Thrusting Against the Quantum Vacuum

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I. INTRODUCTION

This chapter addresses the question of how the properties of the quantum vacuum might be exploited to propel a spacecraft. Quantum electrodynamics (QED), the theory of the interaction of light and matter that has made predictions verified to 1 part in 10 billion [57], predicts that the quantum vacuum, which is the lowest state of the electromagnetic field, contains a fluctuation virtual photon field. This fluctuating field is predicted to produce vacuum forces between nearby surfaces [1][2][57]. Recently these Casimir forces have been measured and found to agree with predictions [3][4][5][7]. If this virtual radiation pressure could be utilized for propulsion, the goal of propellantless propulsion would be achieved. Restrictions due to the conservation of energy and momentum are discussed. A propulsion system based on an uncharged, conducting mirror that vibrates asymmetrically in the vacuum is described. By the dynamic Casimir effect, the mirror produces real photons that impart momentum and result in a net acceleration. The acceleration is very small, but demonstrates that the vacuum can be utilized in propulsion. Technological improvements, some of which are proposed, may be used to increase the accelerating force. Many questions remain about the supporting theory; experiments are needed to probe questions about the quantum vacuum that are far beyond current theory.

Rockets employing chemical or ionic propellants require the transport of prohibitively large quantities of propellant. If the properties of the quantum vacuum could somehow be utilized in the production of thrust, that would provide a decided advantage since the vacuum is everywhere. At this embryonic stage, in the exceedingly brief history of interstellar spacecraft, we are trying to distinguish between what appears possible and what appears impossible within the context of our current understanding of the quantum physics and the fundamental laws of physics, particularly conservation of momentum and energy. Science fiction writers have written about the use of the quantum vacuum to power spacecraft for decades but no research has validated this suggestion. Arthur C. Clark, who proposed geosynchronous communications satellites in 1945, described a "quantum ramjet drive" in 1985 in "Songs of Distant Earth", and observed in the Acknowledgement, "If vacuum fluctuations can be harnessed for propulsion by anyone besides science-fiction writers, the purely engineering problems of interstellar flight would be solved [42]." Australian science fiction writer Ken Ingle described, with my fanciful suggestions, the Casimir Vacuum Drive in his soon to be published book "First Contact."

In the last ten years great progress has been made experimentally in measuring Casimir forces, which arise between closely spaced surfaces due to the quantum fluctuations of the electromagnetic field, the quantum vacuum. Although the forces tend to be small, practical applications of vacuum forces have recently appeared in MEMS (MicroElectroMechancial Systems) devices [55][56][100].

Quantum Electrodynamics or QED predicts the behavior of the quantum vacuum, including vacuum forces and the presence of a vast energy in empty space due to a fluctuating electromagnetic field. Unfortunately we do not know yet have a proven method to to propel a spacecraft by harnessing the vast energy of vacuum fluctuations that QED predicts, and therefore this chapter focuses on general considerations about momentum transfer between the quantum vacuum and a spacecraft. The spacecraft proposed in this paper is described as a "gedanken spacecraft" since its design is not intended as an engineering guide, but just to illustrate possibilities. Indeed, based on our current understanding of quantum vacuum physics, one could reasonably argue that the gedanken spacecraft could be propelled more effectively by simply oscillating a charged mirror that would emit electromagnetic radiation or simply using a flashlight or laser to generate photons. Although the performance of the vacuum powered gedanken spacecraft as presented is disappointing and is no more practical than a spacewarp [43], the discussion illustrates many important ideas about the quantum vacuum, and it suggests the potential role of quantum vacuum phenomena in a macroscopic system like space travel. In fact, with a breakthrough in materials, methods, or our fundamental understanding, this approach could become practical, and we might be able to realize the dream of space travel as presented in science fiction. Physicists have explored various means of locomotion depending on the density of the medium and the size of the moving object. It would be interesting to find an optimum method for moving in the quantum vacuum. Unfortunately we currently have no simple way to mathematically explore various simple possibilities.

II. PHYSICS OF THE QUANTUM VACUUM

A. Historical Background

Quantum mechanics is one of the great scientific achievements of the twentieth century. It provides models that describe many properties of atoms and molecules, such as the optical spectra and transition probabilities. In its original form, as developed by Schrodinger, Heisenberg, Bohr, and others, quantum mechanics is a non-relativistic theory that makes the ad hoc assumption that light is emitted and absorbed by atoms in bundles, called photons. The electromagnetic field, however, is treated as an ordinary classical field that obeys Maxwell's relativistic equations, not as a quantized field. Dirac, Heisenberg, Jordan, Dyson, and others began formulation of a relativistic form of quantum mechanics, and made efforts to quantize the electromagnetic field. This quantized field theory of particles and light theory developed over the next few decades with numerous successful predictions. In 1948, Willis Lamb tested a crucial prediction of the field theory, that the 2s and 2p levels of a hydrogen atom would have precisely the same energy. Lamb sent a beam of hydrogen atoms through a cavity exposed to RF radiation, and he determined that the 2s and 2p energy levels were in fact split by an energy equivalent to 1000 MHz. Within days, Hans Bethe of Cornell realized the problem and published the solution: the theoretical calculation did not consider the effects of the quantum vacuum on the energy levels of the hydrogen atoms. This ushered in the modern formulation of quantum electrodynamics (QED) of Feynmann, Schwinger, and Tomonaga [57].

In QED particles and light are both treated as quantized fields that are fully relativistic. Since the electromagnetic field is quantized, there may be 0, 1, 2, 3 or any number of photons present. Since the fields are relativistic, they can be readily transformed to coordinate systems that are translated or moving uniformly (Lorentz transformations). The entire formalism of QED can be written in tensors that ensure the proper transformation properties of all observables under a Lorentz transformation.

The pervasive and dynamic role of the vacuum state in QED was unexpected to many physicists. The lowest state of the quantized electromagnetic field, which is referred to as the quantum vacuum, was predicted to be filled with photons and electron-positron pairs that appear and disappear continuously, so rapidly that no direct measurement of their presence is possible. Yet these so called "virtual particles" affect measurable properties of atoms, such as the energy levels, magnetic moments, and transition probabilities.

In retrospect, it was clear, arguing from non-relativistic quantum mechanics, from the uncertainly principle, that the vacuum would contain a fluctuating electromagnetic field once the field was treated as a quantized field. The field variables, E_ω and B_ω, representing the electric and magnetic field at a frequency ω, are directly analogous to P_ω, and Q_ω, the position and momentum of a harmonic oscillator of frequency ω. The ground state of the harmonic oscillator has to obey Heisenberg's Uncertainty Principle: $\Delta P_\omega \Delta Q_\omega \geq \hbar$, where ΔP_ω is the uncertainty in the momentum and ΔQ_ω is the uncertainty in the position.. In the lowest state, the oscillator is still vibrating, with an energy $\frac{1}{2}\hbar\omega$. If it weren't vibrating, but was motionless, then the uncertainty in its momentum would be zero. If we knew the approximate position of the oscillator, then this state would violate the uncertainly principle. The energy of the nth excited state of the oscillator is $\left(n+\frac{1}{2}\right)\hbar\omega$.

Similarly quantized electric and magnetic fields cannot vanish, but must, in their lowest state, fluctuate. This isotropic residual fluctuating electromagnetic field, which is present everywhere, at zero Kelvin temperature, with all electromagnetic sources removed, is often called the zero-point electromagnetic field.

Quantum fluctuations occur in the particle fields as well as the electromagnetic field, so the quantum vacuum is filled with virtual electron-positron pairs, as well as virtual photons. Before Lamb's Nobel Prize winning measurement, most physicists felt comfortable ignoring the effects of the quantum fluctuations, assuming that they just shifted the energy zero, but did not have measurable consequences. It turns out that quantum fluctuations affect virtually all physical processes, including the mass, charge, and magnetic moment of all particles, the lifetimes of excited atoms or particles, scattering cross-sections, and the energy levels of atoms. QED, which accounts for all the vacuum processes, has made experimental predictions of magnetic moments and energy levels that have been verified by experiment to 1 part in a ten billion, the most accurate predictions of any scientific theory [9][57].

B. Energy in the Quantum Vacuum

Zero-point field energy density is a simple and inexorable consequence of quantum theory and the uncertainty principle, but it brings puzzling inconsistencies with another well verified theory, general relativity. The energy in the quantum vacuum at absolute zero, which is the lowest energy state of the electromagnetic field, is due to the presence of virtual photons of energy $\frac{1}{2}\hbar\omega_n$ of all possible frequencies:

$$E_0 = \frac{1}{2}\sum_{n=0}^{n\max}\hbar\omega_n \tag{1}$$

Usually a cut-off is used for the high frequencies, such as the frequency corresponding to the Planck Length of $10^{-34}m$ which gives an enormous energy density (about 10^{114} J/m^3 or, in terms of mass, 10^{95} g/cm^3). From the perspective of general relativity, this enormous energy density seems to make no physical sense, and that is why the effects of the quantum fluctuations were neglected for decades. Indeed such a large energy would, according to the General Theory of Relativity, have a disastrous effect on the metric of space-time. For an infinite flat universe, this vacuum energy density would imply an outward zero-point pressure that would rip the universe apart [37]. Astronomical data, on the other hand, indicate that any such cosmological constant must be ~ 4 eV/mm^3, or 10^{-29} g/cm^3 when expressed as mass [38]. The discrepancy here between theory and observation is about 120 orders of magnitude, and is arguably the greatest quantitative discrepancy between theory and observation in the history of science [61][40]! There are numerous approaches to solve this "cosmological constant problem," such as renormalization, supersymmetry, string theory, and quintessence, but as yet this remains an unsolved problem.

