Particle Accelerators, 1991, Vol. 36, pp.1–13
Reprints available directly from the publisher
Photocopying permitted by license only

HELICAL SIBERIAN SNAKES USING DIPOLE MAGNETS

U. WIENANDS
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

(Received 14 September 1990; in final form 20 June 1991)

A family of multi-twist transverse-field spin rotators using discrete bending magnets is described that can be used as Siberian snakes. By varying the number of twists, snakes with quite small orbit excursions can be constructed at only a small penalty in the overall field integral. Examples for a 1/4-twist snake and a 3-twist snake are presented, the first suitable for very high energy machines and the second for use in the proposed TRIUMF KAON Factory.

1. INTRODUCTION

"Siberian snakes" are 180° spin rotators using magnetic fields transverse to the beam direction to rotate the spin. The name, coined by E.D. Courant, refers to their invention in Novosibirsk by Ya.S. Derbenev and A.M. Kondratenko and to the shape of the beam path in these devices.[1] By flipping the spin each turn in a synchrotron, the depolarizing effects of non-vertical magnetic fields in the accelerator can be made to effectively cancel over two turns, thus eliminating depolarizing resonances in the lattice to a high degree.

Recently, Siberian snakes have passed an important milestone with the first successful experimental test of a snake at IUCF.[2] Also, several proposals for inclusion of Siberian snakes in existing or proposed synchrotrons have been made. [3, 4, 5, 6, 7, 8]

Since the first proposal of the Siberian snake, several snake designs have been published. As spin rotation angles in a transverse field are larger than in longitudinal fields by a factor of $\gamma G/(1+G)$ transverse bending magnets are much more efficient than solenoids at high energies. Here γ is the energy in units of the rest energy, and $G = (g-2)/2$ where g is the gyromagnetic ratio; $G = 1.7928$ for protons. However, as the spin precession angle in a transverse field varies only with the inverse velocity $1/\beta$ when measured as a function of position along the orbit, application in high energy accelerators requires a virtually constant magnetic field, and thus the central orbit in the device becomes energy-dependent. This leads to large orbit excursions that can reach tens of centimeters at low energies, necessitating large magnet apertures. Keeping orbit excursions small is therefore a major concern in the design of Siberian snakes.

A snake with very small orbit excursions is the helical snake. Originally proposed in 1978 by Ya.S. Derbenev and A.M. Kondratenko[9] it was recently re-invented by E.D. Courant.[3] In their work, these authors show that a helical magnetic field can lead to

180° spin flip, as well as how, by using helices with several twists, the orbit excursions can be kept small at only modest increase in overall length of the device. As the helical field leads to the central path being offset in one direction, orbit restoration dipoles are used at each end of the helix to create an overall straight-through Siberian snake.

Motivation for the work described in this paper arose from the desire to be able to use a snake in the 30-GeV driver synchrotron of the proposed TRIUMF KAON Factory (KAON stands for Kaons, Antiproton, Other particles, and Neutrinos).[8] At the 3 GeV injection energy the orbit excursions can be quite large, and only a snake of the helical type appears to be practical. This led us to the investigation of helical snakes, but rather than using the continuous field of a special helical magnet we examined the possibility of designing a helix using dipole magnets. It turns out that in this way a whole family of Siberian snakes can be constructed differing from each other not only in the the number of twists but also in the number of magnets used.

The paper is organized as follows. In Section 2 the equations for the helical field and the conditions for 180° spin flip are derived. Orbit restoration schemes are described in Section 3. In Section 4, examples of helical Siberian snakes are presented.

Throughout the paper a right-handed coordinate system is used, where x is the transverse horizontal and z the vertical direction, and y is the direction tangential to the reference orbit. We also define a coordinate system along the axis of the helix, $\tilde{x}, \tilde{y}, \tilde{z} = z$. A positive horizontal bending magnet bends to the right, and the roll angle of the magnetic field about the y axis is measured clockwise from the vertical axis. The particles are assumed to be relativistic protons with $\beta = 1$.

