

Ramón Herrera

Tachyon-Chaplygin inflation on the brane

Received: 30 July 2008 / Accepted: 7 October 2008
© Springer Science+Business Media, LLC 2008

Abstract Tachyon-brane inflationary universe model in the context of a Chaplygin gas equation of state is studied. General conditions for this model to be realizable are discussed. In the high-energy limit and by using an exponential potential we describe in great details the characteristic of this model. Recent observational data from the Wilkinson Microwave Anisotropy Probe experiment are employed to restrict the parameters of the model.

Keywords Inflation, Cosmology

1 Introduction

Inflationary universe models have solved many problems of the Standard Hot Big Bang scenario, for example, the flatness, the horizon, and the monopole problems, among others [1; 2]. In addition, it has provided a causal interpretation of the origin of the observed anisotropy of the cosmic microwave background (CMB) radiation, and also the distribution of large scale structures [3; 4].

In concern to higher dimensional theories, implications of string/M-theory to Friedmann–Robertson–Walker (FRW) cosmological models have recently attracted great deal of attention, in particular, those related to brane-antibrane configurations such as space-like branes [5]. The realization that we may live on a so-called brane embedded in a higher-dimensional Universe has significant implications to cosmology [6; 7; 8; 9; 10; 11]. In this scenario the standard model particles are confined on the brane, while gravitations propagate in the bulk spacetimes. Since, the effect of the extra dimension induces additional terms in the Friedmann equation is modified at very high energies [12; 13; 14], acquiring a quadratic term in the energy density. Such a term generally makes it easier to obtain inflation in the early Universe [15; 16; 17; 18; 19; 20; 21]. For a review, see, e.g., [22].

R. Herrera Instituto de Física Pontificia Universidad Católica de Valparaíso Casilla 4059 Valparaíso, Chile ramon.herrera@ucv.cl

In recent times a great amount of work has been invested in studying the inflationary model with a tachyon field. The tachyon field associated with unstable D-branes might be responsible for cosmological inflation in the early evolution of the universe, due to tachyon condensation near the top of the effective scalar potential [23], which could also add some new form of cosmological dark matter at late times [24]. Cosmological implications of this rolling tachyon were first studied by Gibbons [25] and in this context it is quite natural to consider scenarios in which inflation is driven by the rolling tachyon.

On the other hand, the generalized Chaplygin gas has been proposed as an alternative model for describing the accelerating of the universe. The generalized Chaplygin gas is described by an exotic equation of state of the form $p_{ch} = -A\rho_{ch}^{-\beta}$, where ρ_{ch} and p_{ch} are the energy density and pressure of the generalized Chaplygin gas, respectively [26]. β is a constant that lies in the range $0 < \beta \leq 1$, and A is a positive constant. The original Chaplygin gas corresponds to the case $\beta = 1$ [27; 28]. Inserting this equation of state into the relativistic energy conservation equation leads to an energy density given by [26]

$$\rho_{ch} = \left[A + \frac{B}{a^{3(1+\beta)}} \right]^{\frac{1}{1+\beta}}, \quad (1)$$

where a is the scale factor and B is a positive integration constant.

The Chaplygin gas emerges as an effective fluid of a generalized d-brane in a $(d+1)$ space time, where the action can be written as a generalized Born–Infeld action [26]. These models have been extensively studied in the literature [29; 30; 31; 32; 33; 34; 35; 36; 37; 38; 39; 40; 41; 42; 43; 44; 45; 46; 47; 48; 49; 50].

In the model of Chaplygin inspired inflation usually the scalar field, which drives inflation, is the standard inflaton field, where the energy density given by Eq. (1), can be extrapolate for obtaining a successful inflation period with a Chaplygin gas model [51]. Recently, tachyon-Chaplygin inflationary universe model was considered in [52], and the dynamics of the early universe and the initial conditions for inflation in a model with radiation and a Chaplygin gas was studied in [53] (see also [54; 55]).

A natural extension of [52] is to consider the tachyon field as a degree of freedom on visible three dimensional brane. This work has been extended to include higher order corrections in slow-roll parameters and the formula has been widely used to confront this model with the observations. Moreover, we find constraints on the parameter A and the five-dimensional Planck mass or equivalently the brane tension.

The outline of the paper is as follows. The next section presents a short review of the modified Friedmann equation by using a Chaplygin gas, and we present the tachyon-brane-Chaplygin inflationary model. Section 3 deals with the calculations of cosmological perturbations in general term. In Sect. 4 we use an exponential potential in the high-energy limit for obtaining explicit expression for the model. Finally, Sect. 5 summarizes our findings.

