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W. Wu, A. Beretvas, K. Denisenko, D. Green, J. Marraffino and A. Para

*Fermi National Accelerator Laboratory*
*P.O. Box 500, Batavia, Illinois 60510*


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## Disclaimer

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## Abstract

We have simulated the response of the SDC calorimeter for the decay $Z' \rightarrow$ jet + jet ($M_{Z'} = 10$ TeV). The purpose of the present study is to determine the mass and energy resolutions for the $Z'$. We consider the two cases when the hadron (HAD) calorimeter is made of lead/scintillator or iron/scintillator. We find that mass resolution is essentially the same in both cases and that the energy resolution for lead is only slightly better then for iron, 1.9% versus 2.2%.

## Introduction

Recently the SDC collaboration has decided to build a scintillating Tile/Fiber calorimeter [1]. The SDC calorimeter will contain a 22 radiation length ($X_0$) electromagnetic (EM) calorimeter. The EM calorimeter will consists of 36 pairs of plates. Each pair is made up of 2.5 mm thick scintillator followed by 3.175 mm thick lead. This calorimeter has a total thickness of 0.7 $\lambda_0$ (nuclear interaction lengths). Two options (lead/scintillator, iron/scintillator) are considered for the hadron (HAD) calorimeter. For both options the total thickness is 8.3 $\lambda_0$. We will evaluate these two options in terms of jet physics.

## Physics Motivation and Outline

We have previously studied "Jet Energy Resolution of the SDC Detector" [2]. The present project gave us an opportunity to extend our previous study. At the SSC one will want to study the highest possible energy jet. In our study "Depth requirements in SSC Calorimeters" [3] we concluded that this was roughly a 10 TeV dijet. We have extended the parametrization given in our Monte Carlo program SSCSIM [4]. We now have parametrizations in both the longitudinal and transverse directions for both "photons and hadrons". We have modified our LEGO plot so that now the energy is obtained for both an EM and HAD LEGO plots. The energy smearing routine has been extended to include compensation.

The $Z'$ events were produced as dijets using ISAJET version 6.36. We cluster energy in a cone of radius 0.6 in eta-phi space. The cone is centered on the direction of the parent parton as given by ISAJET. The jet 4-momentum vector was calculated by summing all calorimetric cells within the cone, treating all cells as massless particles.

**Photon Shower Longitudinal Parametrizations**

A common form for the parametrization of longitudinal EM showers is [5]:

$$dE = kt^{a-1}e^{-bt}dt \tag{1}$$

where t is the depth inside the calorimeter expressed in units of radiation lengths $t = z/X_0$, and a, b are energy dependent constants. The scale factor k is obtained by integrating the above equation ($k = \frac{E_0 b^a}{\Gamma(a)}$). The shower maximum occurs at $t_{max} = \frac{(a-1)}{b}$. The Particle Data Group [6], using an EGS4 simulation, gives the following expression:

$$t_{max} = 1.0(\ln y + C) \tag{2}$$

where the scaling variable $y = \frac{E_0}{E_c}$, and the constant $C = 0.5$ (for photon induced cascades).

$E_0$ is the energy of the incident photon. The critical energy is when the loss of energy by ionization is equal to the loss by radiative processes. To calculate the critical energy we use the formula [6]:

$$E_c = \frac{800 \text{ MeV}}{Z + 1.2}. \tag{3}$$

To find the energy deposited in the EM calorimeter we must find where the shower starts ($t_s$) with respect to the front face at $t = 0$ and then integrate the energy deposited up to the end ($t_e$) of the EM calorimeter:

$$E_{EM}(\text{photon}) = \frac{E_0}{\Gamma(a)} b^a \int_0^{t_e - t_s} t^{a-1} e^{-bt} dt. \tag{4}$$

The resulting expression for the energy deposited by a photon induced shower in the EM calorimeter ($t_\tau = t_e$ -$t_s$)) is:

$$E_{EM}(\text{photon}) = \frac{E_0 \Gamma(a, bt_\tau)}{\Gamma(a)}. \tag{5}$$

The energy not deposited in the EM calorimeter (leakage) is deposited in the hadron calorimeter. The expression for leakage energy reduces to a very simple expression when we recall the definition of the incomplete Gamma function:

$$\gamma(a, v_0) = \int_{v_0}^{\infty} v^{a-1} e^{-v} dv \tag{6}$$

$$E_{HAD}(\text{photon}) = \frac{E_0[\Gamma(a) - \Gamma(a, bt_\tau)]}{\Gamma(a)} = \frac{E_0 \gamma(a, bt_\tau)}{\Gamma(a)}. \tag{7}$$

The simulation of the start of the shower is trivial because the probability distribution function is the exponential function:

$$\frac{dN}{dt_s} = e^{-t_s}. \tag{8}$$

**Transverse Photon Shower Parametrization**

The transverse shower parametrization is of the form [7]:

$$\frac{dE}{dr} = Kr\frac{[e^{-\frac{r}{r_1}} + ge^{-\frac{r}{r_2}}]}{r_1^2 + gr_2^2} \tag{9}$$

where $g = 0.0052$, $r_1(cm) = 0.076 + 0.023t$, and $r_2(cm) = 0.052 + 0.089t$.

