

Five Dimensional Little Rip Cosmological Models In General Relativity

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Abstract. In this paper, we have investigated flat Friedmann-Robertson-Walker type Kaluza-Klein model in the presence of ideal fluid with quadratic equation of state(EoS) with time dependent parameters $\omega(t)$ and $\Lambda(t)$. We have investigated the properties of $\omega(t)$ and found that for Little Rip model $\omega(t) = -1$ and for Pseudo Rip model $\omega(t) < -1$. Also, we have studied behaviour of Little Rip(LR) and Pseudo Rip(PR) for dark energy. For obtained LR and PR models we have estimated the disintegration time.

1. Introduction

In recent years many authors have worked on unknown form of negative energy, known as Dark energy, 73% of our universe is completely filled with dark energy(dark fluid). This component can be classified by the equation of state parameter(EoS), which is defined as the ratio of the pressure and the density i.e. $\omega = \frac{p}{\rho}$. $\omega = -1$ represents the Λ CDE, $-\frac{1}{3} < \omega < -1$ represents quintessence DE region and $\omega < -1$ represents phantom dark energy region and corresponds to a dark energy density that strictly increases with time t and the scale factor a .

One of the important features in modern cosmology is, phantom dark energy model can lead to a singularity in which the scale factor(a), the energy density(ρ), and pressure(p) of our universe are all divergent at a finite time, such type of singularity is called a “Big Rip” (Caldwell et al. [1]), while the quintessence dark energy model, can lead to a singularity in which the pressure tends to infinity($p \rightarrow \infty$) at a fixed time, but the scale factor(a) and the energy density(ρ) remain finite, this type of singularity is known as a “sudden singularity”(Frampton et al. [2]). Many authors have investigated on various types of singularities such as big bang, big crunch, big break, big rip, big boost, big freeze, sudden singularities, generalized sudden singularities, ω singularity, inaccessible singularities, and directional singularities (Dabrowski [3], Fernandez-Jambrina [4]).

Generally singularities are not desirable in physics. Therefore, other possible fates of our universe are also considered in the literature, such as the cyclic cosmology. There are some interesting possible scenarios concerning the fate of the universe, including Big Rip(Nojiri and Odinstov [5], Caldwell [6], McInnes [7] and Faraoni [8]), Little Rip(Frampton et al. [2], [9]), Brevik et al. [10]) and Pseudo-Rip(Frampton et al. [11]) models. These models are based on the assumption that the dark energy density ρ is strictly increasing function. Frampton et al. [11] have classified these models into four categories based on the time asymptotic of the Hubble parameter $H(t)$, as



- (i) Big Rip: $H(t) \rightarrow \infty$, when $t \rightarrow t_{rip} < \infty$
- (ii) Little Rip: $H(t) \rightarrow \infty$, when $t \rightarrow \infty$
- (iii) Cosmological Constant: $H(t) = constant$
- (iv) Pseudo-Rip: $H(t) = H_\infty$ when $t \rightarrow \infty$

where H_∞ is a constant.

Little Rip model describes intermediate analysis between an asymptotic de-Sitter expansion and Big Rip. Frampton et al. [12] have explained both LR scenario and PR scenario. Shelote and Khadekar [14] have proposed that both LR and PR are nonsingular. Brevik et al. [13], Khadekar et al. [15] and Astashenok et al. [16] are some of the authors who have worked in these singularities.

In early years, many authors have worked on the quadratic equation of state and also considered in the literature, the main purpose of this quadratic EoS is describing homogeneous and inhomogeneous cosmological models. The quadratic EoS is nothing but the Taylor's expansion of arbitrary barotropic EoS, $p(\rho)$. The expression for quadratic EoS is given by

$$p = p_0 + \alpha\rho + \beta\rho^2 \quad (1)$$

Also, Nojiri and Odinstov [17], Capozziello et al. [18], Ananda et al. [19] have studied on different types of EoS in the dark energy universe, and stated that the quadratic equation of state describes either dark energy or unified dark matter. Brevik et al. [13] have studied on the effect of time dependent parameters $\omega(t)$ and $\Lambda(t)$ in the inhomogeneous equation of state

$$p = \omega(t)\rho + \Lambda(t), \quad (2)$$

upon the event of LR/PR in different cosmological models.

Later, Shelote and Khadekar [14] have proposed inhomogeneous EoS quadratic in density, in which $\omega(t)$ and $\Lambda(t)$ are taken as time dependent parameters and the expression for the quadratic EoS is given by

$$p = (1 + w(t))\rho^2 + \Lambda(t) \quad (3)$$

where p is the pressure and ρ is the energy density.

