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# Models and (some) Searches for CPT Violation: From Early Universe to the Present Era

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**Abstract.** In the talk, I review theoretical models, inspired by quantum gravity, that may violate CPT symmetry. The amount of violation today (which is constrained severely by a plethora of experiments that I will not describe due to lack of space) need not be the same with the one that occurred in the Early Universe,. In certain models, one can obtain a precise temperature dependence of CPT violating effects, which is such that these effects are significant during the radiation era of the Universe, but are damped quickly so that they do not to affect nucleosynthesis and are negligible in the present epoch (that is, beyond experimental detection with the current experimental sensitivity). The CPT Violation (CPTV) in these models may arise from special properties of the background over which the fields of the model are propagating upon and be responsible for the generation of a matter-antimatter asymmetry, where any CP violation effects could only assist in the creation of the asymmetry, the dominant effect being CPTV. However, there are cases, where the CPTV arises as a consequence of an ill-defined CPT operator due to decoherence as a result of quantum gravity environmental degrees of freedom, inaccessible to a low-energy observer. I also discuss briefly the current-era phenomenology of some of the above models; in particular, for the ones involving decoherence-induced CPT violation, I argue that entangled states of neutral mesons (Kaons or B-systems) can provide smoking-gun sensitive tests or even falsify some of these models. If CPT is ill-defined one may also encounter violations of the spin-statistics theorem, with possible consequences for the Pauli Exclusion Principle.

## 1. Introduction and Summary

Invariance of a relativistic (*i.e.* Lorentz Invariant), local and unitary field theory Lagrangian under the combined transformations of Charge Conjugation (C), Parity (spatial reflexions, (P)) and reversal in Time (T), at any order, is guaranteed by the corresponding celebrated theorem [1]. This has important implications for the equality of masses and the absolute values of the various quantum numbers that characterise particles and antiparticles, and the equal amounts of matter and antimatter when they were created in the beginning of the Universe's evolution. On the other hand, today, there is an overwhelming dominance of matter over antimatter in the Cosmos, which calls for an explanation. Assuming the validity of the CPT theorem, A. Sakharov [2] has suggested that the observed matter-antimatter asymmetry in the Universe today is the result of *out of thermal equilibrium* processes in the early universe that violate C, CP and Baryon symmetries. The out of equilibrium assumption is a crucial one so that any asymmetry generated by the violation of C, B and CP symmetries in the expanding Universe is not washed



out but remains to the current epoch, thereby explaining the observed dominance of matter over antimatter today.

Although all of Sakharov conditions are met qualitatively by the Standard Model (SM) of Particle Physics, unfortunately they are not valid quantitatively, meaning that the amount of CP violation within the Standard Model is some ten orders of magnitude smaller than the required one to produce the observable matter-antimatter asymmetry [3]. According to observations, the abundance of baryons over that of antibaryons is of order [4]

$$Y_{\Delta B} = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10} \quad (1)$$

for temperatures  $T \gtrsim 1$  GeV, where  $n_B$  is the number density of baryons,  $n_{\bar{B}}$  is the number density of antibaryons and  $n_\gamma$  is the density of photons (proportional to the entropy density  $s$  of the Universe). This number was determined with accurate measurements of the Cosmic Microwave Background (CMB) radiation [5]. Out of equilibrium processes that generate a baryon asymmetry in the Universe are called collectively *Baryogenesis* [6]. Similarly, the generation of an asymmetry between leptons and antileptons is known as *Leptogenesis* [7], and is expected to be of the same order of magnitude as  $Y_{\Delta B}$ .

Since the amount of CP violation in the SM is not sufficient to produce (1), one should look for models beyond the standard model that contain new sources of CP violation, that can be tuned so as to produce sufficient Baryogenesis and Leptogenesis. There is a plethora of such models in the current literature, ranging from supersymmetric theories, to theories with extra dimensions of space, including strings. There is no experimental evidence as yet for the realisation of such extended models in Nature.

Interesting models [8, 9] of Baryon asymmetry in the Universe involve a two stage process, during which one generates first a lepton asymmetry in the early Universe, by appropriate CP-violating non-equilibrium processes in the early Universe, which is then communicated to the baryon sector, at temperatures lower than those of the electroweak phase transition ( $T \lesssim 100$  GeV), by means of Baryon-minus-Lepton-(B-L)-number-conserving sphaleron processes within the SM sector of the theory. In this important scenario of Leptogenesis as the path to Baryogenesis, pioneered by Fukugita and Yanagida [8, 9], the lepton abundance is produced by the decay of heavy right-handed Majorana neutrinos (and so represents physics beyond the Standard Model (BSM)). The difference in the branching ratios of the channels of production of leptons and antileptons is equal to the imaginary part of the interference term of tree-level and one-loop diagrams for the decay processes. For the interference to generate a non-zero CP violating phase, at least two generations of right-handed neutrinos are needed [8, 7]. In fact, at least two right-handed neutrinos are *also* required in the see-saw mechanism [10] for the generation of light neutrino masses, as necessitated by the observed phenomena of neutrino flavour oscillations among the light neutrinos, which require at least two of the active neutrinos to have a mass. Measurements on solar, atmospheric and reactor neutrinos have established that there are oscillations with distance of neutrino flavours [11]. The model of Fukugita and Yanagida, therefore, connects an explanation of leptogenesis with the see-saw mechanism of light neutrino masses observed in Nature. The model thus represents an economical extension of SM, since, at least for the purpose of generating baryon asymmetry it does not require other particles apart from the massive right-handed neutrinos.

In the framework of ref. [8], the right handed neutrinos are very massive, and thus have decayed today, leaving no other trace apart from the see-saw type masses of the active neutrinos. Shaposhnikov and collaborators [12] have also made a proposal for a Minimal extension of the SM, termed  $\nu MSM$ , involving three generations of right-handed neutrinos, whose masses though are much lower than the corresponding ones in the model of [8]. In fact, in the  $\nu MSM$  the two heavier neutrinos are almost degenerate, with masses of order  $\mathcal{O}(1)$  GeV, while the lightest of

the right-handed neutrinos may have masses of order of a few keV, and has a life time longer than the age of the Universe. In this sense, the model can provide a natural candidate for dark matter (DM), although there is still a long way to go before a satisfactory phenomenology of the dark sector of the Universe, including the dark energy sector, is provided within the framework of  $\nu$ MSM<sup>1</sup>. The relatively light masses of the right-handed neutrinos in the  $\nu$ MSM makes the Baryogenesis process in this model rather complicated [13], and certainly dissociated from the Leptogenesis path of [8]. From our point of view in this work we shall focus our attention to the model of [8], but taking into account of the coupling of the fermions in the model to the gravitational background; this will have interesting consequences.

However, we shall not simply consider the Robertson-Walker Universe. Indeed, an alternative approach to baryon asymmetry, which avoids altogether the Sakharov condition on out of equilibrium processes, is the one in which from the beginning, the Universe has a matter-antimatter asymmetry, as a result of *violations* of the CPT symmetry. Keeping locality and unitarity intact, violation of Lorentz symmetry seems the most plausible possibility for evading the CPT theorem, and in fact this lead to a sort of anti-CPT theorem by Greenberg [15], claiming that the assumption of Lorentz symmetry plays somehow a more fundamental rôle than the other assumptions of the CPT theorem, with the conclusion that CPT violation *implies* necessarily a violation of Lorentz invariance. However, in his proof, Greenberg did assume the existence of a well-defined scattering transfer matrix, and in this sense his anti-CPT theorem is equivalent to the original one on CPT symmetry. Explicit counter examples to these claims, where violations of CPT occur in a Lorentz invariant way, *e.g.* in some non-local theories, have been provided [17]. Moreover there are Lorentz-invariant models with intrinsic decoherence [18], and it is well known that the presence of decoherence may lead [19] in an ill-defined nature of the CPT operator in the effective theory, obtained after tracing over the appropriate environmental degrees of freedom. Such environmentally-induced CPT violation, due to intrinsic decoherence, may lead to interesting and unique effects in entangled states of mesons ( $\omega$ -effect [20]), which we shall review at the end of talk.

Nevertheless, for our purposes, as far as Baryon asymmetry is concerned, we shall restrict ourselves to *spontaneous violations* of Lorentz symmetry, which in turn induce CPT violation. Such violations can be provided by the background geometry, for instance within the context of string-inspired models [21, 22], by certain flux fields which are constant in a given frame (to be identified with the co-moving frame of the observer in an expanding Universe framework).

For early pioneering works on matter-antimatter asymmetry generated by CPT Violating backgrounds within the SME framework we refer the reader to [23]. In that work, it was argued that, under certain circumstances, certain CPT violating terms within an SME effective Lagrangian can produce large baryon asymmetry, at Grand Unified temperatures, which is eventually diluted to the current value by sphaleron processes within the Standard Model sector.

An alternative source of CPT Violating interactions that could lead to matter-antimatter asymmetry is the coupling of the baryon (or B-L) number (anomalous) current to scalar curvature  $\mathcal{R}$  of space-time through a CP violating interaction Lagrangian  $\mathcal{L}$ , that could occur, *e.g.* within some Supergravity theories [24, 25]:

$$\mathcal{L} = \frac{1}{M_*^2} \int d^4x \sqrt{-g} (\partial_\mu \mathcal{R}) J^\mu \quad (2)$$

<sup>1</sup> In this respect, we mention that an O(50) keV right-handed neutrino DM can play an important rôle in explaining galactic structures and resolving some of the tensions between  $\Lambda$ CDM model and observations at small (galactic) scales, especially if appropriate self-interactions among the keV-right-handed neutrinos are introduced [14]. The coincidence in the range of the allowed right-handed neutrino DM mass obtained in this approach, based on purely astrophysical reasons at galactic scales, with the one of the  $\nu$ MSM, induced by particle physics and DM cosmology reasons is intriguing.

where  $M_*$  is a cut-off in the effective field theory and  $J^\mu$  could be the current associated with  $B-L$  ( $L$  being the lepton number). There is an implicit choice of sign in front of the interaction (2), which has been fixed so as to ensure matter dominance. In this context, it has been shown that [24]

$$\frac{n_{B-L}}{s} = \frac{\dot{R}}{M_*^2 T_d}, \quad (3)$$

$T_d$  being the freeze-out temperature for  $B-L$  interactions. To leading order in  $M_*^{-2}$  we have  $R = 8\pi G(1-3w)\rho$  where  $\rho$  is the energy density of matter and the equation of state is  $p = w\rho$  where  $p$  is pressure. For radiation  $w = 1/3$  and so in the radiation dominated era of the Friedmann-Robertson-Walker cosmology  $R = 0$ . However  $w$  is precisely  $1/3$  when  $T_\mu^\mu = 0$ . In general  $T_\mu^\mu \propto \beta(g)F^{\mu\nu}F_{\mu\nu}$  where  $\beta(g)$  is the beta function of the running gauge coupling  $g$  in a  $SU(N_c)$  gauge theory with  $N_c$  colours. This allows  $w \neq 1/3$ . Further issues and progress in gravitationally induced baryogenesis can be found in [25]. Recently some potential problems with the coupling (2) have been pointed out in [26], regarding instabilities of theories with such interactions.

The structure of the talk is as follows: in the next section 2, we discuss a model for matter-antimatter asymmetry in a string-inspired Universe, which is due to a CPT violating space-time geometry with torsion at early epochs. In this model, the torsion decays to negligible values today, in agreement with current phenomenology, but a lepton asymmetry has been frozen at a given temperature where the Universe undergoes a phase transition. The baryon asymmetry is then generated in such scenarios by means of B-L preserving processes in the SM sector of the model. In section 3, we discuss a different scenario for CPT-induced baryon asymmetry, which is based on string Universe models with bulk D-brane defects. The interesting feature of this second class of models is that there is an intrinsic CPT violation in such cases, due to unobserved (by a low-energy observer) degrees of freedom associated with the recoil of the D-brane defects on the brane Universe during their interaction with string matter. In these models, CPT violation is primarily associated with an  $\omega$ -type effect [20], and there are different dispersion relations between particles and antiparticles as they propagate in the “medium” of D-brane defects [27], which can lead to matter-antimatter asymmetry under certain conditions to be discussed. The current Phenomenology of both types of CPT violation is then discussed briefly in section 4, where we also make a few remarks on possible (tiny) violations of the spin-statistics theorem in some models, especially the ones of section 3 entailing  $\omega$ -type CPT-violating effects. Indeed, as we explain, one of the essential underlying assumptions of the spin-statistics theorem is CPT and Lorentz invariance, which if violated or not well-defined may lead to violations, which though may not be describable within a local effective field theory framework. A brief description of current experimental searches for (and bounds of) such spin-statistics violations is also given. Finally, in section 5, we present our conclusions and outlook.

## 2. A string-inspired Model with CPT Violating Geometry in the Early Universe

The basic Lagrangian of this model is based on the order  $\alpha'$  (=Regge slope = string length unit) expansion of a low-energy string effective action, in the Einstein frame, compactified to four space-time dimensions [28]

$$L = (-g)^{1/2} \left\{ \frac{1}{16\pi G} \left\{ -R + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - \frac{1}{6} e^{-2\Phi} H_{\rho\mu\nu} H^{\rho\mu\nu} \right. \right. \\ \left. \left. + \sum_{J=1}^f \bar{\psi}_J \left( i \frac{1}{2} \gamma^\mu \overleftrightarrow{\nabla}_\mu - \frac{1}{4} e^{-\Phi} \gamma_\sigma \gamma^5 \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} \right) \psi_J + \dots \right\} \right\} \quad (4)$$

where  $R$  is the Ricci scalar for the metric  $g_{\mu\nu}$ ,  $H_{\rho\mu\nu} = \partial_{[\rho} B_{\mu\nu]}$ , with the brackets [...] denoting total antisymmetrization of the respective indices, is the field strength of the spin-one antisymmetric tensor (Kalb-Ramond) field  $B_{\mu\nu} = -B_{\nu\mu}$  which, together with the spin two traceless symmetric metric tensor  $g_{\mu\nu}$ , and the spin-zero (scalar) dilaton  $\Phi$  field, constitutes the massless gravitational multiplet of the (bosonic) string spectrum. Newton's gravitational constant  $G$  is related to the string scale  $M_s = 1/\sqrt{\alpha'}$  and the compactification volume  $V^{(c)}$  (in units of  $\sqrt{\alpha'}$ ) via  $8\pi G = \alpha' V^{(c)}$ . The fermions  $\psi_J$  indicate fermions that appear in extensions of the Standard Model of Particle Physics, with (massive) right handed Majorana neutrinos, and the ... denotes all relevant interactions, whose explicit form is of no concern here. The  $\nabla_\mu$  denotes gravitational covariant derivative, as appropriate for the curved-space-time formulation. The coupling of the Kalb-Ramond field strength to the axial currents  $J_\sigma^5 \equiv \bar{\psi}_J \gamma_\sigma \gamma^5 \psi_J$ ,  $J = 1, \dots, f$ , is dictated by the fact that the quantity  $e^{-\Phi} H_{\mu\nu\rho}$  plays the rôle of (totally antisymmetric) torsion, given that one can absorb the  $H^2$  terms in (4) into a generalised curvature scalar of a space-time with a torsionful spin connection [28]:  $\bar{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + e^{-\Phi} H_{\mu\nu}^\rho \neq \bar{\Gamma}_{\nu\mu}^\rho$ , with  $\Gamma_{\mu\nu}^\rho$  the ordinary (torsion-free) Christoffel symbol. In four space-time dimensions, one also has the following duality relation:

$$H_{\mu\nu\lambda} = e^{2\Phi} \epsilon_{\mu\nu\lambda}{}^\sigma \partial_\sigma b, \quad (5)$$

with  $b(x)$  a pseudiscalar (Kalb-Ramond "axion") field.