Gradually, the belief has developed that only changes in the energy density give observable effects [58].

Each virtual photon of frequency ω and wave vector $\vec{\mathbf{k}}, (\mathbf{k} = 2\pi / \lambda)$ has associated with it a momentum $\hbar\vec{\mathbf{k}}$. Since photons are in random directions, the mean momentum of the vacuum fluctuations vanishes, but, just as there are fluctuations in the electric and magnetic fields consistent with the uncertainty principle, there are fluctuations in the root mean square momentum. At finite temperatures, real photons begin to appear in the quantum vacuum, but their contribution to the total energy is much smaller than that of the virtual photons.

C. Casimir Forces Predicted in 1948

About the same time as Lamb's experiment, Heindrick Casimir, director of research at Phillip's Laboratories in the Netherlands, found some disagreements between experiment and his model for precipitation of phosphors used in the manufacture of fluorescent light bulbs. Better agreement between theory and experiment could be obtained if the van der Waal's force between two neutral, polarizable atoms somehow fell off more rapidly at larger distances than had been supposed. A co-worker suggested that this might be related to the finite speed of light, which prompted Casimir and Polder to reanalyze the van der Waals interaction. They found that including the retardation effects caused the interaction to vary as r^{-7} rather than r^{-6} at large intermolecular separations r, which gave agreement with experiment.

Intrigued by the simplicity of the result, Casimir sought a deeper understanding. A conversation with Bohr led him to an interpretation in terms of zero-point energy, and the realization that, by simply considering the changes in vacuum energy arising from the presence of surfaces in the vacuum, forces due to the vacuum fluctuation would appear. To understand this result, consider how inserting two parallel surfaces into the vacuum causes the allowed modes of the EM field to change. This change in the modes that are present occurs since the electromagnetic field must meet the appropriate boundary conditions at each surface. Thus surfaces alter the modes of oscillation and therefore the surfaces alter the energy density corresponding to the lowest state of the EM field. In actual practice, the modes with frequencies above the plasma frequency do not appear to be significantly affected by the metal surfaces since the metal becomes transparent to radiation above this frequency. In order to avoid dealing with infinite quantities, the usual approach is to compute the finite change ΔE_0 in the energy of the vacuum due to the presence of the surfaces:

$$\Delta E_0 = E[\text{energy in empty space}] - E_S[\text{energy in space with surfaces present}] \qquad (2)$$

where the definition of each term is given in brackets. This equation can be expressed as a sum over the corresponding modes:

$$\Delta E_0 = \frac{1}{2}\sum_{n=0}^{n\max} \hbar\omega_n - \frac{1}{2}\sum_{m=0}^{surfaces} \hbar\omega'_m \qquad (3)$$

The quantity ΔE_0 can be computed for various geometries. The forces F due to the quantum vacuum are obtained by computing the change in the vacuum energy for a small change in the geometry and differentiating. For example, consider a hollow conducting rectangular cavity with sides a_1, a_2, a_3. Let $en(a_1, a_2, a_3)$ be the change in the vacuum energy due to the cavity, then the force F_1 on the side perpendicular to a_1, is:

$$F_1 = -\frac{\delta en}{\delta a_1} \qquad (4)$$

where δen represents the infinitesimal change in energy corresponding to an infinitesimal change in the dimension δa_1. This equation also represents the conservation of energy when the wall perpendicular to a_1 is moved infinitesimally:

$$\delta en = -F_1 \delta a_1 \tag{5}$$

Thus, if we can calculate the vacuum energy density as a function of the dimensions of the cavity, we can compute derivatives which give the forces on the surfaces. For uncharged, perfectly conducting, parallel plates with a very large area *A*, very close to each other (separation of d), the tangential component of the electric field must vanish at the surface, and wavelengths longer than twice the plate separation d are excluded. With the appropriate boundary conditions, we can compute the change in vacuum energy and use Eq. 6 to predict an attractive (or negative) force between the plates:

$$F = -\frac{K}{d^4} \tag{7}$$

where

$$K = \frac{\pi^2 \hbar c}{240} \tag{8}$$

This force *F,* commonly called the Casimir force, arises from the *change in vacuum energy density* E_{pp} from the free field vacuum density that occurs between the parallel plates [58][59]:

$$E_{pp}(d) = \frac{-K}{3d^3} \tag{9}$$

Two decades after Casimir's initial predictions, a method was developed to compute the Casimir force in terms of the local stress-energy tensor using quantum electrodynamics [8]. Many innovations have followed. Vacuum forces have been computed for other geometries besides the classic parallel plate geometry, such as a rectangular cavity, a cube, a sphere, a cylinder, a wedge. For a cube or sphere, the Casimir forces are outward or repulsive. For a rectangular cavity, the Casimir forces on the different faces may be inward, outward, or zero, depending on the ratio of the sides.

Situations arise in which there are inward forces on some faces and outward faces on other faces [16]. It is difficult to understand these unusual results intuitively!

The application of Casimir forces in space propulsion is motivated more clearly by the interpretation of the parallel plate Casimir force as arising from radiation pressure, the transfer of momentum from the virtual photons in the vacuum to the surfaces [2]. It is this virtual radiation pressure that we propose to explore as a possible driving force to generate net forces on an object, ultimately to propel a spacecraft. There are very significant advantages if it is possible to use virtual radiation pressure: no propellant may be required, and there is always something to push against.

D. Dynamic Casimir Effect

In the dynamic Casimir effect the parallel plates are imagined to move rapidly, which can lead to an excited state of the vacuum between the plates, meaning the creation of real photons [66]. To understand this process from a physical perspective, imagine that in a real moving conductor, the surface charges must constantly rearrange themselves to cancel out the transverse electric field at all positions. This rapid acceleration of charge can lead to radiation. This effect, generally referred to as the dynamic or adiabatic Casimir effect, has been reviewed but not yet observed experimentally [58][59][68]. The vacuum field exerts a force on the moving mirror that tends to damp the motion. Energy conservation requires the existence of a radiation reaction force working against the motion of the mirror [67]. This dissipative force may be understood as the mechanical effect of the emission of radiation induced by the motion of the mirror. This force of radiation reaction can be used to accelerate the mirror, or a spacecraft attached to the vibrating mirror, as discussed in Section V.

The Hamiltonian is quadratic in the field operators, and formally analogous to the Hamiltonian describing photon pair creation by parametric interaction of a classical pump wave of frequency ω_0 , with a nonlinear medium [69]. Pairs of photons with frequencies $\omega_1+\omega_2=\omega_0$, are created out of the vacuum state. Furthermore the photons have the same polarization, and the components of the corresponding wave vectors $\vec{k}_1$ and $\vec{k}_2$ taken along the mirror surface must add to zero because of the translational symmetry:

$$\vec{k}_1 \cdot \hat{x} + \vec{k}_2 \cdot \hat{x} = \omega_1 \sin\theta_1 + \omega_2 \sin\theta_2 \quad = 0 \tag{10}$$

This last equation relates the angles of emission of the photon pairs with respect to the unit vector $\hat{x}$, which is normal to the surface. It is interesting that the photons emitted by the dynamic Casimir effect are entangled photons. This analysis in terms of the analogous effective Hamiltonian is illuminating but not complete for perfect mirrors, because no consistent effective Hamiltonian can be constructed in this case with the idealized and pathological boundary conditions. More realistic results are obtained assuming that the mirrors are transparent above a plasma frequency.

The dynamic Casimir effect was studied for a single, perfectly reflecting mirror with arbitrary non-relativistic motion and a scalar field in three dimensions in 1982 by Ford and Vilenkin [70]. They obtained expressions for the vacuum radiation pressure on the mirror. In 2001, Barton and Eberlein extended the analysis using a 1 dimensional scalar field to a moving body with a finite refractive index [71]. The vacuum radiation pressure and the radiated spectrum for a non-relativistic, perfectly reflecting, infinite, plane mirror was computed by Neto and Machado for the electromagnetic field in three dimensions, and shown to obey the fluctuation-dissipation theorem from linear response theory [72][67]. This theorem shows the fluctuations for stationary body yield information about the mean force experienced by the body in non-uniform motion. Jaekel computed shifts in the mass of the mirror for a scalar field in two dimensions [73]. The mirror mass is not constant, but rightfully a quantum variable because of the coupling of the mirror to the fields by the radiation pressure. A detailed analysis was done by Barton and Calogeracos for a dispersive mirror in 1 dimension, that includes radiative shift in the mass of the mirror and the radiative reaction force [74]. This model can be generalized to an infinitesimally thin mirror with finite surface conductivity and a normally incident electromagnetic field.

