

MHz gravitational waves from short-term anisotropic inflation

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Abstract. We reveal the universality of short-term anisotropic inflation. As a demonstration, we study inflation with an exponential type gauge kinetic function which is ubiquitous in models obtained by dimensional reduction from higher dimensional fundamental theory. It turns out that an anisotropic inflation universally takes place in the later stage of conventional inflation. Remarkably, we find that primordial gravitational waves with a peak amplitude around $10^{-26} \sim 10^{-27}$ are copiously produced in high-frequency bands 10 MHz~100 MHz. If we could detect such gravitational waves in future, we would be able to probe higher dimensional fundamental theory.

Keywords: inflation, primordial gravitational waves (theory)

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1 Introduction

As is often said, cosmology has entered into the precision area. Therefore, we need to refine predictions of inflation up to the percent level and reveal qualitatively new phenomena. To this aim, we should recall that slight violation of time translation symmetry among de Sitter symmetry gives rise to a tilt of the power spectrum of curvature perturbations of the order of a slow roll parameter. The point is that the slow roll parameter characterizes deviation from the exact de Sitter spacetime. It is interesting to observe that the non-gaussianity is also of the order of the slow roll parameter in conventional inflation. Thus, it is legitimate to infer that the symmetry breaking is a key to seek other qualitatively new phenomena of the order of the slow roll parameter.

Along the above line of thought, it is natural to consider the possibility of violation of rotational symmetry among the de Sitter symmetry, which must lead to the statistical anisotropy in primordial fluctuations. In fact, there were observational hints of statistical anisotropy in the CMB data [1]. Nevertheless, it was not so easy to invent a mechanism to realize the static anisotropy in an inflationary scenario partially because of psychological barrier due to the cosmic no-hair conjecture. Indeed, it had been widely believed that an anisotropy of the universe rapidly decays during inflation. This is the so called the cosmic no-hair conjecture. Indeed, the cosmic no-hair theorem [2, 3] where homogeneous universe with a cosmological constant is assumed supported the cosmic no-hair conjecture. This is because the inflaton potential can mimic the cosmological constant. Historically, there have been challenges to give a counter example to the cosmic no-hair conjecture [4–11]. However, these models suffer from either instability, a fine tuning problem or a naturalness problem [12–16].

Eventually, an anisotropic inflationary model was found [17], which can be regarded as the first counter example to the cosmic no-hair conjecture. More precisely, it is shown that anisotropic inflation occurs in the presence of a gauge kinetic function through which the gauge field is coupled to an inflaton and gives rise to the static anisotropy of the order of a few percent [18–20]. It should be stressed that anisotropic inflation can be naturally realized in the context of supergravity. After the discovery of anisotropic inflation, many cosmological predictions [18–31] and further extension to various models [32–45] have been discussed. Importantly, anisotropic inflation can be tested with observations [46–53].

In previous works, it has been assumed that the gauge kinetic function has a specific relation to the inflaton potential function. Actually, it is not necessary to assume the specific functional form for the presence of short-term anisotropic inflation. Indeed, short-term

anisotropic inflation occurs universally. In this paper, we emphasize this point and study its phenomenological consequence. As a concrete demonstration, we study inflation with the exponential type gauge kinetic function. As is well known, the exponential function is ubiquitous from the point of view of dimensional reduction from higher dimensional fundamental theory. As to the inflaton potential, we consider a monomial potential which can drive chaotic inflation. It turns out that anisotropic inflation occurs in the last several e-folds. In fact, the gauge field starts to grow in the last stage of inflation. We show that this universally happens for any monomial potential.

In the conventional inflation, primordial gravitational waves are produced from vacuum quantum fluctuations. In the present model, due to the coupling between the inflaton and the gauge field, copious gravitational waves are produced by the classical gauge field. Since the production occurs in the last stage of the inflation, the frequency range of gravitational waves is typically 10 MHz~100 MHz. The amplitude of gravitational waves depends on model parameters. In fact, it is easy to exceed the bound coming from the nucleosynthesis. Hence, it is important to observationally explore primordial gravitational waves with MHz frequencies from short-term anisotropic inflation. Although the current sensitivity of observations is too low to detect the gravitational waves [54–56], future detectors [57–63] may achieve the required sensitivity. The recent discovery of gravitational waves [64] encourages us to pursue the study of high frequency gravitational waves.

The paper is organized as follows. In section 2, we discuss the universality of short-term anisotropic inflation. As a demonstration, we show that short-term anisotropic inflation occurs even in a model with the monomial inflaton potential and the exponential type gauge kinetic function. We explain how this occurs in the case of chaotic inflation in detail. In section 3, we discuss quantization of the gauge field during anisotropic inflation and derive mode functions for later calculations. In section 4, we calculate the power spectrum of the gravitational waves induced by the gauge field during anisotropic inflation. It turns out that copious gravitational waves can be produced in the MHz frequency range. The final section is devoted to the conclusion.

2 Universality of short-term anisotropic inflation

In this section, first we review anisotropic inflation [17]. We stress the universality of short-term anisotropic inflation. Then, we investigate a specific exponential type gauge kinetic function to reveal universality of short-term anisotropic inflation.

We consider an action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right], \quad (2.1)$$

where M_p represents the reduced Plank mass, g is the determinant of the metric $g_{\mu\nu}$ and R is the Ricci scalar. Here, we introduced a gauge field A_μ with the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the gauge field is coupled to an inflaton ϕ through a gauge kinetic function $f(\phi)$. For the moment, we do not specify a potential function for the inflaton $V(\phi)$.

Let us discuss the cosmological homogeneous dynamics. The homogeneity imposes $\phi = \phi(t)$. For the gauge field, we can take an ansatz $A_\mu = (0, v_A(t), 0, 0)$ without loss of generality. In the presence of the non-trivial gauge field, we can not take isotropic ansatz for the metric. Instead, we take the anisotropic ansatz

$$ds^2 = -dt^2 + e^{2\alpha(t)} \left[e^{-4\sigma(t)} dx^2 + e^{2\sigma(t)} (dy^2 + dz^2) \right]. \quad (2.2)$$

Now, we can derive equations of motion. It is easy to solve the equation for the gauge field as

$$\dot{v}_A = p_A f_A^{-2} e^{-2\rho_A \frac{\phi}{M_p}} e^{-\alpha-4\sigma}. \quad (2.3)$$

where p_A is an integration constant. Thus, the field equation for the inflaton reads

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - V_{,\phi} + p_A^2 f_{,\phi} f^{-3}(\phi) e^{-4\alpha-4\sigma}, \quad (2.4)$$

From the Einstein equation, we obtain the hamiltonian constraint

$$\dot{\alpha}^2 = \dot{\sigma}^2 + \frac{1}{3M_{\text{pl}}^2} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}p_A^2 f^{-2}(\phi) e^{-4\alpha-4\sigma} \right], \quad (2.5)$$

and the rest of Einstein equations

$$\ddot{\alpha} = -3\dot{\alpha}^2 + \frac{1}{M_p^2} V(\phi) + \frac{1}{6M_p^2} p_A^2 f^{-2}(\phi) e^{-4\alpha-4\sigma}, \quad (2.6)$$

$$\ddot{\sigma} = -3\dot{\alpha}\dot{\sigma} + \frac{1}{3M_p^2} f^{-2}(\phi) e^{-4\alpha-4\sigma}. \quad (2.7)$$

To understand how anisotropic inflation occurs, it is convenient to express the gauge kinetic function in terms of the scale factor $a(t)$ as

$$f(\phi) \propto a^{-n}(t) = e^{-n\alpha}. \quad (2.8)$$

From eq. (2.5), one can see that the energy density of the gauge field grows if $n > 2$. It decays for $n < 2$. In the case of $n = 2$, it is almost constant. Note that σ is always negligible compared with α . In the regime where the inflaton slow rolls and the back reaction from the gauge field is negligible, eqs. (2.4) and (2.5) can be reduced to

$$3\dot{\alpha}\dot{\phi} \simeq -V_{,\phi}, \quad (2.9)$$

$$\dot{\alpha}^2 \simeq \frac{1}{3M_{\text{pl}}^2} V(\phi). \quad (2.10)$$

These equations lead to

$$\frac{d\alpha}{d\phi} = -\frac{V}{V_{,\phi}}. \quad (2.11)$$

Integrating it, we obtain

$$\alpha = -\int \frac{1}{M_{\text{pl}}^2} \frac{V}{V_{,\phi}} + D, \quad (2.12)$$

where D is a constant of integration. Thus, we find functional form of the gauge kinetic function (2.8) as

$$f(\phi) = e^{\frac{n}{M_{\text{pl}}^2} \int \frac{V}{V_{,\phi}} d\phi}. \quad (2.13)$$

Although the expression (2.13) is useful for discussing concrete models of anisotropic inflation, it is too restrictive. In fact, it requires the gauge field is always active during inflation. However, it is interesting to consider the possibility that the gauge field becomes relevant for a certain period during inflation. Actually, as stressed in [46], the condition for the occurrence of anisotropic inflation reads

$$M_{\text{pl}}^2 \frac{f_{,\phi}}{f} \frac{V_{,\phi}}{V} > 2. \quad (2.14)$$

This condition determines the region where short-term anisotropic inflation occurs. In the extreme case (2.13), the gauge field is sustained in the whole period of inflation.

