Some aspects of neutrino phenomenology

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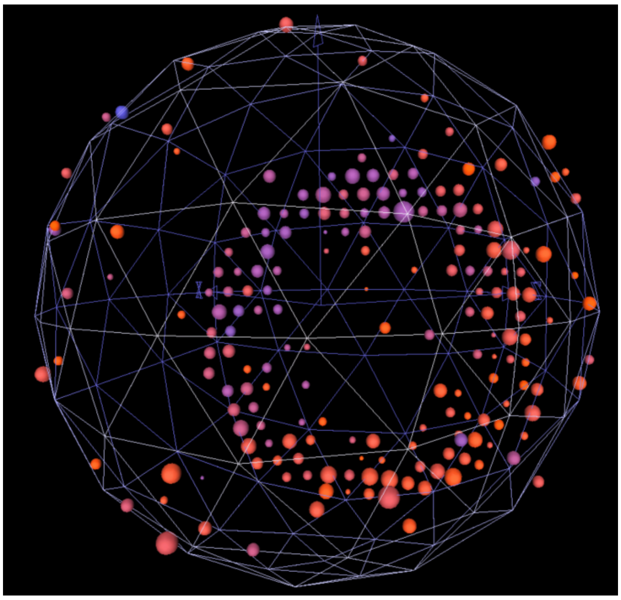

A dissertation submitted to the Universidad Autónoma de Madrid
for the Master degree in Theoretical Physics

Declaration

This Master thesis is the result of my own study, work and research, except where a explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university. The dissertation is adjusted to the asked conditions for the respective Master Degree Court.

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Finally, I would like to add a special thanks to my closest friends, those who know who they are. They are that kind of friends who understand how much I love physics, mathematics and science, those who know why I am doing right now this type of work and study, instead of going out like a common daily life person.

I dedicate this thesis to all these people, those people who really appreciate and love me and those who have supported me even when I was hidden, invisible and missing, out of this world, though just as I have ever been.

Contents

Chapter 1

Introduction

1.1 The Neutrino Window Frontier

Neutrinos [1, 2, 3] provide a wonderful and excellent laboratory for physicists. Their mysterious and challenging character is boosting outstanding investigations in Physics, from both the theoretical and the experimental aside. Since their invention in 1930, their weird behaviour has been shocking the mind and spirit of scientists in various branches of Physics such as Nuclear Physics, High Energy Particle Physics, Astrophysics, Cosmology [80, 81, 82, 83]and Geophysics [5]-[20]. It is precisely this interdisciplinar feature what makes them a powerful tool or probe of the present Standard Model of Particle Physics(SM) [5, 6, 7, 8, 9, 10, 11, 12] and what makes theirselves a core piece in the forthcoming New Physics beyond the Standard Model of interactions and particles. Indeed, they are the first proof of evasive (yet) physics beyond Standard Model (BSM) and they can be used as telescopes and microscopes for new physical phenomena[13, 14, 15, 16, 17, 18, 19, 20]. Some initial questions arise: What are neutrinos? Why are neutrinos so puzzling? Do we undertand them? Are neutrinos important? What are they useful for?

First of all, neutrinos are SM particles but they are also the most enigmatic particles. They feel only the weak interaction at low energies, and it is only at high-energy when they experiment the electroweak interactions through the vector intermediate bosons W^+, W^-, Z. So, from the SM framework, neutrinos are *neutral* (electrically uncharged) *fermions* (spin one-half) that do not feel the strong force. They are leptons. In a more technical language, they are matter field members of the $SU(2)_L \times U(1)_Y$ sector of the SM model, a gauge theory with symmetry group $G = SU(3)_c \times SU(2)_L \times U(1)_Y$. Neutrinos are grouped into families, forming doublets with their corresponding lepton: the electron, the muon and the tau. By the other hand, neutrinos have been puzzling since their first appearance. Invented to explain the continuous spectrum in nuclear β-decay, Pauli himself was afraid to have imagined "a particle that cannot be detected". They are the tiniest known piece of matter in the Universe proved to exist, and they will remain the "tiniest" title, of course, provided there are not some other exotics as axions or very light and eldritch dark matter(DM), e.g., DM very weakly interacting light particles that have been undetected so far. Therefore, its small cross-section was the reason to find *hard* a way to prove their existence. Indeed, they can not be directly detected with conventional detectors and a large or huge detector has to be build to see them, though their presence can be tracked in an indirect way using specifics experiments and huge detectors or searching for missing tranverse energy in the high energy colliders such as LEP, TeVatron or LHC (for instance, seeking evidences for the so-called invisible width of the Z-

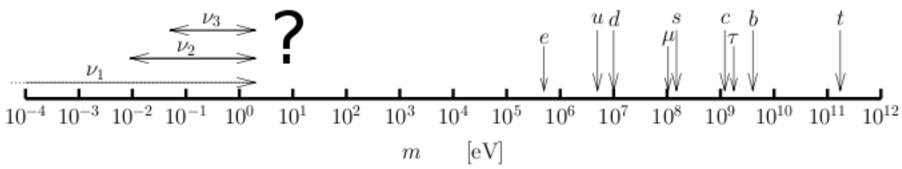

Figure 1.1: SM spectrum of fundamental known subatomic particles in the matter sector. It shows the lepton and quark mass scales excepting those of neutrinos. We must not forget that the origin of mass is likely the most intriguing unsolved question of Physics. And it seems that neutrinos play an important role in the game.