As discussed in [22], among the solutions of the equations of motion of (4), there are those corresponding to a metric tensor  $g_{\mu\nu}$  of a Robertson-Walker expanding universe, a constant dilaton field  $\Phi$ , which without loss of generality is taken to be zero, and some fermion condensates, such that the temporal component of the sum over fermion species of the axial fermion current term is a non-zero constant vacuum expectation value,  $\langle J^{50} \rangle \neq 0$ , and satisfies the following equation of motion for the dual Kalb-Ramond field  $b(x)$ :

$$\partial^\mu \left[ \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} \left( \partial^\sigma b - \frac{1}{4} \langle J^{5\sigma} \rangle + \dots \right) \right] = 0, \quad \langle J^{5\sigma} \rangle = c \delta^{0\sigma} \neq 0, \quad c = \text{constant}. \quad (6)$$

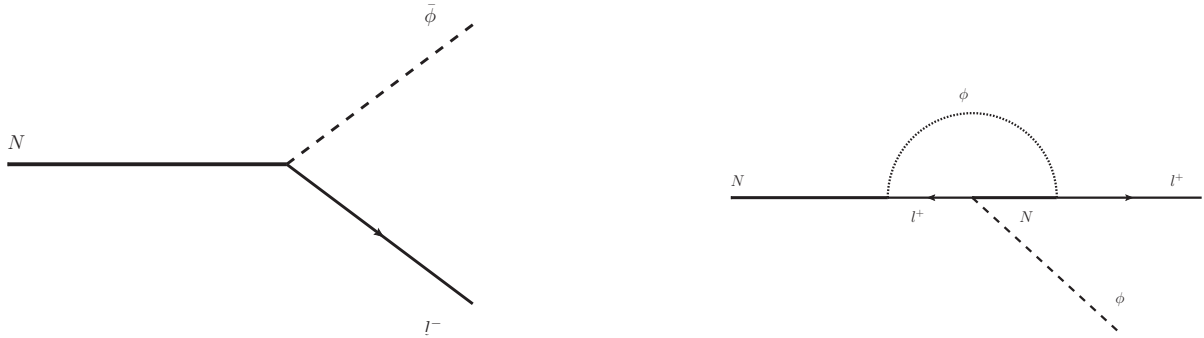
The cosmological solution of (6) is such that  $b(t)$  is a linear function of cosmic time:  $\dot{b} \equiv db/dt = c/4$ . This in turn implies a constant torsion background  $H_{ijk} = \text{constant}$ ,  $i, j, k = 1, 2, 3$  in the frame of a cosmic observer. Such solutions, violate Lorentz invariance, and by the "anti-CPT theorem" of [15] CPT would also be violated. Thus, it is natural to expect matter-antimatter asymmetry generation in such Universes [21, 22].

A few remarks are in order at this point. First, in extensions of the Standard Model with right-handed Majorana neutrinos we are considering here, the axial condensates  $\langle J^{50} \rangle \neq 0$  refer to all other fermion species except the right-handed Majorana neutrinos. Indeed, the condensate of the latter vanishes identically due to the properties of the Majorana spinors [22]. With this in mind, we concentrate from now on to the Heavy neutrino sector of the model (4), which will be responsible for leptogenesis as we shall explain.

The constant torsion background coupling with the axial fermions implies that we should study the following model of the right-handed heavy neutrinos [22]:

$$\mathcal{L} = i\bar{N}\not{\partial}N - \frac{m}{2}(\bar{N}^c N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \tilde{\phi} N + h.c., \quad (7)$$

where  $N$  is the heavy right-handed Majorana field, satisfying the Majorana condition  $N^c = N$ , with the superscript  $c$  denoting Dirac's charge conjugation, and  $L_k$  is the lepton SU(2) left-handed doublet field of the Standard Model, with  $k$  a generation index. The adjoint of the Higgs field is defined by the relation  $\tilde{\phi}_i = \epsilon_{ij} \phi_j$ . Due to our constant H-torsion situation, the axial vector background is  $B^\mu = \dot{b} \delta^{0\mu}$ . This is understood in what follows. For our purposes,



**Figure 1.** Tree- (left) and one-loop (right) decay amplitudes, corresponding to the Yukawa term in (7) that couples a right-handed neutrino to the Standard Model lepton sector. Analogous diagrams describe the decay in anti-leptons. Continuous undirected lines represent right-handed neutrinos, lines with an arrow are used to represent SM leptons, whilst dashed lines correspond to the Standard Model Higgs. The left diagrams are understood to be evaluated in the presence of an axial background field. The right diagram is the standard result of [8], leading to leptogenesis in conventional scenarios, due to CP violation in the lepton sector, in the case of more than one flavour of right-handed neutrinos.

we shall restrict ourselves [22] to *only one generation* of right-handed neutrinos <sup>2</sup>. Notice that the model has a portal of communication of the right-handed neutrino sector with the Standard Model sector via the Yukawa interactions with couplings  $Y_k$ . In this work we do not specify the mechanism for generating a mass  $m$  for the heavy right-handed neutrino. The mass  $M$  will be determined self consistently by the requirement of the model generating sufficient leptogenesis [22].

The important point to notice is that, in the presence of a non-trivial constant (in the Robertson-Walker frame) Lorentz and CPT breaking axial background,  $B^0 = \text{constant} \neq 0$  in (7), which in our case, as explained above, is geometrical in origin, there is a difference in the decay rates [22]

$$\begin{aligned} \Gamma_1(N \rightarrow \ell^- \phi^+) &\neq \Gamma_2(N \rightarrow \ell^+ \phi^-), \\ \Gamma_1 &= \sum_k \frac{|Y_k|^2 m^2 \Omega + B_0}{32\pi^2 \Omega \Omega - B_0}, \quad \Gamma_2 = \sum_k \frac{|Y_k|^2 m^2 \Omega - B_0}{32\pi^2 \Omega \Omega + B_0}, \\ \Omega &\equiv \sqrt{(B^0)^2 + m^2}, \end{aligned} \quad (8)$$

where, since we anticipate the leptogenesis to occur at sufficiently high temperatures, above the electroweak symmetry breaking, the (massless) charged Higgs fields are present in the physical spectrum. It should be stressed that this difference already occurs for a single flavour of right-handed neutrinos, *at tree level* in a quantum field theory framework and will lead to a lepton asymmetry. The situation has to be contrasted with the conventional leptogenesis scenarios [8, 9, 7], based on the seesaw mechanism for generation of light neutrino masses in the Standard Model sector, where the presence of more than two flavours of heavy right handed

<sup>2</sup> This will be sufficient for leptogenesis, but of course in such a case one should consider alternative ways [29] to seesaw mechanism [10] in order to give masses to the active light neutrinos of the Standard Model sector (that are parts of the doublet  $L_k$ ). In case one requires a seesaw type mass generation for neutrinos, then at least two right-handed neutrino generations are needed. In such a case, there are extra sources of CP violation, which lead to a generation of lepton asymmetry [8]. As we shall demonstrate in our model (7), due to the CPT and Lorentz violating background  $B^0$ , lepton asymmetry is generated even with a single generation of right-handed neutrinos.

neutrinos, necessitated by the seesaw mechanism, imply a CP violation, leading to a one-loop difference in the aforementioned decay rates (*cf.* fig. 1).

For weak backgrounds  $B^0 \ll T$ , where  $T$  is the temperature, the difference (8) has been estimated in [22] at the freezeout (decoupling) point,  $T = T_D$ , at which  $H \simeq \Gamma_1 + \Gamma_2 = \sum_k \frac{|Y_k|^2}{16\pi^2} \frac{\Omega^2 + B_0^2}{\Omega}$ . We assumed that standard radiation-era cosmology [30] is not affected much by the presence of the background  $B^0$ , thereby setting  $H = 1,66T_D^2 \mathcal{N}^{1/2} m_P^{-1}$ , where  $\mathcal{N}$  is the effective number of degrees of freedom of all elementary particles and  $m_P$  the Planck mass. From the last equation one can estimate the decoupling temperature  $T_D$  in terms  $\Omega$ ,  $|Y|$  and  $B_0$ .

$$T_D \simeq 6.2 \cdot 10^{-2} \frac{|Y|}{\mathcal{N}^{1/4}} \sqrt{\frac{m_P(\Omega^2 + B_0^2)}{\Omega}} \quad (9)$$

In order for the inverse decay to be suppressed by the Boltzmann factor, we have to impose [8] the further requirement that  $T_D \leq \Omega$  when  $\Gamma \simeq H$ , leading to  $z(\Omega^2 + B_0^2) \leq \Omega^3$ , where  $z = 3.8 \cdot 10^{-3} \frac{m_P |Y|^2}{\mathcal{N}^{1/2}}$ . Upon the requirement that the bound is satisfied for all values of  $B_0$ , one obtains

$$m^2 \geq 1.09 z^2. \quad (10)$$

In general, in our scenarios the Yukawa coupling  $Y$  is a free parameter. If we assume  $|Y| \approx 10^{-5}$ ,  $\mathcal{N} \approx 10^2$ , as in standard leptogenesis models, we get an order of magnitude estimate for the lower bound of the right-handed Majorana neutrino mass:

$$\bar{m} \approx 100 \text{ TeV} . \quad (11)$$

The estimate of [22] for the total lepton number difference between particles and antiparticles  $\Delta L$ , induced by (8), to leading order in the small quantity  $B_0 \ll T_D \simeq m$ , is <sup>3</sup>:

$$\Delta L^{TOT} = (2r - 1)n_N = \frac{2\Omega B^0}{\Omega^2 + (B^0)^2} n_N \simeq \frac{2B^0}{m} n_N, \quad (12)$$

The high-temperature expansion of  $n_N$  yields for small  $B_0 \ll T_D \simeq m$  [22]

$$n_N(T_D) = e^{-\beta \bar{m}} \left( \frac{\bar{m}}{2\pi\beta} \right)^{\frac{3}{2}} + \mathcal{O}(B_0^2) \Big|_{T \simeq T_D} \simeq 0.023 \bar{m}^3. \quad (13)$$

The lepton asymmetry  $\frac{\Delta L^{TOT}}{n_\gamma}$ , where  $n_\gamma$  is the photon density, is expected to be of the same order of magnitude of the baryon asymmetry (1), given that in scenarios of baryogenesis via leptogenesis we are adopting here, the asymmetry in the lepton sector is communicated to the baryon sector via Baryon (B) and Lepton (L) number violating, B-L conserving, sphaleron processes in the Standard Model sector [3]. An order of magnitude estimate of the ratio  $\frac{B_0}{\bar{m}}$

<sup>3</sup> The estimate was based on several simplifying assumptions, for instance the decaying right-handed neutrino is assumed initially at rest, with branching ratios given by  $r = \frac{\Gamma_1}{\Gamma}$  and  $1 - r$ . The decay of a single neutrino produces the lepton number asymmetry  $\Delta L = r - (1 - r) = 2r - 1 = \frac{2\Omega B_0}{\Omega^2 + B_0^2}$ . Multiplying this quantity by the initial abundance of right-handed Majorana neutrinos at the temperature  $T_D$  one gets a rough estimate of the lepton number density. The density of the Majorana neutrinos  $n_N = \sum_\lambda \frac{1}{(2\pi)^3} \int d^3p f(p, \lambda)$  can be estimated by using where  $\lambda$  is the helicity and  $f(p, \lambda)$  Fermi-Dirac distribution function, which at high temperatures, relevant for our case here, is well approximated by the Maxwell-Boltzmann function. Therefore we set  $f(p, \lambda) = e^{-\beta \sqrt{m^2 + (p + \lambda B_0)^2}}$ , with  $\beta = 1/T$  the inverse temperature. The estimate (12) is then obtained upon making an appropriate high temperature expansion, assuming that the right-handed neutrino density distribution follows closely the equilibrium distribution for  $T \geq T_D$  and drops rapidly to zero at lower temperatures  $T \leq T_D$ ; furthermore the density of the sterile neutrino (normalised to the entropy density) is well approximated by a step-function.

can be found making use of the approximation  $T_D \simeq m$  and retaining only first order terms in  $\frac{B_0}{\bar{m}}$ . Recalling that the photon number density is [30]  $n_\gamma \simeq \frac{2\zeta(3)}{\pi^2} T^3 \simeq 0.24 T^3$ , we observe that, in this scenario, phenomenologically relevant baryogenesis (1) induced by a similar-order of magnitude lepton asymmetry,  $\frac{\Delta L}{n_\gamma} \simeq 10^{-10}$ , is achieved provided the ratio of the background field to the mass of the sterile neutrino is of order:

$$\frac{B_0}{\bar{m}} \simeq 5 \cdot 10^{-8}, \quad \text{at } T = T_D \simeq \bar{m} \sim 100 \text{ TeV}. \quad (14)$$

The small value of this ratio also allows us to justify *a posteriori* neglecting higher powers of  $B_0$  in the formulae above. From the lower bound (10) of 100 TeV that has been previously found for the mass, for the case where  $Y = \mathcal{O}(10^{-5})$ , we get an approximation for the smallest possible magnitude of the background field required in order for this mechanism to be effective  $B_0 \simeq 1$  MeV. If other mechanisms contributed to the lepton asymmetry in the universe, or the Yukawa couplings assume smaller values, the minimum value of  $B_0$  would be smaller than the one given here.

It goes without saying, that the proper estimation of the right-handed neutrino abundance, and thus the lepton asymmetry, should of course be obtained by solving the appropriate coupled system of Boltzmann equations for the right-handed neutrino and lepton abundances [7, 8, 9, 30], which get modifications in the presence of  $B_0$  [22]. This is currently in progress. Nevertheless, the above qualitative estimate suffices for demonstrating the non-trivial role of the CPT violating background geometry in inducing a lepton asymmetry. We stress once more that the occurrence of leptogenesis here is due to decay processes at tree level, since the required  $CP$  violation is introduced by the Lorentz and CPT violating background field that enters in the external lines of the Feynman diagrams of fig. 1.

Having established a “torsionful”-geometry-induced matter-antimatter asymmetry, the next step is to discuss mechanisms for a significant reduction of the torsion background field  $B^0$  today, where there are very stringent upper bounds on the magnitude of such Lorentz and CPT violating axial backgrounds. We should also avoid any significant modification of the Big-Bang Nucleosynthesis (BBN) conditions [30], which requires the background  $B_0$  to be sufficiently weak already at BBN temperatures of order  $\mathcal{O}(1 \text{ MeV})$ . Indeed, since the H-field couples universally to all fermionic species in the Standard model sector, it will do so to electrons. In such a case, the form of the corresponding terms in the action (4) will resemble the Standard Model Extension lagrangian of [16], with the  $b_\mu$  coefficient corresponding to  $b_0 = B_0$ , for which there are very stringent upper bounds that we shall discuss later on in section 4. Such stringent limits, imply that one should have a proper mechanism in place, by means of which  $B^0$  today should be very suppressed.

In [22] we remarked that such a situation can be encountered if the (string) Universe undergoes, after decoupling, at temperatures  $T \lesssim T_D$ , a phase transition such that the axial condensate  $\langle J^{50} \rangle \rightarrow 0$ . In such a case, one observes from (6) that the Kalb-Ramond axion field  $b(x)$  no longer varies linearly with the cosmic time but diminishes with the scale factor as  $a^{-3}$ . If one assumes a cooling law for the Universe, of the form  $a \sim T^{-1}$ , then the  $B^0$  torsion field would scale with the temperature as  $T^3$  in this scenario:

$$B_0 = c_0 T^3, \quad c_0 > 0, \quad \text{for } T \lesssim T_D. \quad (15)$$

The parameter  $c_0$  is a phenomenological, and can be constrained by requiring that  $B^0$  today must be at most equal to the experimental upper bounds of the  $b_0$  (temporal) axial Lorentz and CPT violating coefficient of the Standard Model Extension [16].

We remark at this point that models, in which the critical temperature for the destruction of the axial fermion condensate (6) is of the same order of magnitude as the right-handed neutrino

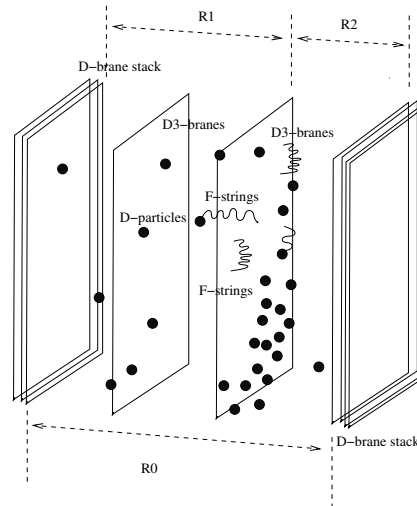
decoupling temperature  $T_D = 100$  TeV, are not constrained at all by the current constraints on the upper bound of the  $b_\mu$  coefficients of the Standard Model extension [16]. Indeed, taking into account the temperature of the Universe today (from the CMB measurements) is  $T_{\text{CMB}} = 2.725$  K = 0.2348 meV, we obtain from (15) and (14) [22]  $c_0 = 1 \text{ MeV}(100 \text{ TeV})^{-3} = 10^{-42} \text{ meV}^{-2}$ . implying a current value of  $B_0$  of order

$$B_{0\text{today}} = \mathcal{O}(10^{-44}) \text{ meV} , \quad (16)$$

way too small for any experimental detection. Moreover, in this case, the value of  $B^0$  at BBN temperature of a few MeV,  $B^0(T = T_{\text{BBN}}) \simeq 10^{-18} \text{ MeV}$ , is also very small, so there are negligible effects of the background on the formation of the material elements. However, as we have already mentioned, in general the temperature at which destruction of the condensate occurs, could be significantly lower than the decoupling temperature for the right handed neutrinos,  $T_D$ ; applying the experimental upper bounds on the magnitude of the background H-torsion field  $B^0$  today, then, we can constrain the parameter  $c_0$  in the cooling law (15). We shall come back to such an analysis in section 4.