From now on, as a concrete demonstration, we consider the gauge kinetic function

$$f(\phi) = e^{\frac{c}{M_{\text{pl}}}\phi}, \quad (2.15)$$

where c is a coupling constant. It should be noted that such an exponential type gauge kinetic function is ubiquitous in models obtained by dimensional reduction from higher dimensional fundamental theory such as superstring theory. Apparently, from the criterion (2.13), anisotropic inflation does not happen. In fact, however, for a certain constant c , the gauge field can grow. We will show it explicitly. For the gauge field to grow, the decreasing rate of the gauge kinetic function must be larger than that of (2.13) with $n = 2$. Note that eq. (2.13) is a decreasing function. When we choose the functional form (2.15), this condition is given by

$$\frac{d}{dt} \left[\exp \left(\frac{c}{M_{\text{pl}}}\phi - \frac{2}{M_{\text{pl}}^2} \int \frac{V}{V_{,\phi}} d\phi \right) \right] < 0. \quad (2.16)$$

Of course, this does not hold in general. Rather, it constrains the region where anisotropic inflation is possible. Indeed, we have the condition

$$c > \frac{2}{M_{\text{pl}}} \frac{V}{V_{,\phi}}, \quad (2.17)$$

where we used the fact that the derivative of the inflaton with respect to time is negative. Notice that this is nothing but the condition (2.14). If the inequality (2.17) is satisfied, the gauge field grows. To be more precise, let us consider a monomial potential

$$V(\phi) \propto \phi^l, \quad (2.18)$$

where l is a real parameter. Then it becomes

$$c > \frac{2}{l} \frac{\phi}{M_{\text{pl}}}. \quad (2.19)$$

Thus, in the region where the condition (2.19) is satisfied, we can realize anisotropic inflation. For example, for the potential (2.18), the inflaton takes the value $\phi \sim \mathcal{O}(10M_{\text{pl}})$ around CMB scales provided the number of e-foldings $50 \sim 60$, and the inflation ends around $\phi \sim \mathcal{O}(1M_{\text{pl}})$. Hence, if $c > \mathcal{O}(1)$ we can see the growth of the gauge field in the late stage of inflation.

In eq. (2.8), we assumed constancy of n . Then, we obtained the functional form (2.13). Since we took the gauge coupling function (2.15) different from the function (2.13), n should be time dependent. Let us try to express the gauge kinetic function (2.15) in the form (2.8) with the time dependent $n(t)$:

$$e^{\frac{c}{M_{\text{pl}}}\phi} \propto a^{-n(t)} = e^{-n(t)H_I t}, \quad (2.20)$$

where H_I is the Hubble constant during inflation. This gives rise to the following relation

$$\frac{c}{M_{\text{pl}}}\dot{\phi} = -\dot{n}H_I t - nH_I. \quad (2.21)$$

Here, we assume the first term of the right-hand side of eq. (2.21) is negligible compared with the second term in a time scale of expansion H_I^{-1} . Then, we get

$$n = -\frac{c}{M_{\text{pl}}H_I}\dot{\phi}. \quad (2.22)$$

From this relation, one can take a derivative of $n(t)$ with respect to the time

$$\frac{\dot{n}}{H_I n} = \frac{\ddot{\phi}}{H_I \dot{\phi}}. \quad (2.23)$$

If the slow roll condition is satisfied, it is much smaller than 1. Therefore we can treat $n(t)$ as an almost constant quantity during slow roll inflation.

Now let us examine a concrete example of short-term anisotropic inflation with the exponential gauge kinetic function and the potential $V = \frac{1}{2}m^2\phi^2$. Here, m is the mass of the inflaton. In this case, eqs. (2.4)–(2.7) lead to

$$\dot{\alpha}^2 = \dot{\sigma}^2 + \frac{1}{3M_{\text{pl}}^2} \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}p_A^2 e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma} \right], \quad (2.24)$$

$$\ddot{\alpha} = -3\dot{\alpha}^2 + \frac{1}{2M_p^2}m^2\phi^2 + \frac{1}{6M_p^2}p_A^2 e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma}, \quad (2.25)$$

$$\ddot{\sigma} = -3\dot{\alpha}\dot{\sigma} + \frac{1}{3M_p^2}p_A^2 e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma}, \quad (2.26)$$

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - m^2\phi + p_A^2 \frac{c}{M_{\text{pl}}} e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma}. \quad (2.27)$$

We solve the above equations numerically. The results are shown in figures 1–3. In the calculations, we have set the mass of inflaton $m = 10^{-5}M_{\text{pl}}$, the initial condition of the inflaton $\phi_0 = 13M_{\text{pl}}$, and $c = 9$. In figure 1, we plotted the evolution of the inflaton. We can see a transition around at $\alpha = 38$. From figure 2, taking a look at the green dotted arrow, one can find that the gauge field start to grow at around $\alpha = 22$ and eventually enter into the attractor around at $\alpha = 38$. From figure 1, the number of e-folds $\alpha = 22$ corresponds to $\phi = 9M_{\text{pl}}$. This is a consequence of the inequality (2.19). This growth of the gauge field made the transition of the inflation to the second inflationary phase around at $\alpha \simeq 38$. In the second phase, the third term of the right-hand side of eq. (2.27) can not be negligible, namely, $p_A^2 \frac{c}{M_{\text{pl}}} e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma} \sim V_{,\phi}$. Then the ratio of the energy density of the gauge field to that of the inflaton field during the second inflationary phase is

$$\mathcal{R} \equiv \frac{\frac{1}{2}p_A^2 e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma}}{V(\phi)} \sim \frac{M_{\text{pl}}}{2c} \frac{V_{,\phi}}{V}. \quad (2.28)$$

In the present case, we have

$$\mathcal{R} \sim \frac{M_{\text{pl}}}{c\phi}. \quad (2.29)$$

Thus, the energy density of the gauge field is negligible in eq. (2.24) even when the gauge field is relevant in eq. (2.27) during the second inflationary phase. Therefore, from eqs. (2.24)

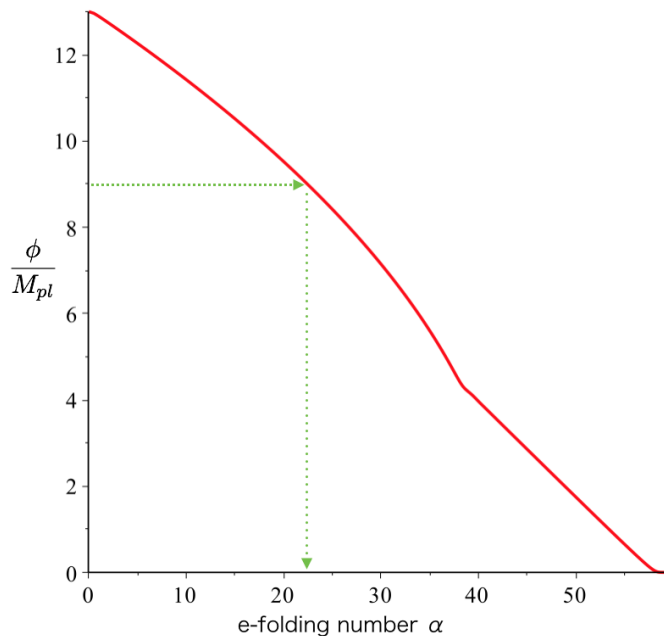


Figure 1. The evolution of the inflaton field is depicted. The green vector indicates the e-folding number corresponding to $\frac{\phi}{M_{\text{pl}}} = 9 (= c)$. The transition to the second inflationary phase occurs around $\alpha \simeq 38$.

and (2.26), we can get

$$\dot{\alpha}^2 \simeq \frac{1}{3M_{\text{pl}}^2} V, \quad (2.30)$$

$$3\dot{\alpha}\dot{\sigma} \simeq \frac{1}{3M_p^2} p_A^2 e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma}, \quad (2.31)$$

where we have assumed $\ddot{\sigma} \ll 3\dot{\alpha}\dot{\sigma}$. From eqs. (2.30) and (2.31), we obtain the relation between the anisotropy $\frac{\dot{\sigma}}{\dot{\alpha}}$ and the ratio \mathcal{R} as follows

$$\frac{\dot{\sigma}}{\dot{\alpha}} \simeq \frac{1}{3} \frac{p_A^2 e^{-2\frac{c}{M_{\text{pl}}}\phi} e^{-4\alpha-4\sigma}}{V} \sim \frac{M_{\text{pl}}}{3c} \frac{V_{,\phi}}{V} \sim \frac{2}{3} \mathcal{R}. \quad (2.32)$$

We note that eq. (2.29) holds during the attractor. Considering the effective potential of the inflaton field, it turns out that the attractor like behavior means the inflaton falls the trough of the effective potential. Numerical result figure 3 tells us that the inflaton moves with the trough. In the second inflationary phase, the rolling of the inflaton is governed by the effective potential corrected by the gauge field, so that the velocity of the inflaton decreases gradually. This result is different from that of conventional anisotropic inflation with the gauge kinetic function (2.13) [17]. From the view of eq. (2.20), we can understand that the difference comes from the growth of the power n .