**Longitudinal Hadron Shower Parametrization**

The hadron shower is parametrized according to Ref. [5]:

$$dE = \frac{E_0}{\Gamma(a)}\{f_0[(bt)^{a-1}e^{-bt}d(bt)] + (1 - f_0)[(dD)^{a-1}e^{-dD}d(dD)]\} \tag{10}$$

where $f_0$ is the fraction of the showers energy that goes into electromagnetic energy ($\pi^0$), and $D = \frac{z}{\lambda_0}$ is the depth in units of nuclear interaction lengths. The constants are:

$$a = 0.62 + 0.32\ln E_0,\ b = 0.22,\ d = 0.91 - 0.024\ln E_0. \tag{11}$$

Although the fraction of energy in $\pi^0$ goes up as a function of the energy in the shower, we use the value 0.46 given in Ref. [5]. We again need to consider the fluctuations in the interaction point:

$$\frac{dN}{dD_s} = e^{-D_s} \tag{12}$$

where $D_s$ is the distance from the front face to the start of the shower in interaction lengths. Again, this is an exponential distribution and thus easy to simulate. We consider two cases, the first of which is when the hadron interacts in the EM calorimeter:

$$E_{EM}(hadron) = E_0\frac{[f_0\Gamma(a, bt_\tau) + (1 - f_0)\Gamma(a, dD_\tau)]}{\Gamma(a)} \tag{13}$$

where $D_\tau = D_e - D_s$ and $t_\tau = t_e - t_s$. The energy not deposited in the EM section will be deposited in the hadron calorimeter (except for leakage). In addition, in most cases, the response of the hadron calorimeter is not the same for hadrons as for photons(electrons). We refer to this difference as non-compensation and use the notation $\pi/e$. If we neglect leakage then:

$$E_{HAD}(\mathrm{hadron}) = E_0[f_0(1 - \frac{\Gamma(a, bt_\tau)}{\Gamma(a)}) + (1 - f_0)(1 - \frac{\Gamma(a, dD_\tau)}{\Gamma(a)}]\frac{\pi}{e}. \tag{14}$$

To model the $\pi/e$ ratio we use an ansatz suggested by D. Groom [8]. The second case occurs when the interaction occurs inside the hadron calorimeter. In this case, clearly no energy is deposited in the EM calorimeter.

$$E_{EM}(\mathrm{hadron}) = 0 \tag{15}$$

$$E_{HAD}(\mathrm{hadron}) = E_0[f_0 + (1 - f_0)(1 - \frac{\Gamma(a, dD_\tau)}{\Gamma(a)})]\frac{\pi}{e} \tag{16}$$

where $D_\tau$ = $D_{ec}$ - $D_s$, and $D_{ec}$ is equal to the total depth in nuclear interaction lengths of the complete calorimeter (9 $\lambda_0$).

### Transverse Hadron Shower Parametrization

We use a parametrization given by F. Binon *et al.* [9].

$$y = a_1 e^{-\frac{|x|}{b_1}} + a_2 e^{-\frac{|x|}{b_2}} \tag{17}$$

where $a_1$ = 0.7, $a_2$ = 0.3, $b_1$ = 2. cm, and $b_2$ = 7. cm.

### Response of Ideal Calorimeter

Our results are based on statistics of 1000 events. Our cell size is 0.05 by 0.05 in eta-phi space for both the HAD and EM calorimeters. Figure 1 shows the ratio of the reconstructed mass to the generated mass for an ideal (i.e. no energy resolution) calorimeter of a realistic cell size. We have also set the $\pi/e$ ratio to 1. To obtain the mean we use a Gaussian fit in a region that contains 75% of the events, a procedure used in our earlier study[2]. We see that the reconstructed mass is 97% of the original mass and the mass resolution is 2.2%. Figure 2 shows the fractional energy error $\frac{E(measured)_{jet} - E(reconstructed)_{jet}}{E(measured)_{jet}}$. The distribution has a sigma of 1.0%.

## Energy Smearing

In our analysis the energy resolution of the detector is approximated by a "stochastic" term (a) due to statistical fluctuations in the shower/detection process and a "constant" term (b) due to non-uniformities in the detector

$$dE = a\sqrt{E} \oplus bE. \tag{18}$$

The constants a and b will be specified for 4 cases. The first case is for a photon shower in an EM calorimeter. For our geometry an EGS4 simulation[10] gives a stochastic term of 13%. These studies also indicate that one can obtain an acceptable light yield and also a uniformity of tile construction consistent with a 1% constant term. The second case is for a photon shower in the HAD calorimeter. The absorber is 8 times thicker in the hadron calorimeter (1 inch) than in the EM calorimeter and the scintillator has the same thickness (2.5 mm) as in the EM. Therefore the stochastic term will be roughly $\sqrt{8}$ times larger (37%). We estimate that the constant term will be 2% or less. The next case is a hadron entering the EM calorimeter. We consider this case to be identical to the first case. Clearly not all hadrons interact (0.7 $\lambda_0$) and only 46% of the energy will go into an electromagnetic shower. The fourth case is that of a hadron entering the HAD calorimeter. We have assumed the "stochastic" term is 70% and the constant term is 2%. The "Hanging File" group will soon have experimental data pertaining to these 4 cases[11]. We will then incorporate the measured resolution into our analysis program.

### Response including Smearing

Figure 3 shows the ratio of reconstructed mass to the generated mass with smearing. The reconstructed mass is 97% of the original mass and the mass resolution is 2.5%. Figure 4 shows that the fractional energy resolution has now been degraded to 1.9%.

### $\frac{\pi}{e}$ Parametrization

In order to specify the actual response of a calorimeter of a given material, we need to indicate how the deposited energy is modified and also how the smearing is modified. As indicated earlier, the deposited energy for hadrons is specified by the ratio of $\pi/e$. We have used the Groom formula[8] to do this:

$$\frac{\pi}{e} = [1.- \mid 1. - \frac{\epsilon_h}{\epsilon_e} \mid (\frac{E_0}{E_{nominal}})^{-0.15}] \tag{19}$$

where $\epsilon_e$ and $\epsilon_h$ are the calorimeter efficiencies for detecting low-energy electronic and hadronic energy deposition. The nominal energy is taken as 1 GeV. This one parameter fit provides a good description of test beam data[8]. In our simulation we use the values of $\frac{\epsilon_h}{\epsilon_e} = 1$ (1.25) for lead (iron). The results for an iron hadron calorimeter, with no smearing are that the reconstructed mass is now 92% of the generated mass, has a mass resolution of 2.5%, and an energy resolution of 2.4%. We have taken the fraction of energy that goes into electromagnetic energy ($\pi^0$) as 46% but in reality this number fluctuates. The fluctuations depend crudely on the square root of the number of $\pi^0$ produced in the first interaction;

$$df_0 = \frac{f_0}{\sqrt{< n_{\pi^0} >}}. \tag{20}$$

An additional constant term needs to be added in quadrature to the smearing:

$$dE = \mid \frac{\epsilon_e}{\epsilon_h} - 1 \mid Edf_0 = \mid \frac{\epsilon_e}{\epsilon_h} - 1 \mid E(0.2) \tag{21}$$

Our estimate of 0.2 is a compromise between that of the experts, Groom (0.14) and Wigmans (0.23). The results for an iron calorimeter with smearing are that the reconstructed mass is now 91% of the generated mass, and has a mass resolution of 3.0% with an energy resolution of 2.8%. These results are shown in Figures 5 and 6.