2 The modified Friedmann equation and the tachyon-brane-Chaplygin Inflationary phase

We consider the five-dimensional brane scenario, in which the Friedmann equation is modified from its usual form, in the following way [14; 27; 28]

$$H^2 = \kappa \rho_\phi \left[1 + \frac{\rho_\phi}{2\lambda} \right] + \frac{\Lambda_4}{3} + \frac{\xi}{a^4}, \quad (2)$$

where $H = \dot{a}/a$ denotes the Hubble parameter, ρ_ϕ represents the matter confined to the brane, $\kappa = 8\pi G/3 = 8\pi/3m_p^2$ (m_p is the four-dimensional Planck mass), Λ_4 is the four-dimensional cosmological constant and the final term represents the influence of bulk gravitons on the brane, where ξ is an integration constant (this term appears as a form of dark radiation). The brane tension λ relates the four and five-dimensional Planck masses via $m_p = \sqrt{3M_5^6/(4\pi\lambda)}$, and is constrained by the requirement of successful nucleosynthesis as $\lambda > (1 \text{ MeV})^4$ [56]. We assume that the four-dimensional cosmological constant is set to zero, and once inflation begins the final term will rapidly become unimportant, leaving us with [22]. Hence, the modified Friedmann equation reads

$$H^2 = \kappa \left[A + \rho_\phi^{(1+\beta)} \right]^{\frac{1}{1+\beta}} \left[1 + \frac{\left[A + \rho_\phi^{(1+\beta)} \right]^{\frac{1}{1+\beta}}}{2\lambda} \right]. \quad (3)$$

Here, ρ_ϕ becomes $\rho_\phi = V(\phi)/\sqrt{1-\dot{\phi}^2}$, and $V(\phi) = V$ is the tachyonic potential. Note that, in the low energy regime $[A + \rho_\phi^{(1+\beta)}]^{1/(1+\beta)} \ll \lambda$, the tachyon-Chaplygin inflationary model is recovered [52], and in a very high-energy regime, the contribution from the matter in Eq. (3) becomes proportional to $[A + \rho_\phi^{(1+\beta)}]^{2/(1+\beta)}$ in the effective energy density.

We assume that the tachyon field is confined to the brane, so that its field equation has the form

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V'}{V} = 0, \quad (4)$$

where dots mean derivatives with respect to the cosmological time and $V' = \partial V(\phi)/\partial \phi$. For convenience we will use units in which $c = \hbar = 1$.

The modification of the Eq. (3) is realized from an extrapolation of Eq. (1), where the density matter $\rho_m \sim a^{-3}$ is introduced in such a way that we may write $\rho_{ch} = [A + \rho_m^{(1+\beta)}]^{\frac{1}{1+\beta}} \longrightarrow [A + \rho_\phi^{(1+\beta)}]^{\frac{1}{1+\beta}}$, and thus, we identifying ρ_m with the contributions of the scalar tachyon field which gives Eq. (3). The generalized Chaplygin gas model may be viewed as a modification of gravity, as described in [57], for chaotic inflation, in [51], and for tachyon-Chaplygin inflationary universe model in the low-energy limit, in [52]. Different modifications of gravity have been proposed in

the last few years, and there has been a lot of interest in the construction of early universe scenarios in higher-dimensional models motivated by string/M-theory [58; 59; 60; 61; 62; 63]. It is well-known that these modifications can lead to important changes in the early universe. In the following we will take $\beta = 1$ for simplicity, which means the usual Chaplygin gas.

During the inflationary epoch the energy density associated to the tachyon field is of the order of the potential, i.e., $\rho_\phi \sim V$. Assuming the set of slow-roll conditions, i.e., $\dot{\phi}^2 \ll 1$ and $\ddot{\phi} \ll 3H\dot{\phi}$ [25; 64], the Friedmann equation (3) reduces to

$$H^2 \approx \kappa \sqrt{A + V^2} \left[1 + \frac{\sqrt{A + V^2}}{2\lambda} \right], \quad (5)$$