In the above example, we took $c = 9$. In this case, at some point the gauge field starts to grow and continues to grow for about 15 e-folds, and enter into the attractor phase of short-term anisotropic inflation which persists about 20 e-folds. For a more moderate parameter c , the duration of short-term anisotropic inflation becomes more short. In the extreme case, there would be no attractor phase and the period where the gauge field is relevant would be about 4 e-folds. It is the case that we are focusing on in the rest of this paper.

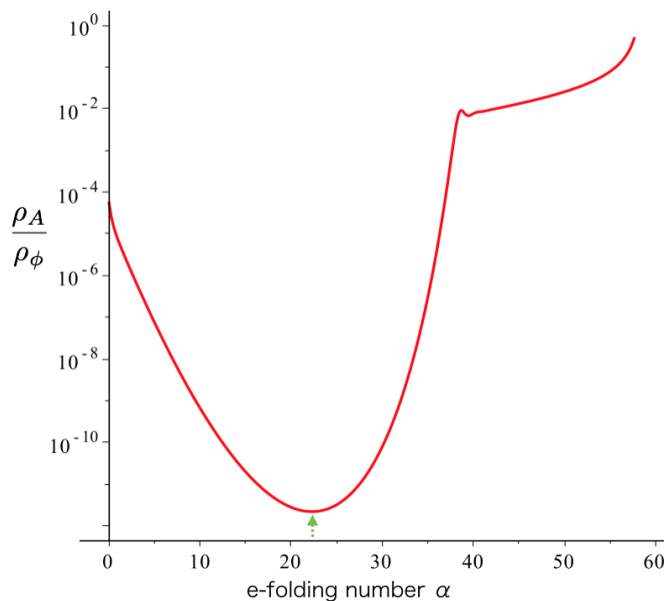


Figure 2. The ratio of the energy density of the gauge field to that of the inflaton field is plotted. The green vector indicates the moment when the inflaton $\frac{\phi}{M_{pl}} = 9(=c)$. After $\alpha \simeq 38$, the ratio becomes nearly a constant.

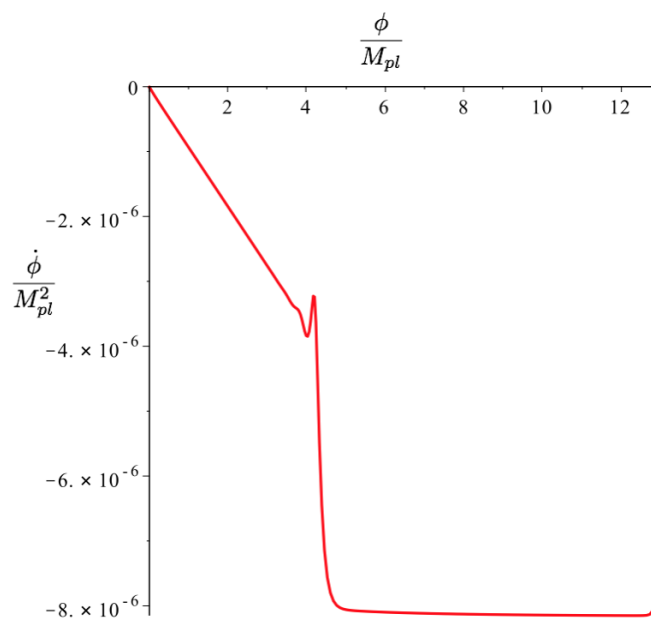


Figure 3. The trajectory of the inflaton field in the phase space is depicted. We took the initial conditions $\phi_0 = 13M_{pl}$ and $\dot{\phi}_0 = 0$. The transition to the second inflationary phase occurs around $\phi \simeq 4M_{pl}$.

3 Quantum gauge field during anisotropic inflation

In this section, we derive the mode function of the gauge field which will be used for calculations of gravitational waves in the next section.

Neglecting the back reaction from the gauge field, we consider the isotropic inflationary background

$$ds^2 = a(\tau)^2 [-d\tau^2 + dx^2 + dy^2 + dz^2] , \quad (3.1)$$

where τ is the conformal time. One can expand the gauge field in Fourier space as

$$\vec{A}(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \hat{A}_{\mathbf{k}}^{(\sigma)}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} . \quad (3.2)$$

Then the part for the gauge field in the action (2.1) can be rewritten as

$$S_{\text{gauge}} = \frac{1}{2} \int d\tau d^3k f^2(\phi) \left[\hat{A}'_{\mathbf{k}} \hat{A}'_{-\mathbf{k}} - k^2 \hat{A}_{\mathbf{k}} \hat{A}_{-\mathbf{k}} \right] , \quad (3.3)$$

where a prime represents a derivative with respect to the conformal time. The Fourier mode of the vector potential can be promoted into the operator and expanded by the creation and annihilation operators satisfying $[\hat{a}_{\mathbf{k}}^{(\sigma)}, \hat{a}_{\mathbf{k}'}^{(\rho)\dagger}] = \delta_{\sigma\rho} \delta(\mathbf{k} - \mathbf{k}')$ as

$$\hat{A}_{\mathbf{k}}^{(\sigma)}(\tau) = \sum_{\sigma=+,-} \vec{e}^{(\sigma)}(\hat{k}) \left[U_{\mathbf{k}}(\tau) \hat{a}_{\mathbf{k}}^{(\sigma)} + U_{\mathbf{k}}^*(\tau) \hat{a}_{-\mathbf{k}}^{(\sigma)\dagger} \right] , \quad (3.4)$$

where σ represents the two polarization degrees of the freedom of the vector potential. The polarization vector $\vec{e}^{(\sigma)}$ satisfies the relations

$$\begin{aligned} \vec{k} \cdot \vec{e}^{(\pm)}(\hat{k}) &= 0 , \\ \vec{k} \times \vec{e}^{(\pm)}(\hat{k}) &= \mp i k \vec{e}^{(\pm)}(\hat{k}) , \\ \left(\vec{e}^{(\sigma)}(\hat{k}) \right)^* &= \vec{e}^{(\sigma)}(-\hat{k}) , \quad \left(\vec{e}^{(\sigma)}(\hat{k}) \right)^* \cdot \vec{e}^{(\rho)}(\hat{k}) = \delta_{\sigma\rho} . \end{aligned} \quad (3.5)$$

The mode function obeys the equation derived from the action (3.3)

$$U_{\mathbf{k}}'' + 2 \frac{f'}{f} U_{\mathbf{k}}' + k^2 U_{\mathbf{k}} = 0 . \quad (3.6)$$

Using the new variable $u_{\mathbf{k}} \equiv f U_{\mathbf{k}}$, we get

$$u_{\mathbf{k}}'' + \left(k^2 - \frac{f''}{f} \right) u_{\mathbf{k}} = 0 . \quad (3.7)$$

Hereafter we consider the exponential type functional form (2.15). Then, we get

$$\frac{f''(\tau)}{f(\tau)} = c\phi''(\tau) + c^2\phi'^2(\tau) . \quad (3.8)$$

Using the cosmic time t , the derivative of the ϕ with respect to τ is rewritten as

$$\phi'(\tau) = a(\tau) \dot{\phi}(t) , \quad (3.9)$$

where an overdot denotes a derivative with respect to the cosmic time. Assuming the slow roll inflation, we obtain

$$a(\tau) \simeq -\frac{1}{H_I \tau}, \quad (3.10)$$

$$\dot{\phi} \simeq -\sqrt{2\epsilon} M_{\text{pl}} H_I, \quad (3.11)$$

where H_I is the Hubble constant during inflation and $\epsilon \equiv -\frac{\dot{H}_I}{H_I^2}$ is the slow roll parameter. From these equations, eq. (3.9) becomes

$$\phi'(\tau) = \frac{\sqrt{2\epsilon} M_{\text{pl}}}{\tau}. \quad (3.12)$$

Since ϵ is almost constant, we can ignore the derivative of ϵ . Hence, we get

$$\phi''(\tau) \simeq -\frac{\sqrt{2\epsilon} M_{\text{pl}}}{\tau^2}. \quad (3.13)$$

Thus, eq. (3.8) reduces to

$$\frac{f''(\tau)}{f(\tau)} \simeq \frac{-c\sqrt{2\epsilon} + 2c^2\epsilon}{\tau^2}. \quad (3.14)$$