2. THE HELICAL FIELD

In the original proposal for the helical snake a corkscrew-like field was used, parameterized as

$$B_x = B \sin ky \tag{1}$$

$$B_z = B \cos ky \tag{2}$$

where $k = 2\pi M/L$ is the wave number; L is the length and M the number of twists in the helix. The field integral necessary for the total spin precession to be 180° is, for a helix with a full number of twists,

$$BL = \pi\sqrt{4M+1}\,\frac{m_p c}{e}\ , \tag{3}$$

where m_p is the proton mass, c the velocity of light, and e the unit charge. The orbit in the snake is also sinusoidal in both planes; *i.e.* for a given bending radius ρ the projection of the orbit on to the $\tilde{x} - z$ plane is circular in shape with a radius

$$r = \frac{1}{\rho k^2}\ . \tag{4}$$

For the dipole helix we approximate the helical field by a series of dipole magnets, which are rolled about the reference orbit. The rotation about the resulting axis is expressed by first transforming the coordinate system such that the rotation axis coincides with the z axis, then carrying out the rotation, and finally transforming back. In spinor algebra the spin transformation through the array of bending magnets is then expressed by the spin transfer matrix:

$$M_{helix} = \prod_{n=1}^{N} \exp\left(i\pi\frac{M(2n-1)}{2N}\sigma_y\right) \exp\left(i\frac{\alpha}{2}\sigma_z\right) \exp\left(-i\pi\frac{M(2n-1)}{2N}\sigma_y\right) , \quad (5)$$

where N is the total number of bending magnets in the helix, α the spin precession angle of each magnet, and $\pi M(2n-1)/N$ the roll angle of each magnet. The $\sigma_{x,y,z}$ are the Pauli spin matrices.

This product can be simplified by combining the roll of successive elements about the longitudinal axis to yield

$$M_{helix} = \exp\left(i\pi\frac{M}{2N}\sigma_y\right)\left[\exp\left(i\frac{\alpha}{2}\sigma_z\right)\exp\left(i\pi\frac{M}{N}\sigma_y\right)\right]^N \quad (6)$$

$$\times \exp\left(-i\pi M\sigma_y\right) \exp\left(-i\pi\frac{M}{2N}\sigma_y\right) . \quad (7)$$

We now introduce a new angle β, so that the expression in [ ] becomes

$$\exp\left(i\left(\vec{\sigma}\cdot\vec{b}\right)\beta\right) \quad (8)$$

with

$$\cos\beta = \cos\frac{\alpha}{2}\cos\pi\frac{M}{N} \quad (9)$$

and the unit vector $\vec{b}$ defined by

$$\frac{b_x}{b} = \frac{\sin\frac{\alpha}{2}\sin\pi\frac{M}{N}}{\sin\beta}$$

$$\frac{b_y}{b} = \frac{\cos\frac{\alpha}{2}\sin\pi\frac{M}{N}}{\sin\beta}$$

$$\frac{b_z}{b} = \frac{\sin\frac{\alpha}{2}\cos\pi\frac{M}{N}}{\sin\beta} \quad (10)$$

The matrix for the helix then becomes

$$M_{helix} = \exp\left(\frac{i}{2}\pi\frac{M}{N}\sigma_y\right)\left[\exp\left(i\left(\vec{\sigma}\cdot\vec{b}\right)N\beta\right)\right. \quad (11)$$

$$\left.\exp\left(-i\pi M\sigma_y\right)\right]\exp\left(-\frac{i}{2}\pi\frac{M}{N}\sigma_y\right) . \quad (12)$$

For 180° spin rotation the trace of the product in [ ] has to vanish, and

$$\cos N\beta\cos\pi M + \frac{b_y}{b}\sin N\beta\sin\pi M = 0 . \quad (13)$$

In the general case of integer N and real M ($N \in \mathbf{N}, M \in \mathbf{R}$), this equation cannot be solved in closed form for α. But we shall give closed solutions for important cases, namely full- and half-twist helices and quarter-twist helices with 4 magnets per twist.

2.1. *Full-twist helices*

For $M \in \mathbb{N}, \sin \pi M = 0$ and Eq. (13) is fulfilled by

$$\beta = \frac{2m+1}{2N}\pi, \qquad m \in \mathbb{N}. \tag{14}$$

Therefore the spin rotation in each magnet is

$$\cos\frac{\alpha}{2} = \frac{\cos\frac{2m+1}{2N}\pi}{\cos\frac{M}{N}\pi}. \tag{15}$$

This is a cosine-like function of m with a period of $2N$, so the smallest value for m that leads to the *rhs* being less than unity, which is $m = M$, will give the solution with the smallest α. Here we restrict ourselves to cases where $M < 2N$; for $M > 2N$ the beam does not describe a helical path but rather is deflected to one side.

2.2. *Half-twist snakes*

For the case $M = (2m-1)/2, m \in \mathbb{N}$, *i.e.* for half-twist snakes, $\sin \pi M = \pm 1, \cos \pi M = 0$ and Eq. (13) becomes

$$\frac{b_y}{b}\sin N\beta = 0, \tag{16}$$

which is solved by

$$\beta = \frac{m}{N}\pi, \tag{17}$$

and therefore

$$\cos\frac{\alpha}{2} = \frac{\cos\pi\frac{2M+1}{2N}}{\cos\pi\frac{M}{N}}, \tag{18}$$

i.e. the same function as Eq. (15) with $m = M$.

