

QCD critical point and correlations

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Abstract. The QCD critical point and its particle correlation signatures are discussed.

1. Preamble

Quantum Chromodynamics is the theory of strong interactions. It is a well-defined quantum field theory, with elegant foundations, and potentially enormous predictive power. In principle, QCD allows one to calculate *ab initio* the properties of matter under extreme conditions where the strong forces are dominant. Nevertheless, despite recent progress, QCD phenomenology at finite temperature and baryon number density is still one of the least known regimes of the theory. There are several experimental windows into such a regime. One is the physics associated with the interior of neutron stars. Another — the subject of the ongoing and planned experimental programs — is the physics of heavy ion collisions.

This brief report summarizes developments to date in the understanding of the finite temperature and baryon density regime of QCD. The emphasis is given to the singular properties of the critical point and their manifestation in the fluctuation/correlation observables in heavy-ion collisions.

2. QCD phase diagram and the critical point

Fig. 1 shows a sketch of the QCD phase diagram as it is perceived by a modern theorist (see, e.g., [1] for review). By a phase diagram we shall mean the information about the location of the phase boundaries (phase transitions) as well as the physics of the phases that these transitions delineate. The phase transitions are the thermodynamic singularities of the system. The system under consideration is a region (in theory, infinite) occupied by strongly interacting matter, described by QCD, in thermal and chemical equilibrium, characterized by the given values of temperature T and baryo-chemical potential μ_B . In practice, it can be a region in the interior of a neutron star, or inside the hot and dense fireball created by a heavy ion collision.

On the phase diagram, the regime of small T and large μ_B is of relevance to neutron star physics. Because of low temperature, a very rich spectrum of possibilities of ordering can be envisaged. The line separating the Color-Flavor-Locked (CFL) phase, predicted in Ref. [2], from the higher temperature disordered phase (quark-gluon plasma, or QGP) is the most simplified representation of the possible phase structure in this region. This regime is also of particular *theoretical* interest because analytical controllable calculations are possible, due to asymptotic freedom of QCD. The reader is referred to the reviews [1, 3, 4, 5, 6] which cover the recent developments in the study of this domain of the phase diagram.

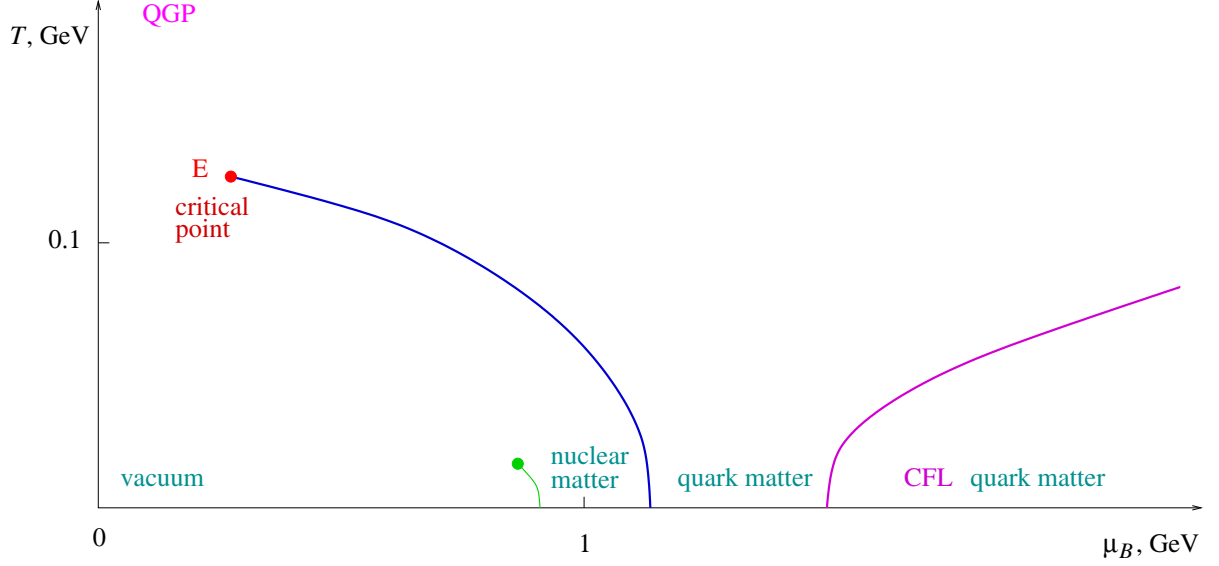


Figure 1. QCD phase diagram (a sketch)

The region of the phase diagram more readily probed by the heavy ion collision experiments is that of rather large $T \sim 100$ MeV, commensurate with the inherent dynamical scale in QCD, and small to medium chemical potential $\mu_B \sim 0 - 600$ MeV. Theorists expect that this region has an interesting feature – the end point of the first order phase transition line, the critical point marked E on Fig. 1. The physics of this point is the focus of the report.