E. Alternative Theories of Casimir Forces

The experimental verification of Casimir's prediction is often cited as proof of the reality of the vacuum energy density of quantum field theory. Yet, as Casimir himself observed, other interpretations are possible:

> The action of this force [between parallel plates] has been shown by clever experiments and I think we can claim the existence of the electromagnetic zero-point energy without a doubt. But one can also take a more modest point of view. Inside a metal there are forces of cohesion and if you take two metal plates and press them together these forces of cohesion begin to act. On the other hand you can start with one piece and split it. Then you have first to break chemical bonds and next to overcome van der Waals forces of classical type and if you separate the two pieces even further there remains a curious little tail. The Casimir force, *sit venia verbo*, is the last but also the most elegant trace of cohesion energy [60].

Several approaches to computing electromagnetic Casimir forces have been developed that are not based on the zero-point vacuum fluctuations directly. In the special case of the vacuum electromagnetic field with dielectric or conductive boundaries, various approaches suggest that Casimir forces can be regarded as macroscopic manifestations of many-body retarded van der Waals forces, at least in simple geometries with isolated atoms[57], [65]. Casimir effects have also been derived and interpreted in terms of source fields in both conventional [57] and unconventional [63] quantum electrodynamics, in which the fluctuations appear within materials instead of outside of the materials. Lifshitz provided a detailed computation of the Casimir force between planar surfaces by assuming that stochastic fluctuations occur in the tails of the wavefunctions of atoms that leak into the regions outside the surface These fluctuating tails can induce dipole moments in atoms in a nearby surface, which leads to a net retarded dipole-induced dipole force between the planar surfaces[47]. These various approaches that are alternatives to conventional QED always postulate the existence of fluctuations in potentials, wave functions, or electromagnetic fields, and give results consistent with QED formulations in the few cases of simple geometries that have been computed[9].

It should be pointed out that all QED calculations must routinely include the effects of the vacuum fluctuations in order to obtain the correct results. For example, the spontaneous emission from excited atoms depends on transitions induced by the vacuum field.

F. Limitations of Current Theoretical Calculations of Vacuum Forces

The parallel plate geometry (and the approximately equivalent sphere-flat-plate geometry or sphere-almost-flat-plate geometry) is essentially the only geometry for which experimental measurements have been conducted and the only geometry for which the vacuum forces between **two separate surfaces** (assumed to be infinite) have been computed. In the calculations with spheres, the radius of curvature of the sphere is very large compared to the separation, so locally, the geometry is a parallel plate geometry. Vacuum forces are known to exist in other experimental configurations between separate surfaces, but rigorous calculations based on QED (quantum electrodynamics) are very difficult and have yet to be completed [10]. Since it is experimentally possible to measure forces between various separate surfaces, with the improvement in experimental techniques, theoreticians may soon see the need for such computations.

Calculations of vacuum stresses for a variety of geometric shapes, such as spheres, cylinders, rectangular parallelepipeds, and wedges are reviewed in [58][59][1]. In general, calculations of vacuum forces become very complex when the surfaces are curved, particularly with right angles. Divergences in energy appear, and there are disagreements about the proper way to deal with these divergences[13]. The material properties, such as the dielectric constant and plasma frequency of the metal and the surface roughness also affect the vacuum forces, and are often not treated realistically in theoretical calculations. Indeed, in the Lizshitz formulation, the Casimir forces depend on the permittivity and permeability as a function of the frequency over the entire frequency range. Because this information is not generally available, approximations have to be made. In addition, usually only a spatial average of the force for a given area for the ground state of the quantum vacuum field is computed, and material properties, such as binding energies, are ignored, a procedure which Barton has recently questioned [14][15][16].

III. MEASUREMENTS OF CASIMIR FORCES

It was not until about 1998, that the parallel plate Casimir force was measured accurately [3][48]. Corrections for finite conductivity and surface roughness have been developed for the parallel plate geometry, and the agreement between theory and experiment is now at about the 1% level for separations of about 0.1-0.7 μm [4]. In actual practice, the measurements are most commonly made with one surface curved and the other surface flat, and using the proximity force theorem to account for the curvature. This experimental approach eliminates the difficulties of trying to maintain parallelism at submicron separations. Mohideen and collaborators have made the most accurate measurements to date in this manner, using an AFM (Atomic Force Microscope) that has a metallized sphere about 250 μm in diameter attached to the end of a cantilever about 200 μm long, capable of measuring picoNewton forces. The deflection of the sphere is measured optically as it is moved close to a flat metallized surface[3]. The more difficult measurement between two parallel plates has been made by Bressi *et al* who obtained results that are consistent with theory[5]. Measurements of the force between two parallel surfaces, each with a small (1 nm) sinusoidal modulation in surface height, have showed that there is a lateral force as well as the usual normal force when the modulations of the opposing surfaces are not in phase [6]. Recent measurements have confirmed the predictions, including effects of finite conductivity, surface roughness, and temperature, uncertainty in dielectric functions, to the 1-2% level for separations from 65 to 300 nm [7].

Parallel plate Casimir forces go inversely as the fourth power of the separation between the plates. The Casimir force per unit area + between perfectly conducting plates is equivalent to about 1 atm pressure at a separation of 10 nm, and so is a candidate for actuation of MEMS (MicroElectroMechanical Systems). The relative strength of Casimir, gravitational, and electrostatic forces for parallel, conducting surfaces is shown in Fig. 1 [52].

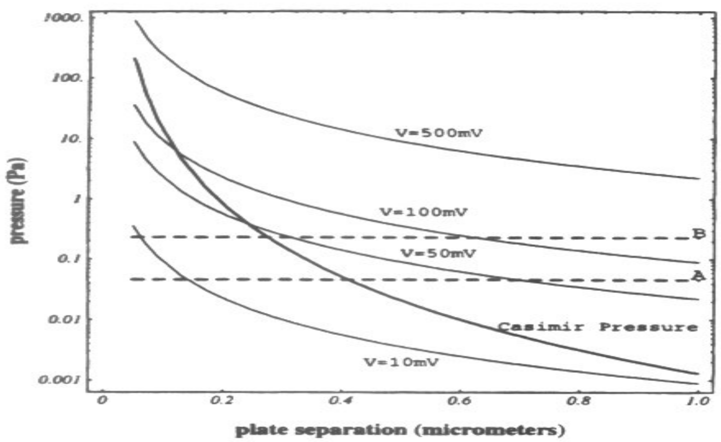

Figure-1. Casimir, electrostatic, and gravitational pressures. A comparison of the attractive pressures due to the Casimir effect and applied electrostatic voltage (V) between two flat parallel plates of conductors in vacuum. Also show are the gravitational pressure on 2 micrometer thick (dashed line A) and 10 micrometer thick (line B) silicon membranes.

In 1995 the first analysis of a dynamic MEMS structure that used vacuum forces was presented by Serry *et al.* [52]. They consider an idealized MEMS component resembling the original Casimir model of two parallel plates, except that one of the plates is connected to a stationary surface by a linear restoring force (spring) and can move along the direction normal to the plate surfaces. The model demonstrates that the Casimir effect could be used to actuate a switch, and might be responsible in part for the "stiction" phenomenon in which micromachined membranes are found to latch onto nearby surfaces during the fabrication process. If the movable surface is vibrating, then an "anharmonic Casimir oscillator" (ACO) results.

In MEMS, surfaces may come into close proximity with each other, particularly during processes of etching sacrificial layers in the fabrication process. To explore stiction in common MEMS configurations, Serry *et al.* computed the deflection of membrane strips and the conditions under which they would collapse into nearby surfaces [53]. Measurements were done by Buks *et al.* on cantilever beams to investigate the role of Casimir forces in stiction[11]. An experimental realization of the ACO in a nanometer-scale MEMS system was recently reported by Chan et al [55]. In this experiment the Casimir attraction between a 500 μm-square plate suspended by torsional rods and a gold-coated sphere of radius 100 μm was observed as a sharp increase in the tilt angle of the plate as the sphere-plate separation was reduced from 300 nm to 75.7 nm. This “quantum mechanical actuation" of the plate suggests “new possibilities for novel actuation schemes in MEMS based on the Casimir force" [55]. In a refinement of this experiment, a novel proximity sensor was demonstrated in which the plate was slightly oscillated with an AC signal, and the deflection amplitude observed with its rapid inverse fourth power behavior gave an indication of the precise location of the nearby sphere[56]. A measurement using a similar torsion oscillator was recently reported using gold on the sphere and chromium on the plate[12].