Motivated by above works, in the present paper we studied flat FRW type Kaluza-Klein universe filled with perfect fluid by taking the time dependent parameters $\omega(t)$ and $\Lambda(t)$. The present paper is organized as follows. In section 2 we have obtained the field equations by taking two different forms for Hubble parameter. In section 3 we have discussed about Pseudo Rip model and found both LR/PR behaviour, further in section 4 we find disintegration time for both LR and PR models, and finally we sum up the results in the last section.

2. The models and field equations:

We consider five dimensional flat FRW type Kaluza-Klein metric in the form

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 + d\psi^2], \quad (4)$$

here, the co-moving coordinates are $(r, \theta, \phi, \psi, t)$, t is the cosmic time, $a(t)$ is the five dimensional scale factor for the model.

The Einstein field equations in mixed form are given by

$$G_j^i = -8\pi G T_j^i, \quad (5)$$

where $G_j^i = R_j^i - \frac{1}{2} R \delta_j^i$ is called Einstein tensor, R is called scalar invariant, T_j^i is the energy momentum tensor, δ_j^i is the Kronecker delta, G is the universal gravitational constant and also, we assume $8\pi G = \epsilon$ then equation (5) becomes,

$$G_j^i = -\epsilon T_j^i, \quad (6)$$

The energy momentum tensor in case of perfect fluid is given by

$$T_j^i = (\rho, -p, -p, -p), \quad (7)$$

and conservation of energy momentum tensor implies

$$T_{j;i}^i = 0, \quad (8)$$

where ρ and p are energy density and pressure of the perfect fluid respectively.

The Einstein field equations (6) for the metric (4) using the energy momentum tensor (7) takes the form

$$3\dot{H} + 6H^2 = -p\epsilon, \quad (9)$$

$$6H^2 = \rho\epsilon, \quad (10)$$

From equation (8) the energy conservation law for this model is given by

$$\dot{\rho} + 4H(\rho + p) = 0. \quad (11)$$

Here, $H = \frac{\dot{a}}{a}$ indicates the Hubble parameter, the overhead dot denotes differentiation with respect to cosmic time t .

Also, we take the different forms of Little Rip model in five dimensional cosmological model with a given Hubble Parameter (H) as

$$\text{case(i)} : \mathbf{H}(\mathbf{t}) = \mathbf{H}_0 e^{\lambda t},$$

$$\text{case(ii)} : \mathbf{H}(\mathbf{t}) = \mathbf{H}_0 e^{c e^{\lambda t}},$$

In both cases H_0, λ and c are constants and as $t \rightarrow \infty, H \rightarrow \infty$.

2.1. Case(i): $H(t) = H_0 e^{\lambda t}, H_0 > 0, \lambda > 0$

In this case the scale factor is obtained as

$$a(t) = e^{\frac{H}{\lambda}}, \quad (12)$$

with the help of equation (12), the metric (4) takes the form

$$ds^2 = dt^2 - (e^{\frac{H}{\lambda}})^2 [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 + d\psi^2], \quad (13)$$

using equations (3), (10) in equation (11), we get

$$\frac{12\lambda H^2}{\epsilon} + 4H \left[(1 + w(t)) \frac{36H^4}{\epsilon^2} + \Lambda(t) + \frac{6H^2}{\epsilon} \right] = 0, \quad (14)$$

solving equation(14) for the value of $\omega(t)$, we get

$$w(t) = \left[-\frac{\lambda\epsilon}{12H^3} - \frac{\Lambda(t)\epsilon^2}{36H^4} - \frac{\epsilon}{6H^2} \right] - 1, \quad (15)$$

If we take Λ independent of the time i.e. $\Lambda(t) = \Lambda_0$, then from equation (15) we get $\omega \rightarrow -1$ asymptotically from below, in the far future. Also, from figure 1 we have observed that $\omega = -1$ which represents Λ CDM and it supports the fact that the present universe is accelerating. Therefore, for perfect fluid obeying equations (11) and (15) the LR scenario was found.

Suppose if ω does not depends on the time $\omega(t) = \omega_0$, then from (14) the value of $\Lambda(t)$ is given by

$$\Lambda(t) = \frac{-3\lambda H}{\epsilon} - \frac{36}{\epsilon^2} (1 + \omega_0) H^4 - \frac{6H^2}{\epsilon}, \quad (16)$$

As $\omega_0 < -1$ it is in accordance with a dark fluid, we see that $\Lambda \rightarrow \infty$ in the far future. The LR behavior in this case is observed.

