Quantum gravity as a low energy effective field theory

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Contents

1 Effective Field Theory
1.1 Quantum Gravity does exist
1.2 What is effective field theory and how does it work?
1.3 An example of an effective field theory
1.4 The rules of effective field theory
2 General Relativity as an Effective Field Theory
2.1 How to do valid quantum corrections in gravity
3 Some low energy theorems of quantum gravity
3.1 The gravitational potential
3.2 Graviton-graviton scattering
3.3 The bending of light
4 Limitations and Frontiers
5 References
6 Pedagogic material

Effective Field Theory

An Effective Field Theory is one which uses only the active degrees of freedom available at some energy. A full quantum field theory treatment is applied. When done properly, the results encode the quantum corrections appropriate to that energy. The perturbative treatment of quantum General Relativity behaves as an effective field theory, and well defined quantum corrections can be calculated. This review discusses effective field theory and its application to general relativity.

Quantum Gravity does exist

The problem of quantum gravity is not what we once thought it was. If you go back to early references, you will find many statements saying that general relativity and quantum mechanics are incompatible, that the combination of general relativity and quantum field theory produces a meaningless theory. It is understandable that some of the pioneers may have thought in this way, as some of our quantum methods are awkward when dealing with general relativity. However, these statements are not correct.

Feynman properly quantized general relativity in the early 1960's (Feynman, 1963), and the formalism was soon put on a solid footing by DeWitt (DeWitt, 1967). Path integral methods prove to be well suited for a theory like general relativity, and the divergence structure of the theory was soon explored (see in particular ('t Hooft and Veltman 1974)). In the meantime, effective field theory methods were developed and applied to other theories, many of which were technically labeled "non-renomalizeable". These techniques apply very straightforwardly to general relativity and they shift the focus from high energy to low energy where the theory is more reliable. We now can see that quantum general relativity has the form of a quantum effective field theory when treated at ordinary energies and scales.

An analogy may be useful. In solids the basic building blocks are atoms and molecules. However, the lowest energy excitations in the system are sound waves. If one is interested in a quantum theory at the lowest energies, one uses the usual quantum rules to quantizes the sound waves as phonons. One can use the quantum theory of phonons to predict thermal properties of solids. However, one does not need to assert that phonons are fundamental - there is no such thing as a phonon field at wavelengths shorter than the inter-atomic distances. We use these degrees of freedom only at the low energies that they are valid.

Likewise we know that gravitational waves exist. The usual quantum rules quantize these as gravitons. General relativity tells us the low energy interactions of the gravitons. At low energies, these quantum degrees of freedom have calculable effects. Of course in a quantum field theory there are also generally divergences from the highest energies. These are not reliable predictions of gravitons, because we do not know if general relativity is the correct high energy theory. But there are effective field theory techniques, discussed below, to separate the unreliable high energy portions from the reliable low energy ones. There can be meaningful predictions at low energy.

We now understand that the quantization of general relativity is not the issue, and we can make some quantum predictions. This is progress. Nevertheless, there remains a problem of quantum gravity. Even the effective field theory points to its own breakdown at high energies or large curvatures. We hope for a more complete theory which is valid at all scales and which reduces to general relativity at low energies. Such efforts occupy much of the effort in quantum gravity. However it is also important to understand the structure and predictions of the quantum theory in effective field theory region.

What is effective field theory and how does it work?

Effective field theory uses only the degrees of freedom that are active at the energy that one is working at. Heavier particles, if known, are integrated out from the theory. Of course, since physics is an experimental science, we may not even know what the heavier particles are or how they interact with the light particles. How then can we make predictions without knowing the full theory?

The answer involves the uncertainty principle. The effects of high energy particles appear to be local when viewed at low energies. These particles cannot propagate far, and so the exchange of high mass particles occurs over a very short distance. The high mass particles can also generate loop effects, including divergences, in the couplings of light fields. But these effects also appear as the renormalization of some parameter in a local Lagrangian. So one accounts

for both known and unknown high energy effects by writing the most general local Lagrangian consistent with the symmetries of the theory. If the high energy effects are known, one can determine the coefficients of the terms in the Lagrangian by direct calculation. If they are unknown, one has to leave the coefficients as free parameters to be determined by experiment.

