^3_\mu - g' B_\mu) \quad \text{with mass} \quad m_Z = \sqrt{g^2 + g'^2}\frac{v}{2}. \tag{3.7}$$

The fourth vector vield, orthogonal to Z^0_μ, remains massless:

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g' A^3_\mu + gB_\mu) \quad \text{with mass} \quad m_A = 0. \tag{3.8}$$

We then identify this field as the electromagnetic vector potential.

3.1.2 Fermion Mass Terms

We now look at the problem of generating the mass terms for fermions, i.e. quarks and leptons. One cannot put ordinary mass terms into the Lagrangian, because the left- and right-handed components of the various fermion fields have different gauge quantum numbers and so simple mass terms violate gauge invariance. To give masses to the quarks and leptons, we must again invoke the mechanism of spontaneous symmetry breaking.

In the SM, the right-handed fermions are assigned as singlets: u^i_R, d^i_R, l^i_R; while the left-handed fermions are assigned as doublets:

$$L_L = \begin{pmatrix} \nu_{l^i} \\ l^i \end{pmatrix}_L , \quad Q_L = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L .$$

As the Lagrangian of mass terms:

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -m\overline{\Psi}\Psi \\ &= -m(\overline{\Psi}_L\Psi_R + \overline{\Psi}_L\Psi_R), \end{aligned} \tag{3.9}$$

we can not construct the mass term. This problem is solved by introducing Higgs particle which we added to the Lagrangian, so that

$$\mathcal{L}_{\text{mass}} = -\lambda\overline{\Psi}\Phi\Psi, \tag{3.10}$$

the parameter λ is a new dimensionless coupling constant. We can write mass terms for the leptons:

$$\Delta\mathcal{L}_l = -\lambda_l \overline{L}^i_L \Phi l^i_R + \text{h.c.}, \tag{3.11}$$

(note that all leptons can have mass except neutrino, because in SM there is no right-handed neutrino). If we replace ϕ in this expression by its vacuum expectation value (3.2), we obtain

$$\Delta\mathcal{L}_e = -\frac{1}{\sqrt{2}}\lambda_e v \overline{e}_L e_R + \text{h.c.} + \cdots \tag{3.12}$$

This is a mass term for the electron. The size of the mass is set by the vacuum expectation value of ϕ, rescaled by the new dimensionless coupling:

$$m_e = \frac{1}{\sqrt{2}}\lambda_e v. \tag{3.13}$$

We can write mass terms for the quarks fields in the same way

$$\Delta\mathcal{L}_q = -\lambda_d \overline{Q}^i_L \Phi d^i_R - \lambda_u \epsilon^{ab} \overline{Q}^i_{La} \Phi^\dagger_b u^i_R + \text{h.c.} \tag{3.14}$$

Substituting the vacuum expectation value of ϕ from Eq.(3.2), these terms become

$$\Delta\mathcal{L}_q = -\frac{1}{\sqrt{2}}\lambda_d v \overline{d}_L d_R - \frac{1}{\sqrt{2}}\lambda_u v \overline{u}_L u_R + \text{h.c.} + \cdots , \tag{3.15}$$

standard mass terms for the d and u quarks. The GWS theory thus gives the relations

$$m_d = \frac{1}{\sqrt{2}}\lambda_d v, \quad m_u = \frac{1}{\sqrt{2}}\lambda_u v. \tag{3.16}$$

As with the electron, the theory parametrizes but does not explain the small values of the d and u quark masses observed experimentally.

3.2 Higgs Mechanism in $SU(6)$ GUT

Now we are using the Higgs Mechanism to the $SU(6)$ GUT as we use it in SM. In $SU(6)$ GUT there are three steps of symmetry breaking:

$$\begin{aligned} SU(6) &\rightarrow SU(3)_{\mathrm{C}} \otimes SU(3)_{\mathrm{H}} \otimes U(1)_{\mathrm{C}} \\ &\rightarrow SU(3)_{\mathrm{C}} \otimes SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{B}} \otimes U(1)_{\mathrm{C}} \\ &\rightarrow SU(3)_{\mathrm{C}} \otimes U(1)_{\mathrm{em}} \ , \end{aligned} \tag{3.17}$$

so we need three types of Higgs particles. What kind of Higgs particle that will be appropriate to break these symmetries ? The answer can be found by the help of group theory.

Let us see again the Lagrangian for fermion mass term with Higgs particle inside

$$\mathcal{L}_{\mathrm{mass}} = -f^{ij}\overline{\Psi}^i_L \Phi^{ij} \Psi^j_R + \mathrm{h.c.}, \tag{3.18}$$

Since in the $SU(6)$ GUT there are two kind of particle assignment, $\{6\}-$plets and $\{15\}-$plets, so we have $\{6\}$ and $\{15\}$ irrep according to the group theory. From these irrep, we can do the combination of direct product to have

$$\begin{aligned} \{6\} \times \{6\} &= \{15\} + \{21\} \\ \{6\} \times \{\overline{6}\} &= \{1\} + \{35\} \\ \{\overline{6}\} \times \{\overline{6}\} &= \text{none} \\ \{6\} \times \{15\} = \{6\} \times \{\overline{15}\} &= \{20\} + \{70\} \\ \{\overline{6}\} \times \{15\} = \{\overline{6}\} \times \{\overline{15}\} &= \text{none} \\ \{15\} \times \{15\} = \{15\} \times \{\overline{15}\} = \{\overline{15}\} \times \{\overline{15}\} &= \{15\} + 2\{105\}. \end{aligned} \tag{3.19}$$

We have to choose three of these kind of possibilities as the number of degree of freedom of Higgs particle. How do we choose them? There is no exact way to choose the appropriate kind of Higgs particles. After some trial, we choose Higgs particles with the least degrees of freedom but the most possible one, they are $\{15\}$, $\{20\}$ and $\{21\}$ Higgs particle.