boson, that is, measuring the cross-section of the Z boson into "nothing" they can be "detected" by collider detectors). Ironically, these ghostly and evasive particles have evolved into a bright science: in spite of the problems there were after their proposal and even after their first indirect finding, they have become now one of the hot topics, if not the hottest, in Modern Particle Physics . Additional motives of their intriguing nature are the neverending list of theoretical problems and experimental anomalies found in their phenomenology. For instance, the parity violating of decaying nuclei forced to introduce the famous $V - A$ structure in the weak lagrangian and hamiltonian (and to embed it into the electroweak SM), the solar neutrino problem drived the physicists to consider the analogue of kaon oscillations: the neutrino oscillations. Neutrino oscillation or mixing imply that neutrinos of a given energy can change their flavor traveling certain "long" distances or interacting with matter. The main result is dramatic for the SM: the neutrinos *are not massless* particles. At least one massive flavor state has a non-null mass and it can be turned into the avalaible different neutrino species. Unlike the quark and gauge boson sectors of the SM, we do not know what is the spectrum of the neutrino, their absolute scale or their ordering(even the complete number of neutrino species is not clear). Joined to the hard task of their detection turn neutrinos somehow into "the dark side" of the SM: we understand neutrinos worst than other known SM particles since we do not know their masses, their absolute scale or their ordering. Lepton and quark masses can be represented in a eV-ruler, given by Figure 1.1. Moreover, we can wonder about what advances can their understanding bring seeing what the spectrum of atoms or nuclei supplied in the past and the technology we have achieved at present time (radiation therapy, nuclear energy, electronics, fotonics, . . .) Some future applications of neutrinos are yet to appear, specially in the High Energy Physics (HEP) world (related areas are, e.g., beta beams, superbeams, neutrino factories, or the future muon colliders) and some other science-fiction-like ideas have been proposed [115, 116, 117] at the edge of the present neutrino knowledge, and it is a good sample of how amazing and exciting can be the neutrino sector. Due to these facts we can fairly say that we don't understand better than other matter and gauge fields but, however, they have drived us to beyond SM(BSM) or New Physics since neutrino mixing imply the existence of massive neutrinos. Neutrino oscillation experiments and the phenomenology of neutrino oscillations experiments have suffered a boom in the last 25 years. In spite of the Dirac vs. Majorana fermionic nature of neutrinos was posed long time ago (about 80 years ago), mainly the analysis of specific neutrino mixing such as Kamiokande and Super-Kamiokande, SNO (solar and atmospheric neutrino experiments), and K2K, T2K, MINOS (long base-line neutrino experiments, LBNE) or CHOOZ, KamLAND (

reactor short base-line neutrino experiments) have launched a frenetic run into neutrino researching, seeking answers to "the big" questions (a complete list would be likely impossible now):

- Are neutrinos Majorana particles? This question is equivalent to ask if neutrinos and antineutrinos are identical particles. To be or not to be a Majorana particle, or conversely a Dirac particle, is a very subtle question. It depends on the detection of double beta decays without neutrinos in the final state (emiting two electrons). Indeed, double beta decay experiments can also provide information on the absolute mass scale of the neutrino spectrum.
- What is the character of the neutrino spectrum? Is the spectrum hierarchical or degenerate? Is the ordering normal or is it inverted?
- Are there sterile neutrino, i.e., neutrinos that do not feel the SM weak force? How many of them do exist$(1, 2, \ldots, \infty)$?
- Why is the neutrino mass smaller than the other leptons? This question points into a different or additional mechanism for mass generation beyond the Higgs mechanism. Seesaw models are the main and natural candidates due to their relation with Gran Unified Theories (GUTs) but they are also other alternatives.
- Is there CP violation ($\not{C}\not{P}$) in the leptonic sector?
- What is the value of the θ_{13} neutrino mixing matrix[1]? Is it non-zero?
- Can we detect the diffuse neutrino flux? Can we observe the coherent elastic neutrino-nuclei scattering with present and future experiments and if so, can we observe them with a good precision? How can we observe the cosmic neutrino background?
- Why are the mixing matrices of quarks and leptons so different?
- Can we detect ultra high-energy cosmic neutrinos?

Finally, we will finish this section with a relatively unknown mysterious formula and its neutrino generalization: **The Koide formula**. The Koide formula is an unexplained relation discovered by Yoshio Koide in 1981[128]. It relates the masses of the three charged leptons so well that it predicted the mass of the tau. Mathematically, the Koide formula for leptons reads:

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \tag{1.1}$$

It is clear that $\frac{1}{3} < Q < 1$. The superior bound follows if we assume that the square roots can not be negative. R. Foot remarked that $\frac{1}{3Q}$ can be interpreted as the squared cosine of the angle between the vector $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ and the vector $(1, 1, 1)$. The mystery is in the physical value and its precision. The masses of the electron, muon and the tau are measured respectively as $0.510998910(13)\text{MeV}, 105.658367(4)\text{MeV}, 1776.84(17)\text{MeV}$, where the digits in parentheses are the measurement uncertainty uncertainties in the last figures. The surprising fact is that taking the two masses as inputs (historically the electron and muon masses was plugged by Koide himself) the remaining lepton mass can be calculated up to a very high level of precision. We do not understand if this is an accident or it hides some secret lepton structure (Koide formula was proposed in the context of preonic models or compositeness of leptons and quarks) just as the Balmer formula for the hydrogen atom was anticipating the Quantum Mechanics almost half-century before its establishment. A modified Koide formula has been proposed for neutrinos by Brannen [128](note

[1] During the writing period of this manuscript evidences for a non-zero value of θ_{13} has increased due to the T2K and MINOS experiments [118, 119, 120].

we do not know the value of any neutrino mass):

$$\frac{\left(-\sqrt{m_{\nu_e}}+\sqrt{m_{\nu_\mu}}+\sqrt{m_{\nu_\tau}}\right)^2}{m_{\nu_e}+m_{\nu_\mu}+m_{\nu_\tau}}=\frac{3}{2} \tag{1.2}$$