Substituting eq. (3.14) into eq. (3.7) and solving it with the Bunch-Davies initial condition, we get the mode function

$$u(\tau, \vec{k}) = \frac{1}{\sqrt{2k}} \sqrt{\frac{-k\tau\pi}{2}} H_\nu^{(1)}(z), \quad (3.15)$$

where $H_\nu^{(1)}(x)$ is the Hankel function of the first kind, $z \equiv -k\tau$ and

$$\nu \equiv c\sqrt{2\epsilon}. \quad (3.16)$$

Now, we define the electric and magnetic fields as

$$\vec{E}(\tau, \mathbf{x}) \equiv -\frac{f}{a^2} \partial_\tau \vec{A}(\tau, \mathbf{x}), \quad \vec{B}(\tau, \mathbf{x}) \equiv \frac{f}{a^2} (\nabla \times \vec{A}(\tau, \mathbf{x})). \quad (3.17)$$

They can be expanded in Fourier space as

$$\vec{E}(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \hat{E}_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (3.18)$$

$$\vec{B}(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \hat{B}_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (3.19)$$

where

$$\hat{E}_{\mathbf{k}}(\tau) = \sum_{\sigma=+,-} \bar{e}^{(\sigma)} \left[\mathcal{E}_{\mathbf{k}} \hat{a}_{\mathbf{k}}^{(\sigma)} + \mathcal{E}_{\mathbf{k}}^* \hat{a}_{-\mathbf{k}}^{(\sigma)\dagger} \right], \quad (3.20)$$

$$\hat{B}_{\mathbf{k}}(\tau) = \sum_{\sigma=+,-} \sigma \bar{e}^{(\sigma)} \left[\mathcal{B}_{\mathbf{k}} \hat{a}_{\mathbf{k}}^{(\sigma)} + \mathcal{B}_{\mathbf{k}}^* \hat{a}_{-\mathbf{k}}^{(\sigma)\dagger} \right]. \quad (3.21)$$

Here, we defined

$$\mathcal{E}_{\mathbf{k}}(\tau) = -\frac{f}{a^2} \partial_\tau U_{\mathbf{k}}(\tau), \quad \mathcal{B}_{\mathbf{k}}(\tau) = \frac{f}{a^2} k U_{\mathbf{k}}(\tau). \quad (3.22)$$

Using the mode function (3.15), we obtain

$$\mathcal{E}_{\mathbf{k}}(\tau) = \sqrt{\frac{\pi}{2}} k^{-3/2} (H_I z)^2 \sqrt{\frac{z}{2}} H_{\nu+1}^{(1)}(z), \quad \mathcal{B}_{\mathbf{k}}(\tau) = \sqrt{\frac{\pi}{2}} k^{-3/2} (H_I z)^2 \sqrt{\frac{z}{2}} H_{\nu}^{(1)}(z). \quad (3.23)$$

In the super horizon limit $|z| \rightarrow 0$, we can use the approximation

$$H_{\nu+1}^{(1)}(z) \simeq -\frac{i\Gamma(\nu)}{\pi} \left(\frac{2}{z}\right)^{\nu}. \quad (3.24)$$

Now, we consider only the case $\nu > 2$. Then the electric field and the magnetic field are

$$\mathcal{E}_{\mathbf{k}}(\tau) = \frac{i\Gamma(\nu + \frac{1}{2})}{\sqrt{\pi}} H_I^2 \left(\frac{2}{k}\right)^{3/2} \left(\frac{2}{z}\right)^{\nu-2}, \quad (3.25)$$

$$\mathcal{B}_{\mathbf{k}}(\tau) = -\frac{i\Gamma(\nu - \frac{1}{2})}{\sqrt{\pi}} H_I^2 \left(\frac{2}{k}\right)^{3/2} \left(\frac{2}{z}\right)^{\nu-3}. \quad (3.26)$$

We can find that the electric field always dominates over the magnetic field and only when $\nu > 2$, it grows in the super horizon regime. Substituting the slow roll parameter by the inflaton potential $\epsilon \simeq \frac{M_{\text{pl}}^2}{2} \left(\frac{V_{,\phi}}{V}\right)^2$ into eq. (3.16), we have

$$\nu \simeq c M_{\text{pl}} \left(\frac{V_{,\phi}}{V}\right). \quad (3.27)$$

Then the condition $\nu > 2$ yields

$$c > \frac{2}{M_{\text{pl}}} \frac{V}{V_{,\phi}}. \quad (3.28)$$

This coincides with the condition (2.17).

4 MHz gravitational waves induced by gauge field

In this section, we consider short-term anisotropic inflation with $c \sim 3$ for which typically there is no an attractor phase. We show that copious gravitational waves can be produced by the gauge field during the phase approaching the attractor. The analysis is quite similar to that used in a different context [65, 66].

Since we consider the situation where the gauge field is negligible in background equations, the energy momentum tensor, namely the nonlinear part, of the gauge field becomes the source of gravitational waves. One can get the tensor sector of the action (2.1) as

$$S_{\text{GW}} = \int d\tau d^3x \left[\frac{M_{\text{pl}}^2}{8} a^2 (h'_{ij} h'^{ij} - \partial_k h_{ij} \partial_k h^{ij}) + \frac{1}{2} a^4 (E_i E_j + B_i B_j) h^{ij} \right], \quad (4.1)$$

where h_{ij} is a transverse traceless tensor and we used the definition of the electric and magnetic fields (3.17). The tensor fluctuation can be expanded in Fourier space as

$$h_{ij} = \sum_{\sigma=+,-} \int \frac{d^3k}{(2\pi)^{3/2}} \hat{h}_{\mathbf{k}}^{(\sigma)} e^{i\mathbf{k}\cdot\mathbf{x}} \Pi_{ij}^{(\sigma)}, \quad (4.2)$$

$$\hat{h}_{\mathbf{k}}^{(\sigma)}(\tau) = V_{\mathbf{k}}(\tau) \hat{a}_{\mathbf{k}}^{(\sigma)} + V_{\mathbf{k}}^*(\tau) \hat{a}_{-\mathbf{k}}^{(\sigma)\dagger}, \quad (4.3)$$

where $\Pi_{ij}^{(\sigma)}$ are polarization tensors constructed by circular polarization vectors as $\Pi_{ij}^{(\sigma)} \equiv e_i^{(\sigma)} e_j^{(\sigma)}$ and we have used creation and annihilation operators. Substituting eqs. (3.18)–(3.21) and (4.2) into eq. (4.1), we obtain

$$S_{\text{GW}} = \sum_{\sigma=+,-} \int d\tau d^3k \left[\frac{M_{\text{pl}}^2}{4} a^2 \left(\hat{h}'_{\mathbf{k}}^{(\sigma)} \hat{h}'_{\mathbf{k}}^{(\sigma)} - k^2 \hat{h}_{\mathbf{k}}^{(\sigma)2} \right) - \frac{a^4}{2} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\hat{E}_{i,\mathbf{k}} \hat{E}_{j,\mathbf{k}-\mathbf{p}} + \hat{B}_{i,\mathbf{k}} \hat{B}_{j,\mathbf{k}-\mathbf{p}} \right) e_i^{*(\sigma)}(\hat{k}) e_j^{*(\sigma)}(\hat{k}) \hat{h}_{\mathbf{k}}^{(\sigma)} \right]. \quad (4.4)$$

Using the normalized variable $v_{\mathbf{k}} \equiv \frac{M_{\text{pl}}}{2} a V_{\mathbf{k}}$, we can get the equation for the mode function of the gravitational waves as

$$v_{\mathbf{k}}''(\tau) + \left(k^2 - \frac{2}{\tau^2} \right) v_{\mathbf{k}}(\tau) = S^{(\sigma)}(\tau, \mathbf{k}), \quad (4.5)$$

where the source term is defined by

$$S^{(\sigma)}(\tau, \mathbf{k}) = -\frac{a^3}{M_{\text{pl}}} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\hat{E}_{i,\mathbf{k}} \hat{E}_{j,\mathbf{k}-\mathbf{p}} + \hat{B}_{i,\mathbf{k}} \hat{B}_{j,\mathbf{k}-\mathbf{p}} \right) e_i^{*(\sigma)}(\hat{k}) e_j^{*(\sigma)}(\hat{k}). \quad (4.6)$$

We define the power spectrum of tensor fluctuations as

$$\langle h^{(\sigma)}(\mathbf{k}) h^{(\sigma)}(\mathbf{k}') \rangle = \frac{2\pi}{k^3} P^{(\sigma)}(k) \delta^{(3)}(\mathbf{k} + \mathbf{k}'). \quad (4.7)$$