A. Forces on Semiconductor Surfaces

One of the potentially most important configurations from the technological viewpoint involves vacuum forces on semiconductor surfaces. The Casimir force for a conducting material depends approximately on the plasma frequency, beyond which the material tends to act like a transparent medium. For parallel plates separated by a distance, *d*, the usual Casimir force is reduced by a factor of approximately $C(a) = \left(1 = \left(8\lambda_p / 3\pi d\right)\right)^{-1}$, where λ_p is the wavelength corresponding to the plasma frequency of the material [30]. Since the plasma frequency is proportional to the carrier density, it is possible to tune the plasma frequency in a semiconductor, for example, by illumination, by temperature, or by the application of a voltage bias. In principle it should be possible to build a Casimir switch that is activated by light, a device that would be useful in optical switching systems. A very interesting measurement of the Casimir force between a flat surface of borosilicate glass and a surface covered with a film of amorphous silicon was done in 1979 by Arnold *et al.* [31]. They observed an increase in the Casimir force when the semiconductor was exposed to light. This experiment has yet to be repeated with modern methods and materials. As a first step, Chen *et al.* have used an AFM to measure the force between a single Si crystal and a 200 μm diameter gold coated sphere, and found good agreement with theory using the Lizshitz formalism [32].

IV. SPACE PROPULSION IMPLICATIONS

A. General Considerations

Conservation of energy and momentum place severe restrictions on what mechanisms may be utilized to propel spacecraft. For example, if a spacecraft is accelerating due to an interaction with the quantum vacuum, then it has to be removing energy from the quantum vacuum. Further the increase in kinetic energy must be equal or be less than the decrease in energy in the quantum vacuum. Some general constraints on using engineering the vacuum for space travel, as well as methods of altering the metric of space-time for space travel, are outlined in the paper by Puthoff *et al.* [33]. In the analysis of any proposed approaches, we need to consider the momentum and energy of the field plus any objects in the field. Consider, for example, a mechanism in a spacecraft that alters the normally isotropic quantum vacuum energy density in a local region surrounding the spacecraft. Let $E(\omega,\vec{r},\vec{r}_S)$ be the change in the vacuum energy density as a function of the frequency ω, the position $\vec{r}$ measured with respect to the center of the sail, given by $\vec{r}_S$, which is measured with respect to some fixed location. If, in actual fact, this function $E(\omega,\vec{r},\vec{r}_S)$ does not depend on $\vec{r}_S$ but has the same shape no matter where the sail is located, then the change in vacuum energy due to the presence of the sail is constant. By the conservation of energy, the sail is moving at a constant velocity, and cannot experience a force due to its interaction with the quantum vacuum. In conclusion, if the change in vacuum energy does not depend on the position of the spacecraft, then the energy and momentum are constant.

B. Sails in the Vacuum

A variety of sail concepts have been proposed [34]. As we mentioned earlier, we can view the vacuum as a source of radiation pressure from virtual photons. The challenge is to design surfaces that alter the symmetry of the free vacuum and produce a net force. Consider for example, a sail made of two different materials on opposite sides, that absorb electromagnetic radiation differently. Can we expect a net force on the sail? A simple classical analysis as shown in Fig. 2 suggests the answer to this question.

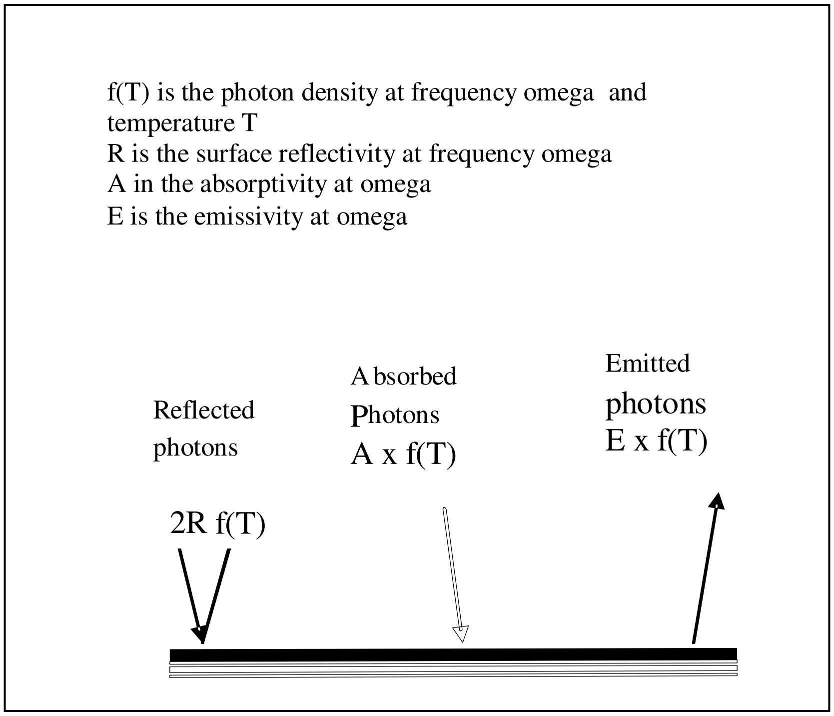

Fig. 2. Schematic of the momentum transfer from zero-point electromagnetic radiation to a sail made from different materials at temperature T on the top and bottom.

For a given frequency, assume the radiation energy density is proportional to $cf(\omega,T)$, the net momentum transfer ΔP_ω to the top surface is

$$\Delta P_\omega = A_\omega f(\omega,T) + E_\omega f(\omega,T) + 2R_\omega f(\omega,T) \tag{11}$$

where A_ω is the absorptivity, E_ω is the emissivity, $R_\omega$the reflectivity, and T the temperature. For a body in thermodynamic equilibrium, $A_\omega = E_\omega$, and by definition, $1 = A_\omega + R_\omega$. Using these restriction, it follows that $\Delta P_\omega = 2f(\omega, T)$, which is independent of the material properties. Therefore, even if the individual A_ω, E_ω, and R_ω are different for the other side, the same relations hold and the force on the opposite side of the sail just cancels this force, and there is no net acceleration. This conclusion holds at every frequency. We assumed the temperature of the sail is the same on both sides because of the intimate contact. If the radiation spectrum corresponds to that at zero temperature, then $f(\omega,0)$ describes the zero-point field, and both sides of the sail would be at zero

Kelvin. On the other hand, if one made a sail in which a temperature gradient was maintained across the sail, a net force might occur, and it would be a function of the energy required to maintain the temperature difference.

There is a complication to this analysis: what happens if the sail is moving? If the radiation density is due solely to the quantum vacuum at zero temperature $\left(cf(\omega,0)=\hbar\omega^3/2\pi^2c^3\right)$, then the spectral energy density the sail sees does not change with motion. The invariance of the spectrum of the zero-point fluctuations with uniform motion is a special property of the zero-point quantum vacuum. Without this property, one could distinguish a unique rest frame for the universe, violating the intent of special relativity. On the other hand, the thermal fields of real photons do not have this unique invariance. Hence uniform motion in a thermal field results in a Doppler shifted spectrum. For a sail, this means that the spectral energy density is different on the opposite sides of the sail, and, provided the integral of the forces over all frequencies were different for the two sides of the sail, it would be possible to obtain a net, thermally generated, force. When one considers the restrictions on the frequency dependence of dielectric coefficients due to causality, it is uncertain if one can generate a net force with this method. The possibility remains unresolved.

Einstein considered this situation for an atom moving in an isotropic thermal field, and showed that the increase in the atom’s kinetic energy upon absorption and emission of radiation is balanced by the drag force if the thermal field follows the usual Planck spectrum. Similarly if the atom is moving in the zero-point vacuum field, there is no net force on the atom. But if the spectrum does not have this form, net forces are possible. By inserting surfaces into the vacuum, we can alter the spectrum of the vacuum fluctuations, which results in net forces. Indeed, wherever there is an inhomogeneous vacuum energy density, there will be a net force on a polarizable particle given by $\frac{1}{2}\alpha\vec{\nabla}\left\langle E(x)^2\right\rangle$ [9]. From a propulsion viewpoint, this suggests the possibility of ejecting particles to generate a propulsive force, an approach that does not yet seem to offer any distinct advantage over more conventional methods.

Friction due to the quantum vacuum has been predicted to exist between two parallel infinite plates that have finite conductivity. The friction arises because of the motion of charges in the surface of the metal moving to maintain the boundary conditions. In conclusion, for a sail to accelerate due to the quantum vacuum, the sail must be removing energy from the vacuum. This prompts the question, by what processes can one remove energy from the vacuum?

C. Inertia Control by Altering Vacuum Energy Density

One can make use of the negative vacuum energy density that arises in a parallel plate structure in an alternative approach to propulsion. Based on the principles of general relativity, one would expect the changes in vacuum energy ΔE to correspond to a change in mass $\Delta E/c^2$. Thus the negative energy density of the vacuum in a parallel plate structure should result in the equivalent of a negative mass object. Proposals have been made to test this hypothesis that a negative vacuum energy leads to a reduction in mass by constructing stacks of capacitors, however the predicted effect is just beyond current measurement capability. From the theoretical perspective, one can estimate the positive and negative mass contributions for a parallel plate capacitor made from plates that are only one hydrogen atom thick and one atom apart, and still the total energy is positive. Thus it appears that a parallel plate Casimir cavity will always have a net positive energy density and cannot be used to create a zero or negative mass spacecraft, or initiate a wormhole. Nevertheless, the negative vacuum energy density may, with more effective approaches, be of use in reducing inertia, which reduces kinetic energy and the amount of work required to accelerate a spacecraft to a given velocity.