In contrast, the light particles in the theory can propagate long distances. They must be treated dynamically. So the local Lagrangians are written in terms of the light fields, and one undertakes a full quantum treatment.

The second organizing principle is the energy expansion. A completely general Lagrangian can have an infinite number of terms. For example if some term in a Lagrangian is invariant under the relevant symmetry, the square of that term and all subsequent powers are also invariant. However, as one increases the mass dimension in an operator basis, the higher dimensioned ones appear in the Lagrangian with inverse powers of a mass, which in practice would be inverse powers of the heavy mass. The idea of an energy expansion is to order the general Lagrangian in powers of the energy, such that the lowest order terms are the ones which are most important at low energy. This allows one to focus only on the important interactions, and the predictions will also come out ordered in an energy expansion.

An example of an effective field theory

A very good example is the transition of the linear sigma model to the non-linear version at low energies. This also displays features relevant for general relativity. The linear sigma model is a renormalizeable field theory defined by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\right] + \frac{\mu^2}{2}(\sigma^2+\vec{\pi}\cdot\vec{\pi}) - \frac{\lambda}{4}(\sigma^2+\vec{\pi}\cdot\vec{\pi})^2. \quad (1)$$

Here, σ and π are the fields of the theory, and the metric signature is $(+,-,-,-)$. After spontaneous symmetry breaking, the σ field is seen to be massive, while the pions are massless Goldstone bosons.

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma} - 2\mu^2\tilde{\sigma}^2\right] + \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \lambda v\tilde{\sigma}(\tilde{\sigma}^2+\vec{\pi}\cdot\vec{\pi}) - \frac{\lambda}{4}(\tilde{\sigma}^2+\vec{\pi}\cdot\vec{\pi})^2 \quad (2)$$

where $v = \sqrt{\mu^2/\lambda}$ and $\sigma = v + \tilde{\sigma}$. h At low enough energy, the pions will be the only active degrees of freedom and we seek a Lagrangian which describes the low energy interactions of the pions. One can show by direct construction that this has the form

$$\mathcal{L}_{eff} = \frac{v^2}{4}\,\mathrm{Tr}\left(\partial_\mu U\partial^\mu U^\dagger\right) \qquad \text{with} \qquad U = \exp(i\vec{\tau}\cdot\vec{\pi}/v), \quad (3)$$

where $\vec{\tau}$ are the SU(2) Pauli matrices. This new form is somewhat amazing in that the original theory has polynomial interactions without derivatives, and the effective Lagrangian only has derivative interactions. But the actual transformations from the full theory to the effective theory are simple to accomplish directly (Donoghue, Golowich, Holstein 2014). One can check that the pionic scattering amplitudes from both versions are the same.

For our purposes here, the important point is that the effective Lagrangian involves the light degrees of freedom only. We can also see the beginning of the energy expansion. The leading Lagrangian involves two derivatives. There are also invariants with four derivatives

$$\mathcal{L} = \ell_1\left[\mathrm{Tr}\left(\partial_\mu U\partial^\mu U^\dagger\right)\right]^2 + \ell_2\mathrm{Tr}\left(\partial_\mu U\partial_\nu U^\dagger\right)\mathrm{Tr}\left(\partial^\mu U\partial^\nu U^\dagger\right). \quad (4)$$

At low enough energies, these will be small corrections to the the leading term. However, in order to reproduce the energy dependence of the pion scattering amplitudes we need these terms also. Of course, yet higher derivatives also eventually are relevant. But at low energies, we can use the energy expansion to order the effective Lagrangian such that we need keep only a small number of terms when working to a given accuracy.

Like general relativity, the effective Lagrangian describes a "non-renormalizeable" theory, in that all powers of the pion field are involved. However this does not hinder the process of quantizing, renormalizing and making predictions. One quantizes by starting with the low energy effective Lagrangian and identifying the quanta - the pions - and their low energy interactions. Loop diagrams produce divergences, which preserve the symmetry and which are equivalent to local effects. In particular one loop divergences can be absorbed by renomalization of the next order parameters ℓ_1 and ℓ_2. The historical phrase "non-renomalizeable" has proven to be misleading as we can directly renormalize the theory. There is a power counting theorem (Weinberg, 1979), which says that one loop effects enter at order E^4, two loops at order E^6, etc. The renormalized parameters are not predictions of the effective theory. In the case of the linear sigma model they can be matched to renormalized parameters of the original theory. After renormalization, loop diagrams can be used to make real predictions, such as pion scattering to one-loop. There is an extensive phenomenology of the equivalent effective field theory for QCD - chiral perturbation theory (Gasser and Leutwyler, 1984).