Let us divide the tensor fluctuations into the two parts. The one comes from vacuum fluctuations and the other comes from the gauge field. Since they are uncorrelated to each other, we can write the tensor power spectrum as the sum

$$P^{(\sigma)}(k) = P_v^{(\sigma)}(k) + P_s^{(\sigma)}(k), \quad (4.8)$$

where $P_v(k) = 2 \times \left(\frac{H_I}{\pi M_{\text{pl}}} \right)^2$. From eqs. (4.5)–(4.8), we can deduce

$$P_s^{(\sigma)}(k) = \frac{k^3}{\pi^2 M_{\text{pl}}^4 a^2} \int \frac{d^3p}{(2\pi)^3} \left(1 + (\hat{k} \cdot \hat{p})^2 \right) \left(1 + (\hat{k} \cdot \widehat{\mathbf{k}-\mathbf{p}})^2 \right) \times \left| \int d\tau' a^3(\tau') G_k(\tau, \tau') \mathcal{E}(\tau', p) \mathcal{E}(\tau', |\mathbf{k}-\mathbf{p}|) \right|^2, \quad (4.9)$$

where we ignored the subdominant contribution of the magnetic field and used an identity $\left| \vec{e}^{(\sigma)}(\hat{k}) \cdot \vec{e}^{(\rho)}(\hat{k}') \right|^2 = \frac{1}{4} \left(1 - \sigma\rho(\hat{k} \cdot \hat{k}') \right)^2$. The green function $G_k(\tau, \tau')$ for eq. (4.5) is given by

$$G_k(\tau, \tau') \simeq \frac{1}{k^3 \tau \tau'} \left[k\tau' \cos(k\tau') - \sin(k\tau') \right] \quad (|k\tau| \ll 1), \\ \simeq -\frac{\tau'^2}{3\tau} \quad (|k\tau|, |k\tau'| \ll 1). \quad (4.10)$$

Substituting eqs. (3.25) and (4.10) into eq. (4.9) and using the new variables $\vec{q} \equiv \frac{\vec{p}}{k}$, $\vec{q}' \equiv \frac{\vec{p}-\vec{k}}{k}$ and $z \equiv -k\tau$, we get

$$\begin{aligned}
 P_s(k) &= 2 \times P_s^{(\sigma)}(k) \\
 &= \frac{1}{36\pi^7} \left(\frac{H_I}{M_{\text{pl}}} \right)^4 \int d^3q \left(1 + (\hat{k} \cdot \hat{q})^2 \right) \left(1 + (\hat{k} \cdot \hat{q}')^2 \right) \\
 &\quad \times \left| \int dz' q^{-\nu+1/2} q'^{-\nu+1/2} \frac{2^{2\nu-1} \Gamma^2(\nu + \frac{1}{2})}{z'^{2\nu-3}} \right|^2, \tag{4.11}
 \end{aligned}$$

where we used the fact that there is no polarization of gravitational waves. Note that ν depends on z' . Since the gauge field becomes relevant as the source of the gravitational waves after starting to grow, we consider the region

$$|\tau_{\text{in}}| > \frac{1}{p}, \frac{1}{|\vec{p}-\vec{k}|}, \tag{4.12}$$

where τ_{in} is the time when the gauge field starts to grow. Multiplying it by k , we get

$$\frac{1}{q_{\text{in}}} > \frac{1}{q}, \frac{1}{q'}, \tag{4.13}$$

where $q_{\text{in}} \equiv |k\tau_{\text{in}}|^{-1}$ represents the infrared cut off of the momentum integral. Since the time integral is carried out in the range where the source has been on super horizon scales, we have the relation

$$|\tau| < |\tau'| < \frac{1}{p}, \frac{1}{|\vec{p}-\vec{k}|}, |\tau_{\text{in}}|. \tag{4.14}$$

From eq. (4.12), it reads

$$|\tau| < |\tau'| < \frac{1}{p}, \frac{1}{|\vec{p}-\vec{k}|}. \tag{4.15}$$

Multiplying k , we obtain

$$z < z' < \min \left[\frac{1}{q}, \frac{1}{q'} \right]. \tag{4.16}$$

The dominant contribution to the momentum integral comes from $q \sim q' \sim q_{\text{in}}$. This means $q \sim q' \sim 1$, namely, the upper limit of the time integral becomes

$$\min \left[\frac{1}{q}, \frac{1}{q'} \right] \sim 1. \tag{4.17}$$

Therefore, we obtain

$$P_s(k) = \frac{2}{9\pi^6} \left(\frac{H_I}{M_{\text{pl}}} \right)^4 \left| \int_z^1 dz' \frac{2^{2\nu-1} \Gamma^2(\nu + \frac{1}{2})}{\sqrt{\nu-2}} \left(\frac{1}{q_{\text{in}}} \right)^{\nu-2} z'^{-2\nu+3} \right|^2. \tag{4.18}$$

To calculate the integration (4.18), we must know the function $\nu(z')$. Taking a look at eq. (3.27), we see an explicit form of the inflaton potential $V(\phi)$ is necessary. Here, we consider a monomial inflaton potential, namely $V(\phi) \propto \phi^l$. Then we obtain

$$\nu = \frac{lcM_{\text{pl}}}{\phi}. \tag{4.19}$$

In this case, eq. (2.12) leads to

$$\alpha = -\frac{1}{2l} \frac{\phi^2}{M_{\text{pl}}^2} + D. \quad (4.20)$$

We set $\alpha = 0$, when $\phi = \frac{clM_{\text{pl}}}{2}$ which is the moment the gauge field starts to grow (see eq. (2.19)). Thus, we obtain the relation

$$\alpha = \frac{1}{2l} \left[\frac{l^2 c^2}{4} - \frac{\phi^2}{M_{\text{pl}}^2} \right]. \quad (4.21)$$

It represents the number of e-foldings counting from the beginning of growth of the gauge field. For convenience, we defined several key moments in figure 4. Using these variables, we can deduce the following expression

$$q_{\text{in}} \equiv \frac{p_{\text{in}}}{k} = e^{-(N_k - N_{\text{in}})}, \quad z' \equiv -k\tau' = e^{-(N - N_k)}, \quad z \equiv -k\tau = e^{-(N_e - N_k)}, \quad (4.22)$$

where N corresponds to the variable z' . Since $\alpha = N - N_{\text{in}}$, we get

$$\alpha = -\ln(z') - \ln(q_{\text{in}}). \quad (4.23)$$

Similarly, we have the relation

$$N_{\text{tot}} \equiv N_e - N_{\text{in}} = -\ln(z) - \ln(q_{\text{in}}). \quad (4.24)$$

Consequently, we have

$$\alpha = N_{\text{tot}} + \ln(z) - \ln(z'). \quad (4.25)$$

Substituting eq. (4.25) into eq. (4.21), we obtain

$$\phi = M_{\text{pl}} \sqrt{\frac{l^2 c^2}{4} - 2l(N_{\text{tot}} + \ln(z) - \ln(z'))}. \quad (4.26)$$

Finally, from eqs. (4.19) and (4.26), we get

$$\nu = \frac{lc}{\sqrt{\frac{l^2 c^2}{4} - 2l(N_{\text{tot}} + \ln(z) - \ln(z'))}}. \quad (4.27)$$

Using this relation, we can carry out the integral (4.18). Before that, we need to determine N_{tot} . Notice that the scalar field at the end of inflation ϕ_{min} is determined by the breakdown of the slow roll condition $\epsilon = \frac{M_{\text{pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 = 1$. This gives $\phi_{\text{min}}^2 = M_{\text{pl}}^2 l^2 / 2$. On the other hand, from eq. (4.26), we find the minimum value of the inflaton is obtained by setting $z = z'$ as

$$\phi_{\text{min}} = M_{\text{pl}} \sqrt{\frac{l^2 c^2}{4} - 2lN_{\text{tot}}}. \quad (4.28)$$

Thus, we can determine N_{tot} as

$$N_{\text{tot}} = \frac{l}{8} (c^2 - 2). \quad (4.29)$$

We are now in a position to calculate the integral (4.18) for several values of l and c . The results are shown in figures 5–7, where we took the Hubble constant to be $H_I = 10^{-5} M_{\text{pl}}$. In

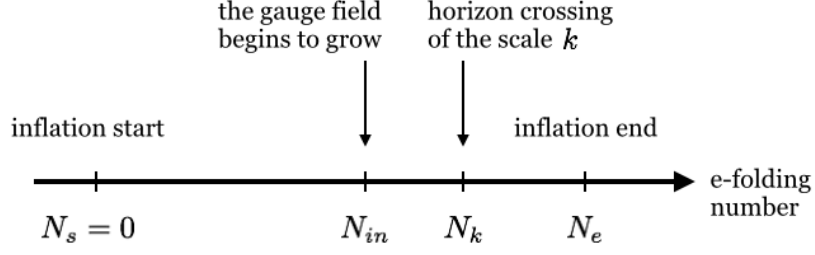


Figure 4. We have defined each relevant moment by the number of e-foldings.