Now we have three kind of Higgs particles, but how do we choose the right Higgs particles for the intended symmetry breakings ? Once again, we have to try any possibilities with some considerations. First, we look at the first symmetry breaking

$$SU(6) \rightarrow SU(3)_{\rm C} \otimes SU(3)_{\rm H} \otimes U(1)_{\rm C}, \tag{3.20}$$

from the above SSB the number of Goldstone bosons (massless) : $8+8+1=17$. From this fact we need Higgs particle with {20} or {21} degrees of freedom, so that we can have massive scalar bosons. After we try all the calculation and consider the whole generation, we choose Higgs particle with {21} degrees of freedom. Then we have massive scalar bosons $21-17=4$.

Now we look at the second symmetry breaking

$$SU(3)_{\rm C} \otimes SU(3)_{\rm H} \otimes U(1)_{\rm C} \rightarrow SU(3)_{\rm C} \otimes SU(2)_{\rm L} \otimes U(1)_{\rm B} \otimes U(1)_{\rm C}, \tag{3.21}$$

from this symmetry breaking we have the number of Goldstone bosons (massless) : $8+3+1+1=13$. From these massless gauge bosons, we can choose Higgs particle with {15} or {20} degrees of freedom, for the appropriate one we choose Higgs particle with {20} degrees of freedom. So we have massive scalar bosons $20-13=7$.

Finally for the last symmetry breaking

$$SU(3)_{\rm C} \otimes SU(2)_{\rm L} \otimes U(1)_{\rm B} \otimes U(1)_{\rm C} \rightarrow SU(3)_{\rm C} \otimes U(1)_{\rm em}, \tag{3.22}$$

From this Symmetry breaking there are $8+1=9$ massless gauge bosons. For this SSB, we have Higgs particle with {15} degrees of freedom. Then we have massive scalar bosons $15-9=6$.

From now on, we know that we have three kinds of Higgs particles, {15}, {20} and {21} degrees of freedom for three steps of symmetry breaking. But, how do we construct these Higgs particles ? First, we know that, In $SU(6)$ GUT, as we see in section 2.4, right-handed particles are assigned in {6}−plets while left-handed particles are assigned in $\{\overline{15}\}$−plets, so in order to to construct all masses we need Higgs particles with {15}−plets or {6}−plets. However after performing some calculation, we have found that the appropriate one is the Higgs particles with {15}−plets.

Now suppose that all Higgs particles have 36 degrees of freedom:

$$\Phi^{15,20,21} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_4 & \phi_5 & \phi_6 \\ \phi_7 & \phi_8 & \phi_9 & \phi_{10} & \phi_{11} & \phi_{12} \\ \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \phi_{18} \\ \phi_{19} & \phi_{20} & \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} \\ \phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} & \phi_{29} & \phi_{30} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} \end{pmatrix}, \tag{3.23}$$

then we should calculate the mass terms in lagrangian for every symmetry breaking

$$\Delta\mathcal{L}_1 = -f^{ij}{\rm Tr}[(\overline{\Psi}_1^6 + \overline{\Psi}_2^6)_R \Phi^{21} (\Psi_1^6 + \Psi_2^6)_L] + {\rm h.c.}, \tag{3.24}$$

$$\Delta\mathcal{L}_2 = -f^{ij}\mathrm{Tr}[(\overline{\Psi}_1^6 + \overline{\Psi}_2^6)_R \Phi^{20}\Psi_L^{15}] + \mathrm{h.c.}, \tag{3.25}$$

$$\Delta\mathcal{L}_3 = -f^{ij}\mathrm{Tr}[\overline{\Psi}_R^{15}\Phi^{15}\Psi_L^{15}] + \mathrm{h.c.}, \tag{3.26}$$

In every term in the equations above one should choose only the the terms with neutral charges. Also in the present case, we consider only the neutral Higgs particles, since only these terms have the vacuum expectation value. The terms with no neutral Higgs particle will only give contribution to the Yukawa interactions which are out of our present interest.

After performing some lengthy calculation, we can write the vacuum expectation values in the form of

$$< \Phi^{21} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{28} & 0 & \phi_{30} \\ 0 & 0 & 0 & \phi_{34} & 0 & \phi_{36} \end{pmatrix}, \tag{3.27}$$

$$< \Phi^{20} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & \phi_5 & \phi_6 \\ 0 & 0 & 0 & 0 & \phi_{11} & \phi_{12} \\ 0 & 0 & 0 & 0 & \phi_{17} & \phi_{18} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3.28}$$

$$< \Phi^{15} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.29}$$

From these results we can see that the Higgs multiplet with 21 degrees of freedom may have 4 VEV's, while Higgs multiplet with 20 degrees of freedom contains only 6 VEV's, and the last one with 15 degrees of freedom may not have any VEV.

Chapter 4

Phenomenological Aspects

4.1 Fermion Masses

Now we are ging to present the fermion mass terms in $SU(6)$ GUT, based on the Higgs multiplets we have chosen in the preceeding section.

We are back to the mass term lagrangian

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= -\text{Tr}[F_{11}\overline{\Psi}^6_{1R}\Phi^{21}\Psi^6_{1L} + F_{12}\overline{\Psi}^6_{1R}\Phi^{21}\Psi^6_{2L} \\ &\quad +F_{13}\overline{\Psi}^6_{2R}\Phi^{21}\Psi^6_{1L} + F_{14}\overline{\Psi}^6_{2R}\Phi^{21}\Psi^6_{2L}] \\ &\quad -\text{Tr}[F_{21}\overline{\Psi}^6_{1R}\Phi^{20}\Psi^{15}_{L} + F_{22}\overline{\Psi}^6_{2R}\Phi^{20}\Psi^{15}_{L}] \\ &\quad -\text{Tr}[F_{31}\overline{\Psi}^{15}_{R}\Phi^{15}\Psi^{15}_{L}] + \text{h.c.},\end{aligned} \tag{4.1}$$