2.3. *Quarter-twist snakes*

For snakes with fractional twists of 1/4 or 3/4 $[M = (4m-2\mp 1)/4]$ Eq. (13) becomes

$$\cos N\beta \pm \frac{b_y}{b}\sin N\beta = 0, \tag{19}$$

so that

$$\cos N\beta \sin\beta \pm \tan\pi\frac{M}{N}\sin N\beta\cos\beta = 0. \tag{20}$$

To solve this equation, we restrict ourselves to helices with 4 magnets per twist, $M/N = 1/4$. Then $\tan \pi M/N = 1$ and

$$\sin(\beta \pm N\beta) = 0 \tag{21}$$

and, thus

$$\beta = \pi \frac{m}{1 \pm N} , \tag{22}$$

$$\cos \frac{\alpha}{2} = \frac{\cos \pi \frac{M+\frac{1}{2}\pm\frac{1}{4}}{N\pm 1}}{\cos \pi \frac{M}{N}} . \tag{23}$$

By defining $M' = M \pm 1/4$ and $N' = N \pm 1$ and noting that $M'/N' = M/N = 1/4$ this can be written in the form

$$\cos \frac{\alpha}{2} = \frac{\cos \pi \frac{2M'+1}{2N'}}{\cos \pi \frac{M'}{N'}} , \tag{24}$$

and $M' = (2m - 1)/2$, *i.e.* the same as M for half-twist helices. Surprisingly half- and quarter-twist helices with four dipoles per twist and the same number of full twists reach 180° spin rotation at the same value for $\cos \alpha/2$ (although the overall spin-rotation axes are different).

Therefore, 1/4-twist helices appear to be most efficient for 180° spin rotation. Full-twist helices do not fit into this scheme for although Eq. (15) is of the same form as Eq. (23) the relationship between M and m is different, leading to different values of $\cos \alpha/2$ for the same m.

2.4. *Hardware parameters*

Once the spin-rotation angle is found the length of the helix can be calculated from the field integral

$$BL = N\alpha \frac{m_p c}{e} . \tag{25}$$

In the limit $N \to \infty$, Eq. (25) converges to the equation for the continuous helix, Eq. (3), as can be shown by expanding the cosines on the *rhs* of Eq. (15) (with $m = M$) to 2^{nd}order:

$$\begin{aligned} \cos \frac{\alpha}{2} &\approx \frac{1 - \pi^2 \frac{(2M+1)^2}{8N^2}}{1 - \pi^2 \frac{M^2}{2N^2}} \\ &\approx 1 - \frac{1}{2} \frac{\pi^2 (4M+1)}{4N^2} . \end{aligned} \tag{26}$$

The *rhs* is just the beginning of the Taylor series of the cosine function, and we can therefore write

$$N\alpha \approx \pi\sqrt{4M+1}, \tag{27}$$

and

$$BL \approx \pi\sqrt{4M+1}\frac{m_p c}{e}. \tag{28}$$

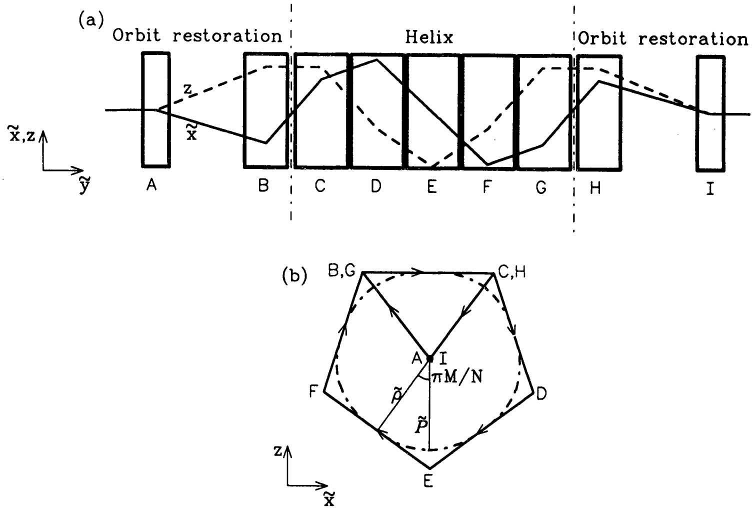

FIGURE 1 Orbit in a dipole helix along the axis (a) and projected on to the $\tilde{x} - z$ plane (b), for thin bending magnets. In Fig. (a) the boxes represent the relative sizes of the dipoles. In Fig. (b) the dot-dashed line indicates the projection for long magnets; sans-serif letters indicate the positions of the magnets.