and Eq. (4) becomes

$$3H\dot{\phi} \approx -\frac{V'}{V}. \quad (6)$$

Introducing the dimensionless slow-roll parameters [65], we write

$$\varepsilon = -\frac{\dot{H}}{H^2} \simeq \frac{m_p^2}{16\pi} \left[\frac{V'^2}{(A + V^2)^{3/2}} \frac{\left(1 + \frac{(A + V^2)^{1/2}}{\lambda}\right)}{\left(1 + \frac{(A + V^2)^{1/2}}{2\lambda}\right)^2} \right], \quad (7)$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \simeq \frac{m_p^2}{8\pi} \left(\frac{V''}{V(A + V^2)^{1/2}} \right) \left[1 + \frac{(A + V^2)^{1/2}}{2\lambda} \right]^{-1}, \quad (8)$$

and

$$\gamma = -\frac{V'\dot{\phi}}{2HV} \simeq \frac{m_p^2}{16\pi} \left(\frac{V'^2}{V^2(A + V^2)^{1/2}} \right) \left[1 + \frac{(A + V^2)^{1/2}}{2\lambda} \right]^{-1}. \quad (9)$$

Note that in the low-energy limit, $\sqrt{A + \rho_\phi^2} \ll \lambda$, the slow-parameters are recovered [52].

The condition under which inflation takes place can be summarized with the parameter ε satisfying the inequality $\varepsilon < 1$, which is analog to the requirement that $\ddot{a} > 0$. This condition could be written in terms of the tachyon potential and its derivative V' , which becomes

$$V'^2 \left[1 + \frac{(A + V^2)^{1/2}}{\lambda} \right] < \frac{16\pi}{m_p^2} (A + V^2)^{3/2} \left[1 + \frac{(A + V^2)^{1/2}}{2\lambda} \right]^2. \quad (10)$$

Inflation ends when the universe heats up at a time when $\varepsilon \simeq 1$, which implies

$$V_f'^2 \left[1 + \frac{(A + V_f^2)^{1/2}}{\lambda} \right] \simeq \frac{16\pi}{m_p^2} (A + V_f^2)^{3/2} \left[1 + \frac{(A + V_f^2)^{1/2}}{2\lambda} \right]^2. \quad (11)$$

However, in the high-energy limit $[A + \rho_\phi^2]^{1/2} \approx [A + V^2]^{1/2} \gg \lambda$ Eq. (11) becomes

$$V_f'^2 \simeq \frac{4\pi}{m_p^2} \frac{(A + V_f^2)^2}{\lambda}.$$

The number of e-folds at the end of inflation is given by

$$N = -\frac{8\pi}{m_p^2} \int_{\phi_*}^{\phi_f} V \frac{\sqrt{A + V^2}}{V'} \left[1 + \frac{\sqrt{A + V^2}}{2\lambda} \right] d\phi, \quad (12)$$

or equivalently

$$N = -\frac{8\pi}{m_p^2} \int_{V_*}^{V_f} V \frac{\sqrt{A + V^2}}{V'^2} \left[1 + \frac{\sqrt{A + V^2}}{2\lambda} \right] dV. \quad (13)$$

Note that in the high-energy limit Eq. (13) becomes $N \simeq -(4\pi/m_p^2 \lambda) \int_{V_*}^{V_f} [V (A + V^2)/V'^2] dV$.

In the following, the subscripts $*$ and f are used to denote the epoch when the cosmological scales exit the horizon and the end of inflation, respectively.

3 Perturbations

In this section we will study the scalar and tensor perturbations for our model. It was shown in [66] that the conservation of the curvature perturbation, \mathcal{R} , holds for adiabatic perturbations irrespective of the form of gravitational equations by considering the local conservation of the energy-momentum tensor. However, we note here that even though the effect of bulk to the cosmological perturbations can not be trivially negligible, it can be shown that the main correction of the spectrum in the brane-world inflation is just the modification of the slow-roll parameters [67] (see also [68; 69; 70; 71; 72]). For a tachyon field the power spectrum of the curvature perturbations is given $\mathcal{P}_{\mathcal{R}} \simeq \left(\frac{H^2}{2\pi\dot{\phi}} \right)^2 \frac{1}{Z_s}$, where $Z_s = V (1 - \dot{\phi}^2)^{-3/2}$ [65]. Under the slow-roll approximation, the power spectrum of curvature perturbations is estimated to be [65]

$$\mathcal{P}_{\mathcal{R}} \simeq \left(\frac{H^2}{2\pi\dot{\phi}} \right)^2 \frac{1}{V} \simeq \frac{128\pi}{3m_p^6} \left(\frac{V (A + V^2)^{3/2}}{V'^2} \right) \left[1 + \frac{(A + V^2)^{1/2}}{2\lambda} \right]^3. \quad (14)$$

Note that in the low-energy limit the amplitude of scalar perturbation given by Eq. (14) coincides with [52].