Evaluating this equation leads to the mass terms for fermions in $SU(6)$ GUT. First, let us see the first symmetry breaking (for the first generation)

$$\begin{aligned}\Delta\mathcal{L}_1 &= -\text{Tr}[F_{11}\overline{\Psi}^6_{1R}\Phi^{21}\Psi^6_{1L} + F_{12}\overline{\Psi}^6_{1R}\Phi^{21}\Psi^6_{2L} \\ &\quad +F_{13}\overline{\Psi}^6_{2R}\Phi^{21}\Psi^6_{1L} + F_{14}\overline{\Psi}^6_{2R}\Phi^{21}\Psi^6_{2L}] + \text{h.c.} \\ &= -F_{11}\overline{N}^{\text{C}}_{4eR}\phi_{28}N_{4eL} - F_{12}\overline{N}^{\text{C}}_{4eR}\phi_{28}\nu_{eL} - F_{13}\overline{\nu}^{\text{C}}_{eR}\phi_{28}N_{4eL} \\ &\quad -F_{14}\overline{\nu}^{\text{C}}_{eR}\phi_{28}\nu_{eL} \\ &\quad -F_{11}\overline{N}_{6eR}\phi_{34}N_{4eL} - F_{12}\overline{N}_{6eR}\phi_{34}\nu_{eL} - F_{13}\overline{N}_{eR}\phi_{34}N_{4eL} \\ &\quad -F_{14}\overline{N}_{eR}\phi_{34}\nu_{eL} \\ &\quad -F_{11}\overline{N}^{\text{C}}_{4eR}\phi_{30}N^{\text{C}}_{6eL} - F_{12}\overline{N}^{\text{C}}_{4eR}\phi_{30}N^{\text{C}}_{eL} - F_{13}\overline{\nu}^{\text{C}}_{eR}\phi_{30}N^{\text{C}}_{6eL} \\ &\quad -F_{14}\overline{\nu}^{\text{C}}_{eR}\phi_{30}N^{\text{C}}_{eL} \\ &\quad -F_{11}\overline{N}_{6eR}\phi_{36}N^{\text{C}}_{6eL} - F_{12}\overline{N}_{6eR}\phi_{36}N^{\text{C}}_{eL} - F_{13}\overline{N}_{eR}\phi_{36}N^{\text{C}}_{6eL} \\ &\quad -F_{14}\overline{N}_{eR}\phi_{36}N^{\text{C}}_{eL} + \text{h.c.}\end{aligned} \tag{4.2}$$

We can write down the result above in a different way

$$(\overline{N}^{\mathrm{C}}_{4e} \quad \overline{\nu}^{\mathrm{C}}_{e} \quad \overline{N}_{6e} \quad \overline{N}_{e})_R \begin{pmatrix} F_{11}\phi_{28} & F_{12}\phi_{28} & F_{11}\phi_{30} & F_{12}\phi_{30} \\ F_{13}\phi_{28} & F_{14}\phi_{28} & F_{13}\phi_{30} & F_{14}\phi_{30} \\ F_{11}\phi_{34} & F_{12}\phi_{34} & F_{11}\phi_{36} & F_{12}\phi_{36} \\ F_{13}\phi_{34} & F_{14}\phi_{34} & F_{13}\phi_{36} & F_{14}\phi_{36} \end{pmatrix} \begin{pmatrix} N_{4e} \\ \nu_e \\ N^{\mathrm{C}}_{6e} \\ N^{\mathrm{C}}_{e} \end{pmatrix}_L \qquad (4.3)$$

Then it is simpler to diagonalize the matrix to obtain N_{4e}, ν_e, N_{6e}, and N_e masses. From the diagonalization we get the masses by evaluating the eigenvalues of the matrix $\lambda_{1,2,3,4}$, i.e. solving the equation below

$$\begin{aligned}
&\lambda^4 + \lambda^3(-F_{11}\phi_{28} - F_{11}\phi_{36} - F_{14}\phi_{28} - F_{14}\phi_{36}) \\
&+\lambda^2(F_{11}^2\phi_{28}\phi_{36} + F_{11}F_{14}\phi_{28}^2 + 2F_{11}F_{14}\phi_{28}\phi_{36} + F_{11}F_{14}\phi_{36}^2 \\
&+F_{14}^2\phi_{28}\phi_{36} + F_{14}^2\phi_{30}^2 - F_{11}^2\phi_{30}\phi_{34}) \\
&+\lambda(-F_{11}^2F_{14}\phi_{28}^2\phi_{36} - F_{11}^2F_{14}\phi_{28}\phi_{36}^2 - F_{11}F_{14}^2\phi_{28}^2\phi_{36} \\
&-F_{11}F_{14}^2\phi_{28}\phi_{36}^2 + F_{11}F_{14}^2\phi_{30}^2\phi_{36} + F_{11}F_{14}^2\phi_{30}^2\phi_{28} \\
&+F_{11}^2F_{14}\phi_{30}\phi_{34}\phi_{36} + F_{11}^2F_{14}\phi_{28}\phi_{30}\phi_{34}) \\
&+F_{11}^2F_{14}^2\phi_{28}^2\phi_{36}^2 + 2F_{12}^2F_{13}^2\phi_{28}\phi_{30}\phi_{34}\phi_{36} \\
&+F_{11}^2F_{14}^2\phi_{30}^2\phi_{34}^2 - F_{12}^2F_{13}^2\phi_{30}^2\phi_{34}^2 - F_{12}^2F_{13}^2\phi_{28}^2\phi_{36}^2 = 0.
\end{aligned} \qquad (4.4)$$

Unfortunately, this equation can be solved only numerically.

Next, let us proceed to the second symmetry breaking

$$\begin{aligned}
\Delta\mathcal{L}_2 &= -\mathrm{Tr}[F_{21}\overline{\Psi}^6_{1R}\Phi^{20}\Psi^{15}_L + F_{22}\overline{\Psi}^6_{2R}\Phi^{20}\Psi^{15}_L] + \mathrm{h.c.} \\
&= -F_{21}\overline{D}_{1dR}\phi_5 d_{1L} - F_{22}\overline{d}_{1R}\phi_5 d_{1L} \\
&\quad -F_{21}\overline{D}_{2dR}\phi_{11} d_{1L} - F_{22}\overline{d}_{2R}\phi_{11} d_{1L} \\
&\quad -F_{21}\overline{D}_{3dR}\phi_{17} d_{1L} - F_{22}\overline{d}_{3R}\phi_{17} d_{1L} \\
&\quad -F_{21}\overline{D}_{1dR}\phi_6 D_{7dL} - F_{22}\overline{d}_{1R}\phi_6 D_{7dL} \\
&\quad -F_{21}\overline{D}_{2dR}\phi_{12} D_{7dL} - F_{22}\overline{d}_{2R}\phi_{12} D_{7dL} \\
&\quad -F_{21}\overline{D}_{3dR}\phi_{18} D_{7dL} - F_{22}\overline{d}_{3R}\phi_{12} D_{7dL} + \mathrm{h.c.}
\end{aligned} \qquad (4.5)$$

We can see that there are only mass terms relevant for d quarks, i.e. the term $F_{22}\overline{d}_{1R}\phi_5 d_{1L}$.