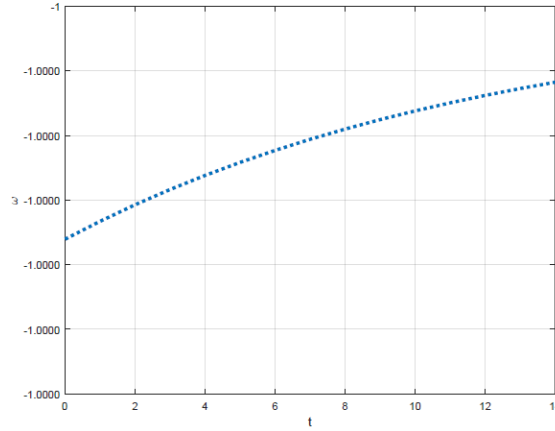


Figure 1. Plot of w versus $time(t)$ for $H_0 = 68 km/sec/Mpc$, $\lambda = 0.04$, $\Lambda_0 = 0.33$, $\epsilon = 0.1$, $t = 13.8 Gyr$.

2.2. Case(ii): $H(t) = H_0 e^{Ce^{\lambda t}}$, $H_0 > 0$, $\lambda > 0$, $C > 0$

In this case(ii) the scale factor is

$$a(t) = e^{\frac{H_0 Ei(Ce^{\lambda t})}{\lambda}}, \quad (17)$$

here Ei is an exponential integral. C , λ are positive constants. Then equation(4) becomes

$$ds^2 = dt^2 - \left(e^{\frac{H_0 Ei(\ln \frac{H}{H_0})}{\lambda}}\right)^2 [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 + d\psi^2], \quad (18)$$

Here H_0 , C and λ are positive constants.

Frampton et al. [12] have investigated this model and it is compatible with observations.

For this case from equation (10), we get

$$\rho = \frac{6H_0^2 e^{2Ce^{\lambda t}}}{\epsilon}, \quad (19)$$

using equations(3), (10) in equation (11), we get

$$\frac{12\lambda H^2 \ln \frac{H}{H_0}}{\epsilon} + 4H \left((1 + \omega(t)) \frac{36H^4}{\epsilon^2} + \Lambda(t) + \frac{6H^2}{\epsilon} \right) = 0, \quad (20)$$

Now, we want to find $\omega(t)$, for this parameter we assume $\Lambda(t)$ in terms of Hubble parameter of the form

$$\Lambda(t) = \gamma H^4, \quad (21)$$

where γ is constant.

By using equation(21) into equation(20), we get $\omega(t)$ as

$$\omega(t) = -\frac{\lambda \epsilon}{12H^3} \ln \frac{H}{H_0} - \frac{\epsilon}{6H^2} - \frac{\gamma \epsilon^2}{36} - 1, \quad (22)$$

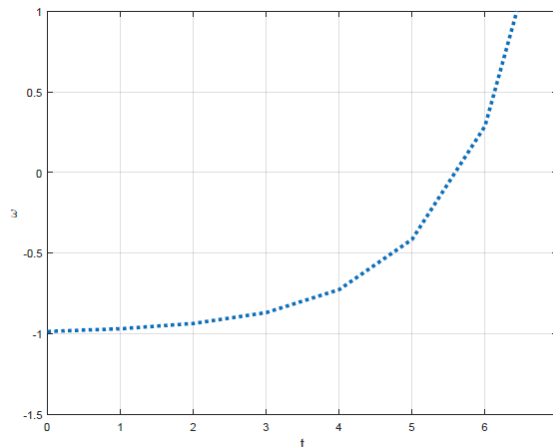


Figure 2. Plot of ω versus $time(t)$ for $H_0 = 68km/sec/Mpc$, $\lambda = 0.04$, $\gamma = 0.5$, $\epsilon = 0.1$, $t = 13.8Gyr$.