The scalar spectral index n_s is given by $n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$, where the interval in wave number is related to the number of e-folds by the relation $d \ln k(\phi) = -dN(\phi)$. From Eq. (14), we get, $n_s \approx 1 - 2(2\varepsilon + \gamma - \eta)$, or equivalently

$$n_s \approx 1 - \frac{m_p^2}{4\pi} (A + V^2)^{-1/2} \left[1 + \frac{(A + V^2)^{1/2}}{2\lambda} \right]^{-1} \times \left(\frac{V'^2}{(A + V^2)} \left[\frac{1 + \frac{(A + V^2)^{1/2}}{\lambda}}{1 + \frac{(A + V^2)^{1/2}}{2\lambda}} \right] + \frac{V'^2}{2V^2} - \frac{V''}{V} \right). \quad (15)$$

One of the interesting features of the 5-year data set from Wilkinson Microwave Anisotropy Probe (WMAP) is that it hints at a significant running in the scalar spectral index $dn_s/d \ln k = \alpha_s$ [3; 4]. From Eq. (15) we get that the running of the scalar spectral index becomes

$$\alpha_s = \left(\frac{4(A + V^2)}{V V'} \right) \left[\frac{\left(1 + \frac{(A + V^2)^{1/2}}{2\lambda} \right)}{\left(1 + \frac{(A + V^2)^{1/2}}{\lambda} \right)} \right] [2\varepsilon_{,\phi} + \gamma_{,\phi} - \eta_{,\phi}] \varepsilon. \quad (16)$$

In models with only scalar fluctuations the marginalized value for the derivative of the spectral index is approximately -0.03 from WMAP 5-year data only [16].

On the other hand, the generation of tensor perturbations during inflation would produce gravitational waves and this perturbations in cosmology are more involved since gravitons propagate in the bulk. The amplitude of tensor perturbations was evaluated in [73; 74]

$$\mathcal{P}_g = 24\kappa \left(\frac{H}{2\pi} \right)^2 F^2(x) \simeq \frac{6}{\pi^2} \kappa^2 (A + V^2)^{1/2} \left[1 + \frac{(A + V^2)^{1/2}}{2\lambda} \right] F^2(x), \quad (17)$$

where $x = H m_p \sqrt{3/(4\pi\lambda)}$ and

$$F(x) = \left[\sqrt{1 + x^2} - x^2 \sinh^{-1}(1/x) \right]^{-1/2}.$$

Here the function $F(x)$ appeared from the normalization of a zero-mode. The spectral index n_g is given by $n_g = \frac{d \mathcal{P}_g}{d \ln k} = -\frac{2x_{,\phi}}{N_{,\phi x}} \frac{F^2}{\sqrt{1 + x^2}}$.

From expressions (14) and (17) we write the tensor–scalar ratio as

$$r(k) = \left(\frac{\mathcal{P}_g}{\mathcal{P}_{\mathcal{R}}} \right) \Big|_{k=k_*} \simeq \left(\frac{8}{3\kappa} \frac{V'^2 F^2(V)}{V (A + V^2) [1 + (A + V^2)^{1/2}/2\lambda]^2} \right) \Big|_{k=k_*}. \quad (18)$$

Here, k_* is referred to $k = Ha$, the value when the universe scale crosses the Hubble horizon during inflation.

Combining WMAP 5-year data [3; 4] with the Sloan Digital Sky Survey (SDSS) large scale structure surveys [75], it is found an upper bound for r given by $r(k_* \simeq 0.002 \text{ Mpc}^{-1}) < 0.28$ (95% CL), where $k_* \simeq 0.002 \text{ Mpc}^{-1}$ corresponds to $l = \tau_0 k \simeq 30$, with the distance to the decoupling surface $\tau_0 = 14,400 \text{ Mpc}$.

The SDSS measures galaxy distributions at red-shifts $a \sim 0.1$ and probes k in the range $0.016 h \text{ Mpc}^{-1} < k < 0.011 h \text{ Mpc}^{-1}$. The recent WMAP 5-year results give the values for the scalar curvature spectrum $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$ and the scalar-tensor ratio $r(k_*) = 0.055$. We will make use of these values to set constraints on the parameters appearing in our model.