Finally, for the last symmetry breaking we get

$$\begin{aligned}
\Delta\mathcal{L}_3 &= -\mathrm{Tr}[F_{31}\overline{\Psi}^{15}_R\Phi^{15}\Psi^{15}_L] + \mathrm{h.c.} \\
&= 0
\end{aligned} \qquad (4.6)$$

which is clear that we can not get any mass terms from this symmetry breaking.

4.2 Gauge Boson Masses

After dealing with the fermion masses, now our task is to generate gauge boson masses in $SU(6)$ GUT, especially to reproduce $W^{\pm}, Z^0$, and A_μ gauge bosons. The mass terms should comeout from the kinetic term of Higgs particles

$$\mathcal{L}_{\text{kin}} = (D_\mu \Phi^k)^\dagger (D^\mu \Phi^k) \tag{4.7}$$

with covariant derivative

$$D_\mu \Phi = (\partial_\mu - i g_6 T_a A^{a\mu})\Phi. \tag{4.8}$$

The terms relevant with gauge boson masses are

$$-g_6^2 \ \text{Tr}[\Phi^{k\dagger}(TaA^{a\mu})^\dagger (T_b A_\mu^b)\Phi^k]. \tag{4.9}$$

For the product $T_a A^{a\mu}$ we get

$$T_a A^{a\mu} = \tfrac{1}{2} \times \tag{4.10}$$
$$\begin{pmatrix} G_3+G_8-H_C & G_1^+ & G_2^+ & X_1^+ & Y_1^+ & Z_1^+ \\ G_1^- & -G_3+G_8-H_C & G_4^+ & X_2^+ & Y_2^+ & Z_2^+ \\ G_2^- & G_4^- & -2G_8-H_C & X_3^+ & Y_3^+ & Z_3^+ \\ X_1^- & X_2^- & X_3^- & H_3+H_B+H_C & H_1^+ & H_2^+ \\ Y_1^- & Y_2^- & Y_3^- & H_1^- & -H_3+H_B+H_C & H_4^+ \\ Z_1^- & Z_2^- & Z_3^- & H_2^- & H_4^- & -2H_B+H_C \end{pmatrix}$$

where

$G_1^{\pm} \equiv A_1 \mp iA_2$	$G_3 \equiv A_3$	$G_2^{\pm} \equiv A_4 \mp iA_5$	$G_4^{\pm} \equiv A_6 \mp iA_7$
$G_8 \equiv \frac{1}{\sqrt{3}} A_8$	$X_1^{\pm} \equiv A_9 \mp iA_{10}$	$X_2^{\pm} \equiv A_{11} \mp iA_{12}$	$X_3^{\pm} \equiv A_{13} \mp iA_{14}$
$Y_1^{\pm} \equiv A_{15} \mp iA_{16}$	$Y_2^{\pm} \equiv A_{17} \mp iA_{18}$	$Y_3^{\pm} \equiv A_{19} \mp iA_{20}$	$Z_1^{\pm} \equiv A_{21} \mp iA_{22}$
$Z_2^{\pm} \equiv A_{23} \mp iA_{24}$	$Z_3^{\pm} \equiv A_{25} \mp iA_{26}$	$H_1^{\pm} \equiv A_{27} \mp iA_{28}$	$H_3^{\pm} \equiv A_{29}$
$H_2^{\pm} \equiv A_{30} \mp iA_{31}$	$H_4^{\pm} \equiv A_{32} \mp iA_{33}$	$H_B \equiv \frac{1}{\sqrt{3}} A_{34}$	$H_C \equiv \frac{1}{\sqrt{3}} A_{35}$

Further, we have to choose one of three Higgs particles that we already have (Φ^{21}, Φ^{20}, and Φ^{15}) to gain, at least $W^{\pm}, Z^0$, and A_μ gauge boson masses. Let us see all possibilities.

First we see the term involves the Higgs particle with 21 degrees of freedom

$$-g_6^2 \ \text{Tr}[\Phi^{21\dagger}(TaA^{a\mu})^\dagger (T_b A_\mu^b)\Phi^{21}]. \tag{4.11}$$

The result is $-g_6^2$ times

$$(\phi_{28}Y_1^- + \phi_{34}Z_1^-)(Y_1^+\phi_{28} + \phi_{34}Z_1^-) + (\phi_{28}Y_2^- + \phi_{34}Z_2^-)(Y_2^+\phi_{28} + \phi_{34}Z_2^-)$$
$$+(\phi_{28}Y_3^- + \phi_{34}Z_3^-)(Y_3^+\phi_{28} + \phi_{34}Z_3^-) + (\phi_{28}H_1^- + \phi_{34}H_2^-)(H_1^+\phi_{28} + \phi_{34}H_2^+)$$
$$+[\phi_{28}(-H_3 + H_B + H_C) + \phi_{34}H_4^-][(-H_3 + H_B + H_C)\phi_{28} + H_4^+\phi_{34}]$$
$$+[\phi_{28}H_4^+ + \phi_{34}(-2H_B + H_C)][H_4^-\phi_{28} + (-2H_B + H_C)\phi_{34}]$$

$$
\begin{gathered}
(\phi_{30}Y_1^- + \phi_{36}Z_1^-)(Y_1^+\phi_{30} + \phi_{36}Z_1^-) + (\phi_{30}Y_2^- + \phi_{36}Z_2^-)(Y_2^+\phi_{30} + \phi_{36}Z_2^-) \\
+(\phi_{30}Y_3^- + \phi_{36}Z_3^-)(Y_3^+\phi_{30} + \phi_{36}Z_3^-) + (\phi_{30}H_1^- + \phi_{36}H_2^-)(H_1^+\phi_{30} + \phi_{36}H_2^+) \\
+[\phi_{30}(-H_3 + H_B + H_C) + \phi_{36}H_4^-][(-H_3 + H_B + H_C)\phi_{30} + H_4^+\phi_{36}] \\
+[\phi_{30}H_4^+ + \phi_{36}(-2H_B + H_C)][H_4^-\phi_{30} + (-2H_B + H_C)\phi_{36}].
\end{gathered}
$$