2.1. Theoretical predictions

Theoretically, finding the coordinates (T, μ_B) of the critical point is a well-defined task. We need to calculate the partition function of QCD and find the singularity corresponding to the end of the first order transition line. The Lagrangian of QCD is known, and the partition function is given by a path integral of the exponent of the QCD action, after Wick rotation to the Euclidean space (with imaginary time compactified on a torus of circumference $1/T$).

Of course, calculating such an infinitely dimensional integral analytically is beyond our present abilities (perturbation theory is not applicable here, in the relevant region of T and μ_B). We are thus left with numerical methods, i.e., lattice Monte Carlo simulation. At zero μ_B this method allows us to determine the equation of state of QCD as a function of T . We expect that the transition at $\mu_B = 0$ is a crossover, not a first order phase transition, based on such numerical calculations [7, 8, 9, 10, 11, 12, 13, 14]. However, at finite μ_B the conventional Monte Carlo method is inapplicable due to the *sign problem*.

The first lattice prediction for the location of the critical point has been reported in Ref. [15]. The assumption is that, although the sign problem becomes exponentially difficult as $V \rightarrow \infty$, in practice, one can get a sensible approximation at finite V . In addition, simulations at finite T might suffer lesser overlap problem [16] because of large thermal fluctuations [17]. One can hope that if the critical point is at a small value of μ_B , the volume V may not need to be too large to achieve a reasonable accuracy. In particular, numerical estimates show [18] that the maximal value of μ_B which one can reach within the same accuracy shrinks only as a power of $1/V$.

However, it is not possible to determine this accuracy, since the exact result is unknown. Normally, one would estimate the error by going to increasingly large volumes V , but the

method becomes prohibitive too quickly (exponentially) in this limit. Ultimately, the result of Ref. [15, 19] might turn out to be a good approximation to the exact answer, but we can only tell once we have an independent result to compare it to. A qualitatively new approach is needed to overcome the QCD sign problem.¹