Thus, the length of the helix is shown to increase only with $\sqrt{M}$, and for this reason a larger number of twists can be used economically to reduce the orbit excursions, as will be shown in Sec. 2.5.

The orientation of the helix about the longitudinal axis determines the direction of the spin-precession axis. The axis will be in the horizontal plane as required for a snake if the vertical field components are symmetric about the center of the helix (cf. Reference 3). The roll angle of the first snake dipole, θ_r, can be determined by finding the amount of rotation of the helix about its axis that sets the vertical component of the rotation axis to zero;

$$\theta_r = \pi \left(\frac{M}{N} - M - \frac{1}{2} \right) . \tag{29}$$

This orientation makes the orbit symmetric in the vertical plane and antisymmetric in the horizontal plane.

2.5. *Orbit excursions*

For the dipole helix the projection of the orbit on to the $\tilde{x} - z$ plane transverse to the central $\tilde{y}$-axis is no longer exactly circular but has N/M-fold symmetry. In the limit of thin bending magnets the shape would be polygonal as shown in Fig. 1. The circular orbit in each dipole projects into an elliptical arc. The radius of the circumscribing circle gives the maximum orbit excursion in the snake.

The orbit in a vertically bending dipole (*e.g.* magnet "E" in Fig. 1) and its projection

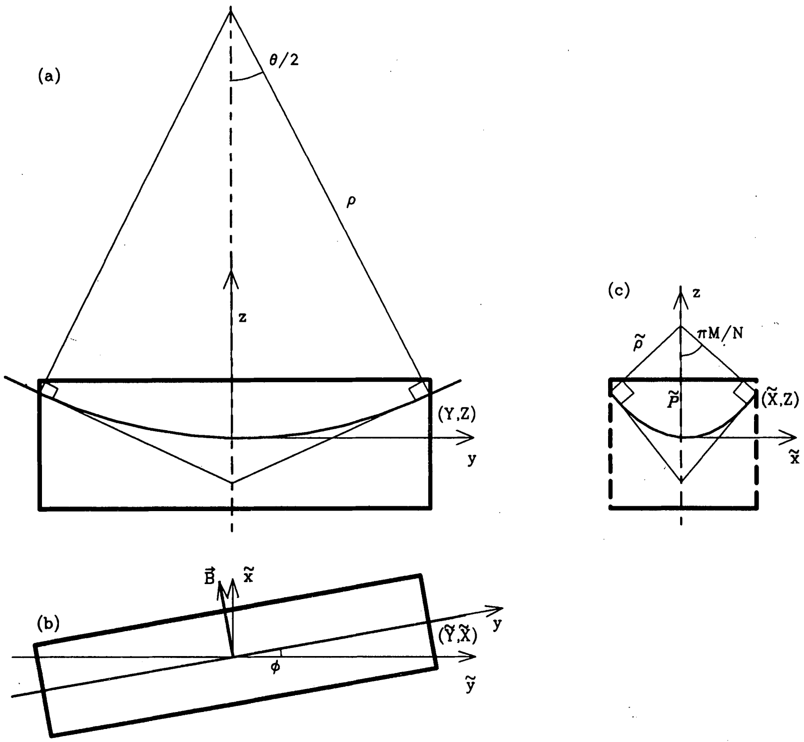

FIGURE 2 Orbit in one bending magnet (a) and its projections on to the $\tilde{x} - \tilde{y}$ plane (b) and $\tilde{x} - z$ plane (c). The vector $\vec{B}$ in (b) indicates the direction of the magnetic field.

are shown in Fig. 2. The orbit coordinates at the end of the magnet are given by

$$\begin{aligned} X &= 0, \\ Y &= \rho \sin \frac{\theta}{2}, \text{ and} \\ Z &= \rho \left(1 - \cos \frac{\theta}{2}\right). \end{aligned} \tag{30}$$

In order to follow the reference orbit the dipole is tilted w.r.t. the central axis of the helix by the angle

$$\phi = \arcsin \frac{\tan \frac{\theta}{2}}{\tan \frac{M}{N} \pi} \tag{31}$$

about the z axis. The projection equation is therefore

$$\tilde{X} = Y \sin \phi = Y \frac{\tan \frac{\theta}{2}}{\tan \frac{M}{N}\ pi} \tag{32}$$

and Eq.(30) becomes

$$\tilde{X} = \tilde{\rho} \sin \pi \frac{M}{N} \tag{33}$$

so that

$$\tilde{\rho} = \frac{Y \tan \frac{\theta}{2}}{\sin \pi \frac{M}{N} \tan \pi \frac{M}{N}} \approx \frac{L \tan \frac{\theta}{2}}{2N \sin \pi \frac{M}{N} \tan \pi \frac{M}{N}} \tag{34}$$

for small θ. By the symmetry of the orbit this is the radius of the inscribing circle.