From this result, we can choose one or more terms to extract $W^{\pm}, Z^0$, and A_μ gauge boson masses. As we can see from the equation above, we can not have photon mass. If we do diagonalization for H_3, H_B, H_C, and H_4 term, we will have two zero mass term, one could become the photon mass but the rest will give rise to another problem.

Finally, we check the Higgs particle with 20 degrees of freedom

$$-g_6^2 \ \mathrm{Tr}[\Phi^{20\dagger}(TaA^{a\mu})^\dagger(T_bA_\mu^b)\Phi^{20}]. \tag{4.12}$$

The result is $-g_6^2$ times

$$
\begin{gathered}
[\phi_5(G_3 + G_8 - H_C) + \phi_{11}G_1^- + \phi_{17}G_2^-][(G_3 + G_8 - H_C)\phi_5 + G_1^-\phi_{11} + G_2^-\phi_{17}] \\
+[\phi_5G_1^+ + \phi_{11}(-G_3 + G_8 - H_C) + \phi_{17}G_4^-][G_1^-\phi_5 + (-G_3 + G_8 - H_C)\phi_{11} + G_4^+\phi_{17}] \\
+[\phi_5G_2^+ + \phi_{11}G_4^+ + \phi_{17}(-2G_8 - H_C)][G_2^-\phi_5 + G_4^-\phi_{11} + (-2G_8 - H_C)\phi_{17}] \\
+[\phi_5X_1^+ + \phi_{11}X_2^+ + \phi_{17}X_3^+][X_1^-\phi_5 + X_2^-\phi_{11} + X_3^-\phi_{17}] \\
+[\phi_5Y_1^+ + \phi_{11}Y_2^+ + \phi_{17}Y_3^+][Y_1^-\phi_5 + Y_2^-\phi_{11} + Y_3^-\phi_{17}] \\
+[\phi_5Z_1^+ + \phi_{11}Z_2^+ + \phi_{17}Z_3^+][Z_1^-\phi_5 + Z_2^-\phi_{11} + Z_3^-\phi_{17}] \\
[\phi_6(G_3 + G_8 - H_C) + \phi_{12}G_1^- + \phi_{18}G_2^-][(G_3 + G_8 - H_C)\phi_6 + G_1^-\phi_{12} + G_2^-\phi_{18}] \\
+[\phi_6G_1^+ + \phi_{12}(-G_3 + G_8 - H_C) + \phi_{18}G_4^-][G_1^-\phi_6 + (-G_3 + G_8 - H_C)\phi_{12} + G_4^+\phi_{18}] \\
+[\phi_6G_2^+ + \phi_{12}G_4^+ + \phi_{18}(-2G_8 - H_C)][G_2^-\phi_6 + G_4^-\phi_{12} + (-2G_8 - H_C)\phi_{18}] \\
+[\phi_6X_1^+ + \phi_{12}X_2^+ + \phi_{18}X_3^+][X_1^-\phi_6 + X_2^-\phi_{12} + X_3^-\phi_{18}] \\
+[\phi_6Y_1^+ + \phi_{12}Y_2^+ + \phi_{18}Y_3^+][Y_1^-\phi_6 + Y_2^-\phi_{12} + Y_3^-\phi_{18}] \\
+[\phi_6Z_1^+ + \phi_{12}Z_2^+ + \phi_{18}Z_3^+][Z_1^-\phi_6 + Z_2^-\phi_{12} + Z_3^-\phi_{18}]
\end{gathered}
$$

Then we choose the third and ninth line for G_2, G_4, G_8, and H_C, and rewrite them in a different form

$$
(G_2 \quad G_4 \quad G_8 \quad H_C)\begin{pmatrix}
\phi_5^2 + \phi_6^2 & \phi_5\phi_{11} + \phi_6\phi_{12} & -2(\phi_5\phi_{17} + \phi_6\phi_{18}) & -\phi_5\phi_{17} - \phi_6\phi_{18} \\
\phi_5\phi_{11} + \phi_6\phi_{12} & \phi_{11}^2 + \phi_{12}^2 & -2(\phi_{11}\phi_{17} + \phi_{12}\phi_{18}) & -\phi_{11}\phi_{17} - \phi_{12}\phi_{18} \\
-2(\phi_5\phi_{17} + \phi_6\phi_{18}) & -2(\phi_{11}\phi_{17} + \phi_{12}\phi_{18}) & 4(\phi_{17}^2 + \phi_{18}^2) & 2(\phi_{17}^2 + \phi_{18}^2) \\
-(\phi_5\phi_{17} + \phi_6\phi_{18}) & -(\phi_{11}\phi_{17} + \phi_{12}\phi_{18}) & 2(\phi_{17}^2 + \phi_{18}^2) & \phi_{17}^2 + \phi_{18}^2
\end{pmatrix}\begin{pmatrix} G_2 \\ G_4 \\ G_8 \\ H_C \end{pmatrix} \tag{4.13}
$$