4 Exponential potential in the high-energy limit

Let us consider a tachyonic effective potential $V(\phi)$, with the properties satisfying $V(\phi) \rightarrow 0$ as $\phi \rightarrow \infty$. The exact form of the potential is $V(\phi) = (1 + \alpha\phi) \exp(-\alpha\phi)$, which in the case when $\alpha \rightarrow 0$, we may use the asymptotic exponential expression. This form for the potential is derived from string theory calculations [23; 76]. Therefore, we simply use

$$V(\phi) = V_0 e^{-\alpha\phi}, \quad (19)$$

where α and V_0 are free parameters. In the following we will restrict ourselves to the case in which $\alpha > 0$. Note that α represents the tachyon mass [64; 77]. In [24] is given an estimation of these parameters in the low-energy limit and $A \rightarrow 0$. Here, it was found $V_0 \sim 10^{-10} m_p^4$ and $\alpha \sim 10^{-6} m_p$. In the following, we develop models in the high-energy limit, i.e., $\sqrt{A + V^2} \gg \lambda$.

From Eq. (13) the number of e-folds results in

$$N = \frac{4\pi}{\lambda \alpha^2 m_p^2} [h(V_*) - h(V_f)], \quad (20)$$

where

$$h(V) = \frac{V^2}{2} + A \ln V. \quad (21)$$

On the other hand, we may establish that the end of inflation is governed by the condition $\varepsilon = 1$, from which we get that the square of the tachyonic potential becomes

$$V(\phi = \phi_f)^2 = V_f^2 = \frac{1}{8\pi} \left[\lambda \alpha^2 m_p^2 - 8\pi A + \sqrt{\lambda \alpha^2 m_p^2 (\lambda \alpha^2 m_p^2 - 16\pi A)} \right], \quad (22)$$

and

$$\dot{\phi}_f = \frac{\alpha m_p}{2V_f} \sqrt{\frac{\lambda}{3\pi}}. \quad (23)$$

Note that in the limit $A \rightarrow 0$ we obtain $V_f = \alpha m_p \sqrt{\lambda}/(2\sqrt{\pi})$ and $\dot{\phi}_f = 1/\sqrt{3}$, which coincides with that reported in [24].

From Eq. (14) we obtain that the scalar power spectrum is given by

$$P_{\mathcal{R}}(k) \approx \frac{16\pi}{3m_p^6} \frac{1}{\alpha^2 \lambda^3} \left[\frac{(A + V^2)^3}{V} \right], \quad (24)$$

Fig. 1 The plot shows the adimensional square of the brane tension $(\lambda/m_p^4)^2$ versus the adimensional scalar potential V_*/m_p^4 . Here, we have used the WMAP 5-year data where $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$, $n_s(k_*) \simeq 0.96$ and $\alpha_s(k_*) \simeq -0.03$

and from Eq. (18) the tensor–scalar ratio becomes

$$r(k) \approx \frac{4m_p^2 \lambda^2 \alpha^2}{\pi} \left[\frac{V}{(A+V^2)^2} F^2(V) \right]. \quad (25)$$

By using, that $V' = -\alpha V$, we obtain from Eq. (15)

$$n_s - 1 \simeq -\frac{m_p^2}{4\pi} \frac{\lambda \alpha^2}{(A+V^2)} \left[\frac{4V^2}{(A+V^2)} - 1 \right], \quad (26)$$

and from Eq. (16) that

$$\alpha_s \simeq -\frac{\lambda^2 \alpha^4 m_p^4}{4\pi^2} \left[\frac{3V^2 - 5A}{(A+V^2)^4} \right] V^2. \quad (27)$$

The Eqs. (24) and (26) has roots that can be solved analytically for the parameters α and A , as a function of n_s , $P_{\mathcal{R}}$, V and λ . The real root solution for m^2 , and A becomes

$$\alpha^2 = 2\pi \left[\frac{4^4 V^5 + 6^2 P_{\mathcal{R}} (n_s - 1) V^2 \lambda^2 m_p^4 + \mathfrak{K} (3 P_{\mathcal{R}} (1 - n_s) \lambda^2 m_p^4 - 8^2 V^3)}{3 P_{\mathcal{R}} \lambda^3 m_p^6} \right], \quad (28)$$

and

$$A = \frac{1}{2} (2V^2 - \mathfrak{K}), \quad (29)$$

where

$$\mathfrak{K} = \sqrt{16V^4 + 3(n_s - 1)P_{\mathcal{R}}V\lambda^2 m_p^4}.$$

From Eq. (29) and since $A > 0$, the ratio V^3/λ^2 satisfies the inequality $V^3/\lambda^2 < (1 - n_s)P_{\mathcal{R}}m_p^4/4$. This inequality allows us to obtain an upper limit for the ratio $V^3(\phi)/\lambda^2$ evaluate when the cosmological scales exit the horizon, i.e., $V_*/\lambda^2 < 2.4 \times 10^{-11}m_p^4$. Here, we have used the WMAP 5-year data where $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$ and $n_s(k_*) \simeq 0.96$.