The rules of effective field theory

Let us attempt a quick summary of the rules of effective field theory:

1) Using only the low energy particles and symmetries of the theory, one constructs the most general Lagrangian consistent with the symmetries, and orders it in an energy expansion.

2) Using the leading Lagrangian at low energy, one quantizes the low energy particles.

3) When calculating loops, the divergences will look like the effect of some local Lagrangian in the most general construction. Renormalization is accomplished by absorbing the divergences into the coefficients of this local Lagrangian.

4) Match or measure. This phrase implies that one determines the renormalized coefficients of the effective Lagrangian either by matching the results to the full theory (if known) or measuring them (if the full theory is not known or not readily calculable).

5) Make predictions order by order in the energy expansion.

General Relativity as an Effective Field Theory

General relativity fits naturally into this effective field theory framework (Donoghue, 1994). Let us go through the various steps.

The basic field is the metric. The symmetry is general coordinate invariance. We use these to construct the most general Lagrangian consistent with the symmetries. The connection

$$\Gamma_{\alpha\beta}^{\ \lambda} = \frac{g^{\lambda\sigma}}{2}\left[\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right] \tag{5}$$

has one derivative of the metric, while the curvatures such as the Riemann tensor

$$R_{\mu\nu\alpha}^{\ \ \ \beta} = \partial_\mu \Gamma_{\nu\alpha}^{\ \ \beta} - \partial_\nu \Gamma_{\mu\alpha}^{\ \ \beta} - \Gamma_{\mu\lambda}^{\ \ \beta}\Gamma_{\nu\alpha}^{\ \ \lambda} - \Gamma_{\nu\lambda}^{\ \ \beta}\Gamma_{\mu\alpha}^{\ \ \lambda} \tag{6}$$

have two. The various contractions of the Riemann tensor are coordinate invariant. The Lagrangian then consists of powers of the curvatures.

The fact that the curvatures have two derivatives, and derivatives correspond to powers of the energy when matrix elements are taken, allows us to order these in an energy expansion. Higher powers of the curvatures are at higher orders in the energy expansion. We then arrive at

$$S_{grav} = \int d^4x\,\sqrt{g}\left[-\Lambda - \frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \ldots + \mathcal{L}_{matter}\right]. \tag{7}$$

Here the terms have zero, two and four derivatives respectively. We can add matter terms to the action also, and a related energy expansion will occur in the matter sector also.

The identity of the first two terms are well known. The first is the cosmological constant. Effective field theory does not have anything special to say about the value of the cosmological constant. Like other parameters, it is not predicted by the effective field theory but must be measured. It is so small on ordinary scales that I will drop it from the discussion below. The parameter κ is related to Newton's constant $\kappa^2 = 32\pi G$. The parameters c_1, c_2 are dimensionless. They are essentially unconstrained, $c_i \leq 10^{+64}$, because the effects of the curvature-squared terms are so tiny at low energy.

Using the Einstein Hilbert action, the most beautiful quantization procedure is the background field method using path integrals, as originally accomplished by ('t Hooft and Veltman, 1974). Renormalization also proceeds straightforwardly. As advertised, the divergences are local, with the one loop effect being equivalent to

$$\Delta\mathcal{L} = \frac{1}{16\pi^2}\,\frac{2}{4-d}\left[\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right], \tag{8}$$

using dimensional regularization in d dimensions. The divergences can then be easily absorbed into renormalized values of the coefficients $c_{1,2}$. The fact that these occur at the order of four derivatives can be seen by counting powers of κ and is completely equivalent to the energy expansion of the linear sigma model described above.