There is another variant of a negative mass drive that deserves mention. A system of charges that has a negative electrostatic potential energy ΔE would also, by the principles of general relativity, be expected to have a negative associated mass $\Delta E/c^2$. Thus in a gravitational field, there would be a levitating force. Pinto has explored this possibility, and suggests the effect may be amplified and made measurable if highly polarizable hydrogen atoms in a magnetic trap are exposed to isotropic laser radiation [88]. Although these technically challenging enhancements may amplify the basic levitating force, it still appears that the effective reduction in mass will be quite small compared to the total mass.

Negative energy drives are discussed more fully in Chapters 4, 5 and 16.

D. Dynamic Systems

Dynamic systems, in which something moves, and interacts with the quantum vacuum, may have the possibility of extracting energy from the vacuum. Hence they may be able to accelerate a spacecraft. The movement might be a macroscopic physical motion, a piezoelectrically driven surface, or the motion of electrons within a semiconductor, possibly altering the plasma frequency or the dielectric constant. We discuss one possible dynamic system in the next Section.

V. VIBRATING MIRROR CASIMIR DRIVE

It is possible to conceive of a vacuum spacecraft that operates by pushing on the quantum vacuum with a vibrating mirror [33]. With a suitable trajectory, the motion of a mirror in vacuum can excite the quantized vacuum electromagnetic field with the creation of real photons.

A. Simple Model based on Energy and Momentum Conservation

The important physical features of using the dynamic Casimir effect to accelerate a spacecraft can be seen in a simplified, heuristic model. Assume that the spacecraft has an energy source, such as a battery, that powers a motor that vibrates a mirror or a system of mirrors in a suitable manner to generate radiation. We will assume that there are no internal losses in the motor or energy source. We assume that at the initial time t_i, the mirrors are at rest. Then the mirrors are accelerated by the motor in a suitable manner to generate a net radiative reaction on the mirror, and at the final time t_f, the mirrors are no longer vibrating, and the spacecraft has attained a non-zero momentum. We can apply the first law of thermodynamics to the system of the energy source, motor, and mirror at times t_i, and t_f:

$$\Delta Q = \Delta U + \Delta W \tag{12}$$

where ΔU represents the change in the internal energy in the energy source, $-\Delta W$ represents the work done on the mirrors moving against the vacuum, and ΔQ represents any heat transferred between the system and the environment. We will assume that we have a thermally isolated system and $\Delta Q = 0$ so

$$0 = \Delta U + \Delta W \tag{13}$$

By the conservation of energy, the energy ΔU extracted from the battery goes into work done on the moving mirror $-\Delta W$. Since the mirror has zero vibrational and kinetic energy and zero potential energy at the beginning and the end of the acceleration period, and is assumed to operate with no mechanical friction, all work done on the mirror goes into the energy of the emitted radiation ΔR; and the kinetic energy of the spacecraft of mass M

$$\Delta W = \Delta R + \frac{M(\Delta V)^2}{2} \tag{13}$$

Thus the energy of the radiation emitted due to the dynamic Casimir effect equals

$$\Delta R = -\Delta U - \frac{M(\Delta V)^2}{2} \quad > 0 \tag{14}$$

The frequency of the emitted photons depends on the Fourier components of the motion of the mirror. We assume that the radiant energy can be expressed as a sum of energies of n_i photons each with frequency ω_i:

$$\Delta R = \sum_i n_i \hbar \omega_i \tag{15}$$

The number of photons emitted depends on the cosine of the angle the photon momentum makes with the normal to the surface, as Neto and Machado [67] show. In this simplified calculation, we will assume that all photons are emitted normally from one side of the accelerating surface. This assumption is not valid, but it allows us to obtain a best-case scenario and illustrates the main physical features. If all photons are emitted normally from one surface, then the photon momentum transfer ΔP is

$$\Delta P = \sum_i n_i \frac{\hbar \omega_i}{c} \quad = \frac{\Delta R}{c} \tag{16}$$

where c is the speed of light. Using Eq. 14, we obtain the result

$$\Delta P = \frac{-\Delta U}{c} - \frac{M(\Delta V)^2}{2c} \tag{17}$$

In a non-relativistic approximation $\Delta P = M\Delta V$ and the change in velocity ΔV of the spacecraft is to second order in $\Delta U/Mc^2$:

$$\frac{\Delta V}{c} = \frac{-\Delta U}{Mc^2} + \left(\frac{\Delta U}{Mc^2}\right)^2 \tag{18}$$

This represents a maximum change in velocity attainable by use of the dynamic Casimir effect (or by the emission of electromagnetic radiation generated by more conventional means) when the energy ΔU is expended. The ratio $\Delta U/Mc^2$ is expected to be a small number, and we can neglect the second term in Eq. 18. As a point of reference, for a chemical fuel, the ratio of the heat of formation to the mass energy is approximately 10^{-10}. With this approximation, we find the maximum value of $\Delta V/c$ equals $\Delta U/Mc^2$, the energy obtained from the energy source divided by the rest mass energy of the spacecraft. It follows that the kinetic energy of the motion of the spacecraft E_{ke} can be expressed as:

$$E_{ke} = \frac{M(\Delta V)^2}{2} \quad = \Delta U \frac{\Delta U}{2Mc^2} \tag{19}$$

This result for the upper limit on the spacecraft kinetic energy shows that the conversion of potential energy ΔU from the battery into kinetic energy of the spacecraft is an inefficient process since $\Delta U/Mc^2$ is a small factor. Almost all of the energy ΔU has gone into photon energy. This inefficiency follows since the ratio of momentum to energy for the photon is $1/c$.

In our derivation, the internal energy of the system is used to create and emit photons from some unspecified process; no massive particles are ejected from the space craft (propellantless propulsion). We have neglected: 1. the change in the mass of the spacecraft as the stored energy is converted into radiation, 2. radiative mass shifts, 3. complexities related to high energy vacuum fluctuations and divergences, 4. all dissipative forces in the system used to make the mirror vibrate. These assumptions are consistent with a heuristic non-relativistic approximation.

In this simplified model, we have not made any estimates about the rate of photon emission and how long it would take to reach the maximum velocity. For configurations considered in the literature, rates of photon emission from the dynamic Casimir effect are estimated to be very low, typically 10^{-5} photons/sec or about 300 photons/year [67]. Also we will have to vibrate the mirror asymmetrically so that more photons are emitted from one side than the other. In the derivation, however, we never made any assumptions about the mechanism by which photons were generated, so the derivation holds quite generally, whether we simply use a battery and a perfect light bulb, or a vibrating charged surface.

1. Use of the static Casimir Effect as an energy Source

The general analysis in the preceding Section can be taken one step further to suggest a spacecraft whose operation is totally based on quantum vacuum properties. The vibrating motor in the spacecraft could be powered by energy removed from the quantum vacuum using an arrangement of perfectly conducting, uncharged, parallel plates. Detailed considerations about extracting energy from the quantum vacuum are presented in Chapter 19. The Casimir energy $U_C(x)$ at zero degrees Kelvin between plates of area A, separated by a distance x is:

$$U_C(x) = -\frac{\pi^2}{720}\frac{\hbar c A}{x^3} \tag{20}$$

If we allow the plates to move from a large initial separation a to a very small final separation b then the change in the vacuum energy between the plates is approximately:

$$U_C(x) = U_C(b) - U_C(a) \tag{21}$$

Substituting Eq. 20 gives the result

$$\Delta Uc \approx -\frac{\pi^2}{720}\frac{\hbar c A}{b^3} \tag{22}$$

The attractive Casimir force has done work on the plates, and, in principle, we can build a device to extract this energy with a suitable, reversible, isothermal process, and use it to accelerate the mirrors. We neglect any dissipative forces in this device, and assume all of the energy ΔU_C can be utilized. Thus the maximum value of $\Delta V_C /c$ obtainable using the energy from the Casimir force "battery" is:

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{720}\frac{1}{Mc^2}\frac{\hbar c A}{b^3} \tag{23}$$

We can make an upper bound for this velocity by making further assumptions about the composition of the plates. Assume that the plate of thickness L is made of a material with a rectangular lattice that has a mean spacing of d, and that the mass associated with each lattice site is m. Then the mass of one plate is:

$$M_P = AL\frac{m}{d^3} \tag{24}$$

The density approximation is good for materials with a cubic lattice, and within an order of magnitude of the correct density for other materials.

In principle, it is possible to make one of the plates in the battery the same as the plate accelerated to produce radiation by the dynamic Casimir effect. As the average distance between the plates is decreased, the extracted energy is used to accelerate the plates over very small amplitudes. If we assume we need to employ two plates in our spacecraft, and that the assembly to vibrate the plates has negligible mass, then the total mass of the spacecraft is $M = 2M_P$ and we obtain an upper limit on the increase in velocity:

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{1400}\frac{\hbar}{Lmc}\frac{d^3}{b^3} \tag{25}$$

The final velocity is proportional to the Compton wavelength $(\hbar / mc)$ of the lattice mass m divided by the plate thickness L. Assume that the final spacing between the plates is one lattice constant ($d = b$), that the lattice mass m equals the mass of a proton m_p and that the plate thickness F is one Bohr radius $a_0 = \hbar^2 / m_e e^2$, then we obtain (α is the fine structure constant with approximate value of 1/137):

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{1400}\frac{\alpha m_e}{m_p} \tag{26}$$

(A real plate made using current technology might easily be three orders of magnitude thicker.) Substituting numerical values we find:

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{1400}\frac{1}{137}\frac{1}{1800} \quad = 2.78 \times 10^{-8} \tag{27}$$

This best-case scenario corresponds to a disappointing final velocity of about 8 m/s about 10^3 times smaller than for a large chemical rocket. As anticipated, the spacecraft is very slow despite the unrealistically favorable assumptions made in the calculation, yet this simple gedanken experiment does demonstrate that it may be possible to base the operation of a spacecraft entirely on the properties of the quantum vacuum. Using an additional energy source can result in higher terminal velocity.