### Corrections to Mass and Energy Resolutions for Iron

We have investigated a few methods for correcting (calibrating) the hadron response of an iron calorimeter [12]. Our method of correcting is based on the Groom formula. As the response of the hadron calorimeter for iron is low, we multiply by the reciprocal $e/\pi$ where this ratio is evaluated for each tower in the LEGO plot at the measured energy found in that tower. This procedure results in a reconstructed mass that is restored to 97% of the generated mass (see Fig. 7). The mass resolution is now 2.8% and the energy resolution is 2.2% (see Fig. 8). Table 1 shows the mass and energy resolutions presented in this report for comparison purposes. Clearly, no major differences in the response of an iron and lead calorimeter are observed in this study.

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TABLE I.

| Di-jet mass resolution and jet energy resolution | | | |
|---|---|---|---|
| case | m(Rec)/m(Gen) | sigma(m)/m | sigma(E)/E |
| Z′ | | | |
| ideal (lead) | 0.968 | 0.022 | 0.010 |
| smearing (lead) | 0.967 | 0.025 | 0.019 |
| ideal (iron) | 0.915 | 0.025 | 0.019 |
| smearing (iron) | 0.913 | 0.030 | 0.028 |
| smearing (iron) e/h-corrected | 0.966 | 0.028 | 0.022 |

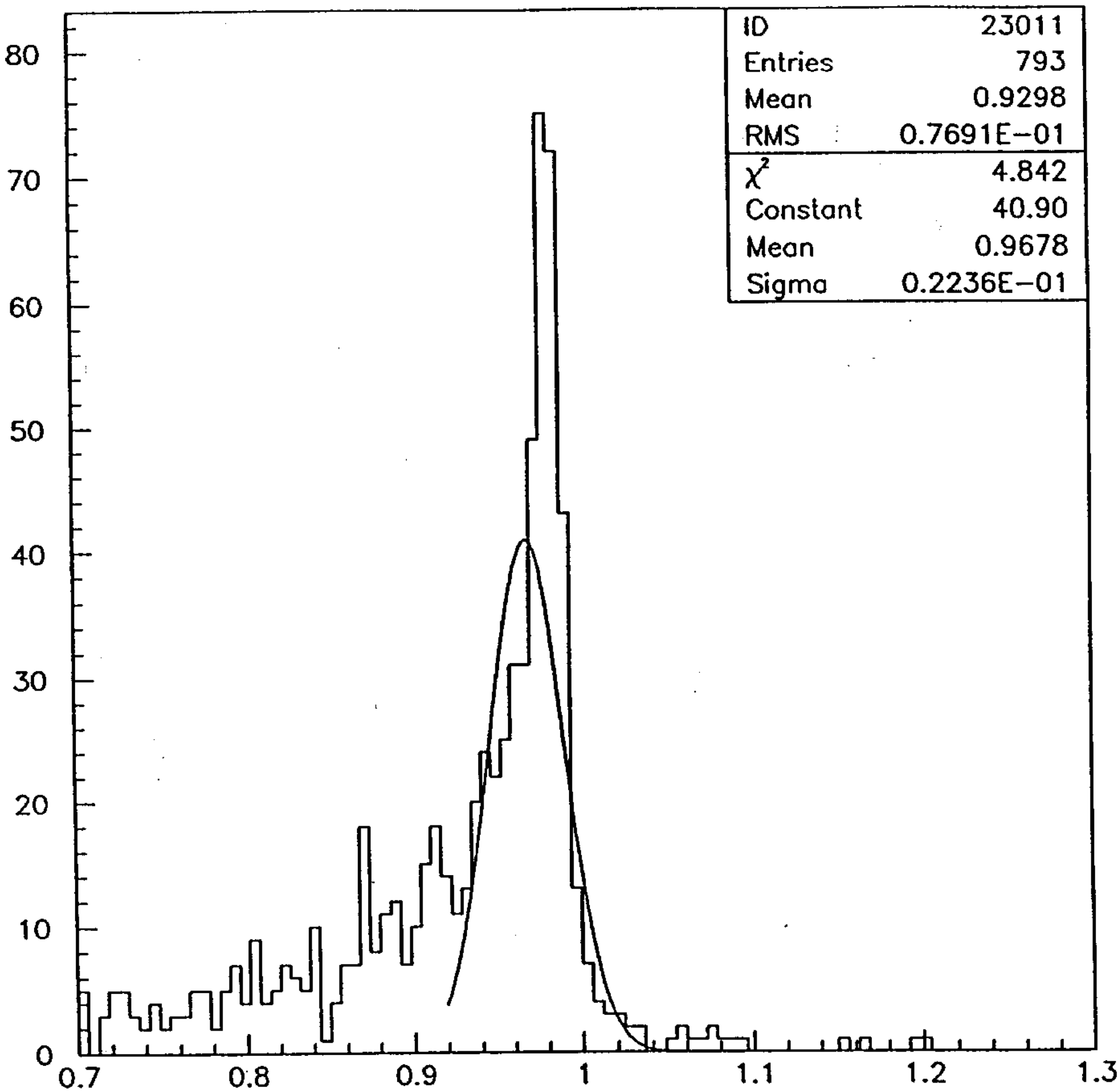


Figure 1: The ratio of the reconstructed mass to the generated mass for a 10 TeV $Z'$. Our simulation is for an ideal (i.e. no energy resolution) lead hadron calorimeter. The mass is reconstructed in a cone of radius $R = 0.6 = \sqrt{(\eta)^2 + (\phi)^2}$.

