Superallowed α-decay transitions

Monika Patial,* R. J. Liotta, and R. Wyss
Royal Institute of Technology (KTH), Alba Nova University Center, SE-10691 Stockholm, Sweden

Introduction

It has been observed that the α-decay process in light Te isotopes carries a preformation factor which is larger than the one in the classic decays of Po isotopes [1]. Such α-decay transitions are called superallowed [2]. This feature is expected because around the ^{100}Sn core the valence nucleons, both neutrons and protons, move in the same single-particle (sp) shells, thus enhancing the neutron-proton interaction. The superallowed character of the α-decay transitions from light Te isotopes was established by examining reduced decay widths [1]. The decay proceeds in two steps: in the first step the four nucleons in the mother nucleus get clustered together to form α-particle. In the second step, α-particle formed penetrates the Coulomb and centrifugal barriers.

In this article, we discuss the first step in detail i.e formation probabilities of the α-particle on the surface of nucleus. Most of the known theories do not evaluate the formation amplitude but rather takes it as a free parameter. We present a full microscopic calculation to analyze in detail the formation probabilities, which includes proton-neutron interaction among the nucleons forming the α-particle.

Theoretical Formalism

We study the systems with two protons and two neutrons outside a double-magic core within the framework of the multistep shell model (MSM)[3] . The MSM dynamical equation for $2p2n$ case is

$$\sum_{\alpha_2'\beta_2'} [\ (W_{\alpha_2} + W_{\beta_2})\, \delta_{\alpha_2,\alpha_2'}\delta_{\beta_2,\beta_2'} + \langle \alpha_2\beta_2;\alpha_4 \mid V_{pn} \mid \alpha_2'\beta_2';\alpha_4\rangle\ ]\ X(\alpha_2'\beta_2';\alpha_4) = W_{\alpha_4} X(\alpha_2\beta_2;\alpha_4), \quad (1)$$

where W_{α_2} (W_{β_2}) is the two-proton (two-neutron) correlated energy and the eigenvalue W_{α_4} is the $2p2n$ energy and the eigenvectors $X(\alpha_2\beta_2;\alpha_4)$. The proton-neutron interaction (V_{pn}) matrix element has the form

$$\begin{aligned}
&\langle \alpha_2\beta_2;\alpha_4 \mid V_{pn} \mid \alpha_2'\beta_2';\alpha_4\rangle \\
&= (-1)^{\beta_2+\beta_2'+\alpha_4}[\alpha_2]^{1/2}[\alpha_2']^{1/2}[\beta_2]^{1/2} \\
&\times[\beta_2']^{1/2}\sum_{ijk}\sum_{pqr}\sum_{\lambda}(-1)^{\lambda+q+i-j+k}[\lambda] \\
&\times\sum_{l}[l]\begin{Bmatrix} \alpha_2 & \alpha_2' & l \\ k & i & j \end{Bmatrix}\begin{Bmatrix} p & i & \lambda \\ k & r & l \end{Bmatrix} \\
&\times\begin{Bmatrix} \beta_2 & \beta_2' & l \\ r & p & q \end{Bmatrix}\begin{Bmatrix} \beta_2 & \beta_2' & l \\ \alpha_2' & \alpha_2 & \alpha_4 \end{Bmatrix} \\
&\times Y(ij;\alpha_2)Y(pq;\beta_2)Y(kj;\alpha_2')Y(rq;\beta_2') \\
&\times\langle ip;\lambda \mid V_{pn} \mid kr;\lambda\rangle\,, \qquad (2)
\end{aligned}$$

where $Y(ij;\alpha_2) = \sqrt{1+\delta_{ij}}X(ij;\alpha_2)$ and $[\alpha_2]$=$2\alpha_2$+1.