Doing the diagonalization, the eigenvalues of this matrix $\lambda_{1,2,3,4}$ will roduce the masses for $W^{\pm}, Z^0$, and A_μ gauge bosons. These eigenvalues can be calculated by solving the equation

$$
\begin{aligned}
&\lambda^4 + \lambda^3(-\phi_5^2 - \phi_6^2 - \phi_{11}^2 - \phi_{12}^2 - 5\phi_{17}^2 - 5\phi_{18}^2)\\
&\lambda^2(\phi_5^2\phi_{11}^2 + \phi_5^2\phi_{12}^2 + \phi_5^2\phi_{17}^2 + \phi_6^2\phi_{11}^2 + \phi_6^2\phi_{12}^2 + \phi_6^2\phi_{18}^2\\
&+5\phi_5^2\phi_{17}^2 + 5\phi_5^2\phi_{18}^2 + 5\phi_6^2\phi_{17}^2 + 5\phi_{11}^2\phi_{18}^2\\
&+4\phi_{11}^2\phi_{17}^2 + 4\phi_{12}^2\phi_{18}^2 + 4\phi_{17}^4 + 4\phi_{18}^4 + 8\phi_{17}^2\phi_{18}^2\\
&-8\phi_5\phi_6\phi_{17}\phi_{18} - 2\phi_{11}\phi_{12}\phi_{17}\phi_{18})\\
&+\lambda(2\phi_5^2\phi_{11}\phi_{12}\phi_{17}\phi_{18} + 2\phi_6^2\phi_{11}\phi_{12}\phi_{17}\phi_{18}\\
&+8\phi_{11}\phi_{12}\phi_{17}^3\phi_{18} + 8\phi_{11}\phi_{12}\phi_{17}\phi_{18}^3 + 8\phi_5\phi_6\phi_{11}^2\phi_{17}\phi_{18}\\
&+8\phi_5\phi_6\phi_{12}^2\phi_{17}\phi_{18} + 8\phi_5\phi_6\phi_{17}^3\phi_{18} + 8\phi_5\phi_6\phi_{17}\phi_{18}^3\\
&-\phi_5^2\phi_{12}^2\phi_{17}^2 - \phi_6^2\phi_{11}^2\phi_{18}^2 - 5\phi_6^2\phi_{12}^2\phi_{17}^2 - 5\phi_5^2\phi_{11}^2\phi_{18}^2\\
&-4\phi_6^2\phi_{11}^2\phi_{17}^2 - 4\phi_{11}^2\phi_{17}^2\phi_{18}^2 - 4\phi_{12}^2\phi_{17}^2\phi_{18}^2\\
&-4\phi_5^2\phi_{12}^2\phi_{18}^2 - 4\phi_5^2\phi_{17}^2\phi_{18}^2 - 4\phi_6^2\phi_{17}^2\phi_{18}^2\\
&-4\phi_6^2\phi_{17}^4 - 4\phi_{12}^2\phi_{17}^4 - 4\phi_5^2\phi_{18}^4 - 4\phi_{11}^2\phi_{18}^4) = 0.
\end{aligned}
\tag{4.14}
$$

Here we have one zero eigenvalue which can be interpreted as the photon masses, because there are no other terms with zero eigenvalues. The other non-zero eigenvalue should give masses for $W^\pm$ and also Z^0 boson, but we must work it out numerically to obtain the specific values for them.

Chapter 5

Result and Discussion

From previous sections we have described explicitely the mechanism of symmetry breaking to generate fermions and gauge bosons masses in $SU(6)$ GUT. According to the scope of present study, here we can conclude that

- From $\{6\}$ and $\{15\}-$plets particle assignment in $SU(6)$ GUT, using group theory, the most minimal Higgs particles we should introduce are Φ^{15}, Φ^{20} and Φ^{21}.

- Using the mass-term lagrangian

$$\mathcal{L}_{\text{mass}} = -f^{ij}\overline{\Psi}^i_L \Phi^{ij} \Psi^j_R + \text{h.c.}, \tag{5.1}$$

  we can determine the VEV's for these Higgs particles are

$$< \Phi^{21} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{28} & 0 & \phi_{30} \\ 0 & 0 & 0 & \phi_{34} & 0 & \phi_{36} \end{pmatrix} , \tag{5.2}$$

$$< \Phi^{20} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & \phi_5 & \phi_6 \\ 0 & 0 & 0 & 0 & \phi_{11} & \phi_{12} \\ 0 & 0 & 0 & 0 & \phi_{17} & \phi_{18} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \tag{5.3}$$

$$< \Phi^{15} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{5.4}$$

- With these VEV's, the relevant lagrangians which are responsible to generate the fermion masses are

$$\Delta\mathcal{L}_1 = -f^{ij}\mathrm{Tr}[(\overline{\Psi}_1^6 + \overline{\Psi}_2^6)_R \Phi^{21} (\Psi_1^6 + \Psi_2^6)_L] + \mathrm{h.c.}, \tag{5.5}$$

$$\Delta\mathcal{L}_2 = -f^{ij}\mathrm{Tr}[(\overline{\Psi}_1^6 + \overline{\Psi}_2^6)_R \Phi^{20} \Psi_L^{15}] + \mathrm{h.c.}, \tag{5.6}$$

$$\Delta\mathcal{L}_3 = -f^{ij}\mathrm{Tr}[\overline{\Psi}_R^{15} \Phi^{15} \Psi_L^{15}] + \mathrm{h.c.}, \tag{5.7}$$

for each step of symmetry breaking. Also we find that only the masses of N_{4e}, ν_e, N_{6e}, and N_e fermions can be generated by solving numerically the equation