The radius of the circumscribing circle, $\tilde{P}$, is given by

$$\tilde{P} = \tilde{\rho} \cos \pi \frac{M}{N} + Z \cdot = Y \left(\frac{\tan \frac{\theta}{2}}{\tan^2 \pi \frac{M}{N}} + \tan \frac{\theta}{4} \right) \tag{35}$$

$$\approx \tilde{\rho} \left(1 + \frac{2 \sin^4 \pi \frac{M}{2N}}{1 - 2 \sin^2 \pi \frac{M}{2N}} \right) \tag{36}$$

again for small θ. Thus, for $M/N = 1/4$ the ratio $\tilde{P}/\tilde{\rho}$ is about 1.06 and approaches unity for small M/N. We can then write, using Eq. (27),

$$\tilde{P} \approx \tilde{\rho} \approx \frac{L\pi\sqrt{4M+1}}{4\gamma G N^2 \tan \pi \frac{M}{N} \sin \pi \frac{M}{N}} \,. \tag{37}$$

For M/N at a constant value, this shows that the orbit radius will decrease approximately as $1/M$ as L increases with $\sqrt{M}$. Keeping M at a fixed value the orbit radius becomes, for $N \to \infty$,

$$\tilde{\rho} \approx \frac{L\sqrt{4M+1}}{4\pi M^2 \gamma G} \tag{38}$$

with the same general dependence on M. Eq. (38) agrees with Eq. (4) for the continuous helix, as can be verified easily. The advantage of multi-twist snakes lies in the orbit excursions being proportional to $1/M$, while the increase in length is only proportional to $\sqrt{M}$. At the same time the number of magnets per twist has only limited effect on the orbit excursions. We can therefore expect helices with 3 or 4 magnets per twist to be efficient devices in terms of orbit excursions as well as in terms of the field integral.

3. ORBIT RESTORATION

Orbit restoration is necessary because the helix itself is not a straight-through device, but rather offsets the beam in one plane. Dipole magnets have to be used on either side of the helix to restore position and angle of the reference orbit outside the helix. If this is done in such a way that the largest orbit excursion to either side is $\tilde{P}$, the orbit excursion is minimized and we call the snake "orbit centered". Two dipoles on either side are needed for this purpose. In addition an optimized orbit restoration system should avoid orbit excursions and magnetic fields in the orbit restoring dipoles larger than the corresponding values in the dipoles of the helix. An orbit restoring system that fulfills these requirements has the first orbit restorer at a center-to-center distance of $2L/N$ from the second orbit restorer, which in turn is at a distance L/N ahead of the first dipole of the helix (c.f. Fig. 1).

It turns out that the analytical treatment of the orbit restoration system is extremely tedious for long bending magnets. The equations given in the following are therefore approximate, using thin dipoles. This is justified as higher-order deviations of the reference orbit lead to corrections to the setting of the orbit restorers that are of the same order of magnitude as the approximations made in deriving these formulae.

The bending angle of the first restorer is

$$\theta_1 = \arctan \frac{\tilde{\rho} N}{2L \cos \pi \frac{M}{N}} \tag{39}$$

in order to achieve the necessary offset of the orbit at the second restorer. Its roll angle is

$$\theta_{r,1} = -\pi \left(\frac{M}{N} + M + \frac{1}{2} \right) ; \tag{40}$$

i.e. the magnet has the same orientation as a hypothetical 0^{th} helix dipole but opposite field polarity.

The second orbit restorer cancels the bending angle from the first and, in addition, matches the beam angle to the entrance angle of the reference orbit w.r.t. the axis of the helix, which is

$$\theta_i = \frac{\phi}{\cos \pi \frac{M}{N}} . \tag{41}$$

Observing that the directions defined by these angles are at an angle $\pi(1/2 - M/n)$ w.r.t. each other we can calculate the bending angle of the second restorer,

$$\theta_2 = \frac{1}{\cos \pi \frac{M}{N}} \sqrt{ \left(\arcsin \frac{\tan \frac{\theta}{2}}{\tan \pi \frac{M}{N}} + \theta_1 \sin \pi \frac{M}{N} \cos \pi \frac{M}{N} \right)^2 + \theta_1^2 \cos^4 \pi \frac{M}{N} } , \tag{42}$$

and its roll angle,

$$\theta_{r,2} = \arctan \frac{\theta_1 \cos^2 \pi \frac{M}{N}}{\arcsin \frac{\tan \frac{\theta}{2}}{\tan \pi \frac{M}{N}} + \theta_1 \sin \pi \frac{M}{N} \cos \pi \frac{M}{N}} - \pi \left(\frac{2M}{N} + M + \frac{1}{2} \right) . \tag{43}$$