Now, we assume the parameter $\omega(t)$ in the form

$$\omega(t) = -1 - \frac{\delta}{36H^4}, \quad (23)$$

where δ is a positive constant

By using equation(23) into equation(20), we get $\Lambda(t)$ as

$$\Lambda(t) = \frac{-3H\lambda \ln \frac{H}{H_0}}{\epsilon} + \frac{\delta}{\epsilon^2} - \frac{6H^2}{\epsilon}, \quad (24)$$

From figure 2 and equation (22) we get $\omega(t) < -1$, i.e. the condition $\omega(t) < -1$ compares to the dark energy density but strictly increases with the time t and the scale factor $a(t)$ and from figure 2 we have observed the phantom region and it suggests that the present universe is accelerating. $\Lambda \rightarrow -\infty$ as $t \rightarrow \infty$, the future behavior of our universe depends on specific model parameters $\omega(t)$ and $\Lambda(t)$. In this way we have studied the equation of state (3), which results the Little Rip. Finally, we observe $\omega < -1$ both theoretically and graphically which is the interesting point in case(ii).

3. Pseudo-Rip Model:

We take the Pseudo Rip model with different behavior of the Hubble parameter(H) given by

$$H(t) = H_0 - H_1 e^{-\lambda t}, \quad (25)$$

where H_0 , H_1 and λ are positive constants, with the condition $H_0 > H_1$ when $t > 0$.

Brevik et al. [13] have proposed that in equation (25) as the second term decreases with increase in t , the universe approaches asymptotically to the de-Sitter universe with Hubble constant H_0 . In this case the scale factor is given by

$$a(t) = e^{H_0 t + \frac{H_1 e^{-\lambda t}}{\lambda}}, \quad (26)$$

here $H_0, H_1 > 0$. Then from equation(26), the metric (4) reduces to

$$ds^2 = dt^2 - (e^{H_0 t + \frac{H_1 e^{-\lambda t}}{\lambda}})^2 [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 + d\psi^2], \quad (27)$$

For this case from equation(10) we get

$$\rho = \frac{6(H_0 - H_1 e^{-\lambda t})^2}{\epsilon}, \quad (28)$$

After differentiating equation(28) with respect to t , we get

$$\dot{\rho} = \frac{12\lambda H(H - H_0)}{\epsilon}, \quad (29)$$

By using equations (3), (10) and (29), the energy conservation equation becomes

$$\frac{12\lambda H(H - H_0)}{\epsilon} + 4H \left((1 + \omega(t)) \frac{36H^4}{\epsilon^2} + \Lambda(t) + \frac{6H^2}{\epsilon} \right) = 0, \quad (30)$$

First, suppose if we consider $\Lambda(t) = \Lambda_0$, then we get

$$\omega(t) = -\frac{\lambda(H - H_0)\epsilon}{12H^4} - \frac{\Lambda_0 \epsilon^2}{36H^4} - \frac{\epsilon}{6H^2} - 1, \quad (31)$$

which displays the PR behavior determined by parameter Λ_0 .

Next, again we consider

$$\Lambda = \gamma H^4, \quad (32)$$

By using equation (32), the equation (31) becomes

$$\omega(t) = -\frac{\lambda(H - H_0)\epsilon}{12H^4} - \frac{\epsilon}{6H^2} - \frac{\gamma \epsilon^2}{36} - 1, \quad (33)$$

Now, solving equation(30) for $\Lambda(t)$ by taking $\omega(t) = \omega_0$, we get

$$\Lambda(t) = \frac{-3\lambda(H - H_0)}{\epsilon} - (1 + \omega_0) \frac{36H^4}{\epsilon^2} - \frac{6H^2}{\epsilon}, \quad (34)$$

again we assume

$$\omega(t) = -1 - \frac{\delta}{36H^4}, \quad (35)$$

Now, by using equation(35), we get $\Lambda(t)$ as

$$\Lambda(t) = \frac{-3\lambda(H - H_0)}{\epsilon} - \frac{\delta}{\epsilon^2} - \frac{6H^2}{\epsilon}, \quad (36)$$

In this case, when $\Lambda(t)$ is constant i.e. $\Lambda(t) = \Lambda_0$, we got $\omega < -1$ from equations (31). If $\Lambda(t)$ is expressed in terms of H we got $\omega < -1$ from equation (33). Thus, we have presented the presence of Little Rip and Pseudo Rip from the quadratic equation of state with time dependent parameters.

4. Disintegration time :

Shelote and Khadekar [14] proposed that physically the scale factor(a) and the density(ρ) in the Little Rip are never infinite at finite time. However, such models usually at a limited time leads to structure disintegration. For models consistent with current supernova observations, such disintegration can occur either earlier or later in a Little Rip model depending on the parameters chosen. Now, we will check the effect of the parameters $\omega(t)$ and $\Lambda(t)$ in the equation of state for dark energy upon the time needed for disintegration in LR/PR models.