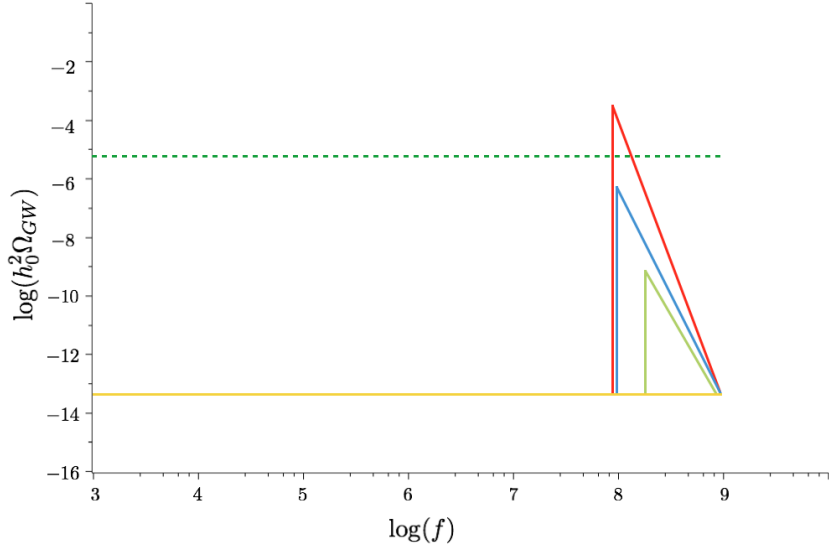


Figure 5. In the case of $l = 2$, the spectrum of gravitational waves are depicted. The red peak represents the case of $c = 3.6$, the blue and green one correspond to $c = 3.45$ and $c = 3.3$, respectively.

figures 5–7, the spectrum of the energy density parameter $h_0^2 \Omega_{\text{GW}}$ (h_0 is the dimensionless Hubble parameter) of gravitational waves are depicted against the frequency of gravitational waves. The yellow line represents conventional vacuum tensor fluctuations constrained by CMB observations. The green dotted line is an upper bound to the primordial gravitational waves from big-bang nucleosynthesis. In the case of $l = 4, 6$ (figures 6, 7), we have assumed that the inflaton potential on CMB scales is different from that near the end of inflation. It is assumed that they are connected with each other smoothly. From the results, we see that gravitational waves can be copiously produced by the gauge field in the $e^{c\phi/M_{\text{pl}}} FF$ model. There exists a peak in the spectrum of the density parameter because the scales which have exited the horizon earlier are more enhanced than the one exited later. Note that the amplitude can exceed the bound of the nucleosynthesis depending on the parameters. As can be seen, gravitational waves can be produced for a short period. It is because that ν of eq. (4.19) grows in the late stage of inflation and it affects the integral (4.18) significantly. As to observations, several works to detect the gravitational waves in MHz~GHz frequency bands have been carried out [67–71]. The current sensitivity for the amplitude of gravitational waves at the frequency 100 MHz is $h_c \sim 10^{-13}$ [54–56], where the characteristic strain is defined by

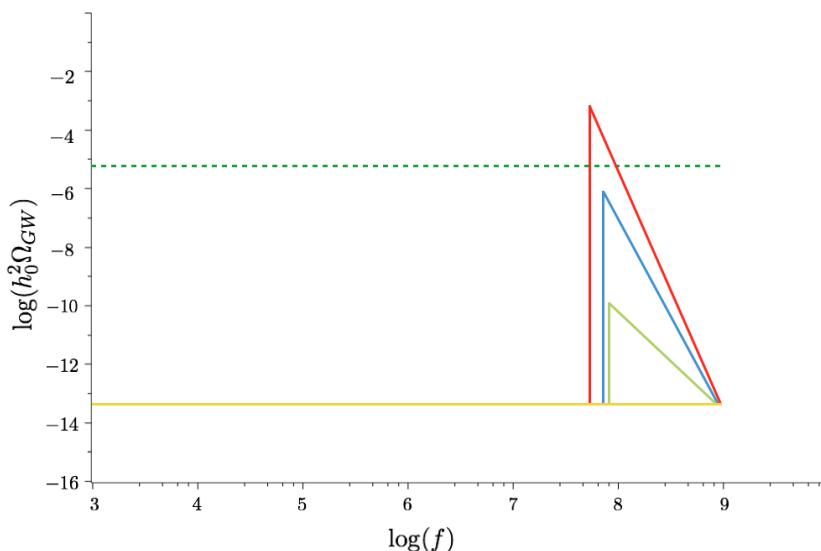


Figure 6. In the case of $l = 4$, the spectrum of gravitational waves are depicted. The red peak represents the case of $c = 3.1$, the blue and green one correspond to $c = 3$ and $c = 2.9$ respectively.

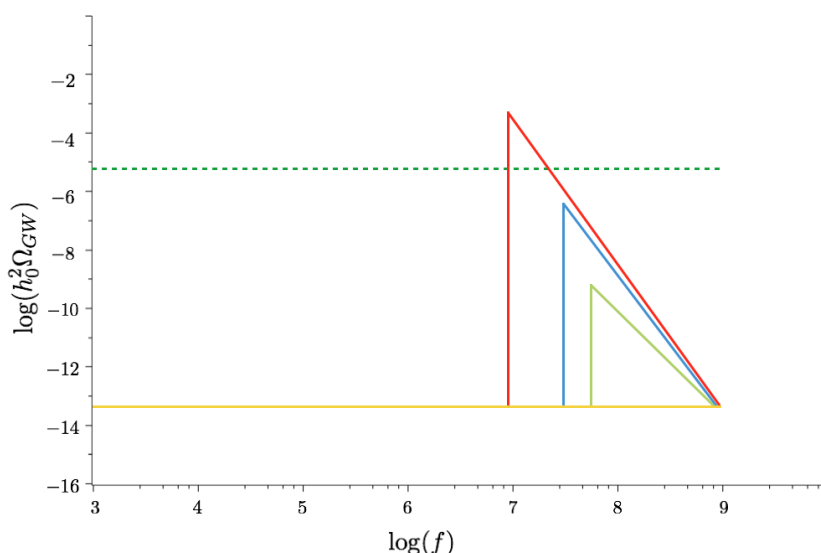


Figure 7. In the case of $l = 6$, the spectrum of gravitational waves are depicted. The red peak represents the case of $c = 2.9$, the blue and green one correspond to $c = 2.8$ and $c = 2.7$ respectively.

$h_c \equiv \sqrt{\frac{3}{2}} \frac{H_0 \Omega_{\text{GW}}^{1/2}}{\pi f}$. On the other hand, our model can reach $10^{-26} \sim 10^{-27}$ in the frequency bands 10 MHz \sim 100 MHz. Although the current sensitivity is too low to detect the signal, recently new methods are suggested and under development [57–63]. Since scalar fluctuations are also produced by the same mechanism, one may worry about primordial black holes. In fact, the mass of primordial black holes depends on the reheating temperature and we need to consider each model separately. However, as long as the reheating temperature is not so low, there is no observable effect due to primordial black holes because they evaporate well before the big-bang nucleosynthesis.

5 Conclusion

We claimed short-term anisotropic inflation is universal in the sense that it appears in models with a broad class of potential and gauge kinetic functions. As a concrete demonstration, we studied anisotropic inflation with an exponential type gauge kinetic function (2.15) which is ubiquitous in fundamental theory. For example, it is obtained by dimensional reduction from higher dimensional fundamental theory. In the $e^{c\phi/M_{\text{Pl}}}FF$ model, it turned out that the gauge field starts to grow in the late stage of inflation and consequently anisotropic inflation commences for a generic parameter $c \sim \mathcal{O}(1)$.

Remarkably, short-term anisotropic inflation provides an interesting phenomenology. Indeed, we have shown that the gauge field during short-term anisotropic inflation produces copious gravitational waves in the frequency band $10 \text{ MHz} \sim 100 \text{ MHz}$. The amplitude of the gravitational waves can reach about $10^{-26} \sim 10^{-27}$. Such gravitational waves could be detectable in near future [57–63] and it enable us to probe fundamental theory. We note that our model is quite different from other models which predict the existence of high-frequency primordial gravitational waves [72–77] in that the spectrum of our model has a peak and a red-tilt rather than a blue-tilt.

There are several applications to be considered. It is interesting to investigate the creation of cosmological magnetic fields in the present model. Since the gauge field begins to grow in the last stage of inflation, we may have a different scenario for primordial magnetic fields [42]. It is also possible to extend the present model to non-abelian cases [43–45] or other inflaton potentials.