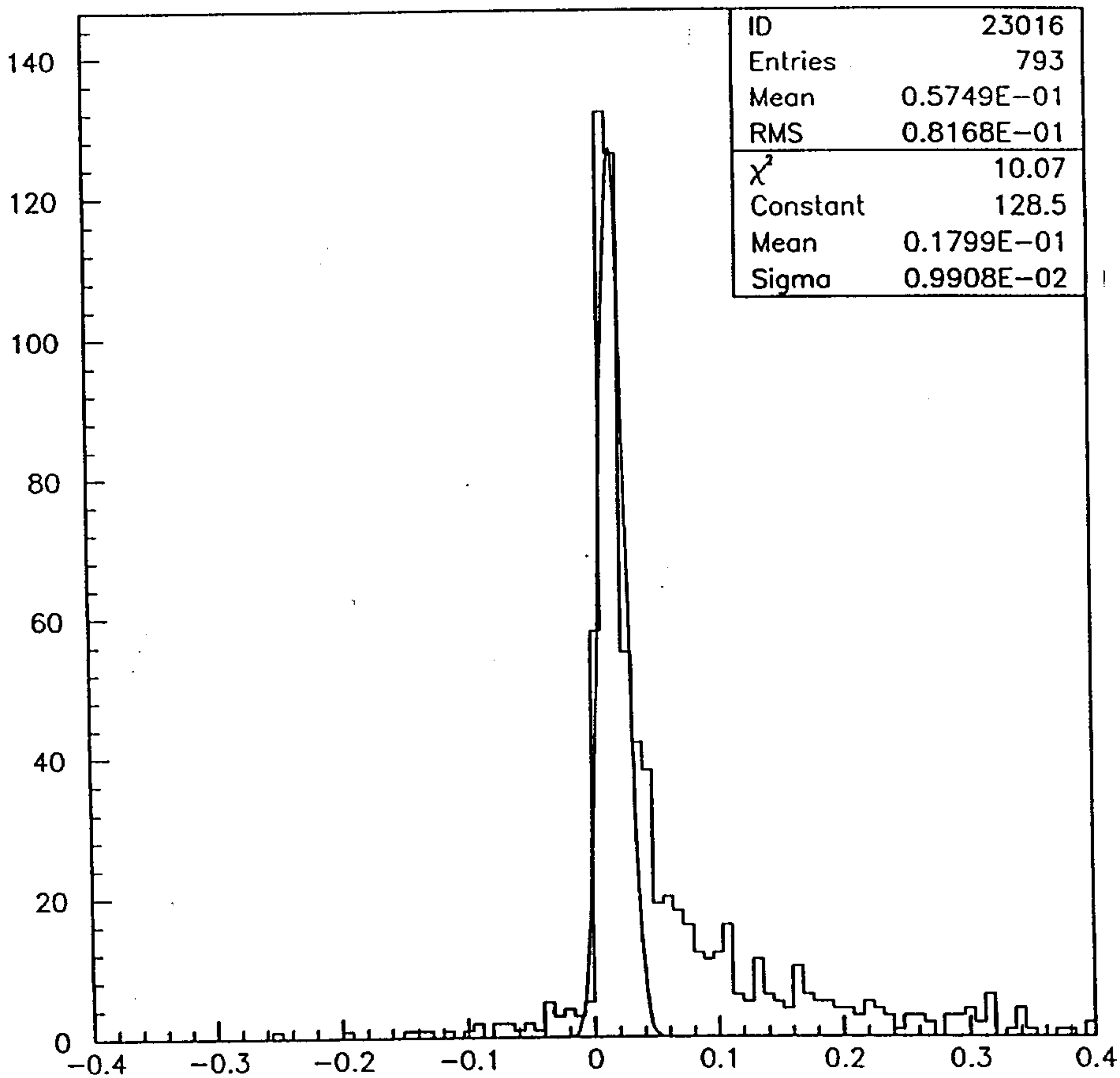


Figure 2: The fractional energy error $\frac{E(measured)_{jet} - E(reconstucted)_{jet}}{E(measured)_{jet}}$ for a cone of radius R = 0.6 for an ideal lead hadron calorimeter.