To find Little Rip time $t = t_{LR}$ for the case $H=H_0e^{\lambda t}$, where $t \rightarrow \infty$ and $H \rightarrow \infty$, $\lambda > 0$ in the quadratic equation of state given by equation(3), we take the values of parameters as constant i.e.

$\omega(t) = \omega_0$, $\omega_0 < -1$, $\Lambda(t) = \Lambda_0$, where ω_0 and Λ_0 are constants.

Let us consider ω_0 in the following form

$$\omega_0 = -1 - \frac{\lambda\epsilon}{12} - \frac{\Lambda_0\epsilon^2}{36} - \frac{\epsilon}{6}, \quad (37)$$

By using equation (37), the energy conservation equation(11) becomes

$$\left(\frac{3\lambda}{\epsilon} + \frac{6}{\epsilon} + \Lambda_0\right)H^4 - \frac{6H^2}{\epsilon} - \frac{3\lambda H}{\epsilon} - \Lambda_0 = 0, \quad (38)$$

Now solving the equation(38) for the disintegration time $t = t_{LR}$, we get

$$t_{LR} = \frac{1}{\lambda} \ln \frac{1}{H_0}, \quad (39)$$

Now we investigate disintegration time for Pseudo Rip model, for this we start with conservation equation given by (11). Here we see the effect of the parameters ω_0 and Λ_0 in the equation of state for dark energy upon the pseudo rip time $t = t_{PR}$,

We assume the values of ω_0 and Λ_0 in the following form

$$\omega_0 = -1 - \frac{\lambda\epsilon}{12} - \frac{\epsilon}{6}, \Lambda_0 = \frac{3\lambda H_0}{\epsilon}. \quad (40)$$

The energy conservation equation(11) reduces to the form

$$(\lambda + 2)H^3 - 2H - \lambda = 0, \quad (41)$$

Now solving the equation(41) for the disintegration time $t = t_{PR}$, we get

$$t_{PR} = -\frac{1}{\lambda} \ln \left(\frac{H_0 - 1}{H_1}\right), \quad (42)$$

and

$$t_{PR} = -\frac{1}{\lambda} \ln \left(\frac{H_0 - C_0}{H_1}\right). \quad (43)$$

where,

$$C_0 = \frac{1}{2} \left(-1 \pm \sqrt{\frac{2-3\lambda}{\lambda+2}}\right). \quad (44)$$

Therefore, the equation (39) gives the span of time t_{LR} and the equations (42) and (44) gives the span of time t_{PR} needed before the system becomes gravitationally unbound.

5. Conclusion:

The recent scenario of cosmic accelerated expansion of the universe is still a challenging problem in modern cosmology. In this paper, we have investigated the flat Friedmann-Robertson-Walker type Kaluza-Klein cosmological model filled with perfect fluid by taking a quadratic equation of state with time dependent parameters $\omega(t)$ and $\Lambda(t)$ in which Little Rip (LR) and Pseudo-Rip (PR) behavior is encountered in far future. Also, we have described properties of $\omega(t)$ through graphically and theoretically. Here, we found two types of scale factors for LR model and one type of scale factor for PR model.

- In case(i) if $\Lambda = \text{constant}$, then $\omega(t) \rightarrow -1$ asymptotically from below, in the far future, while we have observed $\omega = -1$ graphically which represents ΛCDM . Also, we have found Little Rip scenario. This is an interesting point in this case.
- In case(ii), we got $\omega < -1$ compares to dark energy density but monotonically increase with time t and scale factor $a(t)$. At that point when $\Lambda \rightarrow -\infty$ as $t \rightarrow \infty$. The future conduct of our universe can contingent upon the specific model parameters $\omega(t)$ and $\Lambda(t)$. From figure 2 we have observed $\omega(t) < -1$ which represents phantom region and we have found Little Rip scenario. This is an interesting feature in this case.
- In the same manner we have also studied the Pseudo Rip case and got $\omega(t) < -1$ both theoretically and graphically, here we have taken the value of $\Lambda(t)$ as constant in one case and as a variable in next case. Finally we have observed Little Rip and Pseudo Rip scenarios.
- It is of interest to note that the disintegration time of bound structures in the LR/PR models may occur for physically acceptable choice of parameters in the context of Kaluza-Klein model. We have calculated disintegration time in both the cases of Little Rip and Pseudo Rip models.

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