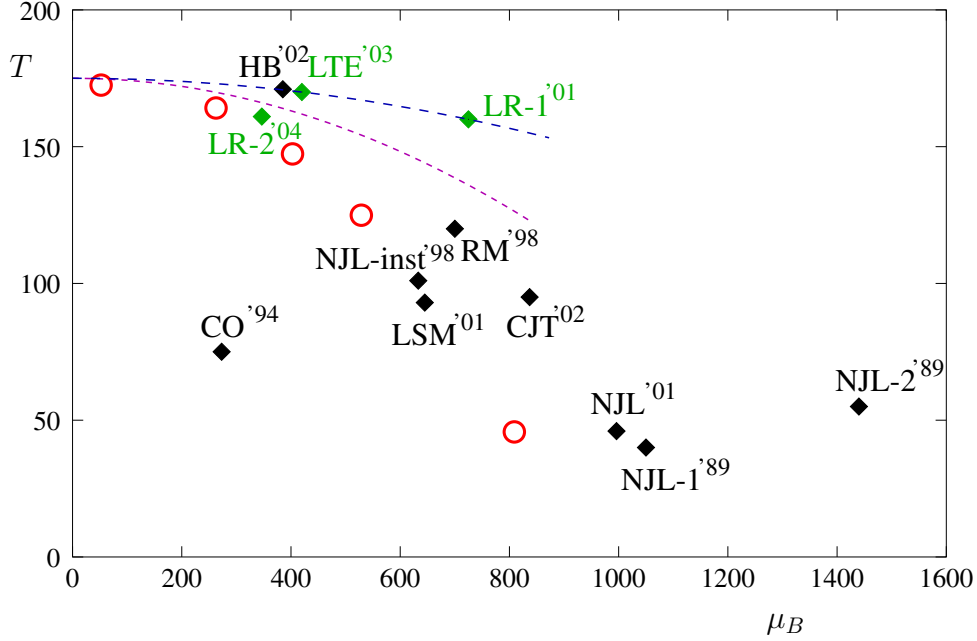


Figure 2. Theoretical (models and lattice) predictions for the location of the critical point. The labels are abbreviations of models/methods used and the publication date (for key and references, see [23]). The two dashed lines indicate the magnitude of the slope $d^2T/d\mu^2$ obtained by lattice Taylor expansion[24]. The upper curve agrees with Ref. [25]. The lower curve corresponds to smaller quark mass. Errors/uncertainties are not shown. The red circles indicate location of freezeout points at various collision energies.

In the absence of a controllable (i.e., systematically improvable) method, one turns to model calculations. Many such calculations have been done [26, 27, 28, 29, 30, 31, 32, 33]. Figure 2 summarizes the results. One can see that the predictions vary wildly. An interesting point to keep in mind is that each of these models is tuned to reproduce vacuum, $T = \mu_B = 0$, phenomenology. Nevertheless, extrapolation to nonzero μ_B is not constrained significantly by this. In a loose sense, the existing lattice methods can be also viewed as extrapolations from $\mu_B = 0$, but finite T .

2.2. Critical behavior: static and dynamic universality class

Determining properties of QCD (equation of state, correlation functions, etc.) near the critical point is difficult, for the same reason as it is difficult to find the location of the critical point. However, as it is the case for any critical point, singular properties, such as critical exponents, can be determined using universality arguments.

According to the scaling postulate, central to the theory of critical phenomena,[34] all singular contributions to the thermodynamic quantities are powers of the correlation length ξ , which

¹ In theories similar, or approximating, the finite density QCD, the sign and/or overlap problems have been tackled recently, using various new methods see, e.g., Refs. [20, 21, 22].

diverges at the critical point. These powers, or critical exponents, are universal, in the sense that they depend only on the degrees of freedom in the theory and their symmetry, but not on the other details of the interactions. Very different physical systems may belong to the same universality class, as far as their critical behavior is concerned.

One should distinguish static and dynamic universality classifications [35]. From the point of view of static critical phenomena, the QCD critical point falls into the universality class of the Ising model. This is a consequence of the fact that at $m_q \neq 0$ no symmetry remains which would require the order parameter to have more than just one component. The field theory which describes the static critical behavior, the one-component ϕ^4 theory in 3 dimensions, has the critical exponents of the Ising model.

What is the nature of this order parameter? A natural choice is the value of the chiral condensate $\langle \bar{\psi}\psi \rangle$, since it is distinct in two phases coexisting across the first order phase transition terminating in the critical point. In the close vicinity of the critical point the static (equal-time) correlation function $\langle \bar{\psi}\psi(\mathbf{x})\bar{\psi}\psi(\mathbf{y}) \rangle$ develops divergent correlation length:

$$\langle \bar{\psi}\psi(\mathbf{x})\bar{\psi}\psi(\mathbf{0}) \rangle_c \sim \begin{cases} \frac{1}{|\mathbf{x}|^{1+\eta}}, & |\mathbf{x}| \ll \xi; \\ e^{-|\mathbf{x}|/\xi}, & |\mathbf{x}| \gg \xi; \end{cases} \quad (1)$$

where $\langle \bar{\psi}\psi(\mathbf{x})\bar{\psi}\psi(\mathbf{0}) \rangle_c \equiv \langle \bar{\psi}\psi(\mathbf{x})\bar{\psi}\psi(\mathbf{0}) \rangle - \langle \bar{\psi}\psi \rangle^2$. The correlation length diverges, $\xi \rightarrow \infty$, at the critical point. For all theories in the Ising universality class $\eta \approx 0.04$.