In the next section 3, we discuss an alternative scenario for induction of a matter-antimatter asymmetry in the Universe [31], which may also provide a detailed string-theoretic mechanism for the destruction of the condensate (6), although the latter feature may not be required. The scenario pertains to brane universes, in which our world is viewed as a D(irichlet)-three-brane, with three large spatial dimensions, propagating in a bulk, which is populated by lower-dimensional D(irichlet) branes appropriately compactified, so that from the point of view of an observer on the three-brane they look effectively point-like defects.

### 3. “D-particle foam” Universe, Matter-Antimatter Asymmetry and Intrinsic ( $\omega$ -effect-type) CPT Violation



**Figure 2.** Schematic representation of a generic D-foam space-time model. In this picture, our brane world is represented as one of the D3-branes (Dirichlet branes with three large spatial dimensions) propagating in a higher-dimensional bulk space, which is punctured by inhomogeneous populations of D-particles. Open fundamental (F-)strings on the D3 branes interact dominantly with the D-particles only if they are electrically neutral.

D-foam models [32, 33] are stringy models of space-time foamy geometries, which involve a

number of parallel brane universes, with three large spatial dimensions, propagating in higher-dimensional bulk geometries (see fig. 2). The required number of parallel brane worlds is determined by the initial target-space supersymmetry [32], which eventually is broken (the motion of the brane worlds in the bulk space is a factor in target-space supersymmetry breaking). One of the three branes represents our observable Universe. The model of ref. [32], acts as a prototype of a D-foam Universe, involving two stacks of D8-branes (which can be eventually compactified to four dimensions), each stack being attached to an orientifold plane. Owing to their special reflective properties, the latter provide a natural compactification of the bulk dimension. The bulk is punctured by effectively point-like D-branes (D-particles), which are either truly point-like (D0-branes), allowed in the type IA string theory, or 3-branes compactified on appropriate 3-cycles, with small radii of order of the string length, which are allowed in (the phenomenologically more relevant) type IIB string theories. The presence of a D-brane is essential due to gauge flux conservation, since an isolated D-particle cannot exist.

For an observer living on our brane world the D-particles will appear as space-time defects. Some of these D-particles will be bounded on the moving brane, others will pass through, and from the point of view of a brane observer they will appear as “flashing on and off” defects (which justifies the name “D-foam”). Open (F-)strings live on the brane world, representing Standard Model (SM) matter and they can interact in a topologically non-trivial way with the D-particle defects in the foam, only if they do not carry electric flux (electrically neutral excitations). Indeed, open strings that interact with D-particles can satisfy Dirichlet boundary conditions on the world-sheet when attached to them. Closed and open strings may be “cut” by D-particles, a process that involves capture of the incident open string and creation of stretched strings between the (recoiling) D-particle and the brane world (string “splitting”), and subsequent re-emission of the open string. In many type IIA string theories the D-particles are stable zero-space-dimensional defects. However for our purposes we will consider them to be present in string theories of phenomenological interest since, even when elementary D-particles cannot exist consistently, as is the case of type IIB string models. In the latter case, there can be “effectively point-like D-particles” formed by the compactification of higher dimensional D-branes [36] (*e.g.* three-branes wrapped around three-cycles, with relatively small radii). D-particles are electrically neutral and thus electric charge would not have been conserved if such processes had taken place. Hence, the D-particle foam is transparent to charged excitations of the Standard Model (or its effects are strongly suppressed on charged particles compared to the neutral excitations in type IIB theories [36]), leaving neutral particles, in particular neutrinos, susceptible to the foam effects. Recoil of the D-particle during such interactions creates appropriate distortion in the space-time geometry, which depend on the momenta of the incident string states.

The D-particles bulk population density is constrained at various epochs of the universe in a different way, given that it is in general *inhomogeneous*. For instance, today, such D-foam Universe cosmologies [34] may be restricted by requiring consistency with observations and in particular with the dark-sector energy budget dictated by  $\Lambda$ CDM fiducial cosmologies [35]. At earlier epochs, the density of D-particles can be different, *e.g.* much denser during the inflationary era [35]. In our context we shall be concerned with the induced asymmetry between matter and antimatter during matter-D-foam interactions, and thus we shall also constrain the density of such D-particles by the amount of CPTV that we observe.

As discussed in detail in [34] the density of D-particles on the brane world is permitted to be relatively large, even at late eras of the universe, given the fact that bulk D-particles exert forces on the brane universe with mixed sign contributions to the brane vacuum energy, depending on the distance of the bulk D-particles from the brane [32]. Such forces are due to stretched strings between the defect and the brane. These energy contributions depend only on the transverse components of the relative velocities of the defect with respect to the brane

worlds. For our purposes in this work we may therefore consider that statistically significant populations of D-particles existed in the early eras of the brane universe. However, as the time elapses, the brane universe, which propagates in the higher-dimensional bulk (*cf.* fig. 2), enters regions characterised by D-particle depletion, in such a way that the late eras cosmology of the universe is not affected. Nevertheless, as we shall discuss below, the early D-particle populations may still have important effects in generating neutrino-antineutrino population differences (asymmetries), which are then communicated to the baryon sector via the standard sphaleron processes [3] or more generally B-L conserving processes in Grand Unified Theories.

To this end, we need to consider the *effective dispersion relation* of a (anti)neutrino field in a brane space-time punctured with statistically significant populations of D-particles <sup>4</sup>. The number density of (anti) neutrinos on the brane world is limited by the requirement that they do not overclose the universe. If neutrinos are assumed to have a chemical potential  $\mu$ , then standard cosmological neutrino models predict that the number densities of a single flavour of relativistic neutrinos and antineutrinos in thermal equilibrium at temperature  $T_\nu$  is estimated by [37]

$$n_{\nu\bar{\nu}} = T_\nu^3 \frac{3\zeta_3}{2\pi^2} \left( 1 + \frac{2 \ln 2 \mu_\nu^2}{3T_\nu^2 \zeta_3} + \frac{\mu_\nu^4}{72T_\nu^4 \zeta_3} + \mathcal{O}\left(\frac{\mu_\nu^6}{T_\nu^6}\right) \right) \quad (17)$$

upon making the standard assumption that  $\mu_\nu \ll T_\nu$  for all neutrino flavours. The quantity  $\xi_\nu \equiv \frac{\mu_\nu}{T_\nu}$  is called the degeneracy parameter and is invariant under cosmic expansion. If we assume that the electron-neutrino chemical potential is the only one with significant presence in the early universe, then BBN constraints imply  $-0.04 < \xi_{\nu_e} < 0.07$ . Thus, the order of magnitude of the neutrino plus antineutrino number density agrees with naive standard estimate  $n_{\nu\bar{\nu}} \sim \frac{3}{11} n_\gamma$ , where  $n_\gamma$  is the photon density. Today, where the temperature of the universe is the CMB temperature, corresponding to an energy of  $k_B T_0 \sim 2.35 \times 10^{-13}$  GeV, the density of neutrinos is found to be of order  $n_{\nu\bar{\nu}}^{(0)} \sim 112 \text{ cm}^{-3}$  and scales roughly with the cubic power of the temperature:  $n_{\nu\bar{\nu}} \sim n_{\nu\bar{\nu}}^{(0)} \left(\frac{T_\nu}{T_0}\right)^3$ . So, for the decoupling temperatures of neutrinos,  $k_B T_d \sim 10^{15} \sim \text{GeV}$ , where we are interested in this work, in order to compute the frozen CPT Violating neutrino-antineutrino population differences, one obtains a number density of neutrino plus antineutrino populations of order

$$n_{\nu\bar{\nu}}(T = T_d \sim 10^{15} \text{ GeV}) \sim 10^{85} \text{ cm}^{-3}. \quad (18)$$

On the other hand, as already mentioned, there are no similar restrictions on the population of the D-particle defects on the brane, in view of the negative contributions on the potential energy of the brane universe by bulk D-particle populations [34]. Thus, at the early universe, at the above neutrino-decoupling temperatures, we may even assume D-particle densities of *one* defect per *string* volume on the three brane world, without overclosing the universe. The assumption that the string length can take on values in the phenomenologically acceptable (post LHC era) range  $10^{-27} - 10^{-32}$  cm, corresponding to string mass scales from 10 TeV to  $10^{18}$  GeV, yields a D-particle number density in the range

$$n_D(T = T_d \sim 10^{15} \text{ GeV}) \sim 10^{54} - 10^{96} \text{ cm}^{-3} \quad (19)$$

respectively. Thus we observe that in order to be able to treat the D-particle populations as providing a more-or-less uniform “medium” over which neutrinos propagate, with non-trivial

<sup>4</sup> Here we are dealing with active neutrino species of the standard model. Heavy right handed Majorana neutrinos, as was the case of the previous section, are their own antiparticles, so their propagation in the D-foam, with which they interact as neutral under the standard model group, although non trivial does lead to this effect [22]. However they may have other effects, for instance can play a rôle in the destruction of the condensate (6) as we shall see below.

effective dispersion relations, we need to have at the decoupling temperature much higher densities of D-particles than those of neutrinos plus antineutrinos. Comparing (18) with (19), we observe that, if one assumes *one* D-particle per *three-dimensional string volume* on the brane, then this latter requirement excludes the low values of the string mass scale, implying an allowed range

$$10^{-5}M_P \leq M_s \leq 10^{-1}M_P , \quad (20)$$

with  $M_P \sim 10^{19}$  GeV the four-dimensional Planck mass. One of course could have much more dense D-particle gases in the early universe, which would allow for lower string scales.

We will now estimate the *modification of the dispersion relations* of neutrinos in such a “media” of D-particles in the early universe [31]. The interaction of a string with a D-particle implies that at least one of the ends of the string is attached to the D-particle defect. Furthermore, the simultaneous creation of virtual strings stretched between the defect and the brane, describes the recoil of the D-particle. During the interaction time, the D-particle undergoes motion characterized by non-trivial velocities,  $u_{\parallel} = \frac{g_s}{M_s} \Delta p_i = \frac{g_s}{M_s} r_i p_i$  along the brane longitudinal dimensions, where  $r_i$  denotes the proportion of the incident neutrino momentum that corresponds to the momentum transfer  $\Delta p_i$  during the scattering, and  $v_{\perp}$  in directions transverse to the brane world [32].

As discussed in [39, 38, 33] the non-trivial capture and splitting of the open string excitation corresponding to neutrino during its interaction with the D-particle, and the recoil of the latter, result in a *local* effective metric distortion of the form:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = (\eta_{\mu\nu} + h_{\mu\nu}) dx^{\mu} dx^{\nu} , \quad h_{0i} = (u_{i\parallel}^a \sigma_a) , \quad (21)$$

where  $u_{i\parallel}$  is the recoil velocity of the D-particle *on* the D-brane world, with  $i = 1, 2, 3$  a spatial space-time index,  $\sigma_a$  are the  $2 \times 2$  Pauli flavour matrices with  $a = 1, 2, 3$  (assuming two-flavour oscillations for simplicity). On average over a population of stochastically fluctuating D-particles including flavour changes, one may have the conditions (26), the second of which in the case of flavour oscillations can be generalised to

$$\ll u_{a,i\parallel}^{\parallel} u_{b,j\parallel}^{\parallel} \gg = \sigma^2 \delta_{ij} \delta_{ab} . \quad (22)$$

(We still assume that  $\ll u_{a,i\parallel}^{\parallel} \gg = 0$  . ) As a result of (22), on average, the flavour change during the interactions of neutrinos with the D-foam can be ignored. In such a case, any flavour structure in the metric (21) is ignored<sup>5</sup>.

On considering string theory scattering amplitudes we find that the four momentum is conserved in the scattering of D-particles and strings. D-particles in the bulk exert forces on the vacuum energy of the brane world of mixed sign, depending on their relative distance. Thus, during the scattering process of a neutrino field with a D-particle, the vacuum energy of

<sup>5</sup> Ignoring the flavour structure, the metric (21) can be written as

$$ds^2 = dt^2 + 2u_i dx^i dt - \delta_{ij} dx^i dx^j , \quad (23)$$

This metric was determined from world-sheet conformal field theory considerations [39] and represents a dragging of the frame by the Galilean (slowly moving) D-particle, which moves on a flat space-time background. As discussed in [31], upon a time coordinate change, this metric becomes the the so-called Gullstrand-Painlevé metric representing the geometry in the exterior of a Schwarzschild black hole, where the falling space into the black hole is represented as a Galilean “river” on a flat space-time in which “relativistic fishes” swim. The river represents the frame of the recoiling D-particle, while the fishes are the relativistic matter strings:

$$ds^2 = dt_{\text{ff}}^2 + 2u_i dx^i dt_{\text{ff}} - \delta_{ij} (dx^i - u^i dt_{\text{ff}}) (dx^j - u^j dt_{\text{ff}}) + \mathcal{O}(u^3) . \quad (24)$$

Here  $t_{\text{ff}}$  is the time of a free-floating observer who is at rest at infinity (compared to the centre of the black hole).

the brane fluctuates by an amount  $\Delta\mathcal{V}$  which depending on the process can be of either sign. From energy-momentum conservation, at each individual scattering event between a neutrino field and a recoiling D-particle, one could thus write:

$$p_{\text{before}}^{\vec{p}} + \vec{p}_{\text{after}} + \frac{M_s}{g_s} \vec{u}_{\parallel} = 0, \quad E_{\text{before}} = E_{\text{after}} + \frac{1}{2} \frac{M_s}{g_s} \vec{u}_{\parallel}^2 + \Delta\mathcal{V} \quad (25)$$

where  $(\vec{p}, E)_{\text{before (after)}}$  denote the incident (outgoing) neutrino momenta, energies respectively and we used the fact that the recoiling heavy D-particle of mass  $M_s/g_s$  (with  $M_s$  the string scale and  $g_s < 1$  the string coupling, assumed weak, so that string perturbation theory applies) has a non-relativistic kinetic energy  $\frac{1}{2} \frac{M_s}{g_s} \vec{u}_{\parallel}^2$ . We have also assumed that the fraction of the neutrino momentum transfer in the direction perpendicular to the brane world is negligible. The importance of the term  $\Delta\mathcal{V}$  not having a fixed sign in each individual scattering process is associated with the possibility of D-particle induced neutrino flavour oscillations [33].

Indeed, upon averaging  $\langle\langle \dots \rangle\rangle$  over a statistically significant number of events, due to multiple scatterings in a D-foam background, we may use the following stochastic hypothesis [33, 31]

$$\langle\langle u_{i\parallel} \rangle\rangle = 0, \quad \langle\langle u_{i\parallel} u_{j\parallel} \rangle\rangle = \sigma^2 \delta_{ij}. \quad (26)$$

implying that Lorentz invariance holds only as an average symmetry over large populations of D-particles in the foam. At a microscopic level, (26) translates to momentum conservation on average in (25), since  $\langle\langle \vec{u}_{\parallel} \rangle\rangle = 0$ . At an individual scattering process, if one represents the energy of the incident neutrino on-shell as  $\sqrt{\bar{p}^2 + m_1^2}$ , where  $\bar{p}$  is the amplitude of the conserved spatial momentum of the neutrino, and the outgoing one as  $\sqrt{\bar{p}^2 + m_2^2}$ , we observe that the energy-conservation equation (25) implies in general  $m_1 \neq m_2$ . Which one is larger depends on the signature of the term  $\frac{1}{2} \frac{M_s}{g_s} \vec{u}_{\parallel}^2 + \Delta\mathcal{V}$ , which as mentioned is not of fixed sign, thereby allowing for neutrino oscillations to take place. The situation is somewhat analogous to the standard Mossbauer effect, where the emitted or absorbed photon from a nucleus of an atom bound in a solid may sometimes be free of nuclear recoil, in contrast to the case of gases, thereby attributing the phenomena of nuclear resonances to such recoil-free fraction of nuclear events. In our case the role of the ‘‘nuclei bound in a lattice’’ is played by the D-particle lattice. In addition to the D-particle recoil energy during scattering with stringy matter, which would lead to energy losses for the neutrinos, there are vacuum energy fluctuations, as a consequence of the motion of bulk particles in the foam, thus the neutrino experiences losses and gains from the vacuum, which results in the induced flavour oscillations. The analogue of resonances in this case would correspond to the loss-and-gain-free fraction of events, in which the neutrino does not oscillate.