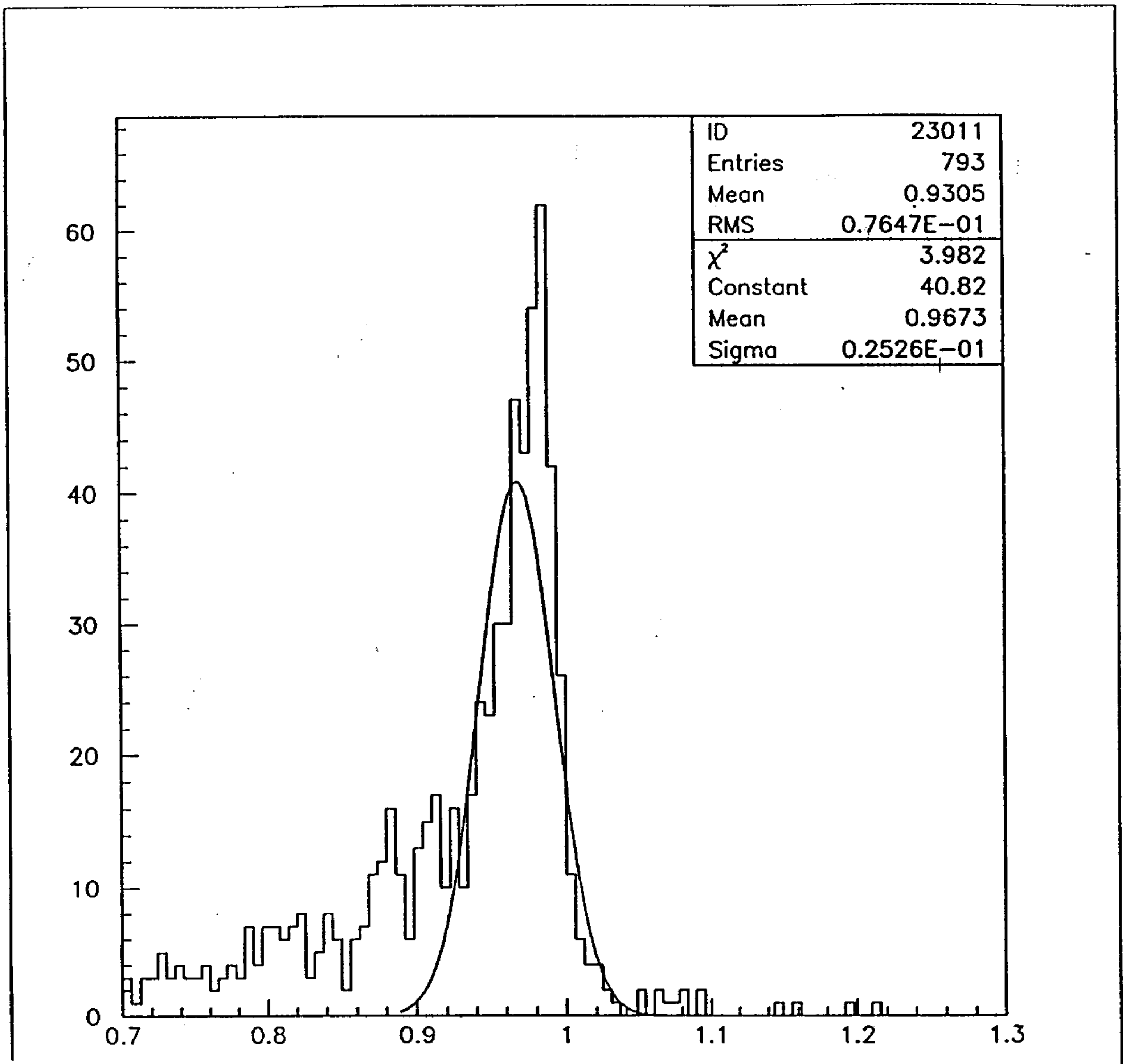


Figure 3: The ratio of the reconstucted mass to the generated mass for a 10 TeV Z′. Our simulation is for a lead hadron calorimeter. We have used an energy smearing dE = $70\%\sqrt{E} \oplus 2\%$.

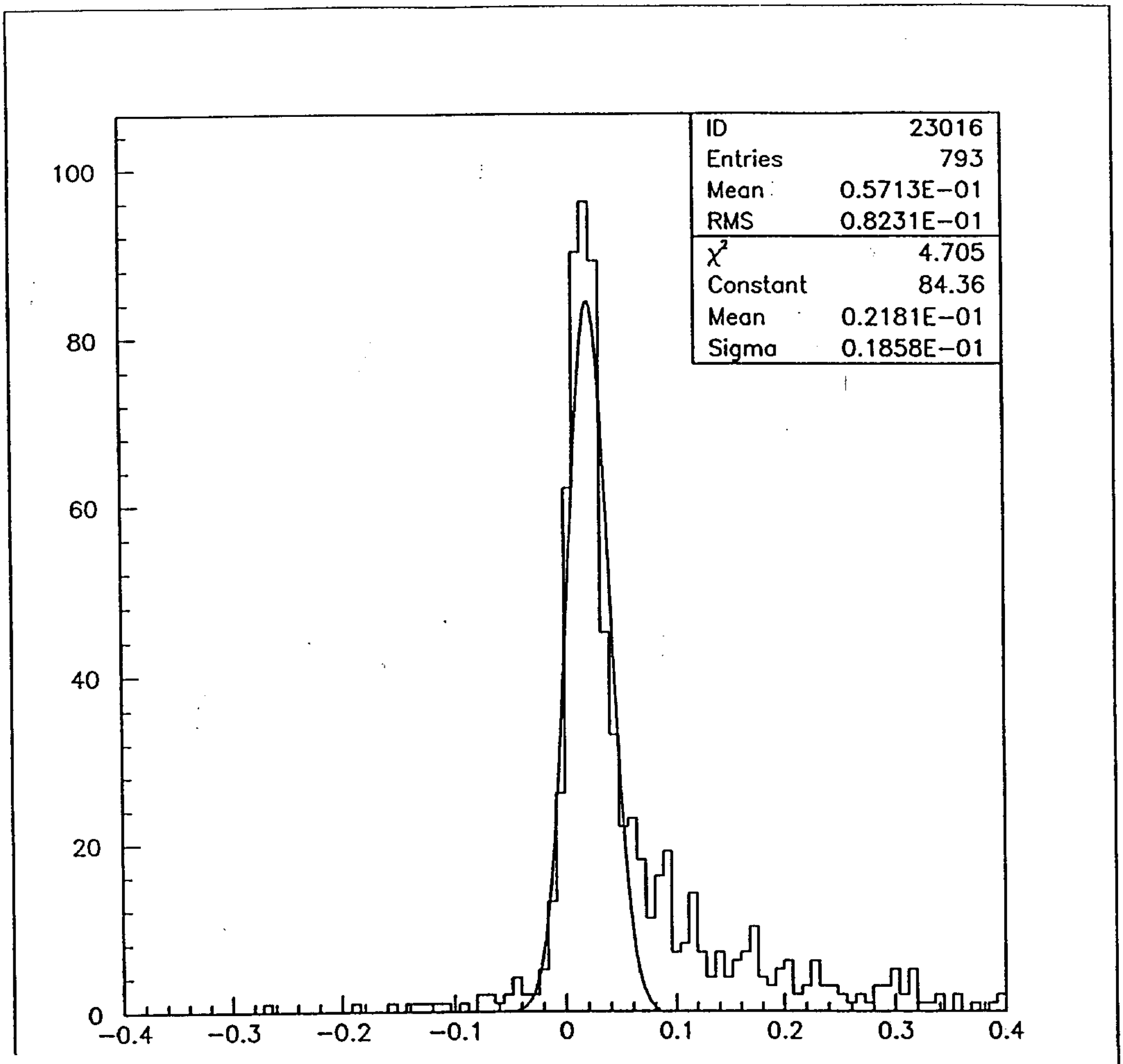


Figure 4: The fractional energy error $\frac{E(measured)_{jet} - E(reconstucted)_{jet}}{E(measured)_{jet}}$ for a cone of radius R = 0.6 for a lead hadron calorimeter. We have used an energy smearing dE = $70\%\sqrt{E} \oplus 2\%$.