One again, note that in the limit $A \rightarrow 0$, the constrains $\alpha \approx 7 \times 10^{-3}M_5$ and $V_* \approx 4 \times 10^{-4}M_5^4$ are recovered [24]. Here, we used the relation $m_p = M_5^3 \sqrt{3/(4\pi\lambda)}$.

In Fig. 1 we have plotted the adimensional quantity λ^2/m_p^8 versus the adimensional scalar tachyon potential evaluated when the cosmological scales exist the horizon V_*/m_p^4 . In doing this, we using Eq. (27) that has roos that can be solved for the brane tension λ , as a function of α_s , m , A and V . For a real root solution for λ , and from Eqs. (28) and (29) we obtain a relation of the form $\lambda = f(V_*)$

Fig. 2 The *plot* shows the tensor–scalar ratio r versus the adimensional parameter A/m_p^8 . Here, we have used the WMAP 5-year data where $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$, $n_s(k_*) \simeq 0.96$ and $\alpha_s(k_*) \simeq -0.03$

Fig. 3 The *plot* shows the tensor–scalar ratio r versus the adimensional parameter α^2/m_p^2 . Here, we have used the WMAP 5-year data where $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$, $n_s(k_*) \simeq 0.96$ and $\alpha_s(k_*) \simeq -0.03$

for a fixed values of α_s , n_s and $P_{\mathcal{R}}$. In this plot we using the WMAP 5-year data where $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$, $n_s(k_*) \simeq 0.96$, $\alpha_s(k_*) \simeq -0.03$. In Fig. 2 we have plotted the tensor–scalar ratio given by Eq. (25) versus the adimensional parameter A/m_p^8 . The WMAP 5-year data favors the tensor–scalar ratio $r \simeq 0.055$ and the from Fig. 2 we obtain that A parameter becomes $A \simeq 2.6 \times 10^{-25} m_p^8$. In Fig. 3 we have plotted the tensor–scalar ratio given by Eq. (25) versus the adimensional parameter α^2/m_p^2 . We note that for $r \simeq 0.055$ we obtain $\alpha^2 \simeq 1.3 \times 10^{-12} m_p^2$.

For these values of the A and α parameters we get the values $V_* \simeq 1.3 \times 10^{-12} m_p^4$, $V_f \simeq 8.9 \times 10^{-14} m_p^4$ and $\lambda \simeq 5.1 \times 10^{-13} m_p^4 \simeq 4 \times 10^{-5} M_5^4$. Also, the number of e-folds, N , becomes of the order of $N \simeq 52.7$. We should note also that the A parameter becomes smaller by two order of magnitude and the α parameter becomes similar when it are compared with the case of tachyon-Chaplygin inflation in the low-energy limit [51].

5 Conclusions

In this work, we have studied the tachyon-Chaplygin inflationary model in the context of a braneworld scenario. In the slow-roll approximation we have found a general relation between the scalar potential and its derivative. This has led us to a general criterium for inflation to occur (see Eq. 10). We have also obtained explicit expressions for the corresponding scalar spectrum index n_s and its running α_s .

By using an exponential potential in the high-energy regime and from the WMAP 5-year data, we found the constraints of the parameters A and α from the tensor–scalar ratio r (see Figs. 2, 3). In order to bring some explicit results we have taken the constraints $A \sim 10^{-25} m_p^8$ and $\alpha \sim 10^{-6} m_p$, from which we get the values $V_* \sim 10^{-12} m_p^4$, $V_f \sim 10^{-13} m_p^4$, $\lambda \sim 10^{-13} m_p^4$ and $N \sim 53$. Here, we have used the WMAP 5-year data where $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$, $n_s(k_*) \simeq 0.96$, $\alpha_s(k_*) \simeq -0.03$ and $r(k_*) \simeq 0.055$. Note that the restrictions imposed by currents observational data allowed us to establish a small range for the parameters that appear in the tachyon-brane-Chaplygin inflationary model.