Being located at a center-to-center distance of L/N before the first helix dipole, the orbit excursion in this magnet does not exceed $\tilde{P}$. In this scheme both orbit restoring dipoles have less bending angle than the dipoles of the helix for 4 or less magnets per twist. For snakes with more magnets per twist the distance of the first orbit restorer may have to be increased as this reduces the angles in both orbit restoring dipoles. Orbit restoration on exit from the helix is obtained from the entrance restoration by reversing the order of magnets and inverting all bending about the vertical axis.

Given that the spin-rotation axis of the helix does not in general coincide with the longitudinal axis the orbit restoration can change the spin-rotation angle of the snake. It is tedious and not particularly instructive to multiply the matrices for the helix and the orbit restoration. With a matrix multiplication program, however, it can be seen very quickly that the effect of the orbit restoration discussed here on the spin-rotation angle is only a few % in most cases. Only for snakes with less than 1 twist was orbit restoration found to lead to deviations of 10–20%. This deviation from 180° is compensated by slightly changing the setting of all dipoles. Orbit restoration also affects the axis of spin rotation of the snake; in fact, it invariably rotates the axis to within a few degrees of the longitudinal direction. As described here, helical snakes are always nearly Siberian snakes of the first kind.

For full-twist helices, a simpler orbit correction scheme using only one horizontal bending magnet at each end of the helix to match the entrance and exit angles is possible. In this case vertical orbit excursions are to one side only, leading to twice the maximum orbit excursion in that plane.

For a realistic dipole at some distance to the helix, the angle has to be reduced slightly from the theoretical value leading to a tilt of the axis of the helix against the straight line connecting the incoming beam with the outgoing beam. This scheme does not change the spin-rotation angle of the snake; it does, however, cause the axis of rotation to be longitudinal just like the scheme described previously.

The equations given above do not take into account the higher-order deviations of the central orbit due to the tilt of the bending magnets. This will result in slightly different magnet settings for the orbit restorers. Also, due to the space needed between magnets the field in the magnets will have to be higher than the "effective" field used in the equations in this paper for the same length of the snake.

4. EXAMPLES

In this section we will present parameters for some helical snakes suitable for specific applications. For each example a numerical calculation was carried out taking into account the gaps necessary between magnets and also higher-order corrections to the central orbit. The parameters obtained from the numerical calculation deviate somewhat from those obtained using the above formulae for these reasons. These examples serve to illustrate the range of properties of helical snakes that can be achieved. We shall name the snakes according to the projection of the central orbit for hypothetical thin bending magnets, i.e., "triangular" snakes, "square" snakes, etc.

4.1. Snakes for very-high-energy machines

According to Section 2, the simplest "helix" is a "square" helix with $1/4$ twist and one dipole, at 90° roll angle. Not surprisingly, the spin-rotation angle in this magnet is 180°. Orbit correction adds 4 dipoles to the array, so the snake has 5 dipole magnets in total. Although M is only $1/4$ the orbit excursions remain small at very high energies. For 3-T magnets, the orbit radius $\tilde{P}$ as given by Eq. (35) is about 0.65 mm at 2 TeV, the SSC injection energy.[10]

The field integral for the "helix" is 5.5 Tm; the orbit restoration systems add 12.6 Tm to this value. Although this is technically speaking an orbit-centered snake; at only $1/4$

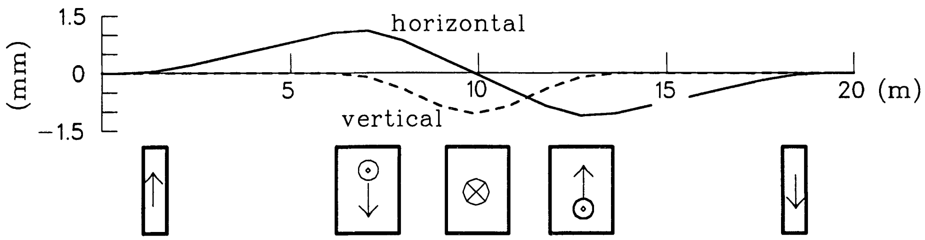

FIGURE 3 Orbit excursions and magnet array for a $\frac{1}{4}$- twist snake for applications at very high energies.

of a twist the beam does not describe a full turn in the helix, and the orbit excursion is therefore asymmetric. Fig. 3 is a diagram of the magnet array of the snake and the numerically calculated reference orbit. A free space of 1 m was left between the 1.85-m-long helix dipoles. Due to the orbit restoration system the bending angle has to be increased by 8% from the analytical values in all dipoles for an overall 180° spin rotation, bringing the total field integral up to 19.5 Tm. Because of this and the space between magnets the orbit excursions have increased to 1.1 mm. The total length is 18.8 m.