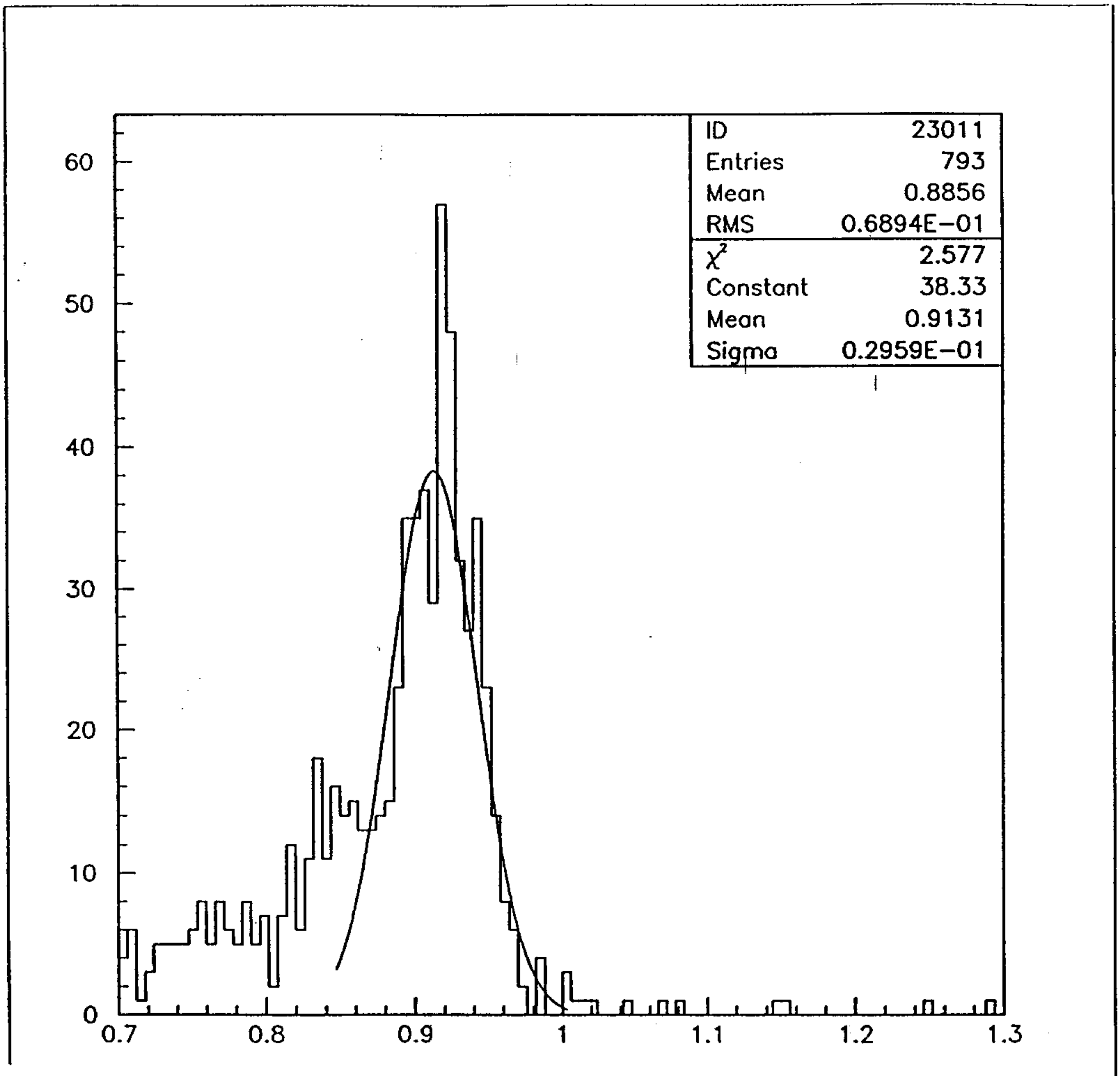


Figure 5: The ratio of the reconstucted mass to the generated mass for a 10 TeV Z′. Our simualtion is for an iron calorimeter ($\frac{\epsilon_h}{\epsilon_e} = 1.25$). We have used an energy smearing dE $= 70\%\sqrt{E} \oplus 2\% \oplus$ b(e/h).