1.2 Neutrino sources

Where do neutrinos come from? Where are they in Nature[121, 122]? They are everywhere. Even in arts and cyberspace[123, 124, 125, 126, 127]. They are right now when you are reading these words, they are out there when you watch the TV, listen to music or read a new paper. There are neutrinos bombarding the Earth from the Sun and from the rest of the Universe. There are neutrinos coming from supernova/hypernova explosions and we think there are high-energy neutrinos out there coming from violent cosmic events. Even, if our present models of Big-Bang are right, there are about 340 "relic" neutrinos per cubic centimeter. Even one more thing: we emit neutrinos ourselves! Our body contains about 20 milligrams of Potassium-40, which is beta radioactive. Neutrinos are, thus, the most common matter particles in the universe. Neutrinos are produced via weak interactions (like beta-decays in atomic nuclei). How many neutrino interactions coming, for instance, from atmospheric neutrinos are we going to expect in our time-life?Since the cross-section for producing a charged lepton averaged on neutrino and antineutrinos in the tested region $1-3\text{TeV}$ is about

$$\sigma \sim 10^{-38}\text{cm}^2 \cdot E_\nu(\text{GeV}) \tag{1.3}$$

then, since the neutrino flux around 1 GeV is isotropic about 1 neutrino per square centimer per second, we get

$$\frac{1\nu}{\text{cm}^2 s}\frac{10^{-38}}{nucleon}\frac{6\cdot 10^{32} nucleon}{kT}\frac{3.15\cdot 10^7\text{s}}{yr}\frac{75yr(human)}{\text{life}}\frac{70\text{kg}}{(human)}\sim 1\nu\frac{interact.}{\text{human life}} \tag{1.4}$$

In number, they exceed the constituents of ordinary matter (electrons, protons, neutrons) by a factor of ten billion. In addition to these facts, we have indirectly observed and used neutrinos from many different origins with different energy ranges to study neutrino oscillations and the properties of neutrino sources. For instance, neutrino number coming from cosmic rays follows an exponential law $N(E)\sim E^{-\gamma}$ with $\gamma\sim 3$. In general, we can classify the neutrino sources into two categories :

1. Natural sources of neutrinos.
2. Artificial or "man-made" sources of neutrinos.

The zoology of natural neutrinos is wide:

- Geoneutrinos, atmospheric neutrinos, solar neutrinos, cosmogenic neutrinos.
- Supernovae neutrinos and other (extra)galactic(ally generated) neutrinos.
- Neutrinos from DM annihilation, relic neutrinos, neutrinos from Ultra High Energy Cosmic Rays(UHECR ν).

The artifitial (un-natural) types of neutrinos, by the other hand, are:

- Reactor neutrinos[2].
- Particle accelerator neutrinos(neutrino beams, beta beams, superbeams, neutrino factories,...).

1.3 Motivations

From section 1.1, it is clear now what neutrinos represent for the past, present and future physics: a completely novel window to new physics, i.e., a new way to observe the Universe just as electromagnetic waves (from the microwaves and the IR to the UV and the gamma rays or the ultra high-energy cosmic rays) are useful at present time. This new laboratory is just beginning and their applications can be so broad that maybe in the near future we will hear about Neutrinology just as at present day we hear about Radiology, Cosmology or Biology.

What is next? We have to argue why neutrino research is interesting. In particular, why to pay attention on the neutrino-nuclei, νN, scattering and the associated neutrino mixing? Again, we find some set of (non-exhaustive) important answers:

- The cross-section for neutrino-nucleon (neutrino-nuclei) scatterings are not so precisely known as for leptonic reactions. This is mainly caused by the poor theoretical knowledge of the nucleon form factors.
- Neutrino-nucleon interactions are essential to determine the Majorana or Dirac character of neutrinos via double beta decay experiments.
- Neutrino-nucleon interactions can be studied and treated in the SM framework. Thus, νN experiments serve as a SM test. How can we understand the forthcoming New Physics if we do not understand better the SM and its predictions? In fact, confronting what SM and quantum field theory (QFT) say about neutrinos has been a very useful guide to learn more stuff about them.
- Some neutrino-nuclei interactions are found to be the important background events involved in DM experiments. We do not want to misidentify neutrinos with DM particles, so we have to understand how they interact with nuclei and the signals they produce at detectors.
- Current and future neutrino experiments are planning on making high statistics measurements of oscillation parameters.
- Cosmic, astrophysical and geophysical neutrinos interact with nuclei at certain rate. We have to calculate and/or estimate the flux of these kind of neutrinos and their interaction rate to be properly identified if possible and present and future experiments.

1.4 The Master Thesis plan

This thesis is organized according to the following scheme:

- In chapter 2, we review the basic SM neutrino-nucleon cross-sections and associated models in the SM framework.

[2]Some interesting new data analysis have been made in the last months from reactor neutrinos [110, 111, 112, 113] that point towards new physics.

- In chapter 3, we go BSM and we introduce the theory of neutrino masses and mixing, the main concepts behind neutrino mixing, we study the different kinds of neutrino masses and neutrino spectra, and we revise some important equations and formulae.
- Next, in chapter 4, we present a summary of neutrino experiments and data, focusing on neutrino oscillations, double beta decay experiments and measurements of the SM neutrino-nucleon cross-sections.
- Finally, in the final chapter 5, we discuss the future of neutrino physics related with the results and topics of this thesis.

This Master thesis *will not directly discuss* important topics: ν-lepton scattering, electromagnetic properties of massive neutrinos, CPT symmetries for Majorana/Dirac spinors, simulations (Monte Carlo,. . .) of cross-sections, the seesaw mechanism or alternative neutrino mass generation mechanisms, SuperNovae neutrinos, UHECRν, neutrino Astrophysics & Cosmology or concrete symmetry models for the neutrino mixing matrix.

Chapter 2

Standard Model, ν's and nucleons

2.1 Standard Model: basic concepts

The electroweak SM is based on a simple set of principles[4, 5, 6, 7, 8, 9, 10, 11, 12]

- Local gauge $SU(2)_L \times U(1)_Y$ invariance for massless fields.
- Unified origin of weak and electromagnetic forces, due to the interchange of gauge bosons W, Z, the force carrier particles.
- Higgs mechanism to generate mass of particles.