However, the effects of the D-foam go beyond the above-mentioned kinematical ones. On assuming isotropic momentum transfer,  $r_i = r$  for all  $i = 1, 2, 3$ . The dispersion relation of a neutrino of mass  $m$  propagating on such a deformed isotropic space-time, then, reads:

$$p^\mu p^\nu g_{\mu\nu} = p^\mu p^\nu (\eta_{\mu\nu} + h_{\mu\nu}) = -m^2 \Rightarrow E^2 - 2E\vec{p} \cdot \vec{u}_{\parallel} - \vec{p}^2 - m^2 = 0. \quad (27)$$

This on-shell condition implies that

$$E = \vec{u}_{\parallel} \cdot \vec{p} \pm \sqrt{(\vec{u} \cdot \vec{p})^2 + \vec{p}^2 + m^2}. \quad (28)$$

We take the average  $\langle\langle \dots \rangle\rangle$  over D-particle populations with the stochastic processes (22), (26). Hence we arrive at the following expression for an average neutrino energy in the D-foam background:

$$\begin{aligned} \langle\langle E \rangle\rangle &= \langle\langle \vec{p} \cdot \vec{u} \rangle\rangle \pm \langle\langle \sqrt{p^2 + m^2 + (\vec{p} \cdot \vec{u})^2} \rangle\rangle \\ &\simeq \pm \sqrt{p^2 + m^2} \left( 1 + \frac{1}{2} \sigma^2 \right), \quad p \gg m, \end{aligned} \quad (29)$$

for the active light neutrino species. The last relation in eq. (29) expresses the corrections due to the space-time distortion of the stochastic foam to the free neutrino propagation. It is this expression for the neutrino energies that should be used in the averaged energy-momentum conservation equation (25) that characterises a scattering event between a neutrino and a D-particle. On further making the assumption for the brane vacuum energy that  $\ll \Delta\mathcal{V} \gg = 0$ , the total combined effect on the energy-momentum dispersion relations, from both capture/splitting and metric distortion, can then be represented as:

$$\ll E_2 \gg = \pm \sqrt{p^2 + m^2} \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \quad (30)$$

Since antiparticles of spin 1/2 fermions can be viewed as “holes” with negative energies, we obtain from (25) and (29) the following dispersion relations between particles and antiparticles in this geometry (for Majorana neutrinos, the roles of particles /antiparticles are replaced by left/right handed fermions):

$$\begin{aligned} \ll E_\nu \gg &= \sqrt{p^2 + m_\nu^2} \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \\ \ll E_{\bar{\nu}} \gg &= \sqrt{p^2 + m_\nu^2} \left( 1 + \frac{1}{2} \sigma^2 \right) + \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \end{aligned} \quad (31)$$

where  $\bar{E} > 0$  represents the positive energy of a physical antiparticle. In our analysis above we have made the symmetric assumption that the recoil-velocities fluctuation strengths are the same between particle and antiparticle sectors<sup>6</sup>.

There can thus be *local* CPTV in the sense that the effective dispersion relation between neutrinos and antineutrinos are different. This is a consequence of the local violation of Lorentz symmetry (LV), as a result of the non-trivial recoil velocities of the D-particle, leading to the LV space-time distortions (21). This difference between neutrino and antineutrino phase-space distribution functions in D-foam backgrounds generates a matter-antimatter lepton asymmetry in the relevant densities

$$\ll n - \bar{n} \gg = g_{d.o.f.} \int \frac{d^3p}{(2\pi)^3} \ll [f(E) - f(\bar{E})] \gg, \quad (32)$$

where  $g_{d.o.f.}$  denotes the number of degrees of freedom of relativistic neutrinos, and  $\ll \dots \gg$  denotes an average over suitable populations of stochastically fluctuating D-particles (26).

As discussed in detail in [33], one may parameterise the momentum transfer by the fraction parameter of the incident momentum  $r$ , which is in turn assumed stochastic, that is

$$u_i = \frac{g_s}{M_s} \Delta p_i \rightarrow g_s r_i \frac{p_i}{M_s}, \text{ no sum over } i, \quad \ll r_i \gg = 0, \quad \ll r_i r_j \gg = \Delta^2 \delta_{ij}. \quad (33)$$

In this case, the dispersion relations (31) are modified by the replacement of

$$\sigma^2 \rightarrow \frac{g_s^2}{M_s^2} \Delta^2 p^2, \quad (34)$$

which is now momentum dependent:

$$\begin{aligned} \ll E_\nu \gg &= \sqrt{p^2 + m_\nu^2} \left( 1 + \frac{g_s^2}{2 M_s^2} \Delta^2 p^2 \right) - \frac{g_s}{2 M_s} \Delta^2 p^2 \\ \ll E_{\bar{\nu}} \gg &= \sqrt{p^2 + m_\nu^2} \left( 1 + \frac{g_s^2}{2 M_s^2} \Delta^2 p^2 \right) + \frac{g_s}{2 M_s} \Delta^2 p^2 \end{aligned} \quad (35)$$

<sup>6</sup> Scenarios for which this symmetry was not assumed have also been considered in an early work [33], and will lead to important phenomena regarding *intrinsic* CPT violation, which we shall discuss later.

In such a case, the lepton asymmetry can be calculated from the integral (32), for  $\Delta^2 < 1$ , but, in contrast to conventional point-like field theory models, where the upper limit of momentum integration can be extended to  $\infty$ , in D-foam models, due to (21), this is extended up to the value for which the D-particle recoil velocity approaches the value of the speed of light *in vacuo*,  $c=1$  in our units, *i.e.*  $p_{\max} \equiv |\vec{p}|_{\max} = \frac{M_s}{g_s \sqrt{\Delta^2}}$ , where  $r$  is the stochastic variable satisfying (33). The resulting integrals in (32) then become:

$$\Delta n_\nu = \frac{g_{d.o.f.}}{2\pi^2} T^3 \int_0^{\frac{M_s}{T g_s \sqrt{\Delta^2}}} d\tilde{u} \left( \frac{1}{1 + e^{\tilde{u} - \tilde{u}^2 \frac{g_s \Delta^2 T}{2M_s}}} - \frac{1}{1 + e^{\tilde{u} + \tilde{u}^2 \frac{g_s \Delta^2 T}{2M_s}}} \right) \simeq \frac{g_{d.o.f.}}{\pi^2} T^4 \frac{\Delta^2 g_s}{M_s} \quad (36)$$

The lepton asymmetry resulting from (36) freezes out at temperature  $T_d$  and is:

$$\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} = \frac{2\Delta^2 g_s T_d}{M_s}. \quad (37)$$

From (37), we observe that for a freeze-out temperature  $T_d \sim 10^{15}$  GeV, the phenomenological value  $\Delta L \sim 10^{-10}$  is attained for

$$\frac{M_s}{g_s} \sim 10^{25} \Delta^2 \text{ GeV}. \quad (38)$$

For  $\Delta^2 \sim 10^{-6}$  a Planck size D-particle mass  $M_s/g_s \sim 10^{19}$  GeV is required so that the D-foam provides the physically observed Lepton and, thus, Baryon Asymmetry. For the unnaturally small  $\Delta^2 < 10^{-21}$  one arrives at  $M_s/g_s \sim 10$  TeV. For  $\Delta^2 \sim \mathcal{O}(1)$  transplanckian D-particle masses are required. We should stress that the above conclusions were based on the assumption that the freeze-out temperature was the temperature at decoupling of neutrinos in standard big-bang cosmology.

Our approach to leptogenesis is distinguished from others in that a local effective field theoretical description is not adopted. Because of D-particle recoil when scattering off matter strings, the background of D-particles can be modelled as a stochastic medium, which goes beyond local field theory frameworks. The underlying string theoretic description provides the rigorous description of the scattering of D-particles. The D-particles backreact (as seen from infra-red divergences in perturbation theory) and change the metric which influences the space in which matter is moving.

The above described model has another important consequence, regarding the induction of an intrinsic CPT Violation ( $\omega$ -effect [20]) which affects entangled states of particles, as discussed in [33], which we review briefly below.

An example of an (unnormalised) initial entangled quantum state  $|i\rangle$  is given by

$$|\psi\rangle = |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} + \xi |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} + \xi' |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} \quad (39)$$

where  $|M_L(\vec{k})\rangle = |k, \uparrow\rangle$  in an actual situation may represent a neutral meson (Kaon  $K^0$  or  $B^0$  meson) and we have taken the momentum  $\vec{k}$  to have only a non-zero component  $k$  in the  $x$ -direction for brevity and concreteness; superscripts label the two separated detectors of the collinear meson pair,  $\xi$  and  $\xi'$  are complex constants and we have left the state  $|\psi\rangle$  unnormalised.

In the case of neutral mesons, which (ignoring other quantum numbers, such as strangeness or beauty) are treated as identical bosons, if the CPT operator was well defined as a quantum mechanical operator, one should have  $\xi = \xi' = 0$  [40]. On the other hand, in case of quantum gravity environments, entailing loss of information for a low-energy observer, the CPT operator may not be well-defined [19], which, in the case of neutral mesons, implies an initial entangled

state (39) with a non-trivial but *small* contamination  $\xi = \xi' = \omega \neq 0$ , with  $\omega = |\omega| e^{i\Omega}$  a complex parameter, so that the antiparticle state is *perturbatively defined*, but the Einstein-Podolsky-Rosen (EPR) correlations associated with the decay products of the initial entangled state (39) get modified. This is the  $\omega$ -effect [20], whose phenomenology will be discussed in the next section.

In our case of matter propagation in D-foam space-times, as a consequence of the effective loss of information for a low-energy observer, due to the recoil degrees of freedom of the stochastically fluctuating D-particles during their interaction with neutral meson matter, we shall face an  $\omega$ -like situation, with  $\omega \neq 0$ , which we shall estimate. To this end, we first remark that, as a consequence of the recoil metric (21), the evolution of this state is governed by a hamiltonian  $\widehat{H}$ : [31]

$$\widehat{H} = g^{01} (g^{00})^{-1} \widehat{k} - (g^{00})^{-1} \sqrt{(g^{01})^2 k^2 - g^{00} (g^{11} k^2 + m^2)} \quad (40)$$

which is the natural generalisation of the standard Klein Gordon hamiltonian in a one particle situation. Moreover  $\widehat{k} |\pm k, \uparrow\rangle = \pm k |k, \uparrow\rangle$  together with the corresponding relation for  $\downarrow$ .

The effect of space-time foam on the initial entangled state of two neutral mesons is conceptually difficult to isolate, given that the meson state is itself entangled with the bath. Nevertheless, in the context of our specific model, which is written as a stochastic hamiltonian, one can estimate the order of the associated  $\omega$ -effect by applying non-degenerate perturbation theory to the states  $|k, \uparrow\rangle^{(i)}$ ,  $|k, \downarrow\rangle^{(i)}$ ,  $i = 1, 2$ . Although it would be more rigorous to consider the corresponding density matrices, traced over the unobserved gravitational degrees of freedom, in order to obtain estimates it will suffice formally to work with single-meson state vectors). Owing to the form of the hamiltonian (40) the gravitationally perturbed states will still be momentum eigenstates. The dominant features of a possible  $\omega$ -effect can be seen from a term  $\widehat{H}_I$  in the single-particle interaction hamiltonian

$$\widehat{H}_I = -(r_1 \sigma_1 + r_2 \sigma_2) \widehat{k} \quad (41)$$

which is the leading order contribution in the small stochastic parameters  $r_i \ll 1$  that satisfy

$$\langle r_i \rangle = 0, \quad \langle r_i r_j \rangle = \Delta_i \delta_{ij}, \quad i, j = 1, 2. \quad (42)$$

In first order in perturbation theory the gravitational dressing of  $|k, \downarrow\rangle^{(i)}$  leads to a state:

$$|k^{(i)}, \downarrow\rangle_{QG}^{(i)} = |k^{(i)}, \downarrow\rangle^{(i)} + |k^{(i)}, \uparrow\rangle^{(i)} \alpha^{(i)} \quad (43)$$

where

$$\alpha^{(i)} = \frac{{}^{(i)}\langle \uparrow, k^{(i)} | \widehat{H}_I | k^{(i)}, \downarrow \rangle^{(i)}}{E_2 - E_1} \quad (44)$$

and correspondingly for  $|k^{(i)}, \uparrow\rangle^{(i)}$  the dressed state is obtained from (44) by exchanging  $|\downarrow\rangle \leftrightarrow |\uparrow\rangle$  and  $\alpha \rightarrow \beta$  where

$$\beta^{(i)} = \frac{{}^{(i)}\langle \downarrow, k^{(i)} | \widehat{H}_I | k^{(i)}, \uparrow \rangle^{(i)}}{E_1 - E_2} \quad (45)$$

Here the quantities  $E_i = (m_i^2 + k^2)^{1/2}$  denote the energy eigenvalues, and  $i = 1$  is associated with the up state and  $i = 2$  with the down state. With this in mind the totally antisymmetric ‘‘gravitationally-dressed’’ state can be expressed in terms of the unperturbed single-particle states as:

$$\begin{aligned}
& |k, \uparrow\rangle_{QG}^{(1)} | -k, \downarrow\rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} | -k, \uparrow\rangle_{QG}^{(2)} = \\
& |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} \\
& + |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) + |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\
& + \beta^{(1)} \alpha^{(2)} |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)}
\end{aligned}$$

It should be noted that for  $r_i \propto \delta_{i1}$  the generated  $\omega$ -like effect corresponds to the case  $\xi = \xi'$  in (39) since  $\alpha^{(i)} = -\beta^{(i)}$ , while the  $\omega$ -effect of [20], specific to neutral mesons, corresponds to  $r_i \propto \delta_{i2}$  (and the generation of  $\xi = -\xi'$ ) since  $\alpha^{(i)} = \beta^{(i)}$ . In the density matrix these cases can be distinguished by the off-diagonal terms.

These two cases are physically very different. In the case of  $\phi$ -factories, the former corresponds to non-definite strangeness in the initial state of the neutral Kaons (seen explicitly when written in terms of  $K_0 - \bar{K}_0$ ), and hence strangeness nonconservation in the initial decay of the  $\phi$ -meson, while the latter conserves this quantum number. We remind the reader that in a stochastic quantum-gravity situation, strangeness, or, in that matter, the appropriate quantum number in the case of other neutral mesons, is not necessarily conserved, and this is reflected in the above-described general parametrisation of the interaction Hamiltonian (41) in ‘‘flavour’’ space.

As we discussed in [31], where we refer the interested reader, the (decoherent) time evolution of these two cases causes the appearance of terms with the opposite effects, as far as the quantum numbers in question are concerned. Namely, the strangeness-conserving initial state leads to the appearance of CPT violating terms with a strangeness violating form, while an initially strangeness-violating combination generates, under evolution in the foam, a strangeness-conserving  $\omega$ -effect of the form proposed in [20].