B. Detailed Model for Propulsion using Vibrating Mirrors

Assume we have a flat, perfectly reflecting, mirror whose equilibrium position is $x = 0$. At a time t where $t_i < t < t_f$ the location of the mirror is given by $x(t)$. Neto has given an expression for the force per unit area $F(t)$ on such a mirror [72]:

$$F(t) = \lim_{\delta x \to 0} \frac{\hbar c}{30\pi^2}\left[\frac{1}{\delta x}\frac{d^4 x(t)}{c^4 dt^4} - \frac{d^5 x(t)}{c^5 dt^5}\right] \tag{28}$$

where δx represents the distance above the mirror at which the stress-energy tensor is evaluated. The second term represents the dissipative force that is related to the creation of travelling wave photons, in agreement with its interpretation as a radiative reaction. In computing the force due to the radiation from the mirror's motion, the effect of the radiative reaction on $x(t)$ is neglected in the nonrelativistic approximation. The divergent first term can be understood in several ways. Physically it is a dispersive force that arises from the scattering of low frequency evanescent waves. The divergence can be related to the unphysical nature of the perfect conductor boundary conditions. Forcing the field to vanish on the surface requires its conjugate momentum to be unbounded. Thus the average of the stress-energy tensor $\langle T_{\mu\nu} \rangle$ is singular at the surface for the same reason that single-particle quantum mechanics would require a position eigenstate to have infinite energy [70]. This divergent term can be lumped into a mass renormalization, and therefore disappears from the dynamical equations when they are expressed in terms of the observed mass of the body [71][73]. We will not discuss this term further in this calculation, although we will return to the general idea of radiative mass shift in our discussion. We will assume that diffraction effects are small for our finite plates.

The total energy radiated per unit plate area E can be expressed

$$E = -\int_{t_1}^{t_2} dt\, F(t)\frac{dx(t)}{dt} \tag{29}$$

Substituting Eq. 28 for $F(t)$ we find

$$E = \frac{\hbar}{30\pi^2}\frac{1}{c^4}\int_{t_1}^{t_2} dt \left(\frac{d^3x(t)}{dt^3}\right)^2 \tag{30}$$

The total impulse I per unit plate area can also be computed as the integral of the force per unit area over time:

$$I = \int_{t_1}^{t_2} dt\, F(t) = -\frac{\hbar}{30\pi^2}\frac{1}{c^4}\left(\left.\frac{d^4x(t)}{dt^4}\right|_{t_2} - \left.\frac{d^4x(t)}{dt^4}\right|_{t_1}\right) \tag{31}$$

The total impulse I equals the mass of the system M per unit area times the change in velocity ΔV in a non-relativistic approximation:

$$I = M\Delta V \tag{32}$$

We want to specify a trajectory for the mirror that will give a net impulse. One of the trajectories that has been analyzed is that of the harmonic oscillator [80][70]. In this case, the mirror motion is in a cycle and we can compute the energy radiated per cycle per unit area and the impulse per cycle per unit area. For a harmonic oscillator of frequency Ω and period $T = 2\pi/\Omega$, there is only one Fourier component of the motion, so the total energy of each pair of photons emitted is $\hbar\Omega = \hbar(\omega_1 + \omega_2)$. For a harmonically oscillating mirror the displacement is

$$x_{ho}(t) = X_0 \sin \Omega t \tag{33}$$

A computation based on Eqs. 30 and 31 shows there will be a net power radiated in a cycle, however, the dissipative force for the harmonic oscillator F_{ho} will average to zero

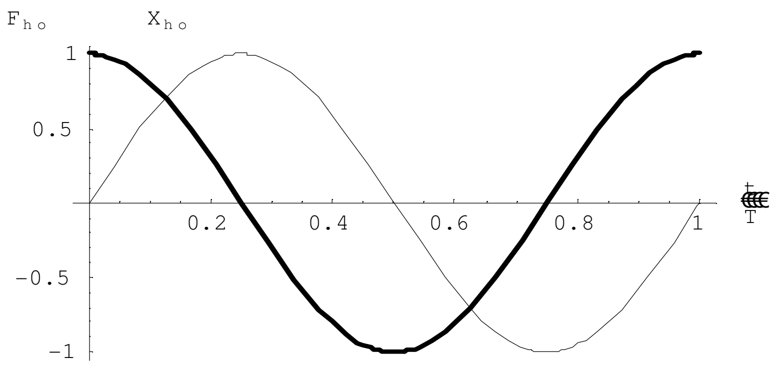

Fig. 3. The displacement x_{ho} and the radiative reaction F_{ho}, bold line, for a harmonically oscillating mirror plotted as a function of the normalized time. For convenience F_{ho} and x_{ho} are normalized to 1.

over the entire cycle as shown in Fig. 3, so there will be no net impulse.

In order to secure a net impulse, we need a modified mirror cycle. One such model cycle can be readily constructed by using the harmonic function $x_{ho}(t)$ over the first and last quadrants of the cycle, where the force F_{ho} is positive, and a cubic function $x_c(t)$ over the middle two quadrants where F_{ho} is negative:

$$x_c(t) = \frac{X_0}{2}\frac{(\Omega t - \pi)^3}{\left(\pi/2\right)^3} - \frac{3X_0}{2}\frac{(\Omega t - \pi)}{\pi/2} \tag{34}$$

The coefficients for the cubic polynomial are chosen so that at Ωt = $\pi/2$, $3\pi/2$ the displacement and the first derivatives of $x_c(t)$ and $x_{ho}(t)$ are equal. As can be seen from Fig. 4, the cubic function $x_c(t)$ matches $x_{ho}(t)$ quite closely in the interval $0.25 < t/T < 0.75$. Of course the higher order derivatives do not match, and that is precisely why the force differs.

The similarity in displacement and the difference in the resulting force is striking. For the mirror displacement $x_m(t)$ in our model we choose:

$$x_m(t) = x_{ho}(t) \qquad \text{for} \quad 0 \le t/T \le 0.25; \quad 0.75 \le t/T \le 1 \tag{35}$$

$$x_m(t) = x_c(t) \qquad \text{for} \quad 0.25 < t/T < 0.75 \tag{36}$$

Fig. 4 shows $x_m(t)$ plotted with the corresponding force per unit area $F_m(t)$ obtained from Eq. 28. The force $F_m(t)$ is positive in the first and last quarter of the cycle, and vanishes in the middle, where the trajectory is described by the cubic. The energy radiated per area per cycle for our model trajectory can be obtained from Eq. 30:

$$E_m = -\frac{\hbar c}{60\pi} X_0^2 \left(\frac{\Omega}{c}\right)^5 \tag{37}$$

The total impulse per area per cycle for our model I_m trajectory is

$$I_m = -\frac{\hbar}{15\pi^2} X_0 \left(\frac{\Omega}{c}\right)^4 \tag{38}$$

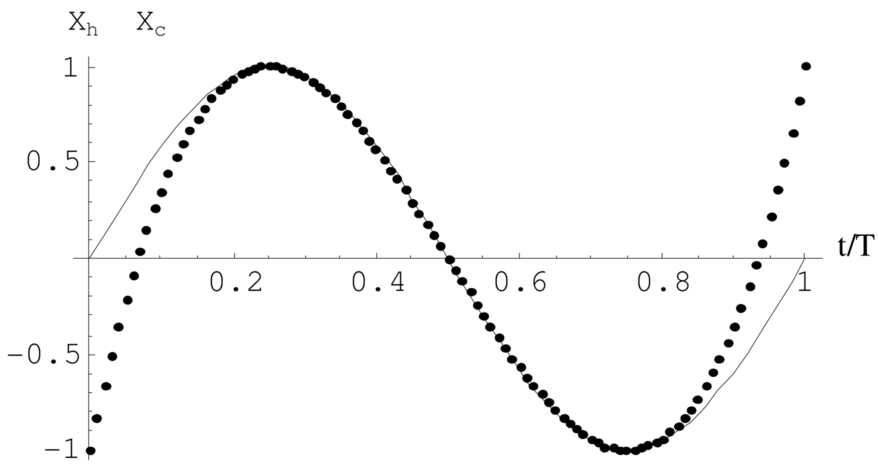

Fig. 4. The normalized displacement $x_{ho}(t)$ for a harmonically oscillating mirror, solid line, and the cubic function $x_{c(t)}$, shown by the bold dotted line.