We have not addressed reheating and transition to standard cosmology in our model (see, e.g., [78; 79; 80; 81; 82; 83]). Specifically, it will be very interesting to know how the reheating temperature in the hight-energy scenario, contributes to establish some constrains on the parameters of the model. We hope to return to this point in the near future.

Acknowledgments This work was supported by the “Programa Bicentenario de Ciencia y Tecnología” through the Grant N^o PSD/06.

References

1. A. Guth (1981) *Phys. Rev. D* **23** 347
2. Albrecht, A., Steinhardt, P.J.: *Phys. Rev. Lett.* **48**, 1220 (1982); A complete description of inflationary scenarios can be found in the book by Linde, A., Particle physics and inflationary cosmology Gordon and Breach, New York (1990)
3. Dunkley, J., et al.: [WMAP Collaboration], arXiv:0803.0586 [astro-ph]
4. Hinshaw, G., et al.: arXiv:0803.0732 [astro-ph].
5. A. Sen (2002) *JHEP* **0204** 048
6. K. Akama (1982) *Lect. Notes Phys.* **176** 267
7. V.A. Rubakov M.E. Shaposhnikov (1985) *Phys. Lett. B* **159** 22
8. N. Arkani-Hamed S. Dimopoulos G. Dvali (1998) *Phys. Lett. B* **429** 263
9. M. Gogberashvili (2000) *Europhys. Lett.* **49** 396
10. L. Randall R. Sundrum (1999) *Phys. Rev. Lett.* **83** 3370
11. L. Randall R. Sundrum (1999) *Phys. Rev. Lett.* **83** 4690
12. P. Binetruy C. Deffayet D. Langlois (2000) *Nucl. Phys. B* **565** 269
13. P. Binetruy C. Deffayet U. Ellwanger D. Langlois (2000) *Phys. Lett. B* **477** 285
14. T. Shiromizu K. Maeda M. Sasaki (2000) *Phys. Rev. D* **62** 024012
15. R. Maartens D. Wands B.A. Bassett I.P.C. Heard (2000) *Phys. Rev. D* **62** 041301
16. J.M. Cline C. Grojean G. Servant (1999) *Phys. Rev. Lett.* **83** 4245
17. C. Csaki M. Graesser C. Kolda J. Terning (1999) *Phys. Lett. B* **462** 34
18. D. Ida (2000) *JHEP* **0009** 014
19. R.N. Mohapatra A. Perez-Lorenzana C.A.de S. Pires (2000) *Phys. Rev. D* **62** 105030
20. R.N. Mohapatra A. Perez-Lorenzana C.A. de S. Pires (2001) *Int. J. Mod. Phys. A* **16** 1431
21. Gong, Y.: arXiv:gr-qc/0005075.
22. Maartens, R.: arXiv:gr-qc/0101059.
23. A. Sen (2002) *Mod. Phys. Lett. A* **17** 1797
24. M. Sami P. Chingangbam T. Qureshi (2002) *Phys. Rev. D* **66** 043530
25. G.W. Gibbons (2002) *Phys. Lett. B* **537** 1
26. M.C. Bento O. Bertolami A. Sen (2002) *Phys. Rev. D* **66** 043507
27. A. Kamenshchik U. Moschella V. Pasquier (2001) *Phys. Lett. B* **511** 265
28. N. Bilic G.B. Tupper R.D. Viollier (2002) *Phys. Lett. B* **535** 17
29. Benaoum, H.B.: arXiv:hep-th/0205140
30. A. Dev J.S. Alcaniz D. Jain (2003) *Phys. Rev. D* **67** 023515
31. G.M. Kremer (2003) *Gen. Rel. Grav.* **35** 1459
32. R. Bean O. Dore (2003) *Phys. Rev. D* **68** 023515
33. Z.H. Zhu (2004) *Astron. Astrophys* **423** 421
34. H. Sandvik M. Tegmark M. Zaldarriaga I. Waga (2004) *Phys. Rev. D* **69** 123524
35. L. Amendola I. Waga F. Finelli (2005) *JCAP* **0511** 009