Thus, the full SM is the theory of spin 1/2 quarks and leptons, spin 1 gauge vector bosons and spin 0 Higgs particles. The lagrangian of theory is built in such a way to include the QCD lagrangian, the $V-A$ lagrangian of the $V-A$ charged current interaction, which describes the β-decay of nuclei, the μ-decay, π-decay, decay of strange particles, and other processes involving neutral currents. In order to ensure the local gauge invariance we must assume that $W^{\pm}, Z$ gauge bosons exist. These bosons acquire masses through spontaneous symmetry breaking (SSB) in the electroweak theory (the photon field remains massless). The only missing particle in the SM from the experimental aside is the Higgs scalar particle that also arises after the SSB.

Important remark: In the original SM, neutrino fields were two-component massless fields. Only left-handed field enter into the lagrangian, and there were no right-handed neutrinos. If the Higgs field is transformed as doublet it is impossible to generate neutrino masses.

As the original (minimal) SM contains massless neutrinos only, we could generate neutrino masses in the same way we do with quarks after SSB(see, e.g., [4]). However, we will now see the problem it poses. After SSB, the mass matrix in the Yukawa piece, we would have to diagonalize certain mass matrix M:

$$\mathcal{L}_m^{\nu} = -\bar{\nu}_L M \nu'_R + h.c. = -\sum_{l,l'} \bar{\nu}'_{l'L} M_{l'l} \nu'_{lR} + h.c. \tag{2.1}$$

and we would get the following (Dirac) neutrino mass term

$$\mathcal{L}_m^{\nu} = -\bar{\nu}(x) m \nu(x) = -\sum_{i=1}^{3} m_i \bar{\nu}_i(x) \nu_i(x) \tag{2.2}$$

Since the $\nu_i(x)$ are the neutrino fields with mass m_i, the flavor neutrino fields, denoted by $\nu_{lL}(x)$ must be linked with the left-handed components of the massive neutrino fields through the mixing matrix U:

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}(x) \quad l = e, \mu, \tau \tag{2.3}$$

The unitary matrix U_{li} is called the Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS), and it is the cause of the neutrino oscillation phenomenon (also called neutrino mixing). For right-handed neutrino fields, by the other hand, we would get (we can choose, a priori, a different unitary matrix):

$$\nu_{lR} = \sum_{i=1}^{3} V_{li}\nu_{iR}(x) \quad l = e, \mu, \tau \tag{2.4}$$

The point is that right-handed neutrinos are not observed, and the original SM is made up left-handed neutrinos. Whenever the neutrino shows to be massive, as the neutrino oscillation does, we have to question about what happen with the right-handed mass terms since they are not protected by any SM symmetry. The electroweak SM unification based upon the local gauge invariance, the Higgs mechanism and a minimal interaction contains as an effective theory the traditional Fermi theory and its phenomenology[10], supplemented by the well-tested predictions of CC and NC interactions based on the existence (proved) of intermediate massive gauge bosons (W's and Z). In addition to this, in order to ensure the renormalizability at the required level in the SM, we also require that the sum of the electric charges of the particles, the fields of which are components of the doublets, is equal to zero:

$$3\left(\frac{2}{3} + \left(-\frac{1}{3}\right)\right) N_f^{quarks} + (0 + (-1))\, N_f^{leptons} = 0 \tag{2.5}$$

This means that the number of quark and lepton families, labelled as N_f^{quarks} and $N_f^{leptons}$ must be the same. That is, renormalizability requires

$$N_f^{quarks} = N_f^{leptons} \tag{2.6}$$

This showed to be useful as the tau particle was found, since it meant there should be another quark family. All the predictions of the SM were perfectly confirmed by experiments.

Present experimental bounds suggest that SM neutrino masses are many order of magnitude smaller than the masses of the quark and leptons. It is very unnatural to assume that the same standard Higgs mechanism for the generation of the masses of the charged leptons, quarks *and the neutrinos*. However, note that Dirac neutrino masses can be generated in the standard Higgs mechanism of the SM, but it is not known if the standard Higgs mechanism is the mechanism of the generation of small neutrino masses due to the small Yukawa couplings it would require. Then, from this viewpoint, small neutrino masses are an evidence of BSM Physics, and the little hierarchy problem that neutrino masses pose: how to understand why, in the SM-Higgs framework, the Yukawa couplings giving masses to neutrinos are several orders of magnitude smaller than those of leptons or quarks.

2.2 SM ν-Nucleon cross-sections

The SM allows us to study the interactions between neutrinos and nucleons (mainly made of neutrons and protons). In this section, we review the main formulae, mechanisms, models and other effects involved in neutrino-nucleon (νN) scattering in the SM framework and some very common models used within Monte Carlo generators.

2.2.1 General background and concepts

The SM establishes, as we have studied in previous sections, the existence of two classes of interactions, CC and NC, between leptons, at scales of energy ranging from a few hundreds of MeV and about hundreds of GeV, where every experiment carried out seems to a reasonable agreement with these interactions. We have to distinguish further categories:

- **Quasielastic and elastic scattering**. The first process is more precisely *CC Quasielastic scattering(CCQE)*. It is a relevant process from energies around hundreds of MeV up to energies around a few GeV, i.e., it is the dominant process at low energies. This type of scattering corresponds to the processes: $\bar{\nu}_l + p^+ \to n + l^+$ and $\nu_l + n \to p^+ + l^-$, where $l = e, \mu, \tau$. In practice, only electron and muon neutrino and antineutrino beams are available in the laboratory. Of course, in the particular case of the electron from the second reaction, it is the celebrated inverse beta decay. The name "quasielastic" is due to the fact the nucleon state changes from a proton to a neutron or vice versa, so it is not a completely elastic process but almost, since the nucleon identity remains untouched (unbroken) in the collision with neutrinos. The second process is *NC elastic scattering (NCE)*. It is associated to the reaction: $\nu + N \to \nu + N$ or $\bar{\nu} + N \to \bar{\nu} + N$, where the nucleon N can be a proton or neutron, of course. Description of some experiments measuring this processes is given by [64, 65, 66, 67, 68, 69, 70, 71, 72]

- **Resonant channel scattering**: one pion(π),... When a neutrino interacts with a targe nucleon, tha hadronic current can be excited into a resonance state[79]. At low energies, the resonances are those of isospin 1/2 (usually referred as N^*) and 3/2(the Δ barion). These resonances, usually decay into single pions, and they are commonly called CCπ reactions. At higher energies, more resonances contribute, and multipion production is also possible. They are an important contribution to the total cross-section up to energies around a few GeV. See,e.g.,again [79].