The impulse is first order in $\hbar$ and is therefore typically a small quantum effect. Thus for our model cycle, the change in velocity per second is $\Delta V/dt$:

$$\Delta V_m \big/ dt = \frac{I_m \Omega}{M} \tag{39}$$

where B is the mass per unit plate area of the spacecraft, and we assume the plate is the only significant mass in the gedanken spacecraft. In order to estimate ΔV_m, we can make some further assumptions regarding the mass of the plate per unit area. As before, we can make a very favorable assumption regarding the mass per unit area of the plates $M = m_p/a_o^2$, which yields the change in velocity per second:

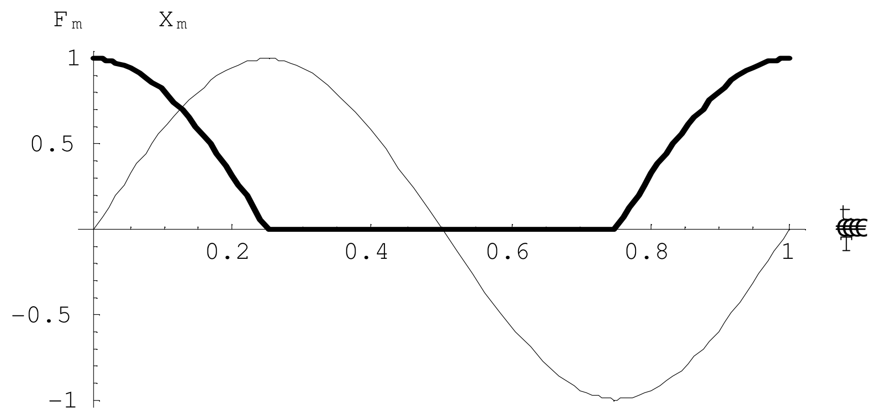

Fig. 5. The normalized displacement $x_m(t)$ and the corresponding normalized radiative force $F_m(t)$, bold solid line, are shown as functions of the time. The force is positive in the first and last quarters, and zero in the middle half of the cycle.

$$\Delta V_m \big/ dt = -\frac{\hbar}{15\pi^2} X_0 \left(\frac{\Omega}{c}\right)^4 \Omega \frac{a_0}{m_p} \tag{40}$$

If we substitute reasonable numerical values[64][80], a frequency of $\Omega = 3\times10^{10}$ s^{-1} and an oscillation amplitude of X_0, = 10^{-9} m, we find that $\Delta V_m/dt$ is approximately 3×10^{-20} m/s^2 per unit area, not a very impressive acceleration. Physically, one would imagine the surface of the mirror vibrating with an amplitude of *just one nanometer*. This conservative limitation in the amplitude arises because the maximum velocity of the boundary is proportional to the elastic deformation, which cannot exceed about 10^{-2} for typical materials. The energy radiated per area $E_m\Omega$ is about 10^{-25} W/m^2. There are a number of methods to increase these values by *many orders of magnitude*, as discussed below.

The efficiency of the conversion of energy expended per cycle in our model E_m into kinetic energy of the spacecraft $E_{ke} = ½M(\Delta V_m)^2 = I_m^2/2M$ is given in the nonrelativistic approximation by the ratio:

$$\frac{E_{ke}}{E_m} = \frac{\hbar}{Mc}\frac{1}{\pi}\left(\frac{\Omega}{c}\right)^3 \tag{41}$$

With our assumptions, the approximate value of this ratio is 10^{-26}, making this conversion an incredibly inefficient process.

C. Methods to Increase Acceleration

The dynamic Casimir effect has yet to be verified experimentally. Hence there have been a number of interesting proposals describing methods designed to maximize the effect so it can be measured. In 1994, Law predicted a resonant response of the vacuum to an oscillating mirror in a one-dimensional cavity [89]. The behavior of cavities formed from two parallel mirrors that can move relative to each other is qualitatively different from that of single plates. For example it is possible to create particles in a cavity with plates separating at constant velocity [90]. The very interesting proposal by Lambrecht *et al.* concludes that if the mechanical oscillation frequency is equal to an odd integer multiple of the fundamental optical resonance frequency, then the rate of photon emission from a vibrating cavity formed with walls that are partially transmitting with reflectivity r_1 and r_2, is enhanced by a factor equal to the finesse $f = 1/\ln(1/r)$ of the cavity [80][64]. For semiconducting cavities with frequencies in the GHz range, the finesse, which can be 10^9, giving our gedanken spacecraft an acceleration of 3×10^{-11} m/s^2 based on Eq. 40. Plunien *et al.* have shown that the resonant photon emission from a vibrating cavity is further increased if the temperature is raised [81]. For a 1 cm cavity, the enhancement is about 10^3 for a temperature of 290K. Assume that one has a gedanken spacecraft with a vibrating cavity operating at about 290K providing a 10^{12} total increase in the emission rate. This would result in an acceleration per unit area of the plates of 3×10^{-8} m/s^2, a radiated power of about 10^{-13} W/m^2 and an efficiency E_{ke}/E_m of about 10^{-14}. After 10 years of operation, the gedanken spacecraft velocity would be approximately 10 m/s, which is about 3 orders of magnitude less than the current speed of Voyager, 17 km/s, obtained after a gravity assist maneuver around Jupiter to increase the velocity. (The burn-out velocity for Voyager at launch in 1977 was 7.1 km/s [91].)

The numerical results for the model obviously depend very strongly on the assumptions made. For the plate mass/area and system to vibrate the plate, we have made unrealistically favorable estimates; for the oscillation frequency and amplitude we have made conservative estimates. It is possible that new materials, with the ability to sustain larger strains, could make possible an amplitude of oscillation orders of magnitude larger than one nanometer. Perhaps the use of nanomaterials, such as carbon nanotubes that support 5 % strain [92], or "super" alloys [93], would allow a much larger effective deformation. If the amplitude was 1 mm instead of 1 nm, the gedanken spacecraft, or some modification with improved coupling to the vacuum, might warrant practical consideration.

Eberlein has shown that density fluctuations in a dielectric medium would also result in the emission of photons by the dynamic Casimir effect [94]. This approach may ultimately be more practical with large area dielectric surfaces driven electrically at high frequencies. More theoretical development is needed to evaluate the utility of this method. Other solid state approaches may also be of value with further technological developments. For example, one can envision making sheets of charge that are accelerated in MOS type structures. Yablonovitch has pointed out that the zero-point electromagnetic field transmitted through a window whose index of refraction is falling with time shows the same phase shift as if it were reflected from an accelerating mirror [95]. To simulate an accelerating mirror, he suggested utilizing the sudden change in refractive index that occurs when a gas is photoionized or the sudden creation of electron hole pairs in a semiconductor, which can reduce the index of refraction from ~ 3.5 to 0 in a very short time. Using subpicosecond optical pulses, the phase modulation can suddenly sweep up low-frequency waves by many octaves. By lateral synchronization, the moving plasma front with its large change in the index of refraction can, in effect, act as a moving mirror exceeding the speed of light. Therefore one can regard such a gas or semiconductor slab as an observational window on accelerating fields, with accelerations as high as ~ 10^{20} m/s [95]. Accelerations of this magnitude will have very high frequency Fourier components. Eq. 40 shows that the impulse goes as the fourth power and the efficiency as the third power of the Fourier component for an harmonic oscillator which suggests that with superhigh accelerations, an optimum time dependence of the field, and an optimum shape of the wavefronts, one might be able to secure much higher fluxes of photons and a much higher impulse/second, with a higher conversion efficiency. On the other hand, a preliminary calculation by Lozovik *et al.* to investigate this approach suggests that the accelerated plasma method may have limitations in producing photons [96].

Ford and Svaiter have shown that it may be possible to focus the fluctuating vacuum electromagnetic field [97]. This capability might be utilized to create regions of higher energy density. This might be of use in a cavity in order to increase the flux of radiated photons. There may be also enhancements due to nature of the index of refraction for real materials. For example, Ford has computed the force between a dielectric sphere, whose dielectric function is described by the Drude model (based on a simple approximate model for free electrons in a metal), and a perfectly reflecting wall, with the conclusion that certain large components of the Casimir force no longer cancel. He predicts a dominant oscillatory contribution to the force, in effect developing a model for the amplification of vacuum fluctuations. Barton and Eberlein have shown that for materials with a fixed index of refraction, the force for a one dimensional scalar field goes as $[(n-1)/n]^2$, which suggest the possibility that one might be able to enhance the force by selection of a material with a small index [71].

Another, albeit improbable, approach to a vacuum facilitated gedanken spacecraft is to consider the possibility of adjusting the radiative mass shift, which we have neglected until this point. There is a very small radiative shift of the mirror due to its interaction with the vacuum, akin to the Lamb shift for an atom [80][74]. To measure a vacuum mass shift, a proposal was made recently to measure the inertial mass shift in a multilayer Casimir cavity, which consists of 106 layers of metal 100 nm thick, 35 cm in diameter, alternating with films of silicon dioxide 5 nm thick [39]. The mass shift is anticipated to arise from the decrease in the vacuum energy between the parallel plates. A calculation shows that the mass shift for the proposed cavity is at or just beyond the current limit of detectability. It appears that if quantum vacuum engineering of spacecraft is to become practical, and the dreams of science fiction writers are to be realized, we may need to develop new methods to be able to manipulate changes in vacuum energy densities that are near to the same order of magnitude as mass energy densities. Then we would anticipate being able to shift inertial masses by a significant amount. Since mass shifts in computations are often formally infinite, perhaps such developments are not forbidden, and, with more understanding (and serendipity!), may be controllable. With large mass shifts one might be able to build a structure that had a small or zero inertial mass, which could be readily accelerated.