Another interesting quantity, both from theoretical and experimental points of view, is the baryon number density $n_B(\mathbf{x})$. Because symmetry (or, rather, the absence of such) allows mixing of $n_B(\mathbf{x})$ with $\bar{\psi}\psi(\mathbf{x})$, the divergence of the baryon number susceptibility is related to the divergence of the correlation length ξ :

$$\frac{\partial n_B}{\partial \mu_B} = \int d^3\mathbf{x} \langle n_B(\mathbf{x})n_B(\mathbf{0}) \rangle_c \sim \int d^3\mathbf{x} \langle \bar{\psi}\psi(\mathbf{x})\bar{\psi}\psi(\mathbf{0}) \rangle_c \sim \xi^{2-\eta}. \quad (2)$$

The baryon number density also jumps across the first order phase transition. One can equally well use n_B as the degree of freedom in the effective theory near the critical point, or any linear combination of $\bar{\psi}\psi$ and n_B (or any other field which can mix with $\bar{\psi}\psi$) which is discontinuous across the first order phase transition. Regardless of the choice, there is only one order parameter, as far as the static critical behavior is concerned.

The situation resembles, but is a little more complicated, if one considers *dynamic* critical behavior, e.g., the singularities of kinetic coefficients, etc. The scaling postulate is similar in this case, but the universality classes are now determined by the effective degrees of freedom which define the effective *hydrodynamic* theory near the critical point.[35] In this case the fundamental difference between $\bar{\psi}\psi$ and n_B fields is that the latter is a *conserved* density. The hydrodynamic equations for n_B are diffusive, while the dynamics of $\bar{\psi}\psi$ is relaxational. Because the two modes mix, there is, again, only one independent hydrodynamic variable, and it is diffusive.[36, 37] This mode involves fluctuations of both $\bar{\psi}\psi$ and n_B in a fixed proportion. The fluctuations of $\bar{\psi}\psi$ alone relax on a finite time scale even at the critical point.²

The complete hydrodynamic theory near the critical point must also involve the energy and momentum densities. Once the hydrodynamic equations are written down, and the mixing between $\bar{\psi}\psi$, n_B and the energy density is taken into account, one finds the theory equivalent to the one describing the liquid-gas phase transition, model H in the classification of Ref. [35]. One consequence of this theory, interesting from phenomenological point of view, is the vanishing of the baryon number diffusion rate at the critical point: $D \sim \xi^{-x_D}$, with exponent $x_D \approx 1$. [37]

² A related observation, that the sigma *pole* mass does not vanish at the critical point in the large- N NJL model, was made in Ref. [31] and confirmed in Ref. [36].

3. Experimental signatures: fluctuations and particle correlations

Even though the exact location of the critical point is not known yet, the available theoretical estimates strongly indicate that the point is within the region of the phase diagram probed by the heavy-ion collision experiments – see Fig. 2. This raises the possibility to discover this point in such experiments [38].

One of the actively pursued signatures of the critical point is the non-monotonous dependence on \sqrt{s} (and thus, on μ_B) of the event-by-event fluctuation observables [38, 39]. The idea can be understood qualitatively by noting that: (1) the susceptibilities diverge at the critical point, and (2) the magnitude of the fluctuations are proportional to the corresponding susceptibilities. For example, for the fluctuations of energy or charge, the well-known relations are

$$\frac{\partial E}{\partial T} = \frac{1}{T^2} \langle (\Delta E)^2 \rangle; \quad \frac{\partial Q}{\partial \phi} = \frac{1}{T} \langle (\Delta Q)^2 \rangle. \quad (3)$$

Ideally, one could determine susceptibilities on the left-hand side by measuring the fluctuations on the right-hand side [40]. However, practically, the measurement of the corresponding fluctuations, ΔE or ΔQ , is not feasible because not all the particles end up in the detector [39, 41]. A more differential measure of the fluctuations needs to be computed in theory and compared to experiment.

3.1. Two-particle correlator

A number of such measures can be obtained starting from a two particle correlator

$$\langle \Delta n_{\mathbf{p}}^{\alpha} \Delta n_{\mathbf{k}}^{\beta} \rangle = \langle n_{\mathbf{p}}^{\alpha} n_{\mathbf{k}}^{\beta} \rangle - \langle n_{\mathbf{p}}^{\alpha} \rangle \langle n_{\mathbf{k}}^{\beta} \rangle \quad (4)$$

where $\Delta n_{\mathbf{p}}^{\alpha} = n_{\mathbf{p}}^{\alpha} - \langle n_{\mathbf{p}}^{\alpha} \rangle$ is the event-by-event fluctuation of the number of particles of the type α in the momentum bin centered around \mathbf{p} . Experts familiar with Hanbury-Brown-Twiss (HBT) interferometry [42] may recognize in (4) the HBT correlation function.