- **CC/NC Deep Inelastic Scattering (DIS)**. Good reviews are [77, 78]. It is corresponds to process to the dominant interactions at energies $E_\nu \gg m_N$ in the laboratory frame. The neutrino collides against the nucleus and "breaks" it. Supposing than the SM holds for arbitrarily high energies, this reaction would be also the most important contribution in the important applied case of high energy neutrino scattering. The *inclusive CC DIS* reaction is:

$\nu_l + N \to l^- + X$ or $\bar{\nu}_l \to l^+ + X$, where N denotes an arbitrar nucleon, X any set of hadrons and l is a lepton. By the other hand, the *inclusive NC DIS* reaction is: $\nu_l + N \to \nu_l + N$ or $\bar{\nu}_l + N \to \bar{\nu}_l + N$. This particular reaction for the muon flavor was important in the Gargamelle experiment in which the existence of NC was established and confirmed. In the parton model, 4-momentum conservation implies that a virtual vector boson with 4-momentum q interacts in such a way that the quarks, in the Breit-Wigner frame, have a 4-momentum equal to a fraction of the nucleon 4-momentum, i.e., $p_i = xp_N$. For each x, the cross-section to find a quark/antiquark is proportional certain probability density $f^N_{q,\bar{q}}$ of finding the nucleon with q_i with $p_i = xp_N$. These probabilities are the parton distribution functions (PDF) of the nucleon and they are related to the so-called structure functions $F_i^{W\pm N}$ in the analytical cross-section since

$$F^{W+}_{i;q_f q_i} = \xi_i |V_{q_f q_i}|^2 f^N_{q_i}(x),\ (N = p, n; i = 1, 2, 3; q_i = d, s, b; q_f = u, c, t) \tag{2.7}$$
$$F^{W+}_{i;\bar{q}_f \bar{q}_i} = \bar{\xi}_i |V_{q_f q_i}|^2 f^N_{\bar{q}_i}(x),\ (N = p, n; i = 1, 2, 3; \bar{q}_i = \bar{u}, \bar{c}, \bar{t}; \bar{q}_f = \bar{d}, \bar{s}, \bar{b}) \tag{2.8}$$
$$\xi_1 = \bar{\xi}_1 = 1; \quad \xi_2 = \bar{\xi}_2 = 2x; \quad \xi_3 = -\bar{\xi}_3 = 2 \tag{2.9}$$
$$F^{W-}_{i;q_f q_i} = \xi_i |V_{q_f q_i}|^2 f^N_{q_i}(x),\ (N = p, n; i = 1, 2, 3; q_i = u, c, t; q_f = d, s, b) \tag{2.10}$$
$$F^{W-}_{i;\bar{q}_f \bar{q}_i} = \bar{\xi}_i |V_{q_f q_i}|^2 f^N_{\bar{q}_i}(x),\ (N = p, n; i = 1, 2, 3; \bar{q}_i = \bar{d}, \bar{s}, \bar{b}; \bar{q}_f = \bar{u}, \bar{c}, \bar{t}) \tag{2.11}$$

The neglecting of the third family mixing allows to derive the known Callan-Gross relation:

$$F_2^{W\pm N}(x) = 2xF_1^{W\pm N}(x) \tag{2.12}$$

Moreover, we also remark that quark and antiquark distributions must satisfy certain sum rules for momentum and quantum numbers (electric charge, baryon number,...) accordingly to the QCD framework. We refer to the interested reader to the bibliography [7, 20].

- **CC/NC Coherent neutrino-nuclei scattering (COS)**. This corresponds to the case in which the neutrino interacts as the nucleus as a whole entity, emitting mainly pions in a coherent way scattered off the nucleus. This is a diffractive process. CC coherent has been measured some years ago, NC coherent elastic scattering has only been observed recently.

Some important ideas and concepts are further commented:

- **CVC and PCAC**. The hadronic matrix element must be a linear combination of a vector and an axial-vector, which can only be constructed from the available kinematical quantities. These are the neutron and proton four-momenta so, we have symbolically

$$\langle p | h_w | n \rangle \tag{2.13}$$

We can separate the vector and pseudovector(axial-vector) contributions

$$h_w = v_w - a_w \tag{2.14}$$

and we get, reintroducing indices

$$v^\rho_w = \bar{u}\gamma^\mu d \tag{2.15}$$
$$a^\rho_w = \bar{u}\gamma^\mu\gamma^5 d \tag{2.16}$$

and taking into account the fact that bilinear spinorial properties, the most general hadronic vector matrix element can be built, in the approximation in which $m_n \simeq m_p \simeq m_N$ as

$$\langle p|v_w^\rho|n\rangle = \bar{u}_p \left[\gamma^\rho F_1(Q^2) + \frac{i\sigma^{\rho\eta}q_\eta}{2m_N} F_2(Q^2) + \frac{q^\rho}{m_N} F_3(Q^2)\right] u_n \tag{2.17}$$

Similarly, the most general axial hadronic matrix element

$$\langle p|a_w^\rho|n\rangle = \bar{u}_p \left[\gamma^\rho\gamma^5 G_A(Q^2) + \frac{q^\rho}{m_N}\gamma^5 G_P(Q^2) + \frac{p_n^\rho + p_p^\rho}{m_N}\gamma^5 G_3(Q^2)\right] u_n \tag{2.18}$$