$$\begin{aligned}
&\lambda^4 + \lambda^3(-F_{11}\phi_{28} - F_{11}\phi_{36} - F_{14}\phi_{28} - F_{14}\phi_{36}) \\
&+\lambda^2(F_{11}^2\phi_{28}\phi_{36} + F_{11}F_{14}\phi_{28}^2 + 2F_{11}F_{14}\phi_{28}\phi_{36} + F_{11}F_{14}\phi_{36}^2 \\
&+F_{14}^2\phi_{28}\phi_{36} + F_{14}^2\phi_{30}^2 - F_{11}^2\phi_{30}\phi_{34}) \\
&+\lambda(-F_{11}^2F_{14}\phi_{28}^2\phi_{36} - F_{11}^2F_{14}\phi_{28}\phi_{36}^2 - F_{11}F_{14}^2\phi_{28}^2\phi_{36} \\
&-F_{11}F_{14}^2\phi_{28}\phi_{36}^2 + F_{11}F_{14}^2\phi_{30}^2\phi_{36} + F_{11}F_{14}^2\phi_{30}^2\phi_{28} \\
&+F_{11}^2F_{14}\phi_{30}\phi_{34}\phi_{36} + F_{11}^2F_{14}\phi_{28}\phi_{30}\phi_{34}) \\
&+F_{11}^2F_{14}^2\phi_{28}^2\phi_{36}^2 + 2F_{12}^2F_{13}^2\phi_{28}\phi_{30}\phi_{34}\phi_{36} \\
&+F_{11}^2F_{14}^2\phi_{30}^2\phi_{34}^2 - F_{12}^2F_{13}^2\phi_{30}^2\phi_{34}^2 - F_{12}^2F_{13}^2\phi_{28}^2\phi_{36}^2 = 0.
\end{aligned} \tag{5.8}$$

using $< \Phi^{21} >$ form the first symmetry breaking. The d quarks masses are obtained from the term $F_{22}\overline{d}_{1R}\phi_5 d_{1L}$ from the second symmetry breaking using $< \Phi^{20} >$, i.e.

$$m_d = F_{22}\phi_5 \tag{5.9}$$

- We also have the gauge boson masses using Lagrangian for the kinetic term on Higgs particle

$$\mathcal{L}_{\mathrm{kin}} = (D_\mu \Phi^k)^\dagger (D^\mu \Phi^k) \tag{5.10}$$

with the term

$$-g_6^2\ \mathrm{Tr}[\Phi^{k\dagger}(TaA^{a\mu})^\dagger (T_b A_\mu^b)\Phi^k]. \tag{5.11}$$

from the equation above we can have $W^{\pm}, Z^0$, and A_μ gauge boson, by evaluating the eigenvalue of the matrix $\lambda_{1,2,3,4}$, which is equal to find the solution for λ from the equation below

$$\begin{aligned}
&\lambda^4 + \lambda^3(-\phi_5^2 - \phi_6^2 - \phi_{11}^2 - \phi_{12}^2 - 5\phi_{17}^2 - 5\phi_{18}^2) \\
&\lambda^2(\phi_5^2\phi_{11}^2 + \phi_5^2\phi_{12}^2 + \phi_5^2\phi_{17}^2 + \phi_6^2\phi_{11}^2 + \phi_6^2\phi_{12}^2 + \phi_6^2\phi_{18}^2 \\
&+5\phi_5^2\phi_{17}^2 + 5\phi_5^2\phi_{18}^2 + 5\phi_6^2\phi_{17}^2 + 5\phi_{11}^2\phi_{18}^2 \\
&+4\phi_{11}^2\phi_{17}^2 + 4\phi_{12}^2\phi_{18}^2 + 4\phi_{17}^4 + 4\phi_{18}^4 + 8\phi_{17}^2\phi_{18}^2 \\
&-8\phi_5\phi_6\phi_{17}\phi_{18} - 2\phi_{11}\phi_{12}\phi_{17}\phi_{18}) \\
&+\lambda(2\phi_5^2\phi_{11}\phi_{12}\phi_{17}\phi_{18} + 2\phi_6^2\phi_{11}\phi_{12}\phi_{17}\phi_{18} \\
&+8\phi_{11}\phi_{12}\phi_{17}^3\phi_{18} + 8\phi_{11}\phi_{12}\phi_{17}\phi_{18}^3 + 8\phi_5\phi_6\phi_{11}^2\phi_{17}\phi_{18} \\
&+8\phi_5\phi_6\phi_{12}^2\phi_{17}\phi_{18} + 8\phi_5\phi_6\phi_{17}^3\phi_{18} + 8\phi_5\phi_6\phi_{17}\phi_{18}^3 \\
&-\phi_5^2\phi_{12}^2\phi_{17}^2 - \phi_6^2\phi_{11}^2\phi_{18}^2 - 5\phi_6^2\phi_{12}^2\phi_{17}^2 - 5\phi_5^2\phi_{11}^2\phi_{18}^2 \\
&-4\phi_6^2\phi_{11}^2\phi_{17}^2 - 4\phi_{11}^2\phi_{17}^2\phi_{18}^2 - 4\phi_{12}^2\phi_{17}^2\phi_{18}^2 \\
&-4\phi_5^2\phi_{12}^2\phi_{18}^2 - 4\phi_5^2\phi_{17}^2\phi_{18}^2 - 4\phi_6^2\phi_{17}^2\phi_{18}^2 \\
&-4\phi_6^2\phi_{17}^4 - 4\phi_{12}^2\phi_{17}^4 - 4\phi_5^2\phi_{18}^4 - 4\phi_{11}^2\phi_{18}^4) = 0
\end{aligned} \tag{5.12}$$

From this equation, we have one zero eigenvalue, which is the photon mass, while the rest non-zero eigenvalues should be calculated numerically.

- From all calculations, we conclude also that unfortunately anoter exotic fermions masses can not be generated simultaneously using this kind of ordinary Higgs mechanism. We should guess that the mode requires either more additional Higgs multiplets or another alternate mechanisms as extra dimension etc.

Chapter 6

Conclusion

From our results above we can conclude that Higgs mechanism has not succeeded in generating the fermion and gauge boson masses simultaneously in the SU(6) GUT model. This conclusion has been obtained by introducing the most general but minimal Higgs multiplets allowed in the model.

We have shown that only the masses of N, N_4, N_6 and ν fermions which could be reproduced. We also have the mass term for d quark fermion, $F_{22}\phi_5$ as well. For gauge boson masses we are able to reproduce the already known $W^{\pm}, Z^0$ and A_μ gauge bosons masses. Of course, this terms lead alto to another masses of another gauge bosons beyond the Standard Model.

However, the up-quark masses should be worke out using another mechanisms, as adding more Higgs multiplets or for instance the extra dimension like model.