Pure gravity is one-loop finite. This is because the equations of motion for pure gravity (not including the cosmological constant) is $R_{\mu\nu} = 0$, so that both of the divergent counter terms vanish when treated as a perturbation. This is an interesting and useful fact, although in the real world it does not imply any special finiteness to the theory because in the presence of matter the counterterms are physically relevant.

How to do valid quantum corrections in gravity

The divergences described in the previous section are not reliable predictions of the effective field theory. They come from the high energy portion of the theory, which we expect to be incorrect. However, the low energy content of the theory is the part that is describe well by general relativity. Therefore we need to change our focus from the UV to the IR and isolate the low energy quantum effects.

Here again the uncertainty principle comes to our rescue. If high energy effects are necessarily local, those of low energy will in general be non-local. That is, at low energy fields propagate long distances. Since we normally do our calculations in momentum space, we are not used to thinking about local vs non-local in QFT. But the effective Lagrangians make this easy. Since they catalog the most general local Lagrangian, anything found in a specific calculation which can be differentiated from the effective Lagrangian is a non-local effect. In particular there are some noteworthy signals. Non-analytic behavior in scattering amplitudes, such as $\log(-q^2)$ or $\sqrt{-q^2}$ are always associated with long distance propagation of massless fields - these terms can never arise from local Lagrangians. So the non-analytic momentum dependence often plays a special role in low energy predictions.

In practice, one calculates with the Feynman rules, renormalizes the parameters through matching or measuring, and the remaining aspects of the calculations are predictions at low energy. Many field theory calculations in the literature fit into this description without mentioning the name of effective field theory. In these cases, one is simply doing field theory using general relativity and looking for finite effects at low energy. However, effective field theory is a way of thinking about field theory that allows us to better understand what is calculable in a theory like quantum general relativity.

Some low energy theorems of quantum gravity

The phrase "low energy theorem" is used to describe a prediction that must be true at low energy independent of the structure of the high energy structure of the theory. Low energy theorems for General Relativity have been known for tree amplitudes (Weinberg, 1964) (Weinberg, 1965) (Gross and Jackiw, 1968) but also can be found for quantum loop processes. In gravity we do not know the high energy structure. But still we can make low energy theorems because at the lowest energies the massless particles are gravitons and their interactions are those of general relativity. We just need to make sure that the observable itself is sensitive to only these properties. As mentioned above, this is easily accomplished because the high energy effects are contained in the local coefficients, c_i, so that quantum corrections independent from these parameters come from low energy.

The gravitational potential

The leading quantum correction to the gravitational potential is a low energy theorem independent of the ultimate high energy theory. This example lets one see the nature of the energy expansion. Because $\kappa^2 \sim G \sim 1/M_P^2$, the loop corrections will involve higher powers of the momentum, which in this case is the transverse momentum q^2. The types of corrections that are expected (and found) are

$$-i\mathcal{M} \sim \frac{GMm}{q^2}\left[1 + aG(M+m)\sqrt{-q^2} + bGq^2\log(-q^2) + cGq^2\right]. \tag{9}$$

Here we see the expansion in the energy. One also sees the non-analytic corrections which can only come from long range propagation. Fourier transforming tells us that the corresponding results in coordinate space are

$$\frac{1}{q^2} \sim \frac{1}{r}\,, \qquad \frac{1}{q^2}\times\sqrt{q^2} \sim \frac{1}{r^2}\,, \qquad \frac{1}{q^2}\times q^2\ln q^2 \sim \frac{1}{r^3}\,, \qquad \frac{1}{q^2}\times q^2 \sim \delta^3(x). \tag{10}$$

The analytic term $[Gq^2]/q^2 \sim$ const. gives a purely short distance effect - the delta function. All of the local terms in the Lagrangian, and all divergences, have to go into this analytic term. This tells us that the unknown high energy modifications of quantum gravity go into the delta-function term. The non-analytic ones give long distance effects. These are necessarily finite, because the divergences were local. Thus these power law corrections are pure predictions from the effective field theory and cannot be modified by any high energy completion to gravity.