Hence, the dependence on Q^2 of the form factors is practically irrelevant in neutron decay. On the other hand, the same form factors enter in the CCQE reactions where Q^2 can be large and the dependence on Q^2 of the form factors is important. It can be shown that the invariance under time reversal of strong interactions implies that the six form factors. Moreover, the invariance of strong interactions under isospin transformations implies that $F_3(Q^2) = G_3(Q^2) = 0$. These are the form factors of the so-called *second-class currents*, whose absence has been verified by experiments realized until now. Therefore, we get the hadronic matrix terms:

$$\text{VC term: } \langle p|v_w^\rho|n\rangle = \bar{u}_p \left[\gamma^\rho F_1(Q^2) + \frac{i\sigma^{\rho\eta}q_\eta}{2m_N} F_2(Q^2)\right] u_n \tag{2.19}$$

$$\text{AC term: } \langle p|a_w^\rho|n\rangle = \bar{u}_p \left[\gamma^\rho\gamma^5 G_A(Q^2) + \frac{q^\rho}{m_N}\gamma^5 G_P(Q^2)\right] u_n \tag{2.20}$$

where the nuclear form factors $F_1(Q^2), F_2(Q^2), G_A(Q^2), G_P(Q^2)$ are called, respectively, Dirac, Pauli, axial, and pseudoscalar weak charged-current form factors of the nucleon. The isospin symmetry allows us to express the form factors $F_1(Q^2), F_2(Q^2)$ in terms of the electromagnetic form factors of the nucleons, whose values for $q^2 = 0$ are known. Noether's theorem implies that the isovector currents

$$v_a^\rho(x) = \bar{Q}(x)\gamma^\rho T_a Q(x) \tag{2.21}$$

are conserved

$$\partial_\rho v_a^\rho(x) = 0 \tag{2.22}$$

This property, which was formulated as an hypothesis in the 1950s, is called the *conserved vector current (CVC) hypothesis.* It is now understood that it is a consequence of the *isospin invariance of the strong interaction QCD Lagrangian.* However, the axial part behaviour is completely different. In the rest frame of the neutron in a beta decay we get the matrix element

$$\langle p|h_w|n\rangle \simeq \bar{u}_p\gamma^\rho \left(g_V - g_A\gamma^5\right) u_n \tag{2.23}$$

The parameter g_V is close to unity but the g_A is different from the unity, thus provoking an anomalous non-conserved axial current. This is expected however, since the pion has a finite mass(range). Gell-Mann and Levy introduced the solution to this issue introducing the so-called *partially converved axial current (PCAC)*

$$\partial_\rho v_a^\rho(x) = f_\pi m_\pi^2 \pi^-(x) \tag{2.24}$$

- **The quark-parton model**. The current interpretation of DIS is based on this model of hadrons. The main hypothesis of this model are: i) the nucleon is a composite system made of elementary quarks called valence quarks and a sea of quark-antiquarks of every possible flavor, ii) the interactions between constituent quarks can be neglected in the DIS due to asymptotic freedom, iii) in the so-called Breit-Wigner frame (where $Q^2 \gg m_N$ and $p_N \cdot q \gg m_N$) the constituent quarks have 3-momenta in the same direction that the nucleon and transver momenta is neglected, iv) constituent quark masses are neglected in the Breit-Wigner frame.

- **The axial mass**, M_A. The fall-off of the form-factor strength with increasing Q^2 is traditionally parameterized using an effective axial-vector mass M_A. Its value, M_A, is known to be around 1 GeV to an accuracy of 5%. This value agrees with the theoretically-corrected value from pion electroproduction. However, uncertainty in the value of axial mass contributes directly to uncertainty in the total quasi-elastic cross-section and future experiment would benefit if we could measure it better.

- **Nuclear general effects in neutrino scatterings**. In most neutrino scattering experiments, massive nuclear target/detectors are necessary to obtain useful reaction rates. Neutrino experiments, despite the extremely intense beams designed for them, must also use very massive iron, water or other (usually heavy) nuclear target/detectors, since they are located hundreds of kilometers from the production point. Analysis of neutrino reactions within nuclear media requires an understanding of certain processes which are absent in neutrino scattering on free nucleons; these processes involve the so-called "nuclear effects". There are two general categories of effects. A *first category* include the nucleon target motion within the nucleus (usually modeled as a Fermi gas, since nuclei are fermions, or more preferably modeled by nucleon spectral functions), the exclusion of certain states in the final states due to "Pauli blocking", and final state interactions (FSI) resulting from the final state (such as rescattering or absorption). These interactions may significantly alter the observed final-state configuration and measured energy. The first two effects in this category are either already included in Monte Carlos or are currently being examined in collaboration with nuclear theorists and will soon be included. The third effect (FSI) is perhaps the most troublesome for current and future neutrino experiments. These effects are likely to be sizable for neutrino energies producing a large fraction of elastic and resonant final states. A *second category* of nuclear effects are those by which the neutrino interaction probability on nuclei is modified relative to that for free nucleons. Nuclear effects of this type have been extensively studied in deep-inelastic scattering (DIS) measurements of structure functions using muon and electron beams.

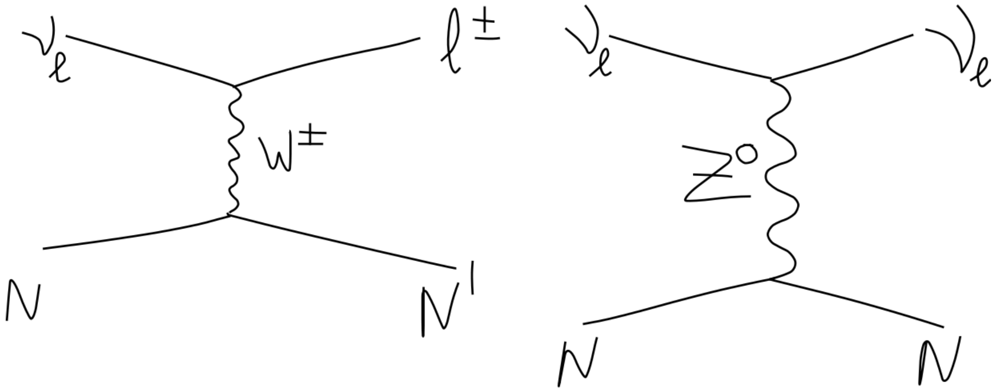

Figure 2.1: CCQE and NCE scattering.