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References

- [1] D.J. Schwarz, C.J. Copi, D. Huterer and G.D. Starkman, *CMB Anomalies after Planck*, [arXiv:1510.07929](#) [INSPIRE].
- [2] R.M. Wald, *Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant*, *Phys. Rev. D* **28** (1983) 2118 [INSPIRE].
- [3] S.W. Hawking and I.G. Moss, *Supercooled Phase Transitions in the Very Early Universe*, *Phys. Lett. B* **110** (1982) 35 [INSPIRE].
- [4] L.H. Ford, *Inflation driven by a vector field*, *Phys. Rev. D* **40** (1989) 967 [INSPIRE].
- [5] N. Kaloper, *Lorentz Chern-Simons terms in Bianchi cosmologies and the cosmic no hair conjecture*, *Phys. Rev. D* **44** (1991) 2380 [INSPIRE].
- [6] J.D. Barrow and S. Hervik, *Anisotropically inflating universes*, *Phys. Rev. D* **73** (2006) 023007 [[gr-qc/0511127](#)] [INSPIRE].
- [7] J.D. Barrow and S. Hervik, *Simple Types of Anisotropic Inflation*, *Phys. Rev. D* **81** (2010) 023513 [[arXiv:0911.3805](#)] [INSPIRE].
- [8] L. Campanelli, *A Model of Universe Anisotropization*, *Phys. Rev. D* **80** (2009) 063006 [[arXiv:0907.3703](#)] [INSPIRE].

- [9] A. Golovnev, V. Mukhanov and V. Vanchurin, *Vector Inflation*, *JCAP* **06** (2008) 009 [[arXiv:0802.2068](#)] [[INSPIRE](#)].
- [10] S. Kanno, M. Kimura, J. Soda and S. Yokoyama, *Anisotropic Inflation from Vector Impurity*, *JCAP* **08** (2008) 034 [[arXiv:0806.2422](#)] [[INSPIRE](#)].
- [11] N.E. Groeneboom, M. Axelsson, D.F. Mota and T. Koivisto, *Imprints of a hemispherical power asymmetry in the seven-year WMAP data due to non-commutativity of space-time*, [arXiv:1011.5353](#) [[INSPIRE](#)].
- [12] B. Himmetoglu, C.R. Contaldi and M. Peloso, *Instability of anisotropic cosmological solutions supported by vector fields*, *Phys. Rev. Lett.* **102** (2009) 111301 [[arXiv:0809.2779](#)] [[INSPIRE](#)].
- [13] B. Himmetoglu, C.R. Contaldi and M. Peloso, *Instability of the ACW model and problems with massive vectors during inflation*, *Phys. Rev. D* **79** (2009) 063517 [[arXiv:0812.1231](#)] [[INSPIRE](#)].
- [14] B. Himmetoglu, C.R. Contaldi and M. Peloso, *Ghost instabilities of cosmological models with vector fields nonminimally coupled to the curvature*, *Phys. Rev. D* **80** (2009) 123530 [[arXiv:0909.3524](#)] [[INSPIRE](#)].
- [15] A. Golovnev, *Linear perturbations in vector inflation and stability issues*, *Phys. Rev. D* **81** (2010) 023514 [[arXiv:0910.0173](#)] [[INSPIRE](#)].
- [16] G. Esposito-Farese, C. Pitrou and J.-P. Uzan, *Vector theories in cosmology*, *Phys. Rev. D* **81** (2010) 063519 [[arXiv:0912.0481](#)] [[INSPIRE](#)].
- [17] M.-a. Watanabe, S. Kanno and J. Soda, *Inflationary Universe with Anisotropic Hair*, *Phys. Rev. Lett.* **102** (2009) 191302 [[arXiv:0902.2833](#)] [[INSPIRE](#)].
- [18] A.E. Gumrukcuoglu, B. Himmetoglu and M. Peloso, *Scalar-Scalar, Scalar-Tensor and Tensor-Tensor Correlators from Anisotropic Inflation*, *Phys. Rev. D* **81** (2010) 063528 [[arXiv:1001.4088](#)] [[INSPIRE](#)].
- [19] T.R. Dulaney and M.I. Gresham, *Primordial Power Spectra from Anisotropic Inflation*, *Phys. Rev. D* **81** (2010) 103532 [[arXiv:1001.2301](#)] [[INSPIRE](#)].
- [20] M.-a. Watanabe, S. Kanno and J. Soda, *The Nature of Primordial Fluctuations from Anisotropic Inflation*, *Prog. Theor. Phys.* **123** (2010) 1041 [[arXiv:1003.0056](#)] [[INSPIRE](#)].
- [21] M.-a. Watanabe, S. Kanno and J. Soda, *Imprints of Anisotropic Inflation on the Cosmic Microwave Background*, *Mon. Not. Roy. Astron. Soc.* **412** (2011) L83 [[arXiv:1011.3604](#)] [[INSPIRE](#)].
- [22] R. Emami, H. Firouzjahi, S.M. Sadegh Movahed and M. Zarei, *Anisotropic Inflation from Charged Scalar Fields*, *JCAP* **02** (2011) 005 [[arXiv:1010.5495](#)] [[INSPIRE](#)].
- [23] N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, *Anisotropic power spectrum and bispectrum in the $f(\phi)F^2$ mechanism*, *Phys. Rev. D* **87** (2013) 023504 [[arXiv:1210.3257](#)] [[INSPIRE](#)].
- [24] R. Emami and H. Firouzjahi, *Curvature Perturbations in Anisotropic Inflation with Symmetry Breaking*, *JCAP* **10** (2013) 041 [[arXiv:1301.1219](#)] [[INSPIRE](#)].
- [25] A.A. Abolhasani, R. Emami, J.T. Firouzjaee and H. Firouzjahi, *δN formalism in anisotropic inflation and large anisotropic bispectrum and trispectrum*, *JCAP* **08** (2013) 016 [[arXiv:1302.6986](#)] [[INSPIRE](#)].
- [26] J. Ohashi, J. Soda and S. Tsujikawa, *Observational signatures of anisotropic inflationary models*, *JCAP* **12** (2013) 009 [[arXiv:1308.4488](#)] [[INSPIRE](#)].
- [27] M. Shiraishi, E. Komatsu and M. Peloso, *Signatures of anisotropic sources in the trispectrum of the cosmic microwave background*, *JCAP* **04** (2014) 027 [[arXiv:1312.5221](#)] [[INSPIRE](#)].
- [28] B. Chen and Z.-w. Jin, *Anisotropy in Inflation with Non-minimal Coupling*, *JCAP* **09** (2014) 046 [[arXiv:1406.1874](#)] [[INSPIRE](#)].

- [29] X. Chen, R. Emami, H. Firouzjahi and Y. Wang, *The TT, TB, EB and BB correlations in anisotropic inflation*, *JCAP* **08** (2014) 027 [[arXiv:1404.4083](#)] [[INSPIRE](#)].
- [30] A. Naruko, E. Komatsu and M. Yamaguchi, *Anisotropic inflation reexamined: upper bound on broken rotational invariance during inflation*, *JCAP* **04** (2015) 045 [[arXiv:1411.5489](#)] [[INSPIRE](#)].
- [31] R. Emami and H. Firouzjahi, *Clustering Fossil from Primordial Gravitational Waves in Anisotropic Inflation*, *JCAP* **10** (2015) 043 [[arXiv:1506.00958](#)] [[INSPIRE](#)].
- [32] R. Emami, H. Firouzjahi, S.M. Sadegh Movahed and M. Zarei, *Anisotropic Inflation from Charged Scalar Fields*, *JCAP* **02** (2011) 005 [[arXiv:1010.5495](#)] [[INSPIRE](#)].
- [33] T.Q. Do, W.F. Kao and I.-C. Lin, *Anisotropic power-law inflation for a two scalar fields model*, *Phys. Rev. D* **83** (2011) 123002 [[INSPIRE](#)].
- [34] T.Q. Do and W.F. Kao, *Anisotropic power-law inflation for the Dirac-Born-Infeld theory*, *Phys. Rev. D* **84** (2011) 123009 [[INSPIRE](#)].
- [35] K. Yamamoto, M.-a. Watanabe and J. Soda, *Inflation with Multi-Vector-Hair: The Fate of Anisotropy*, *Class. Quant. Grav.* **29** (2012) 145008 [[arXiv:1201.5309](#)] [[INSPIRE](#)].
- [36] K. Yamamoto, *Primordial Fluctuations from Inflation with a Triad of Background Gauge Fields*, *Phys. Rev. D* **85** (2012) 123504 [[arXiv:1203.1071](#)] [[INSPIRE](#)].
- [37] J. Ohashi, J. Soda and S. Tsujikawa, *Anisotropic power-law k-inflation*, *Phys. Rev. D* **88** (2013) 103517 [[arXiv:1310.3053](#)] [[INSPIRE](#)].
- [38] N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, *Anisotropy in solid inflation*, *JCAP* **08** (2013) 022 [[arXiv:1306.4160](#)] [[INSPIRE](#)].
- [39] J. Ohashi, J. Soda and S. Tsujikawa, *Anisotropic Non-Gaussianity from a Two-Form Field*, *Phys. Rev. D* **87** (2013) 083520 [[arXiv:1303.7340](#)] [[INSPIRE](#)].
- [40] A. Maleknejad, M.M. Sheikh-Jabbari and J. Soda, *Gauge Fields and Inflation*, *Phys. Rept.* **528** (2013) 161 [[arXiv:1212.2921](#)] [[INSPIRE](#)].
- [41] A. Ito and J. Soda, *Designing Anisotropic Inflation with Form Fields*, *Phys. Rev. D* **92** (2015) 123533 [[arXiv:1506.02450](#)] [[INSPIRE](#)].
- [42] S. Kanno, J. Soda and M.-a. Watanabe, *Cosmological Magnetic Fields from Inflation and Backreaction*, *JCAP* **12** (2009) 009 [[arXiv:0908.3509](#)] [[INSPIRE](#)].
- [43] K. Murata and J. Soda, *Anisotropic Inflation with Non-Abelian Gauge Kinetic Function*, *JCAP* **06** (2011) 037 [[arXiv:1103.6164](#)] [[INSPIRE](#)].
- [44] K.-i. Maeda and K. Yamamoto, *Inflationary Dynamics with a Non-Abelian Gauge Field*, *Phys. Rev. D* **87** (2013) 023528 [[arXiv:1210.4054](#)] [[INSPIRE](#)].
- [45] K.-i. Maeda and K. Yamamoto, *Stability analysis of inflation with an SU(2) gauge field*, *JCAP* **12** (2013) 018 [[arXiv:1310.6916](#)] [[INSPIRE](#)].
- [46] J. Soda, *Statistical Anisotropy from Anisotropic Inflation*, *Class. Quant. Grav.* **29** (2012) 083001 [[arXiv:1201.6434](#)] [[INSPIRE](#)].
- [47] L. Campanelli, P. Cea, G.L. Fogli and L. Tedesco, *Cosmic Parallax in Ellipsoidal Universe*, *Mod. Phys. Lett. A* **26** (2011) 1169 [[arXiv:1103.6175](#)].
- [48] A.R. Pullen and C.M. Hirata, *Non-detection of a statistically anisotropic power spectrum in large-scale structure*, *JCAP* **05** (2010) 027 [[arXiv:1003.0673](#)] [[INSPIRE](#)].
- [49] N. Bartolo, E. Dimastrogiovanni, M. Liguori, S. Matarrese and A. Riotto, *An estimator for statistical anisotropy from the CMB bispectrum*, *JCAP* **01** (2012) 029 [[arXiv:1107.4304](#)] [[INSPIRE](#)].