36. P.F. Gonzalez-Diaz (2003) *Phys. Lett. B* **562** 1
37. L.P. Chimento (2004) *Phys. Rev. D* **69** 123517
38. L.P. Chimento R. Lazkoz (2005) *Phys. Lett. B* **615** 146
39. U. Debnath A. Banerjee S. Chakraborty (2004) *Class. Quant. Grav.* **21** 5609
40. W. Zimdahl J.C. Fabris (2005) *Class. Quant. Grav.* **22** 4311
41. P. Wu H. Yu (2007) *Class. Quant. Grav.* **24** 4661
42. M.R. Setare (2007) *Phys. Lett. B* **654** 1
43. M. Bouhmadi-Lopez R. Lazkoz (2007) *Phys. Lett. B* **654** 51
44. M.C. Bento O. Bertolami A. Sen (2003) *Phys. Lett. B* **575** 172
45. M.C. Bento O. Bertolami A.A. Sen (2003) *Phys. Rev. D* **67** 063003
46. M. Makler S.Q. de Oliveira I. Waga (2003) *Phys. Lett. B* **555** 1
47. Fabris, J.C., Goncalves, S.V.B., de Souza, P.E.: arXiv:astro-ph/0207430
48. M. Biesiada W. Godlowski M. Szydlowski (2005) *Astrophys. J.* **622** 28
49. Y. Gong C.K. Duan (2004) *Mon. Not. Roy. Astron. Soc.* **352** 847
50. Y. Gong (2005) *JCAP* **0503** 007
51. O. Bertolami V. Duvvuri (2006) *Phys. Lett. B* **640** 121
52. S. del Campo R. Herrera (2008) *Phys. Lett. B* **660** 282
53. G.A., Monerat (2007) *Phys. Rev. D* **76** 024017
54. R. Herrera (2008) *Phys. Lett. B* **664** 149
55. S. del Campo R. Herrera (2008) *Phys. Lett. B* **665** 100
56. J.M. Cline C. Grojean G. Servant (1999) *Phys. Rev. Lett.* **83** 4245
57. T. Barreiro A.A. Sen (2004) *Phys. Rev. D* **70** 124013
58. L. Randall R. Sundrum (1999) *Phys. Rev. Lett.* **83** 4690
59. T. Shiromizu K. Maeda M. Sasaki (2000) *Phys. Rev. D* **62** 024012
60. D. Dvali G. Gabadadze M. Porrati (2000) *Phys. Lett. B* **485** 208
61. K. Freese M. Lewis (2002) *Phys. Lett. B* **540** 1
62. R. Maartens (2004) *Lect. Notes Phys.* **653** 213
63. A. Lue (2006) *Phys. Rept.* **423** 1
64. M. Fairbairn M.H.G. Tytgat (2002) *Phys. Lett. B* **546** 1
65. J.C. Hwang H. Noh (2002) *Phys. Rev. D* **66** 084009
66. S. Tsujikawa D. Parkinson B.A. Bassett (2003) *Phys. Rev. D* **67** 083516
67. K. Koyama D. Langlois R. Maartens D. Wands (2004) *JCAP* **0411** 002
68. K. Koyama A. Mennim D. Wands (2005) *Phys. Rev. D* **72** 064001
69. K. Koyama S. Mizuno D. Wands (2005) *JCAP* **0508** 009
70. A. Cardoso K. Koyama A. Mennim S.S. Seahra D. Wands (2007) *Phys. Rev. D* **75** 084002
71. K. Koyama A. Mennim V.A. Rubakov D. Wands T. Hiramatsu (2007) *JCAP* **0704** 001
72. K. Koyama A. Mennim D. Wands (2008) *Phys. Rev. D* **77** 021501
73. D. Langlois R. Maartens D. Wands (2000) *Phys. Lett. B* **489** 259
74. M. Sami N. Savchenko A. Toporensky (2004) *Phys. Rev. D* **70** 123528
75. M., Tegmark (2004) *Phys. Rev. D* **69** 103501
76. D. Kutasov M. Marino G.W. Moore (2000) *JHEP* **0010** 045
77. A. de la Macorra U. Filobello G. German (2006) *Phys. Lett. B* **635** 355
78. B. Feng M. Li (2003) *Phys. Lett. B* **564** 169
79. Campuzano, C., del Campo, S., Herrera, R.: *Phys. Rev. D* **72**, 083515 (2005), [Erratum-ibid. *D* **72**, 109902 (2005)]

-
80. C. Campuzano S. del Campo R. Herrera (2006) *Phys. Lett. B* **633** 149
 81. C. Campuzano S. del Campo R. Herrera (2006) *JCAP* **0606** 017
 82. S. del Campo R. Herrera (2007) *Phys. Rev. D* **76** 103503
 83. J.C. Bueno Sanchez K. Dimopoulos (2007) *JCAP* **11** 007