The final form of α-particle formation amplitude is

$$\begin{aligned}
&F_{\alpha_4}(R) \\
&= \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\alpha_4)\sum_{N_\pi} T(\alpha_2 N_\pi)\sum_{N_\nu} T(\beta_2 N_\nu) \\
&\times\sum_{N_\alpha}\langle N_\pi\alpha_2 N_\nu\beta_2;\alpha_4 \mid 00N_\alpha\alpha_4;\alpha_4\rangle\mathcal{R}_{N_\alpha\alpha_4}(R), \qquad (3)
\end{aligned}$$

where $\langle N_\pi\alpha_2 N_\nu\beta_2;\alpha_4 \mid 00N_\alpha\alpha_4;\alpha_4\rangle$ is a Moshinsky bracket and $\mathcal{R}_{N_\alpha\alpha_4}(R)$ is the ra-

*Electronic address: monikapatial@gmail.com, patial@kth.se; Present Address: Department of Physics, Worcester Polytechnic Institute, Worcester, MA-01609

dial wave function corresponding to the outgoing α-particle. The detailed formalism and notations are explained in Ref. [4].

Results and discussions

We analyze in detail the nuclear structure and the α-formation probabilities in ^{212}Po and ^{104}Te.

^{212}**Po**: The proton sp states correspond to major shell $N = 5$ and the neutron sp states correspond to major shell $N = 6$ are taken into account. All the two-particle states (α_2, β_2) provided by this set of sp states are included in the MSM calculations [5]. The dynamical equation (1) is diagonalized to obtain eigen values and wavefunctions. The calculated energies are shown in Table I, which agree with the experimental values within few keV. However, looking at the wavefunctions [4], it shows *pn* interaction is not dominant in this case. This is expected because in ^{212}Po, the protons and neutrons move in shells having different parities .

^{104}**Te**: We consider all the sp states in the major shell $N = 4$ for both proton and neutron. In this case we use, two particle states described by Morten interaction [6]. As in ^{212}Po, we solve (1) and the energies obtained are mentioned in Table I. It is noticed [4] that, the *pn*-interaction plays a dominant role in ^{104}Te, which can be attributed to the fact that the protons and neutrons move in the same sp shells.

TABLE I: Calculated excitation energies(in MeV) in ^{104}Te:$\left[^{102}\text{Te}(\alpha_2) \otimes^{102} \text{Sn}(\beta_2)\right]$ and ^{212}Po :$\left[^{210}\text{Po}(\alpha_2) \otimes^{210} \text{Pb}(\beta_2)\right]$ and the corresponding available experimental data.

I_1^π	^{104}Te	^{212}Po
0_1^+	0	0
2_1^+	0.61	0.78(0.73*)
4_1^+	1.31	1.15(1.13*)
6_1^+	2.17	1.30(1.36*)
8_1^+	3.60	1.40(1.48*)

*Expt. energies

Using the wavefunctions evaluated, we calculate the α-formation amplitude (3) in ^{212}Po and ^{104}Te. The α-particle is formed on the surface of the daughter nucleus. The α-formation probability in the region R around the nuclear surface in ^{104}Te is found to be

$$F(R)_{^{104}\text{Te}} = 4.85 \times F(R)_{^{212}\text{Po}}.$$

The large formation amplitude in ^{104}Te explains the superallowed character of α-transition. This is because of enhanced proton-neutron correlations in Te region compared to Po region.

In conclusion the evaluated formation probabilities show that in ^{104}Te there is indeed a superallowed α-decay transition.

Acknowledgments

This work was supported by the Royal Institute of Technology, Stockholm, and the Swedish Research Council (VR) under Grants No. 621-2012-3805 and No. 621-2013-4323.

References

[1] S. N. Liddick et al., Phys. Rev. Lett. **97**, 082501 (2006).
[2] R. D. Macfarlane and A. Siivola, Phys. Rev. Lett. **14**, 114 (1965).
[3] R. J. Liotta and C. Pomar, Nucl. Phys. A **382**, 1 (1982).
[4] Monika Patial, R. J. Liotta, and R. Wyss, Phys. Rev. C. **93**, 054326 (2016).
[5] T. T. S. Kuo, G. H. Harling, *Two-Particle and Two-Hole States in the Pb Region* (Defense Technical Information Center, Washington, DC, 1971).
[6] M. Hjorth-Jensen, T. T. S. Kuo, and E. Osnes, Phys. Rep. **261**, 125 (1995).