Appendix A

$SU(6)$ Generators

Here, we provide a complete set of matrices which forms generators for $SU(6)$ group.

$$\lambda_1 = \left(\begin{array}{ccc|c} 0 & 1 & 0 & \\ 1 & 0 & 0 & (0)_{3\times3} \\ 0 & 0 & 0 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right) \qquad \lambda_2 = \left(\begin{array}{ccc|c} 0 & -i & 0 & \\ i & 0 & 0 & (0)_{3\times3} \\ 0 & 0 & 0 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right)$$

$$\lambda_3 = \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & -1 & 0 & (0)_{3\times3} \\ 0 & 0 & 0 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right) \qquad \lambda_4 = \left(\begin{array}{ccc|c} 0 & 0 & 1 & \\ 0 & 0 & 0 & (0)_{3\times3} \\ 1 & 0 & 0 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right)$$

$$\lambda_5 = \left(\begin{array}{ccc|c} 0 & 0 & -i & \\ 0 & 0 & 0 & (0)_{3\times3} \\ i & 0 & 0 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right) \qquad \lambda_6 = \left(\begin{array}{ccc|c} 0 & 0 & 0 & \\ 0 & 0 & 1 & (0)_{3\times3} \\ 0 & 1 & 0 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right)$$

$$\lambda_7 = \left(\begin{array}{ccc|c} 0 & 0 & 0 & \\ 0 & 0 & -i & (0)_{3\times3} \\ 0 & i & 0 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right) \qquad \lambda_8 = \frac{1}{\sqrt{3}}\left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & (0)_{3\times3} \\ 0 & 0 & -2 & \\ \hline & (0)_{3\times3} & & (0)_{3\times3} \end{array}\right)$$

$$\lambda_9 = \left(\begin{array}{ccc|ccc} & & & 1 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right) \qquad \lambda_{10} = \left(\begin{array}{ccc|ccc} & & & -i & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline i & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right)$$

$$\lambda_{11} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 1 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right) \qquad \lambda_{12} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & -i & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & i & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right)$$

$$\lambda_{13} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right) \qquad \lambda_{14} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & -i & 0 & 0 \\ \hline 0 & 0 & i & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right)$$

$$\lambda_{15} = \left(\begin{array}{ccc|ccc} & & & 0 & 1 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 1 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right) \qquad \lambda_{16} = \left(\begin{array}{ccc|ccc} & & & 0 & -i & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & i & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right)$$

$$\lambda_{17} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 1 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 1 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right) \qquad \lambda_{18} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & -i & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & i & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right)$$

$$\lambda_{19} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 1 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right) \qquad \lambda_{20} = \left(\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & -i & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & i & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{array}\right)$$

$$\lambda_{21} = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{matrix}0&0&1\\0&0&0\\0&0&0\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\1&0&0\end{matrix} & (0)_{3\times3} \end{array}\right) \qquad \lambda_{22} = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{matrix}0&0&-i\\0&0&0\\0&0&0\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\i&0&0\end{matrix} & (0)_{3\times3} \end{array}\right)$$

$$\lambda_{23} = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{matrix}0&0&0\\0&0&1\\0&0&0\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\0&1&0\end{matrix} & (0)_{3\times3} \end{array}\right) \qquad \lambda_{24} = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{matrix}0&0&0\\0&0&-i\\0&0&0\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\0&i&0\end{matrix} & (0)_{3\times3} \end{array}\right)$$

$$\lambda_{25} = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{matrix}0&0&0\\0&0&0\\0&0&1\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\0&0&1\end{matrix} & (0)_{3\times3} \end{array}\right) \qquad \lambda_{26} = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{matrix}0&0&0\\0&0&0\\0&0&-i\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\0&0&i\end{matrix} & (0)_{3\times3} \end{array}\right)$$

$$\lambda_{27} = \left(\begin{array}{c|c} (0)_{3\times3} & (0)_{3\times3} \\ \hline (0)_{3\times3} & \begin{matrix}0&1&0\\1&0&0\\0&0&0\end{matrix} \end{array}\right) \qquad \lambda_{28} = \left(\begin{array}{c|c} (0)_{3\times3} & (0)_{3\times3} \\ \hline (0)_{3\times3} & \begin{matrix}0&-i&0\\i&0&0\\0&0&0\end{matrix} \end{array}\right)$$

$$\lambda_{29} = \left(\begin{array}{c|c} (0)_{3\times3} & (0)_{3\times3} \\ \hline (0)_{3\times3} & \begin{matrix}1&0&0\\0&-1&0\\0&0&0\end{matrix} \end{array}\right) \qquad \lambda_{30} = \left(\begin{array}{c|c} (0)_{3\times3} & (0)_{3\times3} \\ \hline (0)_{3\times3} & \begin{matrix}0&0&1\\0&0&0\\1&0&0\end{matrix} \end{array}\right)$$

$$\lambda_{31} = \left(\begin{array}{c|c} (0)_{3\times3} & (0)_{3\times3} \\ \hline (0)_{3\times3} & \begin{matrix}0&0&-i\\0&0&0\\i&0&0\end{matrix} \end{array}\right) \qquad \lambda_{32} = \left(\begin{array}{c|c} (0)_{3\times3} & (0)_{3\times3} \\ \hline (0)_{3\times3} & \begin{matrix}0&0&0\\0&0&1\\0&1&0\end{matrix} \end{array}\right)$$

$$\lambda_{33} = \left(\begin{array}{c|ccc} (0)_{3\times 3} & & (0)_{3\times 3} & \\ \hline & 0 & 0 & 0 \\ (0)_{3\times 3} & 0 & 0 & -i \\ & 0 & i & 0 \end{array}\right) \qquad \lambda_{34} = \frac{1}{\sqrt{3}}\left(\begin{array}{c|ccc} (0)_{3\times 3} & & (0)_{3\times 3} & \\ \hline & 1 & 0y & 0 \\ (0)_{3\times 3} & 0 & 1 & 0 \\ & 0 & 0 & -2 \end{array}\right)$$

$$\lambda_{35} = \frac{1}{\sqrt{3}}\left(\begin{array}{c|c} (-1)_{3\times 3} & (0)_{3\times 3} \\ \hline (0)_{3\times 3} & (1)_{3\times 3} \end{array}\right).$$

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