A similar snake with only 5 magnets can be constructed using a helix with one twist and three magnets and the simple orbit restoration involving only one magnet at each end. Again, the spin rotation in each magnet of the helix has to be 180°. For the same field as above $\tilde{P}$ is 0.4 mm at 2 TeV, because the orbit is offset to one side the excursions are given by $2\tilde{P}$. Such a snake has already been proposed by Ya.S. Derbenev and A.M. Kondratenko in 1977;[9] it is identified here as a member of the family of helical snakes.

4.2. *A Siberian snake for the TRIUMF KAON Factory*

The proposed TRIUMF KAON Factory[8] consists of the existing 500 MeV TRIUMF cyclotron, a 3 GeV booster synchrotron cycling at 50 Hz, and a 30 GeV "driver" synchrotron cycling at 10 Hz. The system will provide proton beams of up to 100μA at 30 GeV energy. In addition, an optically pumped polarized ion source is being commissioned that is currently delivering about 5μA of H^- ions extracted from the cyclotron. Due to its fast cycling rate, depolarization in the Booster will be very small, less than 10%. For the driver synchrotron, the use of a Siberian snake is more attractive than resonance correction. However, a Siberian snake for the KAON Factory driver will need to have reasonable orbit excursions at the injection energy of 3 GeV.

Of the many possible designs, "square" snakes have the advantage of having only two bend planes, tilted at 45° for full- and half-twist snakes but straight for quarter-twist snakes. This makes the quarter-twist snake more attractive. As stated in Sec. 2.3, snakes with a one-quarter fractional twist are more economical and therefore preferable over three-quarter-twist snakes.

The beam size in the snake will be about 5 cm. In order to keep the orbit excursions at the same value as the beam size, $3\frac{1}{4}$ or $4\frac{1}{4}$ twists would be needed. This would result

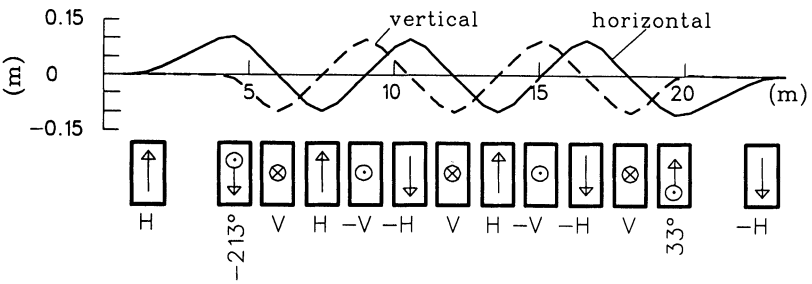

FIGURE 4 Orbit excursions and magnet array for a $2\frac{1}{4}$- twist snake for the TRIUMF KAON Factory.

in 17 or 21 quite short magnets, which is not cost-effective. A more practical design is a $2\frac{1}{4}$-twist snake with 13 magnets of about 1 m length each, having 2 T magnetic field. The orbit radius $\tilde{P}$ is then 6.4 cm at 3 GeV, and the snake is orbit-centered. Fig. 4 shows the orbit obtained from the numerical calculation and the magnet array of the snake. The length of the snake is 22.5 m.

The maximum orbit excursion has increased to 10 cm because of the 0.5-m space between each magnet. The total field integral is 23.6 Tm. Due to the orbit restoration system the magnet setting is 2.7% higher than given by the analytical calculation.

Such a snake was successfully matched into the straight section of the driver lattice. Transforming sections at each end facilitate matching of the lattice functions while maintaining the phase advance of the section. The matching conditions vary with energy requiring the transformers to be programmed. Figure 5 shows the lattice functions at 3 GeV.