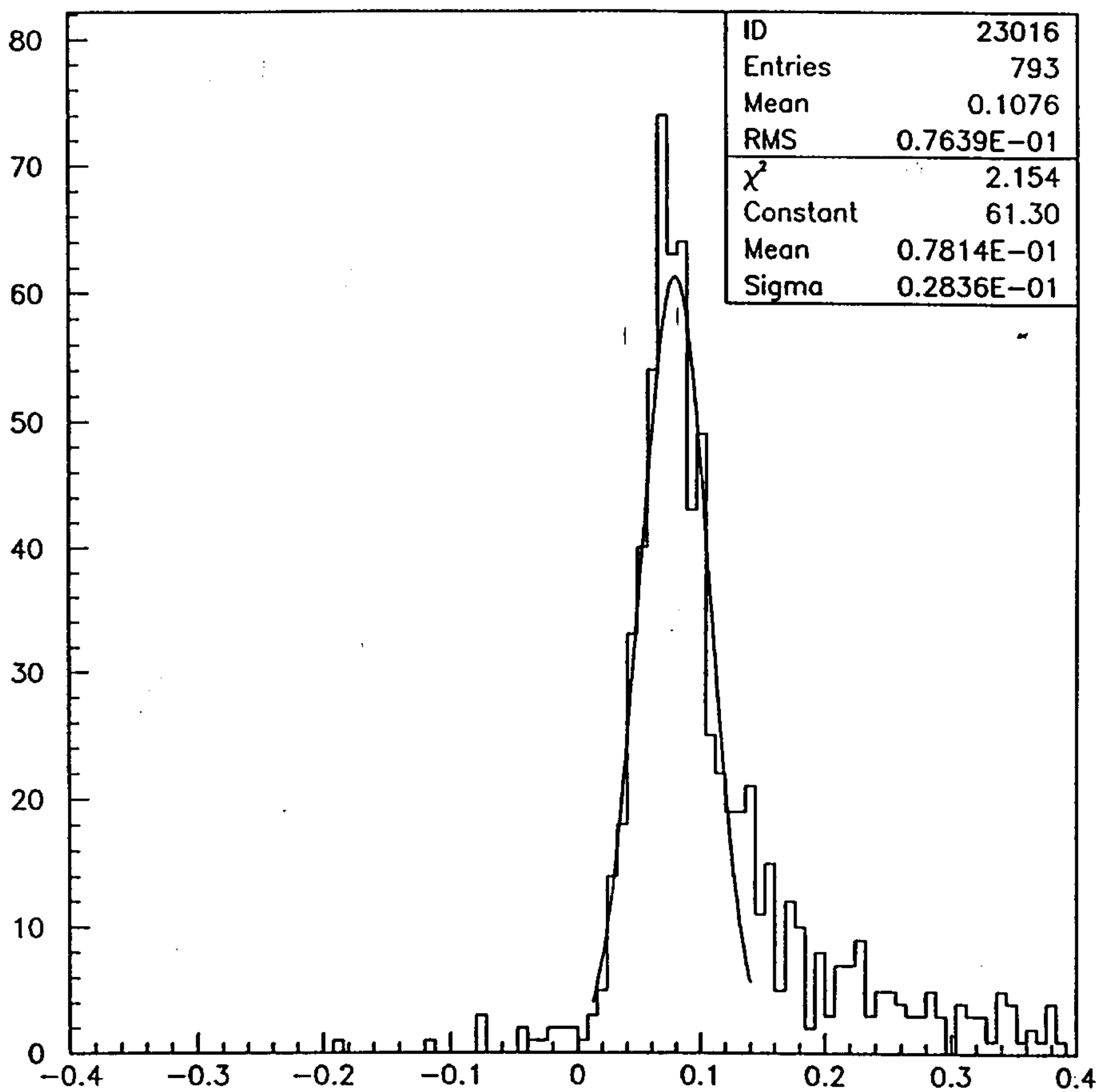


Figure 6: The fractional energy error $\frac{E(measured)_{jet} - E(reconstucted)_{jet}}{E(measured)_{jet}}$ for a cone of radius R = 0.6 for an iron calorimeter ($\frac{\epsilon_h}{\epsilon_e} = 1.25$). We have used an energy smearing dE = 70%$\sqrt{E}$ $\oplus$ 2% $\oplus$ b(e/h).