The two-particle correlator (4) can be directly measured. However, for such a function of many variables, it might be difficult to represent the result of this measurement. A useful representation, for example, is obtained by limiting (projecting) the variables to transverse components of \mathbf{p} and \mathbf{k} . The resulting plot of a function of two arguments, p_T and k_T , is often referred to as a ‘Trainor plot’ (see, e.g., Ref. [43]). Interesting information can be also obtained by projecting onto the rapidities of \mathbf{p} and \mathbf{k} . If in addition, one weights each particle with its charge, the resulting correlator, as a function of the rapidity difference $y_{\mathbf{p}} - y_{\mathbf{k}}$, is essentially the balance function introduced in [44].

There also exist many *cumulative* measures, proposed by theorists and/or used by experimentalists, [45, 46, 39, 47, 48, 41, 49, 50, 51] that can be expressed in terms of correlator (4). As an example, the fluctuation of electric charge is given by summing over momenta \mathbf{p} and \mathbf{k} of *all* particles in the experimental acceptance window and weighting each particle with its charge q^{α} :

$$\Delta Q = \sum_{\mathbf{p}, \alpha} q^{\alpha} \Delta n_{\mathbf{p}}^{\alpha}; \quad \text{thus} \quad \langle (\Delta Q)^2 \rangle = \sum_{\mathbf{p}, \alpha} \sum_{\mathbf{k}, \beta} q^{\alpha} q^{\beta} \langle \Delta n_{\mathbf{p}}^{\alpha} \Delta n_{\mathbf{k}}^{\beta} \rangle. \quad (5)$$

The same applies to the fluctuations of the baryon number, with q^{α} substituted by the baryon number of the particles. Similar equation (see Eq.(7)) also applies to the fluctuations of the mean transverse momentum p_T , in which case q^{α} should be replaced with $p_T - \overline{p_T}$ – the deviation of the momentum p_T from the all-event (inclusive) mean $\overline{p_T}$.

The correlator (4) can, in principle, be calculated, under assumption of thermal equilibrium, once the relevant interactions are known. In the case of the critical point, we need to concern ourselves with the interactions which can lead to singular contribution to the correlator (and, as a consequence, to susceptibilities) as the critical point is approached.

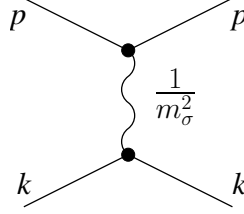


Figure 3. Diagrammatic representation of the singular contribution to the correlator $\langle \Delta n_{\mathbf{p}} \Delta n_{\mathbf{k}} \rangle$.

In a non-interacting gas in thermal equilibrium the correlator (4) vanishes unless $\mathbf{p} = \mathbf{k}$ and $\alpha = \beta$.³ The hadrons, however, are interacting. One can ask a question: what is the effect of the interaction on the correlator (4)? The answer can be found to leading order [52]. The contribution is proportional to the amplitude of the forward scattering $\mathcal{A}_{pk \rightarrow pk}$ of the particles with momenta \mathbf{p} and \mathbf{k} . This is easy to understand using the following argument. The amplitude of the forward scattering shifts the energy of the 2-particle state relative to the sum of single particle energies. The statistical weight of the two particle state is therefore changed relative to the product of the single-particle weights. The difference is the two-particle correlator:

$$\langle n_p n_k \rangle - \langle n_p \rangle \langle n_k \rangle = f_p f_k (e^{-\beta E_I} - 1) \approx f_p f_k (-\beta E_I) \sim f_p f_k \beta \mathcal{A}_{pk \rightarrow pk}. \quad (6)$$

where f_p is the equilibrium distribution function and E_I is the interaction energy. The exact formula, obtained using diagrammatic analysis, [52] contains additional factors $(1 + f_{\mathbf{p}})(1 + f_{\mathbf{k}})$, which can be understood as Bose enhancement (stimulated emission) factors (or, in the case of fermions, $(1 - f_{\mathbf{p}})(1 - f_{\mathbf{k}})$ – Pauli blocking).

Near the critical point the most singular contribution comes from the exchange of the sigma field quanta in the t channel.⁴ Since, by kinematics, the quanta carry zero momentum, the singular contribution is proportional to $1/m_\sigma^2$, which equals ξ^2 – the square of the sigma field correlation length.