- [50] J. Kim and E. Komatsu, *Limits on anisotropic inflation from the Planck data*, *Phys. Rev. D* **88** (2013) 101301 [[arXiv:1310.1605](#)] [[INSPIRE](#)].
- [51] S.R. Ramazanov and G. Rubtsov, *Constraining anisotropic models of the early Universe with WMAP9 data*, *Phys. Rev. D* **89** (2014) 043517 [[arXiv:1311.3272](#)] [[INSPIRE](#)].
- [52] G.I. Rubtsov and S.R. Ramazanov, *Revisiting constraints on the (pseudo)conformal universe with Planck data*, *Phys. Rev. D* **91** (2015) 043514 [[arXiv:1406.7722](#)] [[INSPIRE](#)].
- [53] R. Emami, *Anisotropic Inflation and Cosmological Observations*, [arXiv:1511.01683](#) [[INSPIRE](#)].
- [54] K. Kuroda, W.-T. Ni and W.-P. Pan, *Gravitational waves: Classification, Methods of detection, Sensitivities and Sources*, *Int. J. Mod. Phys. D* **24** (2015) 1530031 [[arXiv:1511.00231](#)] [[INSPIRE](#)].
- [55] T. Akutsu et al., *Search for a stochastic background of 100-MHz gravitational waves with laser interferometers*, *Phys. Rev. Lett.* **101** (2008) 101101 [[arXiv:0803.4094](#)] [[INSPIRE](#)].
- [56] A.M. Cruise, *The potential for very high-frequency gravitational wave detection*, *Class. Quant. Grav.* **29** (2012) 095003 [[INSPIRE](#)].
- [57] H. Wen, F. Li and Z. Fang, *Electromagnetic response produced by interaction of high-frequency gravitational waves from braneworld with galactic-extragalactic magnetic fields*, *Phys. Rev. D* **89** (2014) 104025 [[INSPIRE](#)].
- [58] F. Li, H. Wen and Z. Fang, *Quasi-B-mode generated by high-frequency gravitational waves and corresponding perturbative photon fluxes*, [arXiv:1505.06546](#) [[INSPIRE](#)].
- [59] J. Li, L. Zhang, K. Lin and H. Wen, *The advanced system for the electromagnetic response of high-frequency gravitational waves*, [arXiv:1411.1811](#) [[INSPIRE](#)].
- [60] R. Woods, R. Baker, F. Li, G. Stephenson, E. Davis and A. Beckwith, *A New Theoretical Technique for the Measurement of High-Frequency Relic Gravitational Waves*, *J. Mod. Phys.* **2** (2011) 498.
- [61] F. Li, R.M.L. Baker, Jr., Z. Fang, G.V. Stephenson and Z. Chen, *Perturbative Photon Fluxes Generated by High-Frequency Gravitational Waves and Their Physical Effects*, *Eur. Phys. J. C* **56** (2008) 407 [[arXiv:0806.1989](#)] [[INSPIRE](#)].
- [62] N.I. Kolosnitsyn and V.N. Rudenko, *Gravitational Hertz experiment with electromagnetic radiation in a strong magnetic field*, *Phys. Scripta* **90** (2015) 074059 [[arXiv:1504.06548](#)] [[INSPIRE](#)].
- [63] M.B. Mensky and V.N. Rudenko, *High-frequency gravitational wave detector with electromagnetic-gravitational resonance*, *Grav. Cosmol.* **15** (2009) 167.
- [64] VIRGO, LIGO SCIENTIFIC collaborations, B.P. Abbott et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, *Phys. Rev. Lett.* **116** (2016) 061102 [[arXiv:1602.03837](#)] [[INSPIRE](#)].
- [65] N. Barnaby, R. Namba and M. Peloso, *Observable non-Gaussianity from gauge field production in slow roll inflation and a challenging connection with magnetogenesis*, *Phys. Rev. D* **85** (2012) 123523 [[arXiv:1202.1469](#)] [[INSPIRE](#)].
- [66] K. Choi, K.-Y. Choi, H. Kim and C.S. Shin, *Primordial perturbations from dilaton-induced gauge fields*, *JCAP* **10** (2015) 046 [[arXiv:1507.04977](#)] [[INSPIRE](#)].
- [67] A. Nishizawa et al., *Laser-interferometric Detectors for Gravitational Wave Background at 100 MHz: Detector Design and Sensitivity*, *Phys. Rev. D* **77** (2008) 022002 [[arXiv:0710.1944](#)] [[INSPIRE](#)].
- [68] A.M. Cruise, *An electromagnetic detector for very-high-frequency gravitational waves*, *Class. Quant. Grav.* **17** (2000) 2525 [[INSPIRE](#)].

- [69] A.M. Cruise and R.M.J. Ingley, *A correlation detector for very high frequency gravitational waves*, *Class. Quant. Grav.* **22** (2005) S479 [[INSPIRE](#)].
- [70] P. Bernard, G. Gemme, R. Parodi and E. Picasso, *A detector of small harmonic displacements based on two coupled microwave cavities*, *Rev. Sci. Instrum.* **72** (2001) 2428 [[gr-qc/0103006](#)] [[INSPIRE](#)].
- [71] R. Ballantini et al., *Microwave apparatus for gravitational waves observation*, [gr-qc/0502054](#) [[INSPIRE](#)].
- [72] M. Giovannini, *Production and detection of relic gravitons in quintessential inflationary models*, *Phys. Rev. D* **60** (1999) 123511 [[astro-ph/9903004](#)] [[INSPIRE](#)].
- [73] P.J.E. Peebles and A. Vilenkin, *Quintessential inflation*, *Phys. Rev. D* **59** (1999) 063505 [[astro-ph/9810509](#)] [[INSPIRE](#)].
- [74] A. Riazuelo and J.-P. Uzan, *Quintessence and gravitational waves*, *Phys. Rev. D* **62** (2000) 083506 [[astro-ph/0004156](#)] [[INSPIRE](#)].
- [75] M. Gasperini and G. Veneziano, *String Theory and Pre-big bang Cosmology*, [hep-th/0703055](#) [[INSPIRE](#)].
- [76] M. Gasperini and G. Veneziano, *The pre-big bang scenario in string cosmology*, *Phys. Rept.* **373** (2003) 1 [[hep-th/0207130](#)] [[INSPIRE](#)].
- [77] M. Gasperini and M. Giovannini, *Dilaton contributions to the cosmic gravitational wave background*, *Phys. Rev. D* **47** (1993) 1519 [[gr-qc/9211021](#)] [[INSPIRE](#)].