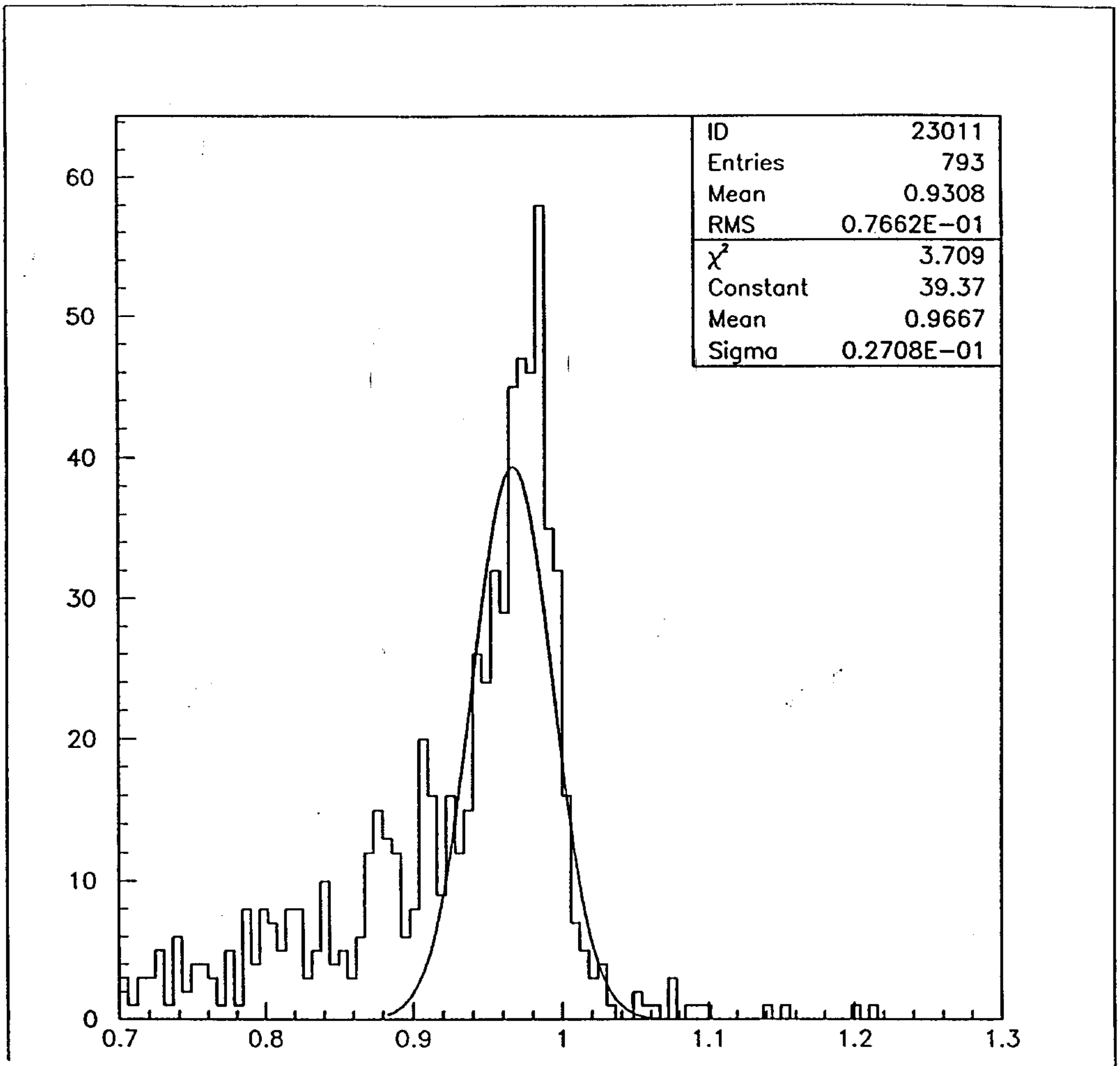


Figure 7: The ratio of the reconstucted mass to the generated mass for a 10 TeV Z′. Our simulation is for an iron calorimeter ($\frac{\epsilon_h}{\epsilon e} = 1.25$). The energy smearing used was dE = $70\%\sqrt{E} \oplus$ b(e/h). We have corrected the response by multiplying the energy measured in each tower by $\frac{e}{\pi}$ (Groom's formula).

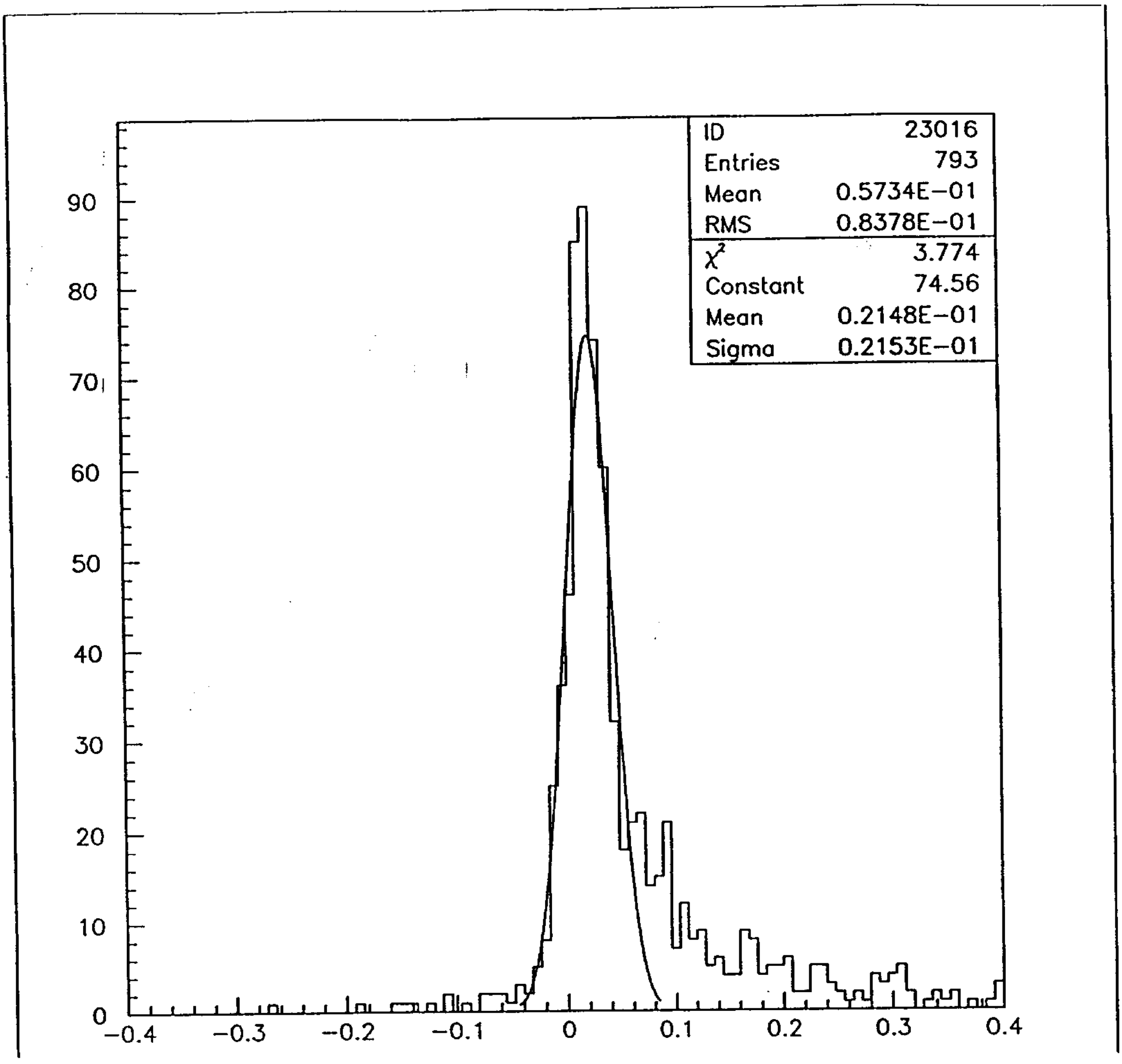


Figure 8: The fractional energy error $\frac{E(measured)_{jet} - E(reconstucted)_{jet}}{E(measured)_{jet}}$ for a cone of radius R = 0.6 for an iron calorimeter ($\frac{\epsilon_h}{\epsilon_e}$ = 1.25). We have used an energy smearing = dE = $70\%\sqrt{E} \oplus 2\% \oplus$ b(e/h). We have corrected the response by multiplying the energy measured in each tower by $\frac{e}{\pi}$ (Groom's formula).