The most direct calculation is that of the scattering potential. The scattering amplitude is calculated in momentum space, and in the non-relativistic limit is can be Fourier-transformed to generate a potential. (One can alternatively define a different but related potential using the Schwinger-Keldysh or "in-in" formalism (Park and Woodard, 2010). An early treatment of the gravitational potential is found (Radkowski, 1970).) The result for the potential of gravitational scattering of two heavy masses is (Bjerrum-Bohr, Donoghue and Holstein, 2003) (Khriplovich and Kirilin, 2002)

$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]. \tag{11}$$

where the last term is the quantum correction and the term preceding that is the classical post-Newtonian correction. The non-analytic terms in momentum space have become power-law dependence in coordinate space. The square-root non-analyticity is associated with the classical correction to the potential, while the logarithm gives the quantum correction. It is interesting that the classical correction arises from a loop diagram (Holstein and Donoghue, 2004). As expected the quantum correction is finite and independent of any unknown parameters. It is a demonstration of how a non-renormalizeable effective field theory can make meaningful predictions.

This example also can be used to demonstrate a soft theorem of a different sort. Not only is the quantum correction calculable but it is universal. We know of universal soft theorems for tree amplitudes. For example in both QED and in gravity, the low-energy Compton amplitude is universal and independent of spin of the particle. However, modern calculational methods allow one to extend this universality to loop level. These unitarity-based methods build loop corrections out of products of tree amplitudes. The tree amplitude universality then extends to the low energy behavior of loops, such that the quantum correction is universal. This was first found by Holstein and Ross by direct Feynman diagram calculation (Holstein and Ross, 2007), and then connected to the tree amplitude universality by the unitarity methods (Bjerrum-Bohr, Donoghue and Vanhove, 2014)

Graviton-graviton scattering

Perhaps the most elementary prediction of quantum general relativity is the scattering of two gravitons. This was worked out to one-loop by (Dunbar and Norridge, 1995). The form is

$$\mathcal{A}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}\left(1 + \frac{\kappa^2 stu}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} \times \left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st} + \frac{\ln(-t)}{su} + \frac{\ln(-s)}{tu}\right) + \frac{1}{s^2}f\left(\frac{-t}{s}, \frac{-u}{s}\right) + 2\left(\frac{\ln(-u)\ln(-s)}{su} + \frac{\ln(-t)\ln(-s)}{tu} + \frac{\ln(-t)\ln(-s)}{ts}\right)\right]\right) \tag{12}$$

and

$$\mathcal{A}(++;--) = -i\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right) \qquad \mathcal{A}(++;+-) = -\frac{1}{3}\mathcal{A}(++;--) \tag{13}$$

where

$$\begin{aligned} f\left(\frac{-t}{s}, \frac{-u}{s}\right) = {} & \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right) \\ & + \frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u} \\ & + \frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}\,, \end{aligned} \tag{14}$$

and where the $+,-$ refer to the graviton helicities, $\epsilon = 2 - d/2$, and where s, t, u are the usual Mandelstam variables. Here again $\mathcal{A}(++;++)$ shows the nature of the energy expansion for general relativity most clearly. It is the only amplitude with a tree level matrix element - the others all vanish at tree level. The tree amplitude is corrected by terms at the next order in the energy expansion, i.e. by factors of order κ^2(Energy)^2 relative to the leading contribution. Note also the nonanalytic logarithms. Another interesting feature, despite the presence of $1/\epsilon$ terms in the formulas, is that these results are finite without any unknown parameters. Because the counterterms vanish in pure gravity, as noted above, the scattering amplitudes cannot depend on the coefficients of the higher order terms. The $1/\epsilon$ terms have been shown to be totally infrared in origin, and are canceled as usual by the inclusion of gravitational bremsstrahlung (Donoghue and Torma, 1999) , as would be expected from general principles (Weinberg, 1965).