We next remark that on averaging the density matrix over the random variables  $r_i$ , we observe that only terms of order  $|\omega|^2$  will survive, with the order of  $|\omega|^2$  being

$$|\omega|^2 = \mathcal{O} \left( \frac{1}{(E_1 - E_2)} (\langle \downarrow, k | H_I | k, \uparrow \rangle)^2 \right) = \mathcal{O} \left( \frac{\Delta_2 k^2}{(E_1 - E_2)^2} \right) \sim \frac{\Delta_2 k^2}{(m_1 - m_2)^2} \quad (46)$$

for the physically interesting case in which the momenta are of order of the rest energies (*i.e.* masses of the constituent states). The variance  $\Delta_2$  (and also  $\Delta_1$ ) is of the order of the square of the momentum transfer during the scattering of the single particle state off a space-time-foam defect [39] (*cf.* (34)), *i.e.*

$$\Delta_2 = g_s^2 \frac{\Delta^2 k^2}{M_s^2}, \quad (47)$$

where  $M_s/g_s$  the D-particle defect rest mass (with  $M_s$  the string mass scale, and  $g_s < 1$  the (perturbative) string coupling). The parameter  $\Delta^2$  is at present a phenomenological parameter, which is proportional to the probability of interaction of the string matter with the D-particles, and therefore the total cross section of such processes. It cannot be further determined due to the lack (at present) of a complete theory of (string/brane) quantum gravity. Thus, we arrive at the following estimate of the order of  $\omega$  in this model of foam [31]:

$$|\omega|^2 \sim g_s^2 \frac{\Delta^2 k^4}{M_s^2 (m_1 - m_2)^2} \quad (48)$$

We shall perform a phenomenological analysis of this phenomenon in the next section.

Before doing so, we would like to make a final remark on the baryogenesis via leptogenesis if we embed the models of the previous section 2, employing Majorana right handed massive neutrinos to initiate lepton asymmetry, in the current (D-foam) framework. Since Majorana

neutrinos are their own antiparticles, it is only the corresponding helicity states that would exhibit differences between matter and antimatter à la (35), which however are not associated with a lepton asymmetry, as they do not have a well-defined lepton number [22]. In such a case therefore, the thermal equilibrium matter-antimatter asymmetries (35) could only refer to light neutrino species. Such asymmetries could co-exist with the mechanism of leptogenesis of [22], described in the previous sector, which however we consider the dominant one. In such models, the D-particle bulk foam could provide a way for destruction of the condensate (6), at a given era of the Universe, which can be fine tuned so as the current epoch axial CPT- and Lorentz-violating background  $B^0$  is consistent with the stringent phenomenological bounds, to be examined below, and also with the BBN constraints.

#### 4. Current Phenomenology of CPT Violation, inclusive of Spin-Statistics Violation searches

In this section we shall discuss some aspects of the current era phenomenology of the types of CPT Violation (CPTV) motivated by early Universe matter-antimatter asymmetry generation in the previous two sections. Above we examined two basic types of CPTV, one (section 2) induced by Lorentz violating backgrounds, which can be studied within the framework of effective field theories, and in particular the Standard Model Extension (SME) [16] parametrization. The other (section 3) goes beyond local effective field theories, and is linked to situations of environmental decoherence in which the CPT operator is not well-defined, leading to  $\omega$ -type modifications of the Einstein-Podolsky-Rosen (EPR) correlators of entangled boson (specifically, meson) states in appropriate meson factories ( $\phi$ -factories [41] or  $B$ -factories [42]).

##### 4.1. CPT Violation within Standard Model Extension Framework

Let us commence our discussion with the phenomenology of CPTV within the SME framework [16]. There is a plethora of precision tests which are tabulated and regularly updated in ref. [43], where we refer the interested reader for details. For our purposes here we shall only concentrate on bounds of the axial background Lorentz and CPT Violating coefficient  $b_\mu$  in the notation of [16, 43], and in particular its temporal component, which coincides with our axial background vector  $B_0$  in (7). The first few terms in the fermion sector of the SME read [16, 43]:

$$\begin{aligned}\mathcal{L}_{SME} &\ni \frac{1}{2}i\bar{\psi}\Gamma^\nu\partial_\nu\psi - \bar{\psi}\mathcal{M}\psi, \\ \mathcal{M} &\equiv m + a_\mu\gamma^\mu + b_\mu\gamma^5\gamma^\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}, \quad \sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu], \\ \Gamma^\nu &\equiv \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + if^\nu\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}\sigma_{\lambda\mu}, \quad .\end{aligned}\tag{49}$$

For our purposes we note that the terms proportional to  $a_\mu$  and  $b_\mu$  violate both Lorentz and CPT symmetries, unlike the  $c_{\mu\nu}$ ,  $d_{\mu\nu}$  and  $H_{\mu\nu}$  terms that violate only Lorentz symmetries. Analogous observations can be made for the various terms inside the  $\Gamma^\mu$  structure of (49). The main assumption behind the form of such operators is that an unknown physics at high energy scales could lead to a spontaneous breaking of Lorentz invariance by giving an expectation value to certain tensorial fields, which are not in the Standard Model (SM) spectrum. The interaction of these fields with operators composed from the SM fields, which are fully Lorentz-symmetric before the spontaneous breaking, will manifest itself as effective LV terms, which below the scale of the LV condensation would have the schematic form:

$$O_{\mu\nu\dots}^{\text{SM}} C^{\mu\nu\dots} \rightarrow O^{\text{SM}} \langle C^{\mu\nu\dots} \rangle, \tag{50}$$

where  $C^{\mu\nu\dots}$  is an external field that undergoes condensation and  $O^{\text{SM}}$  is a SM field operator that transforms properly under the Lorentz group. The classification of [44] requires that the independent dimension-5 operators must be gauge invariant, Lorentz invariant after contraction with the background tensors  $\langle C^{\mu_1\mu_2\dots} \rangle$ , not reducible to total derivatives or to lower-dimension operators by the use of equations of motion, and they should couple to an irreducible background tensor. Several experiments, of diverse origin, can be used in order to impose stringent constraints on the relevant SME coefficients, that range from searches for forbidden atomic transitions in precision experiments and studies of low-energy antiprotonic atoms and antimatter factories, to high-energy cosmic rays, nuclear spin precession and atomic and nuclear Electric Dipole Moments (EDM) measurements, as well as data on neutrino oscillations. In the following we shall discuss some phenomenological consequences of some of the above coefficients, especially in the context of neutral mesons, forbidden transitions in (anti)hydrogen molecules or atomic dipole moment measurements. Then we shall scale the appropriate bounds on  $b_\mu$  back in time, in the context of our CPTV stringy Universe of section 2, in order to see whether (some of) our leptogenesis scenarios discussed above can be falsified.

We commence with SME tests in antiprotonic atoms [45], in particular antihydrogen ( $\bar{\text{H}}$ ) of interest in this conference. Motivated by the theoretical microscopic models of section 2, I shall restrict myself to constraining the  $b_\mu$  coefficients of the SME (49) using spectroscopy, in particular looking for forbidden transitions, *e.g.*  $1s \rightarrow 2s$ . Within H spectroscopic measurements, the presence of a  $b_\mu$  coefficient in the SME (49) leads to the relevant transition of the electron in the H atom. The sensitivity of the tests depend crucially whether the atoms are free or trapped in an external magnetic field. In the case of free H (and  $\bar{\text{H}}$ ), the frequency shift of the  $1s$ - $2s$  transition is a higher-loop quantum effect in the SME/Quantum-Electrodynamics (QED) lagrangian, and thus the effect is suppressed by the square of the fine structure constant,  $\alpha^2$ :  $\delta_{1s-2s}\nu^{\text{H}} \simeq -\alpha^2 b_3^e/8\pi$ , *i.e.* the pertinent sensitivity of such experiments would be about five orders of magnitude smaller compared to tests involving the corresponding transitions in trapped H and  $\bar{\text{H}}$ . However, in the latter tests, the corresponding frequency shifts are proportional to the difference  $b_3^e - b_3^p$  of the third spatial component of  $b_\mu$  between electrons (e) and protons(p) (in a frame where the direction of the external magnetic field is along the  $z$  axis). In view of the universal character of  $B^\mu$  vectors due to background space-time geometries discussed in section 2, for this model the above difference would vanish. To cover ourselves against such cases, it is therefore imperative to either measure the sum of the coefficients  $b_\mu^{e,p}$ , or isolate them experimentally. The former can be achieved by examining hyperfine structure transitions in atomic (anti)matter. Indeed, within  $1s$  transitions of H or  $\bar{\text{H}}$ , one can determine the relevant energy shifts induced by  $b_\mu$  [45]:

$$\Delta_{a \rightarrow b}^{\text{H}} \simeq (b_\mu^e + b_\mu^p)/\pi + \dots \quad (51)$$

where the  $\dots$  denote contributions from the rest of the SME coefficients (49), which are not written explicitly here. Hyperfine transitions within the  $1S$  level of H can be measured with accuracies exceeding 1 mHz in masers. So transitions of this type in trapped H and  $\bar{\text{H}}$  are interesting candidates for performing tests of Lorentz or CPT symmetry, although to achieve resolutions of 1 mHz in trapped antihydrogen does not seem feasible in the foreseeable future.

Another possibility would be to measure [45] radio-frequency transitions between states within the triplet of hyperfine levels in H and  $\bar{\text{H}}$ , in particular the so called  $d\rangle_1 \rightarrow |c\rangle_1$  transition at external magnetic fields of order  $B \simeq 0.65$  Tesla. The corresponding frequency shifts depend solely on  $b_3^p$ :

$$\Delta_{c \rightarrow d}^{\text{H}} \simeq -b_3^p/\pi, \quad \Delta_{c \rightarrow d}^{\bar{\text{H}}} \simeq +b_3^p/\pi \quad (52)$$

where we took into account that under the action of CPT operation, which exchanges H and  $\bar{\text{H}}$ , the coefficient of the  $b_3^p$  changes sign. Thus, comparison of the above spectroscopic measurement

between trapped H and  $\bar{\text{H}}$  would yield immediately a bound (or a value !) on  $b_3^p$ . If a frequency resolution of 1 mHz could be attained (which at present is far from being plausible), then, one could obtain  $|b_3^p| \leq 10^{-27}$  GeV. Still such bounds are about four orders of magnitude smaller than the ones coming from masers. We also note that, although, clock-comparison experiments are able to resolve spectral lines to about 1  $\mu\text{Hz}$ , nevertheless, isolating  $b_i^p$  is very complicated due to the complex structure of the nuclei involved.

The above experiments are sensitive only to spatial components of Lorentz-violating couplings. Sensitivity to timelike couplings,  $b^0$ , would require appropriate boosts. On the other hand, in the context of the model (7) of section 2, such experiments can bound the combinations  $\gamma \vec{v}_3 B^0$ , where  $\vec{v}$  is the current-era relative velocity of us (as local observers) with respect to the CMB (or Friedmann-Robertson-Walker) frame. Currently, we can quote the following bounds on  $b_\mu$  coefficients for electrons [43]:

$$b^0 \leq 0.02 \text{ eV} , \quad |\vec{b}| \leq 10^{-21} \text{ eV} . \quad (53)$$

It should be mentioned that the stringent limits on  $|\vec{b}|$  have been obtained in measurements using torsion pendulum containing macroscopic numbers of polarised electrons [46]. New interactions, such as the above-mentioned Lorentz- and CPT-violating ones are then searched for (and bounded) by looking at the corresponding effects on the electron spin. Such tests may also be performed in man-made antihydrogen [47] or other anti-atoms, with the aim of providing direct comparison of CPT properties and thus tests of CPT invariance.

Next we describe the situation governing the constraints on the relevant dimension-5 terms of the SME lagrangian coming from EDM. These are generically found to be of order [44]  $\leq 10^{-25}$  e cm. The overall expression for the total EDM, due to the CP Violating conventional QED terms and the CPT Violating terms due to the presence of an appropriate Lorentz-violating background vector  $n^\mu$ , is obtained from the effective Lagrangian

$$\mathcal{L}_{\text{EDM}} = -i \frac{1}{2} d_{\text{CP}} \bar{\psi} \sigma^{\mu\nu} F_{\mu\nu}(A) \psi + d_{\text{CPT}} \bar{\psi} \gamma^\mu \gamma^5 F_{\mu\nu}(A) n^\nu \psi , \quad (54)$$

where  $F_{\mu\nu}$  is the Maxwell field strength. The currently null result on the neutron dipole moment imposes the constraint  $d_{\text{CP}} + d_{\text{CPT}} = 0$ . The lagrangian (54) should be completed with the  $a^\mu$  and  $b^\mu$  SME terms (49), as well as the appropriate dimension-5 operators from the QED sector of the SME [44]:

$$\mathcal{L}_5 = \sum_{\text{fermion species}} \left[ c^\mu \bar{\psi} \gamma^\lambda F_{\lambda\mu} \psi + d^\mu \bar{\psi} \gamma^\lambda \gamma^5 F_{\lambda\mu} \psi + g^\mu \bar{\psi} \gamma^\lambda \tilde{F}_{\lambda\mu} \psi + f^\mu \bar{\psi} \gamma^\lambda \gamma^5 \tilde{F}_{\lambda\mu} \psi \right] , \quad (55)$$

where  $\tilde{F}_{\mu\nu}$  is the dual of the Maxwell tensor. The various terms in (55) have different transformation properties under the action of the discrete symmetries C, P and T, which, together with the corresponding terms of (49), are indicated in figure fig. 3, on the assumption that the vector backgrounds are time-like and invariant under C,P and T reflections [44].

Experimentally [48], one can disentangle CP-odd from CPT-odd operators, because of different suppression scales. Specifically, the former require helicity flip and are thus represented by dimension-six operators in the SME effective lagrangian, with suppression by the CP breaking scale of order  $1/\Lambda_{\text{CP}}^2$ . Such operators imply spin precession in a magnetic field relative to the direction of  $\mathbf{B} \times \mathbf{v}$ . On the other hand, the CPT-odd operators are of dimension 5, as they do not require helicity flip, *e.g.* in the quark sector such operators are of the form  $\bar{q}_{R(L)} \gamma^\nu \gamma^5 F_{\nu\mu} q_{R(L)}$ , and  $\bar{q}_L \gamma^\nu \gamma^5 F_{\nu\mu}^a \tau^a q_L$ , where  $\tau^a$ ,  $a = 1, 2, 3$  are the SU(2) generators of the weak interaction standard model group, and  $F_{\mu\nu}$  and  $F_{\mu\nu}^a$  are the U(1) and SU(2) gauge field strengths respectively. These operators are suppressed linearly by the CPT-breaking scale,  $1/\Lambda_{\text{CPT}}$ .

Coefficient	Operator	$C$	$P$	$T$
$a^0$	$\bar{\psi}\gamma_0\psi$	-	+	+
$b^0$	$\bar{\psi}\gamma_0\gamma_5\psi$	+	-	+
$c^0$	$F_{\lambda 0}\bar{\psi}\gamma^\lambda\psi$	+	+	-
$d^0$	$F_{\lambda 0}\bar{\psi}\gamma^\lambda\gamma^5\psi$	-	-	-
$f^0$	$\tilde{F}_{\lambda 0}\bar{\psi}\gamma^\lambda\gamma^5\psi$	-	+	+
$g^0$	$\tilde{\tilde{F}}_{\lambda 0}\bar{\psi}\gamma^\lambda\psi$	+	-	+

**Figure 3.** Transformation properties of the various terms in (55) under the action of the discrete symmetries C, P and T. A + (-) indicates an even (odd) function. From ref. [44].

EDMs have been bounded with high precision in several occasions [48]:

- (i) neutrons, with the bound  $d_n < 3 \times 10^{-26}$  e cm ,
- (ii) diamagnetic atoms (such as Hg, Xe, ...) : their EDMs are induced by the EDMs of the valence nucleons; for the case of mercury EDM, one has the (approximate) relation:  $d_{\text{Hg}} \simeq -5 \times 10^{-4}(d_n + 0.1 d_p) \sim -5 \times 10^{-4} d_n$ . The last approximate relation implies that a signal consistent with CPT violation would occur, if a non zero  $d_n, d_{\text{Hg}}$  were found.
- (iii) paramagnetic atoms (such as Tl, Cs, ...): their EDM are extremely suppressed as a result of the absence of a CPT-odd electron EDM.

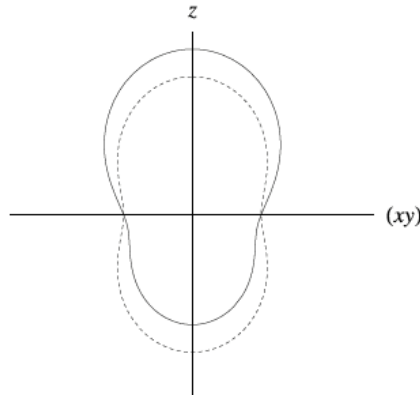
In general, theoretical estimates of dimension-three operators induced by multiloop CP violating corrections in the standard model, imply the following bounds of the SME coefficients in (49) [48]

$$a^\mu, b^\mu \sim d^\mu \left( 10^{-20} - 10^{-18} \right) \text{GeV}^2, \quad (56)$$

providing sensitivity to  $d^\mu \leq 10^{-12} \text{GeV}^{-1}$  and thus  $\Lambda_{\text{CPT}} \sim (10^{11} - 10^{12}) \text{GeV}$ , if one takes into account the current bounds on  $b_\mu$  [43] (*cf.* (53) below).