The absolute strength of the singularity depends on the coupling of the critical mode sigma to the corresponding hadron in Fig. 3, which is difficult to estimate reliably. Order of magnitude estimates have been made for coupling to pions [39] and to protons [53].

As an example of the singular contribution in Fig. 3 consider baryon number susceptibility. Let Q in equation (5) be the net baryon number. Then one can see that the $1/m_\sigma^2$, or ξ^2 , singularity from Fig. 3 for scattering two baryons results in the divergence of the baryon number susceptibility (2) (the critical exponent $\eta = 0$ at this order). If only charged baryons are detected, the total baryon number cannot be measured event by event, but the number of protons is measurable. Since, according to Fig. 3, the proton number fluctuations should also be singular at the critical point, measurement of such fluctuations may provide a signal of the critical point [53].

In principle, knowing the correlator (4) one could make quantitative predictions for fluctuation measures used in experiment. In practice, calculating the correlator is a very difficult task (what interactions should be included and what is their strength?). Non-equilibrium effects make this task even more difficult. Near the critical point these complications become somewhat less relevant since, as long as we limit ourselves to the singular effects, we only need to consider contributions such as in Fig. 3.

³ We are not considering HBT correlations, which are a finite size effect.

⁴ Strictly speaking, what we call here, for simplicity, “sigma” is a mixture (a linear combination) of chiral condensate, baryon density and energy density fluctuations.

3.2. Fluctuations, correlations, and acceptance

Cumulative measures of fluctuations are often used to represent experimental results. These measures suffer an important drawback – they depend on the size and shape of the acceptance window of the detector. This makes comparison of different experiments, as well as an experiment to a theory, difficult. However, knowing certain properties of the correlator (4), it is possible to correct for acceptance in such comparisons.

As an illustration consider event-by-event fluctuations of the mean transverse momentum p_T per particle. Most commonly used fluctuation measures are based on the width of the distribution of the event mean p_T , σ_{ebe}^2 . Similar to (5), σ_{ebe}^2 can be expressed through the correlator (4) [39]:

$$\sigma_{\text{ebe}}^2 = \frac{1}{\langle N \rangle^2} \sum_{\mathbf{p}\mathbf{k}} \Delta p_T \Delta k_T \langle \Delta n_{\mathbf{p}} \Delta n_{\mathbf{k}} \rangle, \quad (7)$$

where $\Delta p_T \equiv p_T - \overline{p_T}$ and $\langle N \rangle$ is the average multiplicity of accepted particles. In the thermodynamic limit $\langle N \rangle \rightarrow \infty$ the fluctuation σ_{ebe}^2 vanishes as $1/\langle N \rangle$. Thus in this limit, the quantity $\langle N \rangle \sigma_{\text{ebe}}^2$ does not depend on the size of the system $\langle N \rangle$ and is therefore a natural subject of theoretical predictions.

To make a closer comparison to experiment, it is useful to exclude the diagonal terms $\mathbf{p} = \mathbf{k}$ from the sum in (7), since they give the trivial statistical contribution $\langle N \rangle^{-1} \sigma_{\text{inc}}^2$, where σ_{inc} is the r.m.s. width of the inclusive distribution of p_T . The remaining off-diagonal terms in (7) give the nontrivial “dynamical fluctuation”, experimentally obtained after the subtraction:

$$\sigma_{\text{dyn}}^2 \equiv \sigma_{\text{ebe}}^2 - \langle N \rangle^{-1} \sigma_{\text{inc}}^2. \quad (8)$$

In an experiment, the sum in (7) is limited to \mathbf{p} and \mathbf{k} which fall within detector acceptance. Assume, for clarity, that the acceptance is limited in rapidity, i.e., $y_{\mathbf{p}}$ and $y_{\mathbf{k}}$ belong to an interval $[y_{\text{min}}, y_{\text{max}}]$. The cumulative measure σ_{ebe}^2 , or σ_{dyn}^2 , will then depend on $y_{\text{acc}} \equiv y_{\text{max}} - y_{\text{min}}$. This dependence simplifies in two regimes of y_{acc} . The boundary between the two regimes is determined by the characteristic range y_{corr} of the rapidity correlator of the fluctuations:

$$\langle N \rangle \sigma_{\text{dyn}}^2 \Big|_{y_{\text{acc}}} = \begin{cases} O(y_{\text{acc}}), & \text{for } y_{\text{acc}} \ll y_{\text{corr}}; \\ \langle N \rangle \sigma_{\text{dyn}}^2 \Big|_{\infty}, & \text{for } y_{\text{acc}} \gg y_{\text{corr}}. \end{cases} \quad (9)$$