The importance of this processes for the SM can be understood, for instance, from the following Paschos-Wolfenstein formulae:

$$R^- = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{NC}}{\sigma^{\nu}_{CC} - \sigma^{\bar{\nu}}_{CC}} = \frac{1}{2} - \sin^2\theta_w \tag{2.25}$$

$$R^+ = \frac{\sigma^{\nu}_{NC} + \sigma^{\bar{\nu}}_{NC}}{\sigma^{\nu}_{CC} + \sigma^{\bar{\nu}}_{CC}} = \frac{1}{2} - \sin^2\theta_w + \frac{10}{9}\sin^4\theta_w \tag{2.26}$$

from where the value of s_w can be extracted and compared with other SM predictions and results[1] and that, for instance, CC/NC DIS provide complementary information about the nucleon PDFs to those obtained in charged DIS on nucleons, due to the electromagnetic interaction.

The Feynmann graphs corresponding to these interactions are sketched in the following figures:

2.2.2 CC Quasielastic scattering(CCQE)

Following [20, 21], the CCQE differential cross section reads:

$$\frac{d\sigma^{\nu_l n, \bar{\nu}_l p}_{CC}}{dQ^2} = \frac{G_F^2 |V_{ud}|^2 m_N^4}{8\pi\,(P_\nu \cdot P_{N_i})^2}\left[A(Q^2) \pm B(Q^2)\frac{(s-u)}{m_N^2} + C(Q^2)\frac{(s-u)^2}{m_N^4}\right] \tag{2.27}$$

or equivalently

$$\frac{d\sigma^{\nu_l n, \bar{\nu}_l p}_{CC}}{dQ^2} = \frac{G_F^2 m_N^4}{8\pi E_\nu^2}\left[A(Q^2) \pm B(Q^2)\frac{(s-u)}{m_N^2} + C(Q^2)\frac{(s-u)^2}{m_N^4}\right] \tag{2.28}$$

[1] NuTeV experiment main goal was to measure R^-.

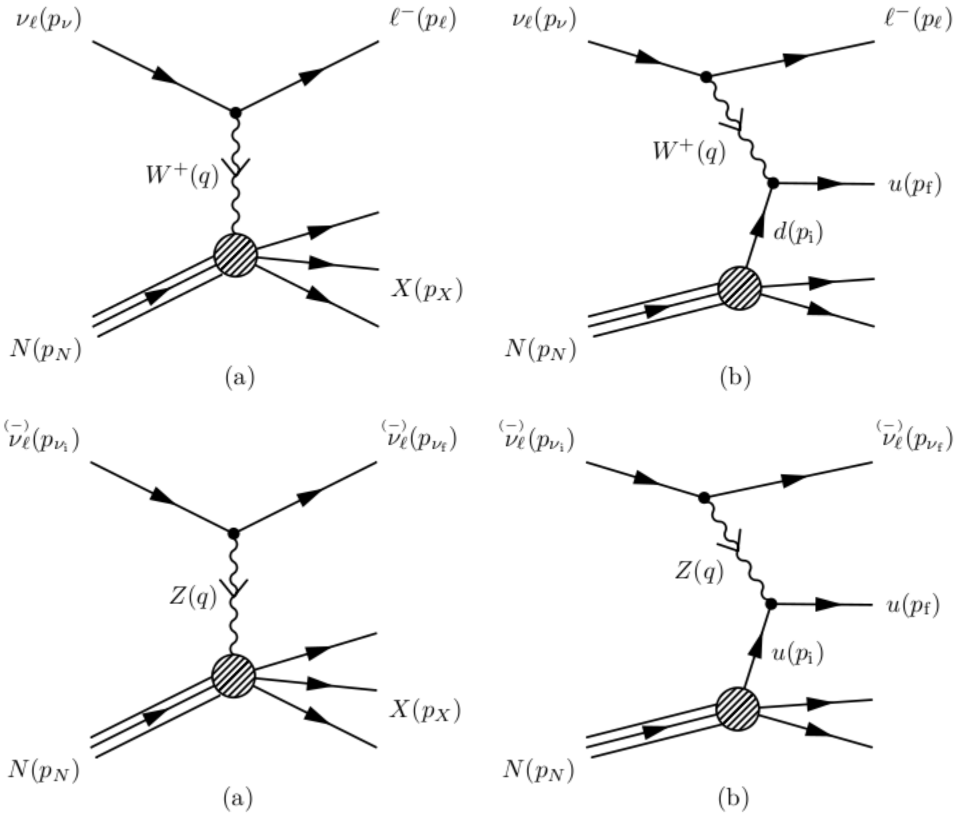

Figure 2.2: DIS processes.

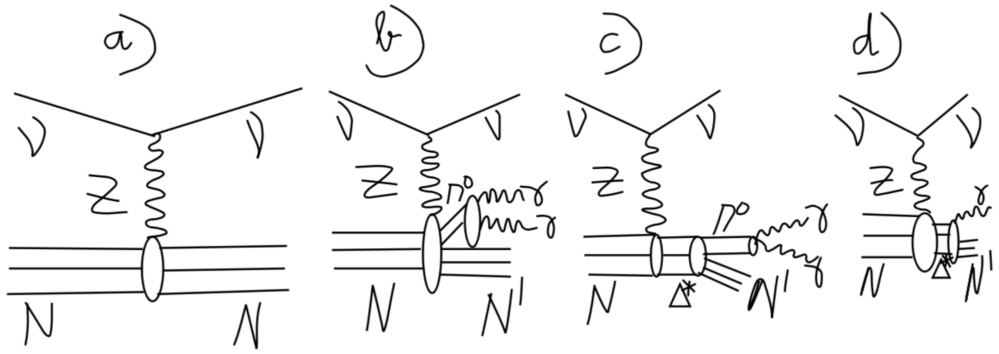

Figure 2.3: Coherent and resonant scattering.a)Elastic (purely) coherent scattering, b) One pion resonant scattering, c) Delta resonant scattering with one pion emission, and d)Delta resonant scattering.