The bending of light

Another classic gravitational effect is the bending of light by a massive object. The calculation of the quantum correction is in principle similar to that of the scattering of two masses, except that one of the masses vanishes. However this makes a great difference in the actual calculation, there are soft singularities which arise and the Feynman diagrams are more complicated to evaluate. However, the process has now been calculated using unitarity based methods. These examine the unitarity cut formed by two on-shell tree amplitudes and use this to reconstruct the full diagram. The scattering amplitude has been calculated this way (Bjerrum-Bohr et al. 2015). In the limit of small momentum transfer t

$$\mathcal{M} \simeq \mathcal{N}\frac{(M\omega)^2}{4}\times\left[\frac{\kappa^2}{t}+\kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}}+\hbar\kappa^4\frac{15}{512\pi^2}\log\left(\frac{-t}{M^2}\right)-\hbar\kappa^4\frac{bu^i}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right)+\hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right)+\kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\right]. \quad (15)$$

Here $\mathcal{N}$ is a normalization factor and the coefficients bu^i are $bu^{(0)}=3/40$ for a massless spin zero particle and $bu^{(1)}=-161/120$ for a photon. Again the square-root is a classical correction and the logarithms give quantum effects. One notable feature is that it is *not* universal. Even in the limit of small momentum transfer, it is different for a massless scalar and a massless photon. This is in contrast to the universal behavior of the potential between two masses mentioned above. This is because the tree level low energy theorems for gravitational Compton scattering hold in the non-relativistic limit, but there are not comparable theorems for ultra-relativistic particles.

One can calculate from this amplitude the bending angle in terms of the impact parameter b, using either eikonal techniques or using an effective semi-classical potential. In both methods one arrives at the bending angle

$$\theta \simeq \frac{4GM}{b}+\frac{15}{4}\frac{G^2M^2\pi}{b^2}+\frac{8bu^i+9+48\log\frac{b}{2r_o}}{\pi}\frac{G^2\hbar M}{b^3}\,. \quad (16)$$

Here r_o is an infrared factor related to the soft divergences in the amplitude. The first two terms give the well known classical value of the bending angle, including the first post-Newtonian correction. The last term is a quantum gravity effect of order $G^2\hbar M/b^3=\ell_P^2 r_S/(2b^3)$ and involves the product of the Planck length and the Schwarzschild radius of the massive object divided by the cube of the impact parameter.

It is interesting that the bending angle depends on the type of massless particles. The particles no longer simply follow geodesics. This violates some ways that we talk about the equivalence principle. However it is not a violation in a fundamental sense. It is a manifestation of the fact that in quantum loops, massless particles propagate long distances. The long-distance propagation of massless photons and gravitons is not localized, and consequently can be interpreted as a tidal correction in that the massless particle is no longer be describable as a point source. There is then no requirement from the equivalence principle that such non-local effects be independent of the spin of the massless particle. However, it is interesting that quantum effects predict such a difference, without any free parameter, modifying one of the key features of classical general relativity.

Limitations and Frontiers

A weakness of the type of work described in the previous section is that it is only describing scattering amplitudes defined on flat spacetime. Some gravitational physics, like the potential and light bending, can be described using such an amplitude. But some of the most interesting gravitational physics involves classical solutions that are not just small deviations from flat space. The fact that the effective field theory has first been applied to scattering amplitudes is not an intrinsic weakness of the effective field theory. It is just a reflection of the fact that the techniques of quantum field theory are well-developed and simple for the calculation of amplitudes. Quantum corrections around classical solutions is a more difficult topic within QFT, especially when combined with the general coordinate invariance of general relativity. The effective field theory is capable of describing such corrections. However, this is still a topic that has only been explored lightly and more work is needed.

One technique that appears to have some promise in this effort is that of non-local effective actions, pioneered in gravity by Barvinsky and Vilkovisky and collaborators (Barvinsky and Vilkovisky, 1985, 1990) .(See also (Buchbinder, Odintsov, Shapiro, 1992).) These non-local actions are written in an expansion in the curvatures, and in principle include the effects of one loop quantum corrections. The leading quantum correction in momentum-space amplitudes is something which can be written as $\log q^2$, as described above. If one is trying to incorporate this into a position space effective Lagrangian, it would be represented by an operator $\log\Box$. This is intrinsically a non-local operator. In flat spacetime it has the representation

$$\langle x|\ln\left(\frac{\Box}{\mu^2}\right)|y\rangle \equiv L(x-y)=\int\frac{d^4q}{(2\pi)^4}\,e^{-iq\cdot(x-y)}\ln\left(\frac{-q^2}{\mu^2}\right). \quad (17)$$