Higher Lorentz-violating background tensors, *e.g.* terms in SME effective lagrangian of the form  $\mathcal{D}_{\mu\nu\rho}\bar{e}\gamma^\rho\gamma^5 e F_{\mu\nu}$  can also be bounded experimentally with high accuracy, by looking [48] for corrections to the spin precession frequency of the form  $(\mathcal{D}^{i[0k]} + \mathcal{D}^{k[0i]}) E_i B_k$ , which changes sign under the reversal of the electric field  $E_i$ . The relative signal changes during the day as a result of the change of the Laboratory orientation relative to the tensor background.

We close this section by mentioning the interesting suggestion of ref. [49] on further tests of CPT symmetry due to the CPT-odd axial vector background  $b^\mu$ , which has been of interest to us in section 2. According to this work, within the framework of Lorentz-violating extended electrodynamics, the Dirac equation for a bound electron in an external electromagnetic field has been considered, assuming the interaction with the background field  $b_\mu$ . A Foldy-Wouthysen quasi-relativistic ( $1/c$ )-series expansion (truncated to order  $1/c^2$ ) has been applied to obtain an effective Hamiltonian for the hydrogen atom and through this the relativistic Dirac eigenstates in a spherically-symmetric potential to second order in  $b^0$ . The  $b^0$ -induced CPT-odd corrections



**Figure 4.** Angular distribution for spontaneous radiation for the atomic transition  $2p_{1/2,1/2} \rightarrow 1s_{1/2,-1/2}$  in the presence of a CPT-odd SME axial background vector  $b^\mu$ . The dashed line indicates the standard electrodynamics  $b^\mu = 0$  case. From ref. [49].

to the electromagnetic dipole moment operators of a bound electron have been calculated. Such corrections contribute to the anapole moment of the atomic orbital and may cause a specific asymmetry of the angular distribution of the radiation of a hydrogen atom, in particular the  $2p_{1/2,1/2} \rightarrow 1s_{1/2,-1/2}$  (*cf.* fig. 4). The non-observation currently of such asymmetries leads to bounds of the magnitude of  $|b^0|$ :  $|b^0| \leq 2 \times 10^{-8} m_e c^2 \simeq 10^{-11}$  GeV, which are consistent with the general bounds (53) for the SME coefficient  $b_\mu$  for electrons [43].

Finally we mention that, further tests of CPT invariance can be made by direct measurements of particle antiparticle mass and charge differences, which we are not going to discuss here. However, in the spirit of our cosmological model discussed in section 2, we do mention that, if the observed matter/antimatter asymmetry were due to a mass difference between particle and antiparticles, then, one may make the reasonable assumption that baryogenesis could be due to mass differences between quarks and antiquarks [50]. The latter may depend linearly with temperature,  $m_q(T) \sim gT$ , as a consequence of known high-temperature properties of Quantum Chromodynamics (QCD). Furthermore, it is reasonable (although not strictly necessary) to assume that the quark-antiquark differences today are bound by the current bound on proton-antiproton mass difference, which is of order  $7 \times 10^{-10}$  GeV, as provided in 2011 by the ASACUSA Collaboration [47]. Scaling back in temperature such differences, up to the respective decoupling temperature of the quarks, lead to baryon asymmetries that are much smaller than the observed one [50].

In this sense the model of [21] can still survive, given that, even if a  $B^0 < 0.02$  eV is observed today, according to the current SME limits, the Universe may have undergone such a (or series of) phase transition at  $T \sim 10^9$  GeV towards a smaller (or zero) H-torsion background. This is an (crude) example of how one can use current SME bounds to fit early universe cosmologies. In a similar spirit, the model of leptogenesis through CPT and CP violating decays of heavy right-handed neutrinos, discussed in section 2, based on the Lagrangian (7), with the assumption that the temperature of the destruction of the condensate (6) is of the same order as the decoupling temperature of right-handed neutrinos,  $T_D \sim 100$  TeV (14), lies comfortably within the limits (53), given the scaling (15) of the background  $B^0 = b^0$  and its current value (16). Reversing the logic, we can assume the upper bound of  $B^0$  today (53), scale back in time with the scaling (15) up to BBN temperatures,  $T_{\text{BBN}} \sim \mathcal{O}(1)$  MeV, and then constrain the coefficient  $c_0$  by the requirement that the BBN conditions are not disturb. The one can continue scaling back in the cosmic time, to check at which temperature range sufficient leptogenesis is produced, if at all.

Such procedures require of course detailed models of baryogenesis via leptogenesis, which we reserve for future studies.

#### 4.2. Intrinsic CPT Violation in Quantum Decoherence Models - the $\omega$ -effect

We now come to examine the current phenomenology of the  $\omega$ -effect [20], which is associated with a second type of CPTV, already discussed in the context of the D-foam model in section 3, in which the quantum CPT operator is not well-defined. We stress again that this latter type of CPTV goes beyond local effective field theories, and its most sensitive bounds can be placed in experimental facilities involving entangled states of neutral mesons, such as neutral Kaon( $\Phi$ ) factories [41] or  $B - \bar{B}$  meson factories [42].

In case of loss of information for a low energy observer, carried by degrees of freedom (d.o.f.) inaccessible to him/her due to quantum gravity environments (*e.g.* the gravitational recoiled d.o.f. in the D-foam example of section 3), the quantum operator that generates CPT symmetry may not be well defined [19]. The proof is obtained by *reductio ad absurdum*, that is by first assuming the existence of a well-defined *unitary* and *invertible* CPT operator acting on density matrices (antiunitary if acting on state vectors):  $\Theta$ , such that (a bar above an operator denotes a quantity pertaining to anti-matter states, obtained via the action of the CPT transformation):

$$\rho_{\text{out}} = \$ \bar{\rho}_{\text{in}} \quad (57)$$

where the subscript “in” and “out” denotes asymptotic states (from now on we ignore the time arguments for brevity), and the density matrix is defined as  $\rho = \text{tr}|\psi\rangle\langle\psi|$ , where the trace operation “tr” is over quantum states inaccessible to a low energy observer.

. Using the “superscattering matrix”  $\$$  (which is a linear operator acting on density matrices without an inverse, due to the existence of information loss in the problem), we may write:

$$\rho_{\text{out}} = \$ \rho_{\text{in}} \Rightarrow \Theta \rho_{\text{in}} = \$ \Theta^{-1} \bar{\rho}_{\text{out}} \Rightarrow \bar{\rho}_{\text{in}} = \Theta^{-1} \$ \Theta^{-1} \bar{\Theta}_{\text{out}} . \quad (58)$$

However, since  $\bar{\rho}_{\text{out}} = \$ \bar{\rho}_{\text{in}}$ , the last relation on the right-hand-side of (58) implies

$$\bar{\rho}_{\text{in}} = \Theta^{-1} \$ \Theta^{-1} \$ \bar{\rho}_{\text{in}} . \quad (59)$$

But this is *impossible*, as it would imply that the superscattering operator  $\$$  has an inverse  $\Theta^{-1} \$ \Theta^{-1}$ , that contradicts the initial assumption of information loss<sup>7</sup>. This is the so called *strong form of (intrinsic) CPT violation*, which would imply a *microscopic time arrow*.

Nevertheless in nature there could be [19] a *weak form* of CPT invariance, according to which the microscopic time arrow does not show in any scattering experiments. Indeed, in such a case the experimentalist would be able to prepare initial pure quantum mechanical state vectors, and there should a well defined transition probability  $\mathcal{P}$  from the initial pure state  $|\psi\rangle$  to the final state  $|\phi\rangle$ , such that

$$\mathcal{P}(\psi \rightarrow \phi) = \mathcal{P}(\theta^{-1} \phi \rightarrow \theta \psi) \quad (60)$$

where the (antiunitary) CPT operator  $\theta$  acts on “in” and “out” Hilbert spaces  $\mathcal{H}$  vectors now,  $\theta : \mathcal{H}_{\text{in}} \rightarrow \mathcal{H}_{\text{out}}$ , and is such that

$$\Theta \rho = \theta \rho \theta^\dagger , \quad \theta^\dagger = -\theta^{-1} , \quad (61)$$

<sup>7</sup> In a theory without information loss, of course,  $\$$  would factorise to the Heisenberg scattering matrix  $S$  as  $\$ = S S^\dagger$ , and would have a well-defined inverse,  $\$^{-1} = \$^\dagger$ , in which case the CPT operator would be well defined satisfying  $\Theta = \$ \Theta^{-1} \$$ .

which in terms of the  $\$$  matrix can be written as

$$\$\dagger = \Theta^{-1} \$ \Theta^{-1} . \quad (62)$$

Whether there exists such a situation of weak form of CPT invariance is in general an experimental question. The  $\omega$ -effect [20] is one way to answer this question experimentally, and, as we have seen in 3, it characterises the D-foam example. We next proceed to the phenomenology of this effect in entangled states of mesons, which, if observed, would constitute a “smoking-gun” evidence for such an intrinsic CPT violation.

We commence our discussion by briefly mentioning direct tests of Time reversal invariance within the Lorentz invariant standard model theory, using entangled neutral mesons, independently of CP and CPT violation. These have been initially proposed in [52], leading to the recent observation of *direct* T violation by the Ba-Bar collaboration [42], through the exchange of initial and final states in transitions that can only be connected by a T -symmetry transformation. For example, the transition  $\bar{B}^0 \rightarrow B^-$  for the second B to decay, at time  $t_2$ , once the first B (entangled with the second) has been tagged at time  $t_1$ , is identified by reconstructing events in the time-ordered final states  $(\ell^+ X, J/\psi K_s^0)$ . The rate of this transition is then compared to that of the  $B^- \rightarrow \bar{B}^0$  transition, that exchanges initial and final states, which is identified by the reconstruction of the final states  $(J/\psi K_L^0, \ell^- X)$ . Any observed difference between these two rates, would thus indicate direct observation of T violation, independent of CP properties [53]. This would also imply an independent test of CPT symmetry within the standard Model. Similar tests of T violation in entangled Kaon  $\Phi$  factories have also been suggested [54], by identifying the appropriate reactions that exchange initial and final states.

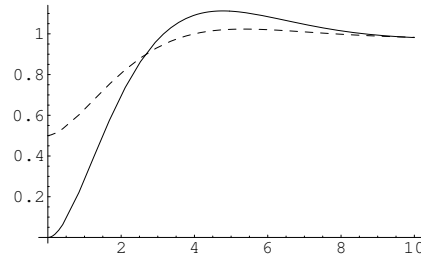
However, if CPT is *intrinsically* violated, in the sense of being not well defined due to decoherence [19] induced by quantum gravity [56], the above-mentioned direct observation of T violation cannot constitute a test of decoherence-induced CPT breaking. This is because in such a case a distinct phenomenon, associated with the ill-defined nature of CPT operator, emerges, termed  $\omega$ -effect [20]. If the  $\omega$  effect were present, such direct T-violation tests using entangled states of B-mesons [55] would allow the experimenter to disentangle it from conventional CPT violating effects in the Hamiltonian, within the SME framework, and also to measure independently  $\text{Im}\omega$  and  $\text{Re}\omega$ . We shall comment briefly on this later in the section.

For the moment, let us concentrate first to the neutral Kaon system, where the effects as we shall see are dominant, although conceptually our analysis applies equally [57] to entangled B-meson factories as well, such as those of [42]. In a quantum-gravity induced decohered situation, the Neutral mesons  $K^0$  and  $\bar{K}^0$  should *no longer* be treated as *identical particles*. As a consequence [20], the initial entangled state in  $\Phi$  factories  $|i\rangle$ , after the  $\Phi$ -meson decay, assumes the form:

$$|i\rangle = \mathcal{N} \left[ \left( |K_S(\vec{k}), K_L(-\vec{k})\rangle - |K_L(\vec{k}), K_S(-\vec{k})\rangle \right) + \omega \left( |K_S(\vec{k}), K_S(-\vec{k})\rangle - |K_L(\vec{k}), K_L(-\vec{k})\rangle \right) \right] , \quad (63)$$

where  $\omega = |\omega|e^{i\Omega}$  is a complex parameter, parametrizing the intrinsic CPTV modifications of the EPR correlations [20]. The  $\omega$ -parameter controls the amount of contamination of the final C(odd) state by the “wrong” (C(even)) symmetry state. The appropriate observable (*c.f.* fig. 5) is the “intensity”  $I(\Delta t) = \int_{\Delta t \equiv |t_1 - t_2|}^{\infty} |A(X, Y)|^2$ , with  $A(X, Y)$  the appropriate  $\Phi$  decay amplitude [20], where one of the Kaon products decays to the final state  $X$  at  $t_1$  and the other to the final state  $Y$  at time  $t_2$  (with  $t = 0$  the moment of the  $\Phi$  decay).

It must be noticed that in Kaon factories there is a particularly good channel, the one with bi-pion states  $\pi^+\pi^-$  as final decay products, which *enhances the sensitivity* to the  $\omega$ -effect by three orders of magnitude. This is due to the fact that the relevant terms [20] in the intensity



**Figure 5.** A characteristic case of the intensity  $I(\Delta t)$  (vertical axis) as a function of  $\Delta t$  (horizontal axis), with  $|\omega| = 0$  (solid line) vs  $I(\Delta t)$  (dashed line) with  $|\omega| = |\eta_{+-}|$ ,  $\Omega = \phi_{+-} - 0.16\pi$ , for definiteness [20].

$I(\Delta t)$  (c.f. fig. 5) contain the combination  $\omega/|\eta_{+-}|$ , where  $\eta_{+-}$  is the relevant CP-violating amplitude for the  $\pi^+\pi^-$  states, which is of order  $10^{-3}$ . The KLOE experiment bounds of the  $\omega$  parameter are [51]:

$$\text{Re}(\omega) = (-1.6_{-2.1}^{+3.0} \text{stat} \pm 0.4_{\text{syst}}) \times 10^{-4}, \quad \text{Im}(\omega) = (-1.7_{-3.0}^{+3.3} \text{stat} \pm 1.2_{\text{syst}}) \times 10^{-4}. \quad (64)$$

At least an order of magnitude improvement is expected for upgraded facilities such as KLOE-2 at (the upgraded) DAΦNE-2 [51].

This sensitivity is not far from certain optimistic models of space time foam leading to  $\omega$ -like effects [31]. Indeed, let us comparing these bounds to the D-foam case (48). First, we recall that successful leptogenesis from this class of models requires heavy D-particles masses (20). Assuming for definiteness D-particle masses of order of the Planck mass,  $M_s/g_s \sim M_P$ , we recall that the variance  $\Delta^2 \sim O(10^{-6})$  (38) in order to have phenomenologically acceptable leptogenesis. In the case of neutral kaons, with momenta of the order of the rest energies ( $\sim 1$  GeV) and mass differences  $m_1 - m_2 \simeq 10^{-15}$  GeV, we observe from (48) that  $|\omega| \sim 10^{-4}|\Delta|$  (whilst for B-mesons we have  $|\omega| \sim 10^{-6}|\Delta|$ ). For  $1 > \Delta \geq 10^{-3}$  these values for  $\omega$  are not far below the sensitivity of current facilities, such as KLOE-2 at DAΦNE, and thus such leptogenesis CPTV models may be constrained experimentally in the foreseeable future.