VI. UNRESOLVED PHYSICS

Casimir effects are typically small and difficult to measure. In fact, measurements have only been made for the simplest geometries, such as the parallel plate geometry or the sphere plate geometry. No measurements have ever been made that validate the dynamic Casimir effect. Assuming the effect is verified experimentally, then the question is, is it possible to amplify these effects and bring them into the useful range? This is certainly one of the challenges of vacuum engineering. Experiments are needed that explore some of the issues that are beyond the present calculational ability of QED, for example, the effect of complex geometries on vacuum forces, or the effect of massive fields or dense, moving nuclear matter on the quantum vacuum. Is it possible to make a stable vacuum field that has a large variation in energy density? What is the effect of changes in vacuum energy on the index of refraction? Can energy density gradients be found on a length scale that's useful to humans? We need to greatly increase our knowledge of the quantum vacuum. The development of a very sensitive small probe that provided a frequency decomposition of the local vacuum energy density would certainly be VERY helpful!

From the status of current research in Casimir forces, it is clear that we are at the tip of the iceberg describing the properties of the quantum vacuum for real systems with real material properties. For example, there is no general agreement with regard to the calculations of static vacuum forces for geometries other than infinite parallel plates of ideal or real metals at a temperature of absolute zero. Non-zero temperature corrections for flat, real metals are uncertain [98][99]. There are fundamental disagreements about the computation of vacuum forces for spheres or rectangular cavities, about how to handle real material properties and curvature in these and other geometries [71]. Indeed, it is very difficult to calculate Casimir forces for these simple geometries and to relate the calculations to an experiment. Calculations have yet to be done for more complex, and potentially interesting geometries. The usual problems in QED, such as divergences due to unrealistic boundary conditions, to curvature, to interfaces with different dielectric coefficients, *etc.,* abound [36].

A. Possible Discriminating Tests

As discussed above, to make practical spacecraft based on engineering the quantum vacuum, it seems we need to find new boundary conditions for the vacuum that can alter the vacuum energy density orders of magnitude more than with our current boundary conditions, which are primarily metal or dielectric surfaces. Perhaps the use of new materials, for example with a negative index of refraction, or an ultra high electrical carrier density (steady state or transient), or novel superconducting materials may open the door to new Casimir phenomena. Recently the use of negative index materials was proposed to make a repulsive Casimir force [101]. With significantly increased funding of research, some breakthroughs might be possible.

There are some important experiments that can help our understanding of vacuum energy and Casimir forces, and which may lead to significant improvements in our engineering capability. Experiments to verify the adiabatic Casimir effect have been suggested in the literature. This is an important theoretical issue that has ramifications in different fields, including astrophysics and elementary particle physics. Within the next five years, some clever experimental approaches will probably be developed to explore the adiabatic Casimir effect. Experiments measuring the Casimir forces for semiconductor surfaces would be helpful in the development of new applications of vacuum forces and demonstrating that it is possible to alter the Casimir force by altering the carrier density. The measurement of Casimir forces and energies for different geometry and composition objects, such as rectangular cavities or spheres, would provide some badly needed answers for theoretical modeling. Measurements of Casimir forces between separate non-planar surfaces are also needed. There may be surfaces that have larger forces than the classic parallel plates, but we have yet to measure anything but parallel plates.

Tests to determine if negative vacuum energy yields a negative force in a gravitation field have been proposed. This is a fundamental question. Order of magnitude estimates suggest that a negative mass object is not possible, but that the total mass may be reduced a measurable amount by contributions from negative vacuum energy.

B. Application Implications

Because Casimir forces require small separations, they have been utilized in micro and nanoelectromechanical systems (MEMS/NEMS). As microfabrication technology develops, nano-devices with shorter working distances and more varied materials will be developed, resulting in more applications for Casimir forces. An excellent recent survey by Capasso *et al.* discusses a variety of devices, including the anharmonic Casimir oscillator and a surface with a Pd film, whose electrical properties change in the presence of hydrogen, which could lead to a measurable change in the Casimir force [100]. Also reported were repulsive Casimir forces for certain planar material combinations as predicted by the Lifshitz formulation, for example, a gold covered planar surface and a silica covered surface when ethanol is the medium in between. An ultrasensitive magnetometer was proposed using the repulsive Casimir force to levitate a disc with a magnetic dipole in alcohol. Also proposed was a demonstration of quantum torque, analogous to the Casimir force in origin. In this system, the birefringent material is floated by the repulsive Casimir force above a fixed birefringent plate. Steady progress in the application of Casimir forces in MEMS is to be expected.

VII. CONCLUSION

One objective in this paper is to illustrate some of the unique properties of the quantum vacuum and how they might be utilized in propulsion of a spacecraft. We have outlined some of the considerations for the use of vacuum energy to propel a spacecraft and pointed out some directions in which some helpful discoveries may lie.

We have demonstrated that it is possible in principle to propel a spacecraft using the dissipative force an accelerated mirror experiences when photons are generated from the quantum vacuum. Further we have shown that one could in principle utilize energy from the vacuum fluctuations to operate the vibrating mirror assembly required. The application of the dynamic Casimir effect and the static Casimir effect may be regarded as a proof of principle, with the hope that the proven feasibility will stimulate more practical approaches exploiting known or as yet unknown features of the quantum vacuum. A model gedanken spacecraft with a single vibrating mirror was proposed which showed a very unimpressive acceleration due to the dynamic Casimir effect of about 3×10^{-20} m/s^2 with a very inefficient conversion of total energy expended into spacecraft kinetic energy. Employing a set of vibrating mirrors to form a parallel plate cavity and raising the cavity temperature to about 290ºK increases the output by a factor of the finesse of the cavity times approximately 10^3 yielding an acceleration of about 3×10^{-8} m/s^2 and a conversion efficiency of about 10^{-14}. To put this into perspective, after 10 years at this acceleration, the spacecraft would have only attained a 10 m/s velocity. At least we have computationally suggested (pending experimental verification of the phenomena predicted by QED), that it is possible to use the quantum vacuum to propel a spacecraft. This represents progress.

The vacuum effects that we computed scale with Planck's constant and therefore are very small. In order to have a practical spacecraft based on quantum vacuum properties, it would be preferable that the vacuum effects scale as $\hbar^0$, meaning that the effects are essentially independent of Plank's constant and consequently may be much larger. By itself this requirement does not guarantee a large enough magnitude, but it certainly helps [80]. New methods of modifying the quantum vacuum boundary conditions may be needed to generate the large changes in the free field vacuum energy or momentum required if "vacuum engineering" as proposed in this paper is ever to be practical. For example, the vacuum energy density difference between parallel plates and the region outside them in free space is simply not large enough in magnitude for our engineering purposes. Energy densities, positive or negative, that are orders of magnitude greater are required. Such energy density regions may be possible, at least in some cases. For example, a region appeared in the 1 dimensional dynamic system in which the energy density was below that of the Casimir parallel plate region [89]. Similarly, more effective ways of transferring momentum to the quantum vacuum than using photons generated with the adiabatic Casimir effect are probably necessary if a spacecraft is to be propelled using the vacuum.

Although the results of our calculations using the vibrating mirrors to propel a gedanken rocket are very unimpressive, it is important to not take our conclusions regarding the final velocity in our simplified models too seriously. The choice of numerical parameters can easily affect the final result by 4 orders of magnitude. The real significance of the result is that a method has been described that illustrates the possibility of propelling a rocket by coupling to the vacuum. It is possible that there may be vastly improved methods of coupling. There are numerous potential ways in which the ground state of the vacuum electromagnetic field might be engineered for use in technological applications, a few of which we have mentioned here. As the technology to fabricate small devices improves, as the theoretical capability of calculating quantum vacuum effects increases, as experiments help us understand the issues, it will be interesting to see which possibilities prove to be useful and which remain curiosities.

I have not mentioned optical applications of vacuum engineering, such as fabricating lasers in cavities to control spontaneous emission. I have not mentioned the current astrophysical conundrums about dark energy and the cosmological constant, which may relate to vacuum energy. We are not very good at predicting the development of technology. In the 1980's we thought AI was going to revolutionize the world, but it didn't. In the 1960s, manufacturers were hard put to think of any reason why an individual would want a home computer and today we wonder how we ever survived without them. In about 1900 an article was published in Scientific American proving that it was impossible to send a rocket, using a conventional propellant, to the moon. The result was based on the seemingly innocuous assumption of a single stage rocket. Hopefully, a paper on vacuum propulsion written a hundred years from now, will also find amusing our failure to perceive the key issues and see clearly how quantum propulsion should be done.

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