If we are using this in a gravitational setting, this operator would need to involve the covariant $\Box$ and would need to be acting on covariant objects, i.e. curvatures. Thus the leading non-local actions in the Barvinsky-Vilkovisky framework have the form

$$S_{QL}=\int d^4x\sqrt{g}\left(\alpha R\log\left(\frac{\Box}{\mu_\alpha^2}\right)R+\beta R_{\mu\nu}\log\left(\frac{\Box}{\mu_\beta^2}\right)R^{\mu\nu}+\gamma R_{\mu\nu\alpha\beta}\log\left(\frac{\Box}{\mu_\gamma^2}\right)R^{\mu\nu\alpha\beta}\right) \quad (18)$$

where α,β,γ are numerical coefficients. Or in a different basis, using the Weyl tensor $C_{\mu\nu\alpha\beta}$ we could rewrite this in the equivalent form

$$S_{QL}=\int d^4x\,\sqrt{g}\,\left[\bar\alpha R\log\left(\frac{\Box}{\mu_1^2}\right)R+\bar\beta C_{\mu\nu\alpha\beta}\log\left(\frac{\Box}{\mu_2^2}\right)C^{\mu\nu\alpha\beta}+\bar\gamma\left(R_{\mu\nu\alpha\beta}\log(\Box)R^{\mu\nu\alpha\beta}-4R_{\mu\nu}\log(\Box)R^{\mu\nu}+R\log(\Box)R\right)\right]. \quad (19)$$

where $\bar\alpha,\bar\beta,\bar\gamma$ are a related set of coefficients. These coefficients are calculable at one loop and depend on the spin of the particle in the loop. They are given in Table I of (Donoghue and El-Menoufi, 2014). The non-local effective action can be computed in a couple of ways. Barvinsky and Velkovisky use a non-local heat kernel expansion in the background field formalism. Equivalently, knowing the form of the general answer, one can match on to a perturbative calculation of the gravitational action for the small fluctuations around flat space, which can be referred to as a non-linear completion of the action.

There remains an uncertainty on the best way to define the generally covariant log $\Box$ in curved space. This has not been fully resolved and remains an important issue for the future. Depending on that resolution, there are also non-local actions that are third order in the curvatures. These are quite complicated and are not well explored.

An early phenomenological exploration of the effect of the non-local actions has been in FRLW cosmology (Espriu, Multamaki, Vagenas, 2005)(Donoghue and El-Menoufi, 2014). The quantum corrections modify the classical evolution only as the Hubble constant approaches the Planck scale. In a contracting universe, they seem to indicate that the cosmology avoids the singularity and bounces. This will be interesting to explore more. Also for future exploration is the quantum corrections to classical solutions like black holes.

Other limitations of the effective field theory are fundamental. As the energy or the curvature gets large, of order the Planck scale, all the terms in the effective Lagrangian become of the same order. The energy expansion breaks down. The standard expectation in such a situation is that a new theory takes over, with new particles and new interactions. So the effective field theory predicts its own demise at high energy.

In specific calculations there is another limitation, at least with present techniques. This is that even when gravity is weak, its effects can build up over long distances. As an example, with a large black hole, the curvature is small everywhere outside and even at the horizon, and local gravitational effects are small. However, the net result of gravity between the horizon and spatial infinity generates a major effect, that of the horizon. Classical techniques for dealing with this are simple - we look for solutions to the Einstein equations. With quantum methods we have more difficulty. Coordinates which are well behaved in one region cannot always be used arbitrarily far away. We would have a hard time describing propagation past a black hole for example. It is possible that these represent a fundamental limitation of the effective field theory, in that there is a second expansion parameter - roughly the integrated curvature - besides the energy expansion which controls the convergence of the theory. Some of the debate on quantum black hole physics can be viewed in this light. Or it could be that the limitation is not fundamental, but only due to the limits of present QFT techniques. For example, one can imagine patching together different spatial regions using well-behaved coordinates in each, in order to extend the reach of present techniques into the far-infrared.

Given the interest in quantum gravity, we have identified a portion of the theory - the low energy/low curvature limit over ordinary distance scales - where the general relativity can be treated as a quantum effective field theory. The result is a subtle and interesting quantum field theory. We can identify some firm predictions of quantum gravity and also have interesting challenges for future work.

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Pedagogic material

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