In B-factories one can look for similar  $\omega$ -like effects. Although in this case there is no particularly good channel to lead to enhancement of the sensitivity, as in the  $\Phi$ -factories, nevertheless one gains in statistics, and hence interesting limits may also be obtained [57]. The presence of a quantum-gravity induced  $\omega$ -effect in B systems is associated with a theoretical limitation on flavour tagging, namely the fact that in the absence of such effects the knowledge that one of the two-mesons in a meson factory decays at a given time through a flavour-specific channel determines unambiguously the flavour of the other meson at the same time. This is not true if intrinsic CPT Violation is present. One of the relevant observables [57] is given by the CP-violating semi-leptonic decay charge asymmetry (in equal-sign dilepton channel), with the first decay  $B \rightarrow X\ell^\pm$  being time-separated from the second decay  $B \rightarrow X'\ell^\pm$  by an interval  $\Delta t$ . In the absence of  $\omega$ -effects, the intensity at equal decay times vanishes,  $I_{\text{sl}}(\ell^\pm, \ell^\pm, \Delta t = 0) = 0$ , whilst in the presence of a complex  $\omega = |\omega|e^{i\Omega}$ ,  $I_{\text{sl}}(\ell^\pm, \ell^\pm, \Delta t = 0) \sim |\omega|^2$ . In such a case, the asymmetry observable exhibits a peak, whose position depends on  $|\omega|$ , while the shape of the curve itself depends on the phase  $\Omega$  [57]. The analysis of [57], using the above charge asymmetry method and comparing with currently available experimental data, leads to the following bounds:

$$-0.0084 \leq \text{Re}(\omega) \leq 0.0100, \quad \text{at 95\% C.L.} \quad (65)$$

Such tests for intrinsic CPT violation may be performed simultaneously with the above-mentioned observations of direct T violation, as they are completely independent. Quite recently,

we have embarked [55] on a detailed study of  $\omega$ -effects in  $B_d$ -system, using the experimental procedure suggested in [52], and implemented in [42], for tests of T violation in entangled meson systems independent of CP violation. We have identified how to probe the complex  $\omega$  parameter in the entangled  $B_d$ -system using Flavour(f)-CP(g) eigenstate decay channels: the connection between the Intensities for the two time-ordered decays (f, g) and (g, f) is lost in the presence of a non-zero  $\omega$ . Appropriate observables have been constructed allowing independent experimental determinations of  $\text{Re}(\omega)$  and  $\text{Im}(\omega)$ , disentangled from CPT violation in the evolution Hamiltonian,  $\text{Re}(\theta)$  and  $\text{Im}(\theta)$ , which parametrise CPTV within the SME local effective field theory frameworks. The general analysis of [55] has found

$$\begin{aligned} \text{Im}(\omega) &= \pm(6.40 \pm 2.80) \times 10^{-2}, & \text{Re}(\omega) &= (1.09 \pm 1.60) \times 10^{-2}, \\ \text{Im}(\theta) &= \pm(6.11 \pm 3.45) \times 10^{-2}, & \text{Re}(\theta) &= (0.99 \pm 1.98) \times 10^{-2}, \end{aligned} \quad (66)$$

where the  $2.4 \sigma$  deviations from  $\text{Im}(\omega) = 0$  and  $\text{Re}(\theta) = 0$  are interpreted as upper bounds. These  $2\sigma$  tensions have been shown to be uncorrelated [55].

At this juncture, I would like to point out that an observation of the  $\omega$ -effect in both the  $\Phi$  and B-factories could also provide an independent test of Lorentz symmetry properties of the intrinsic CPT Violation, namely whether the effect respects Lorentz symmetry. This is because, although the  $\Phi$  particle in neutral Kaon factories is produced at rest, the corresponding  $\Upsilon$  state in B-factories is boosted, and hence there is a frame change between the two experiments. If the quantum gravity  $\omega$ -effect is Lorentz violating, as it may happen in certain models [31], then a difference in the value of  $\omega$  between the two experiments should be expected.

Finally, since Lorentz Violation has been mentioned, I also point out that bounds of the LV SME coefficients  $a^\mu$  (*cf.* eq. (49)) can be placed by measurements in the entangled Kaon  $\Phi$  factories [51]. In particular by adopting the relevant SME terms to the quark sector, relevant for Kaon physics, one can bound differences  $\Delta a^\mu = a_{q_1}^\mu - a_{q_2}^\mu$ , where  $q_i$ ,  $i = 1, 2$  denote appropriate quark states. The current experimental limits for the coefficients  $\Delta a^\mu$  are: from the KTeV Collaboration  $\Delta a_X, \Delta a_Z < 9.2 \times 10^{-22}$  GeV, while from the the KLOE Collaboration in the DAΦNE  $\Phi$  factory [51] are less competitive but with the advantage that entangled meson factories have sensitivity to all four coefficients  $\Delta a^\mu$ , in particular:  $\Delta a^0 = (0.4 \pm 1.8) \times 10^{-17}$  GeV, from KLOE, with expected sensitivity at KLOE-2 in upgraded DAΦNE facilities for  $\Delta a^{X,Y,Z} = \mathcal{O}(10^{-18})$  GeV. Unfortunately, entangled meson factories have only sensitivity to differences  $\Delta a^\mu$  rather than absolute coefficients  $a^\mu$ . Of course, if gravity acts universally for all quark species, such differences may be zero.

#### 4.3. Spin Statistics & Ill-defined CPT Violation - Searches for Pauli Principle Violations

Before closing the talk I would like to make some remarks on another potential effect of quantum-gravity-decoherence-induced CPTV, namely tiny violations of the Spin-Statistics theorem [1] and in particular Pauli exclusion principle and related searches [58]. As with the CPT theorem, Spin-Statistics theorem is based on several underlying assumptions, which if relaxed, could lead to violations. One of the most dramatic consequences of such violations would be the evasion of Pauli principle, and recently there are dedicated experimental searches for it by the VIP, VIP2 collaborations [58], looking for spontaneous x-ray emissions of atoms (*e.g.* forbidden transitions  $2p \rightarrow 1s$  in Copper atoms, which test the PEP for electrons).

The *Spin-Statistics Theorem* states that the wave function of a system of identical integer-spin particles has the same value when the positions of any two particles are swapped. Particles with wave functions symmetric under exchange are called bosons. The wave function of a system of identical half-integer spin particles changes sign when two particles are swapped. Particles with wave functions antisymmetric under exchange are called fermions. The theorem was first

proposed by Fierz in 1939, by Pauli in a more systematic formulation in 1940, and in a rigorous mathematical formalism, using quantum field theory path integrals, by Schwinger in 1950, where the underlying mathematical assumptions were made clear. It is this latter proof we shall follow in this talk, in order to discuss possible violations.

An important consequence of the spin-statistics theorem is that the wavefunction of two identical fermions is zero, hence two identical fermions (i.e. with all quantum numbers the same) cannot occupy the same state, which is the celebrated *Pauli exclusion Principle* (PEP), which was postulated by Pauli in 1925, without knowledge of the spin-statistics theorem at the time.

Another important consequence of the theorem is that in quantum field theory, Bosons obey commutation relations, whilst fermions obey anticommutation ones. Schwinger's proof of the Spin-Statistics theorem, using quantum field theory machinery, requires the following assumptions:

- (1) The theory has a Lorentz and CPT invariant Lagrangian and relativistic causality.
- (2) The vacuum is Lorentz-invariant (can be weakened).
- (3) The particle is a localized excitation. Microscopically, it is not attached to a string or domain wall.
- (4) The particle is propagating (has a non-infinite mass).
- (5) The particle is a real excitation, meaning that states containing this particle have a positive-definite norm and has positive energy.

*Proof:*

Consider generic quantum relativistic fields  $\phi(x)$ , where  $x$  denotes a generic space-time point. The object of interest is the two-point correlator

$$\mathcal{G}(x) = \langle 0 | \phi(-x) \phi(0) | 0 \rangle \quad (67)$$

Let us denote the matrix corresponding to the rotation by  $\pi$  of the spin polarization of the field by  $\mathcal{R}(\pi)$ . Following Schwinger, we take the following steps towards the proof of the Spin-Statistics theorem:

**STEP I :** Formulate the quantum field theory at hand in Euclidean space time where the path integral makes rigorous sense. In this case spatial Lorentz transformations are ordinary rotations, but Boosts become also rotations in imaginary time, and hence a rotation by  $\pi$  in the plane (x(space) - t(time)) plane in Euclidean space-time is a CPT transformation in the language of Minkowski spacetime.

CPT transformation, if well defined, takes states in a path integral into their conjugates so, CPT invariance of the theory implies that the two-point correlator:

$$\langle 0 | \mathcal{R} \phi(x) \phi(-x) | 0 \rangle \quad (68)$$

must be positive-definite at  $x=0$  according to the positive-norm-state assumption (5) of the spin-statistics theorem. Propagating states, i.e. finite mass states, implies that this correlator is non-zero at space-like separations. Special relativity is needed to define space-like intervals of course, hence the Lorentz invariance assumptions (1) + (2) of the theorem.

**STEP II:** Lorentz Invariance allows fields to be transformed according to their spin, such that:

$$\langle 0 | \mathcal{R} \mathcal{R} \phi(x) \mathcal{R} \phi(-x) | 0 \rangle = \pm \langle 0 | \phi(-x) \mathcal{R} \phi(x) | 0 \rangle, \quad (69)$$

where  $+(-)$  is for bosons/integer spin (fermions/half-integer spin).

**STEP III :** Using CPT invariance (which is equivalent to also assuming a well-defined CPT operator and which, as we have already mentioned, in Euclidean space-time is equivalent to

rotational invariance) we may equate the rotated correlation function (69) to  $\mathcal{G}(x)$  (67), thus obtaining:

$$\langle 0 | \left( \mathcal{R} \phi(x) \phi(y) - \phi(y) \mathcal{R} \phi(x) \right) | 0 \rangle = 0, \quad (70)$$

for integer spins, and

$$\langle 0 | \left( \mathcal{R} \phi(x) \phi(y) + \phi(y) \mathcal{R} \phi(x) \right) | 0 \rangle = 0, \quad (71)$$

for half-integer spins.

This completes the proof of the Spin-Statistics theorem. The theorem essentially implies that, since the operators are space-like separated, a different order can only create states that differ by a phase. The argument fixes the phase to be  $\pm 1$  or  $\pm i$  according to the spin. Since it is possible to rotate the space-like separated polarizations independently by local perturbations; the phase should not depend on the polarization in appropriately chosen field coordinates.

We should remark at this point that when a violation of the spin-statistics theorem appears, it is one or more of the above assumptions that they are violated. For instance, spinless anticommuting fields, which could exist in condensed matter models are not relativistic invariant; ghost fields in gauge theories are spinless fermions but they have negative norm. In 2+1 dimensional Chern-Simons theory has anyons (fractional spin) but in such a case the wave function of the planar system splits between the bulk and the boundary, and hence is somehow delocalised. One remark concerns quarks: Despite being attached to a confining string, QCD quarks can have a spin-statistics relation proven at short distances (ultraviolet limit) due to asymptotic freedom.

We have seen above that the rôle of CPT and Lorentz invariance is crucial for the validity of the spin-statistics theorem. Although spontaneous violation of Lorentz and CPT symmetry may be tolerated (see assumption (2), which may be relaxed), nevertheless if the CPT is ill defined, and there is a strong (intrinsic) form of CPT violation, as is the case of quantum-gravity induced decoherence (or D-foam situations, as in section 3), then there may be tiny violations of the spin-statistics theorem. In this respect, we recall that it was the Bose-Statistics requirement of the neutral mesons that resulted in the antisymmetric initial state (63) when  $\omega = 0$ . In D-foam situation, for instance, the matter excitation is dressed by open strings stretched between the defect and the brane world, and in this sense, assumption (3) of the theorem is violated, together with assumption (1), due to the ill-defined nature of the CPT operator. In such cases, there are hidden d.o.f. in a particle state, and thus an evasion, in case of fermions, of the PEP may be understood by the fact that the otherwise looking identical quantum states entering the PEP formulation, actually differ by hidden quantum numbers. Thus we believe that searches for intrinsic CPT violation could also be complemented by searches of PEP. The current searches of PEP [58] are mainly for charged particles (electrons), for which, as we have discussed previously, the effects of the D-particle foam are expected to be strongly suppressed for type IIB theories. Nevertheless, this is only one model and experiment should be independent of any theoretical assumptions, hence searches of PEP are equally encouraged as searches of intrinsic CPT violation.

## 5. Conclusions and Outlook

In the talk I discussed theoretical motivations for CPT Violation, both within the SME local effective field theory framework and outside of it, as a result of ill-defined CPT generators in theories of quantum gravity entailing decoherence of quantum matter. We have been concerned with generation mechanisms of matter-antimatter asymmetry in the Universe by means of either early geometries with CPT Violation, e.g. the string-inspired Kalb-Ramond torsion models discussed in section 2, or space-time foamy models of brane worlds propagating in bulk

geometries punctured with D-particle (effectively) point-like defects, leading to a decoherence-induced  $\omega$ -effect in entangled states of mesons (section 3).

By using detailed models of the early Universe with CPTV, and knowing the scaling law of the relevant CPTV quantities entering the induced matter-antimatter asymmetry with the temperature of the Universe, I have used the current stringy bounds of such parameters to constrain the models. Specifically, I argued that the presence of Lorentz and CPT Violating geometries in the early universe, rather than quantum gravity, may be responsible for the emergence of  $b^\mu$ -like axial vector SME backgrounds. Such vectors have been argued to be responsible for the observed matter/antimatter asymmetry in the Universe. From this perspective, having small remnants of such vector backgrounds today is a not so unrealistic possibility, given that the Universe may have undergone a phase transition at a certain temperature during an early era. Hence, it makes perfect sense to search for or bound such SME coefficients by precision atomic spectroscopy or other methods, such as EDMs, including comparison of the relevant properties of matter with antimatter, especially now that we have available man-made antimatter.

In the talk I also reviewed some of such tests, with direct interest to this conference, including a brief discussion on direct observations of T violation in entangled particle states, independently of CP properties. In addition I discussed a novel phenomenon that may characterise certain quantum gravity models, namely “intrinsic CPT violation” as a result of the fact that, due to the associated decoherence of matter propagating in a quantum space-time foam environment, the CPT operator is perturbatively ill-defined: although the anti particle exist, nevertheless the properties of the CPT operator when acting on entangled states of particles lead to modified EPR correlators. Such modifications imply a set of well-defined observables, which can be measured in current or upcoming facilities, such as  $\Phi$  or B-factories.

The signatures of quantum-gravity induced decoherence in entangled states of mesons are rather unique, and in this sense they constitute “smoking-gun” evidence for this type of CPT Violation, if realised in Nature. The other important advantage of such searches is that they are virtually cost free, in the sense that the relevant tests can be performed in facilities that have already been or are to be built for other purposes at no extra cost, apart from minor modifications/adjustments in the relevant Monte-Carlo programmes to take proper account of these quantum-gravity effects. As a related topic to cases of CPTV decoherence, where the CPT operator is not well defined, I also discussed searches for violations of the Pauli equivalence principle, and more generally the spin statistics theorem.

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## References

- [1] R. F. Streater and A. S. Wightman 2000, *PCT, spin and statistics, and all that* (Princeton, USA: Princeton Univ. Press) and references therein.
- [2] A. D. Sakharov 1967, *Pisma Zh. Eksp. Teor. Fiz.* **5**, 32 [*JETP Lett.* **5**, 24 (1967)] [*Sov. Phys. Usp.* **34**, 392 (1991)] [*Usp. Fiz. Nauk* **161**, 61 (1991)].
- [3] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov 1985, *Phys. Lett. B* **155**, 36; M. B. Gavela, P. Hernandez, J. Orloff and O. Pene 1994, *Mod. Phys. Lett. A* **9**, 795 doi:10.1142/S0217732394000629 [hep-ph/9312215]; M. B. Gavela, P. Hernandez, J. Orloff, O. Pene and C. 1994, *Nucl. Phys. B* **430**, 382 doi:10.1016/0550-3213(94)00410-2 [hep-ph/9406289].
- [4] For a review see: W. Buchmuller, R. D. Peccei and T. Yanagida 2005, *Ann. Rev. Nucl. Part. Sci.* **55**, 311 [hep-ph/0502169], and references therein.
- [5] D. N. Spergel *et al.* [WMAP Collaboration] 2003, *Astrophys. J. Suppl.* **148**, 175 [astro-ph/0302209].