5. CONCLUSION

The scaling properties of helical snakes make it possible to trade a small increase in length of the snake for a large relative decrease in orbit excursions. This facilitates the design of snakes with orbit excursions sufficiently small for use in synchrotrons with moderate injection energy of a few GeV. As demonstrated in the example of a snake for the proposed TRIUMF KAON Factory, reasonable snakes using conventional bending magnets can be designed using the formulae derived in this paper. On the other hand, trading orbit excursion for length and simplicity, fairly compact snakes with only 5 dipoles can be designed that are suitable for machines with high injection energies where orbit excursions are of no concern.

ACKNOWLEDGEMENT

The author would like to thank Prof. M.K. Craddock for carefully reading the manuscript and in particular for his suggestions regarding the orbit calculation.

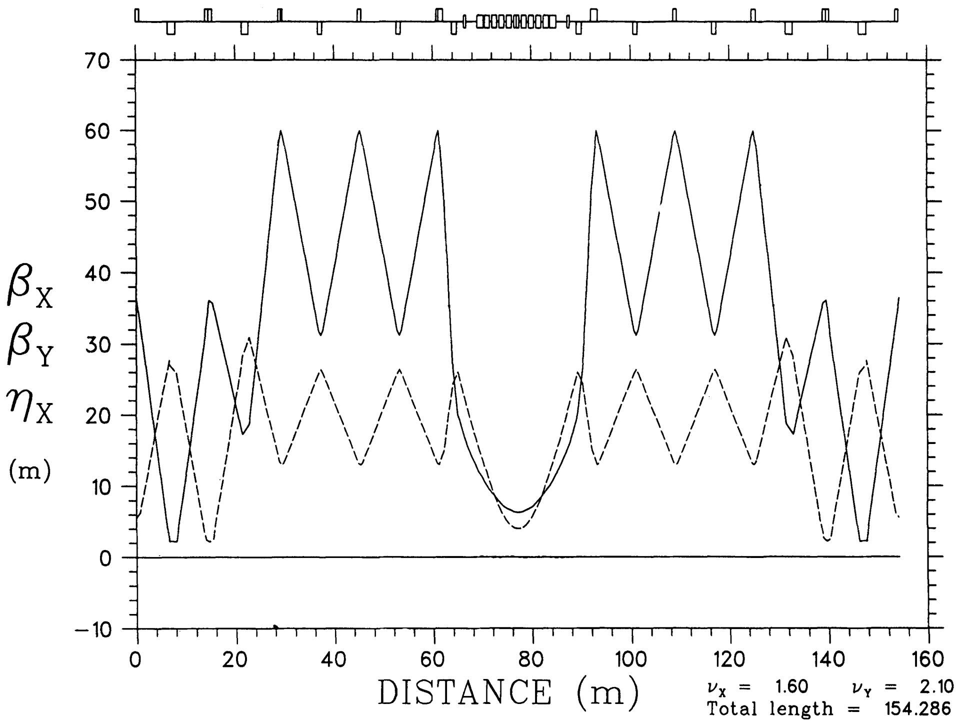

FIGURE 5 Lattice functions of the straight section of the KAON Factory Driver synchrotron with the snake, at 3 GeV.

REFERENCES

1. Ya.S. Derbenev and A.N. Kondratenko, *in Proceedings of the 10th Conf. on High Energy Accelerators*, Protvino, USSR, 70 (1977), and Ya.S. Derbenev and A.N. Kondratenko, *Part. Accel.* **8**, 115 (1978).
2. A.D. Krisch *et al.*, *Phys. Rev. Lett.* **63**, 1137 (1989).
3. E.D. Courant, *EHF Internal Report 86-07 and 87-09* (Mainz, F.R.G., 1986 and 1987, unpublished), and E.D. Courant, *Proc.* 8^{th} *Int. Symposium on High Energy Spin Physics*, Minneapolis, MN, (1988), p. 1085*ff.*
4. J.F. Crawford, Ed., *Proposal for a European Hadron Facility* (Mainz, Germany, May 1987, unpublished).
5. U. Wienands, *EHF Internal Report 87-10* (Mainz, Germany, 1987, unpublished).
6. K.G. Steffen, *DESY HERA 87-11* (Hamburg, Germany, 1987, unpublished).
7. A. Pisent, *AHF Technical Note 88-015* (Los Alamos, NM, 1988, unpublished).
8. *KAON Factory Study Accelerator Design Report* (TRIUMF, Vancouver, BC, Canada, 1990, unpublished).
9. Ya.S. Derbenev and A.N. Kondratenko, *in Proceedings of the Int. Conf. on High Energy Physics with Polarized Beams and Polarized Targets*, (Argonne, IL, USA, 1978) 292*ff.*
10. SSC Central Design Group, *Conceptual Design of the Superconducting Super Collider, SSC-SR 2020* (Berkeley, CA, 1986).