In other words, cumulative measure $\langle N \rangle \sigma_{\text{dyn}}^2$ grows linearly with y_{acc} for small acceptance windows and saturates at its thermodynamic limit value when the acceptance range exceeds the correlation range. In most current experiments, the width of the rapidity window y_{acc} is smaller or at most comparable to the typical range of the rapidity correlator $y_{\text{corr}} \sim 1$. This means that in a typical experiment, for a cumulative measure, normalized to be finite in the thermodynamic limit, the experimentally observed magnitude is roughly proportional to the acceptance window size [48, 54, 55].

3.3. Experimental results and concluding remarks

As an example of the QCD phase diagram scan, the plot in Fig.4 shows the results of the measurements of the p_T fluctuations using a cumulative measure Σ_{p_T} described in Ref. [56]. No clear non-monotonous signal, which one would expect if the experiments probed the vicinity of the critical point, was found.

It is also interesting to compare the magnitude of the observed fluctuations to the singular contribution expected near the critical point [39]. After correcting for acceptance using the method outlined in the previous section one finds:

$$\Sigma_{p_T} \sim 2\% \times \left(\frac{G}{300 \text{ MeV}} \right)^2 \left(\frac{\xi}{3 \text{ fm}} \right)^2, \quad (10)$$

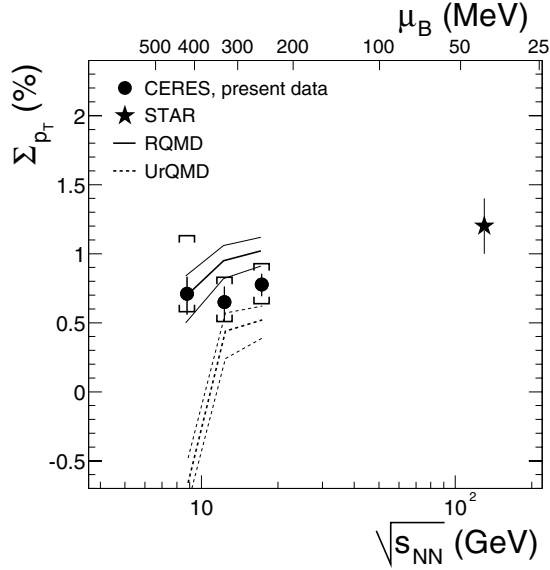


Figure 4. CERES and STAR results for different collision energies [56]. The corresponding values of μ_B at freezeout are determined using statistical model analysis [57].

where G is the magnitude of the $\sigma\pi\pi$ coupling in the diagram Fig. 3 and $\xi = 1/m_\sigma$.

It is important to note, that observation of a large magnitude of fluctuations would not by itself constitute the signal of the critical point. There are many possible contributions to the fluctuations, which are difficult to estimate. The distinct signature of the critical point is the non-monotonous behavior of fluctuation observables.

Another important experimental variable is the transverse momentum window of acceptance. The two-particle correlations induced by critical point have most significant effect on particles with small transverse momenta (soft part of the spectrum). These correlations are thermodynamic and affect particles with typical thermal momenta, i.e., $p \sim 3T \sim 400 - 500$ MeV. For comparison, the results reported in [56] include particles in the range of $0.1 < p_T < 1.5$ GeV. The interesting (from the point of view of critical fluctuations) signal can be enhanced by restricting this window to, e.g., $p_T < 500$ MeV, eliminating potential contributions from correlations among higher momentum particles, which have completely different origin (e.g., jets).

Experiments at other energies, at CERN SPS, RHIC, and future GSI facility, will be able to provide a complete scan of the reachable domain on the QCD phase diagram and either discover or rule out the presence of the critical point in this domain.

This review focused mainly on the signatures of the QCD critical point based on the event-by-event fluctuations. Further study of the properties of the critical point may reveal other, perhaps, even more sensitive and experimentally cleaner signatures [58]

Finally, the lack of a controllable and reliable theoretical method to calculate coordinates of the critical point impairs our ability to perform a more focused search. It is hard to overemphasize the importance of such a theoretical method.

Acknowledgments

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