- P. A. R. Ade *et al.* [Planck Collaboration] 2014, *Astron. Astrophys.* **571**, A16 [arXiv:1303.5076 [astro-ph.CO]].
- [6] R. N. Mohapatra and P. B. Pal 1998 [2004], *World Sci. Lect. Notes Phys.* **60**, 1 [World Sci. Lect. Notes Phys. **72**, 1 (2004)]. M. Trodden 1999, *Rev. Mod. Phys.* **71**, 1463 [hep-ph/9803479]; W. Buchmuller, arXiv:0710.5857 [hep-ph].
- [7] M. C. Chen 2007, *TASI 2006 Lectures on Leptogenesis*, hep-ph/0703087 [HEP-PH]. A. Pilaftsis 2009, 2013, *J. Phys. Conf. Ser.* **171**, 012017 [arXiv:0904.1182 [hep-ph]]; *J. Phys. Conf. Ser.* **447**, 012007.
- [8] M. Fukugita and T. Yanagida 1986, *Phys. Lett. B* **174**, 45.
- [9] M. A. Luty 1992, *Phys. Rev. D* **45**, 455.
- [10] P. Minkowski 1977, *Phys. Lett. B* **67**, 421; M. Gell-Mann, P. Ramond and R. Slansky 1979, in *Supergravity*, eds. D.Z. Freedman and P. van Nieuwenhuizen (North-Holland, Amsterdam); T. Yanagida 1979, in *Proc. of the Workshop on the Unified Theory and the Baryon Number in the Universe*, Tsukuba, Japan, eds. O. Sawada and A. Sugamoto; R. N. Mohapatra and G. Senjanovic 1980, *Phys. Rev. Lett.* **44**, 912; J. Schechter and J. W. F. Valle 1980, *Phys. Rev. D* **22**, 2227; G. Lazarides, Q. Shafi and C. Wetterich 1981, *Nucl. Phys. B* **181**, 287; doi:10.1016/0550-3213(81)90354-0
- [11] M. C. Gonzalez-Garcia and M. Maltoni 2008, *Phys. Rept.* **460**, 1 (2008) [arXiv:0704.1800 [hep-ph]].
- [12] T. Asaka, S. Blanchet and M. Shaposhnikov 2005, *Phys. Lett. B* **631**, 151; doi:10.1016/j.physletb.2005.09.070 [hep-ph/0503065]. For cosmological/astrophysical applications of this model see: A. Boyarsky, O. Ruchayskiy and M. Shaposhnikov 2009, *Ann. Rev. Nucl. Part. Sci.* **59**, 191 doi:10.1146/annurev.nucl.010909.083654 [arXiv:0901.0011 [hep-ph]]; M. Shaposhnikov, *Prog. Theor. Phys.* **122**, 185 (2009). doi:10.1143/PTP.122.185
- [13] L. Canetti, M. Drewes, T. Frossard and M. Shaposhnikov, *Phys. Rev. D* **87**, 093006 (2013) doi:10.1103/PhysRevD.87.093006 [arXiv:1208.4607 [hep-ph]].
- [14] C. R. Argüelles, N. E. Mavromatos, J. A. Rueda and R. Ruffini 2016, *JCAP* **1604**, no. 04, 038 (2016) doi:10.1088/1475-7516/2016/04/038 [arXiv:1502.00136 [astro-ph.GA]]; N. E. Mavromatos, C. R. Argüelles, R. Ruffini and J. A. Rueda 2016, *Int. J. Mod. Phys. D* **26**, no. 03, 1730007; doi:10.1142/S0218271817300075, and references therein.
- [15] O. W. Greenberg 2002, *Phys. Rev. Lett.* **89**, 231602 doi:10.1103/PhysRevLett.89.231602 [hep-ph/0201258].
- [16] D. Colladay and V. A. Kostelecky 1998, *Phys. Rev. D* **58**, 116002 [hep-ph/9809521].
- [17] M. Chaichian, K. Fujikawa and A. Tureanu 2012, 2013, *Phys. Lett. B* **712**, 115 (2012) doi:10.1016/j.physletb.2012.04.048 [arXiv:1203.0267 [hep-th]]; *Eur. Phys. J. C* **73**, no. 3, 2349 doi:10.1140/epjc/s10052-013-2349-2 [arXiv:1205.0152 [hep-th]].
- [18] G. J. Milburn 2006, *New J. Phys.* **8**, 96 doi:10.1088/1367-2630/8/6/096 [gr-qc/0308021].
- [19] R. M. Wald 1980, *Phys. Rev. D* **21**, 2742 doi:10.1103/PhysRevD.21.2742
- [20] J. Bernabeu, N. E. Mavromatos and J. Papavassiliou 2004, *Phys. Rev. Lett.* **92**, 131601 doi:10.1103/PhysRevLett.92.131601 [hep-ph/0310180]; E. Alvarez, J. Bernabeu, N. E. Mavromatos, M. Nebot and J. Papavassiliou 2005, *Phys. Lett. B* **607**, 197 doi:10.1016/j.physletb.2004.12.032 [hep-ph/0410409]; J. Bernabeu, N. E. Mavromatos, J. Papavassiliou and A. Waldron-Lauda 2006, *Nucl. Phys. B* **744**, 180 doi:10.1016/j.nuclphysb.2006.03.028 [hep-ph/0506025].
- [21] J. Ellis, N. E. Mavromatos and S. Sarkar 2013, *Phys. Lett. B* **725**, 407 doi:10.1016/j.physletb.2013.07.016 [arXiv:1304.5433 [gr-qc]].
- [22] M. de Cesare, N. E. Mavromatos and S. Sarkar 2015, *Eur. Phys. J. C* **75**, no. 10, 514 doi:10.1140/epjc/s10052-015-3731-z [arXiv:1412.7077 [hep-ph]].
- [23] O. Bertolami, D. Colladay, V. A. Kostelecky and R. Potting 1997, *Phys. Lett. B* **395**, 178 doi:10.1016/S0370-2693(97)00062-2 [hep-ph/9612437].
- [24] H. Davoudiasl, R. Kitano, G. D. Kribs, H. Murayama and P. J. Steinhardt, *Phys. Rev. Lett.* **93**, 201301 (2004) doi:10.1103/PhysRevLett.93.201301 [hep-ph/0403019].
- [25] B. Mukhopadhyay, *Mod. Phys. Lett. A* **20**, 2145 (2005) doi:10.1142/S0217732305017640 [astro-ph/0505460]. U. Debnath, B. Mukhopadhyay and N. Dadhich, *Mod. Phys. Lett. A* **21**, 399 (2006) doi:10.1142/S0217732306019542 [hep-ph/0510351]. G. Lambiase and S. Mohanty, *Phys. Rev. D* **84**, 023509 (2011) doi:10.1103/PhysRevD.84.023509 [arXiv:1107.1213 [hep-ph]]. G. Lambiase, *Phys. Lett. B* **642**, 9 (2006) doi:10.1016/j.physletb.2006.09.037 [hep-ph/0612212]. S. Capozziello and G. Lambiase, *Frascati Phys. Ser.* **58**, 17 (2014) [arXiv:1409.3370 [gr-qc]]. J. I. McDonald and G. M. Shore 2015, *Phys. Lett. B* **751**, 469 doi:10.1016/j.physletb.2015.10.075 [arXiv:1508.04119 [hep-ph]]. *Phys. Lett. B* **766**, 162 (2017) doi:10.1016/j.physletb.2016.11.057 [arXiv:1604.08213 [hep-ph]].
- [26] E. V. Arbuzova and A. D. Dolgov 2017, arXiv:1702.07477 [gr-qc]; arXiv:1612.06206 [gr-qc].
- [27] J. Bernabeu, N. E. Mavromatos and S. Sarkar 2006, *Phys. Rev. D* **74**, 045014 doi:10.1103/PhysRevD.74.045014 [hep-th/0606137]; N. E. Mavromatos and S. Sarkar 2013, *Eur. Phys. J. C* **73**, no. 3, 2359 doi:10.1140/epjc/s10052-013-2359-0 [arXiv:1211.0968 [hep-ph]].

- [28] D. J. Gross and J. H. Sloan 1987, Nucl. Phys. B **291**, 41 doi:10.1016/0550-3213(87)90465-2 R. R. Metsaev and A. A. Tseytlin 1987, Nucl. Phys. B **293**, 385 doi:10.1016/0550-3213(87)90077-0.
- [29] N. E. Mavromatos and A. Pilaftsis 2012, Phys. Rev. D **86**, 124038 doi:10.1103/PhysRevD.86.124038 [arXiv:1209.6387 [hep-ph]].
- [30] E. W. Kolb and M. S. Turner 1990, *The Early Universe*, Front. Phys. **69**, 1-547.
- [31] N. E. Mavromatos and S. Sarkar 2013, Eur. Phys. J. C **73**, no. 3, 2359 doi:10.1140/epjc/s10052-013-2359-0 [arXiv:1211.0968 [hep-ph]].
- [32] J. R. Ellis, N. E. Mavromatos and M. Westmuckett 2004, 2005, Phys. Rev. D **70**, 044036 (2004) doi:10.1103/PhysRevD.70.044036 [gr-qc/0405066]; *ibid.* Phys. Rev. D **71**, 106006 doi:10.1103/PhysRevD.71.106006 [gr-qc/0501060]; J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and M. Westmuckett 2006, Int. J. Mod. Phys. A **21**, 1379 (2006) doi:10.1142/S0217751X06028990 [gr-qc/0508105].
- [33] J. Bernabeu, N. E. Mavromatos and S. Sarkar 2006, Phys. Rev. D **74**, 045014 doi:10.1103/PhysRevD.74.045014 [hep-th/0606137].
- [34] N. E. Mavromatos, V. A. Mitsou, S. Sarkar and A. Vergou 2012, Eur. Phys. J. C **72**, 1956 doi:10.1140/epjc/s10052-012-1956-7 [arXiv:1012.4094 [hep-ph]].
- [35] N. E. Mavromatos, M. Sakellariadou and M. F. Yusaf 2013, JCAP **1303**, 015 doi:10.1088/1475-7516/2013/03/015 [arXiv:1211.1726 [hep-th]]. T. Elghozi, N. E. Mavromatos, M. Sakellariadou and M. F. Yusaf 2016, JCAP **1602**, no. 02, 060 doi:10.1088/1475-7516/2016/02/060 [arXiv:1512.03331 [hep-th]].
- [36] T. Li, N. E. Mavromatos, D. V. Nanopoulos and D. Xie 2009, Phys. Lett. B **679**, 407 doi:10.1016/j.physletb.2009.07.062 [arXiv:0903.1303 [hep-th]].
- [37] G. G. Raffelt 2003, Proc. Int. Sch. Phys. Fermi **152**, 161 doi:10.3254/978-1-61499-008-6-161 [hep-ph/0208024].
- [38] J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and G. Volkov 2000, Gen. Rel. Grav. **32**, 1777 doi:10.1023/A:1001980530113 [gr-qc/9911055].
- [39] I. I. Kogan, N. E. Mavromatos and J. F. Wheeler, Phys. Lett. B **387**, 483 (1996) doi:10.1016/0370-2693(96)01067-2 [hep-th/9606102]; J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Int. J. Mod. Phys. A **13**, 1059 (1998) doi:10.1142/S0217751X98000470 [hep-th/9609238].
- [40] J. Bernabeu, F. J. Botella and J. Roldan 1989, FTUV/89-35, IFIC/89-11, Proc. Blois Conference on CP Violations, pp. 389-400
- [41] A. Di Domenico 2016, *Search for CPT and Lorentz-Symmetry Violation in Entangled Neutral Kaons*, arXiv:1608.00241 [hep-ph]; A. De Santis [KLOE-2 Collaboration and DAFNE Team] 2015, *DAΦNE and KLOE-2*, Phys. Scripta T **166**, 014015 doi:10.1088/0031-8949/2015/T166/014015 [arXiv:1503.06002 [physics.ins-det]] and references therein.
- [42] J. P. Lees *et al.* [BaBar Collaboration] 2012, Phys. Rev. Lett. **109**, 211801 [arXiv:1207.5832 [hep-ex]]; G. Eigen [BaBar Collaboration] 2015, *Recent BABAR Results*, J. Phys. Conf. Ser. **631**, no. 1, 012033 doi:10.1088/1742-6596/631/1/012033 [arXiv:1503.02867 [hep-ex]] and references therein.
- [43] V. A. Kostelecky and N. Russell 2011, *Data Tables for Lorentz and CPT Violation*, Rev. Mod. Phys. **83**, 11 doi:10.1103/RevModPhys.83.11 [arXiv:0801.0287 [hep-ph]].
- [44] P. A. Bolokhov and M. Pospelov 2008, Phys. Rev. D **77**, 025022 [hep-ph/0703291 [HEP-PH]], and references therein.
- [45] N. Russell 2002, hep-ph/0209251; M. Hori and J. Walz 2013, Prog. Part. Nucl. Phys. **72**, 206 (2013) doi:10.1016/j.pnpnp.2013.02.004 [arXiv:1304.3721 [physics.atom-ph]], and references therein.
- [46] B. R. Heckel, E. G. Adelberger, C. E. Cramer, T. S. Cook, S. Schlamminger and U. Schmidt 2008, Phys. Rev. D **78**, 092006 [arXiv:0808.2673 [hep-ex]].
- [47] M. Hori *et al.* [ASACUSA Collaboration] 2011, Nature **475**, 484; doi:10.1038/nature10260; M. Hori *et al.* [ASACUSA Collaboration] 2016, Science **354**, no. 6312, 610 doi:10.1126/science.aaf6702
- [48] P. A. Bolokhov, M. Pospelov and M. Romalis 2008, Phys. Rev. D **78**, 057702 [hep-ph/0609153] and references therein.
- [49] O. G. Kharlanov and V. C. Zhukovsky 2007, J. Math. Phys. **48**, 092302 [arXiv:0705.3306 [hep-th]].
- [50] See, *e.g.*: A.D. Dolgov 2010, Phys. Atom. Nucl., **73**, 588, arXiv: 0903.4318, and references therein.
- [51] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **642**, 315 (2006); A. Di Domenico 2016, arXiv:1608.00241 [hep-ph].
- [52] M. C. Banuls and J. Bernabeu 2000, Nucl. Phys. B **590**, 19 [hep-ph/0005323]; J. Bernabeu, M. C. Banuls and F. Martinez-Vidal 2001, PoS HEP **2001**, 057 [hep-ph/0111073]; J. Bernabeu, F. Martinez-Vidal and P. Villanueva-Perez 2012, JHEP **1208**, 064 [arXiv:1203.0171 [hep-ph]]; See also: E. Alvarez and A. Szykman 2008, Mod. Phys. Lett. A **23**, 2085 [hep-ph/0611370].
- [53] J. Bernabu, F. J. Botella and M. Nebot 2016, JHEP **1606**, 100 doi:10.1007/JHEP06(2016)100 [arXiv:1605.03925 [hep-ph]].

- [54] J. Bernabeu, A. Di Domenico and P. Villanueva-Perez 2013, Nucl. Phys. B **868**, 102 [arXiv:1208.0773 [hep-ph]].
- [55] J. Bernabeu, F. J. Botella, N. E. Mavromatos and M. Nebot 2016, arXiv:1612.05652 [hep-ph].
- [56] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki 1984, Nucl. Phys. B **241**, 381; J. R. Ellis, J. L. Lopez, N. E. Mavromatos and D. V. Nanopoulos 1996, Phys. Rev. D **53**, 3846. P. Huet and M. E. Peskin 1995, Nucl. Phys. B **434**, 3. For entangled states, the requirement of complete positivity implies a different parametrization for the foam effects (in some cases one may consider  $\alpha = \gamma, \beta = 0$  in the parameterization of Ellis *et al.*): F. Benatti and R. Floreanini 1997, Nucl. Phys. B **488**, 335. The experiment can independently measure all three decoherence parameters  $\alpha, \beta, \gamma$  of Ellis *et al.* and hence test the assumption of complete positivity, which notably may not be a property of quantum gravity.
- [57] E. Alvarez, J. Bernabeu, N. E. Mavromatos, M. Nebot and J. Papavassiliou 2005, Phys. Lett. B **607**, 197 ; E. Alvarez, J. Bernabeu and M. Nebot 2006, JHEP **0611**, 087.
- [58] J. Marton *et al.* 2017, arXiv:1703.01615 [quant-ph]; A. Pichler *et al.* 2016, J. Phys. Conf. Ser. **718**, no. 5, 052030 doi:10.1088/1742-6596/718/5/052030 [arXiv:1602.00898 [physics.ins-det]]; C. Curceanu *et al.* 2016, Found. Phys. **46**, 263 doi:10.1007/D10701-015-9923-4, 10.1007/s10701-015-9923-4 [arXiv:1601.06617 [quant-ph]]. H. Shi *et al.* [VIP Collaboration] 2016, J. Phys. Conf. Ser. **718**, no. 4, 042055 doi:10.1088/1742-6596/718/4/042055 [arXiv:1601.05828 [physics.ins-det]]. C. Curceanu *et al.* 2015, J. Phys. Conf. Ser. **631**, no. 1, 012068 doi:10.1088/1742-6596/631/1/012068