with $\pm$ referring to neutrino/antineutrino[2]. The angular differential cross-section is:

$$\frac{d\sigma_{CC}^{\nu_l n,\bar{\nu}_l p}}{d\cos\theta} = -\frac{G_F^2 m_N^2 p_l}{4\pi E_\nu}\left[A(Q^2) \pm B(Q^2)\frac{(s-u)}{m_N^2} + C(Q^2)\frac{(s-u)^2}{m_N^4}\right] \tag{2.29}$$

The Mandelstam variables are defined such as

$$s = (P_\nu + P_N)^2 \tag{2.30}$$
$$t = (P_\nu - P_N)^2 = q^2 = -Q^2 \tag{2.31}$$
$$u = (P_l - P_N)^2 \tag{2.32}$$
$$s - u = 4m_N E_\nu - Q^2 - m_l^2 = 4m_N E_\nu - 2E_\nu(E_l - p_l\cos\theta) \tag{2.33}$$

The expression for the three functions A, B, C are, with $\tau = Q^2/4m_N^2$ (up to numerical conventional or normalization factors):

$$A(Q^2) = \left[\frac{m_l^2}{m_N^2} + 4\tau\right] a(Q^2) \tag{2.34}$$
$$a(Q^2) = \left[(1+\tau)\,G_A^2 - (1-\tau)\,(F_1^2 - \tau F_2^2) + 4\tau F_1 F_2\right] + -\frac{m_l^2}{4m_N^2}\left[(F_1+F_2)^2 + (G_A + 2G_P)^2 - 4\,(1+\tau)\,G_P^2\right]$$
$$B(Q^2) = 4\tau G_A\,(F_1 + F_2) \quad C(Q^2) = \frac{1}{4}\left(G_A^2 + F_1^2 + \tau F_2^2\right) \tag{2.35}$$

The vector form factors are given by:

$$F_1(Q^2) = \frac{1 + \tau(1 + \mu_p - \mu_n)}{(1+\tau)\left(1 + \frac{Q^2}{M_V^2}\right)^2} \quad F_2(Q^2) = \frac{(\mu_p - \mu_n)}{(1+\tau)\left(1 + \frac{Q^2}{M_V^2}\right)^2} \tag{2.36}$$

with the axial-vector and the pseudo-scalar form factors:

$$G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \quad G_P(Q^2) = \frac{2m_N^2}{M_\pi^2 + Q^2} G_A \tag{2.37}$$

Here, $g_A = -1.25$, $m_V = 0.84$GeV is the vector mass and $m_A = 1.03$ is the axial mass, M_π is the pion mass and μ_p, μ_n are the anomalous magnetic moments for the proton and the neutron. We remark firstly that the appearing of the form factors in the differential cross-section is caused by the nuclear compositeness. Secondly, the vector form factors are determined using empirical fits and are relatively well known. However, the important fact is to realize that the nuclear axial-vector and pseudo-scalar form factors are not so accurately known. Indeed, the above expression for the expression is based on the crude or naive hypothesis that the exponential function represents the electromagnetic charge distribution for the nucleus.

[2] It is conventional from the definition of B and it may cause some confusion from different references. Take care! Note that the cross-section for the neutrino-neutron scattering has to be larger than the one for antineutrino-proton to disentangle the mind.

2.2.3 NC elastic scattering (NC)

The result is very similar to the CCQE scattering with some minor differences:

$$\frac{d\sigma_{NC}^{\nu_l n,\bar{\nu}_l p}}{dQ^2} = \frac{G_F^2|V_{ud}|^2 m_N^4}{8\pi\left(P_\nu \cdot P_{N_i}\right)^2}\left[A_N(Q^2) \pm B_N(Q^2)\frac{(s-u)}{m_N^2} + C_N(Q^2)\frac{(s-u)^2}{m_N^4}\right] \tag{2.38}$$

or equivalently

$$\frac{d\sigma_{CC}^{\nu_l n,\bar{\nu}_l p}}{dQ^2} = \frac{G_F^2 m_N^4}{8\pi E_\nu^2}\left[A_N(Q^2) \pm B_N(Q^2)\frac{(s-u)}{m_N^2} + C_N(Q^2)\frac{(s-u)^2}{m_N^4}\right] \tag{2.39}$$

and now we define the functions

$$A_N(Q^2) = 4\tau a_N(Q^2) \tag{2.40}$$

$$a_N(Q^2) = \left[(1+\tau)\left(G_A^{ZN}\right)^2 - (1-\tau)\left(\left(F_1^{ZN}\right)^2 - \tau\left(F_2^{ZN}\right)^2\right) + 4\tau F_1^{ZN}F_2^{ZN}\right] \tag{2.41}$$

and

$$B_N(Q^2) = 4\tau G_A^{ZN}\left(F_1^{ZN} + F_2^{ZN}\right) \tag{2.42}$$

$$C_N(Q^2) = \frac{1}{4}\left(\left(G_A^{ZN}\right)^2 + \left(F_1^{ZN}\right)^2 + \tau\left(F_2^{ZN}\right)^2\right) \tag{2.43}$$

Now the vector form factors read

$$F_i^{ZN} = \pm\left(F_i^p - F_i^n\right) - 2\sin^2\theta_w F_i^N - \frac{1}{2}F_i^{sN} \quad i = 1, 2 (N = p, n) \tag{2.44}$$

with the plus (minus) sign for the proton(neutron) and the F_i^{sN} being the strange form factors (it is though they are the dominant contribution). Hence, the vector hadronic NC matrix element is determined by the nucleon electromagnetic form factors, whose values are reasonably well known and the strange vector form factors. Although hard, an improvement and better knowledge of NCE scattering has an invaluable bonus. For instance, it means a better understanding of the main background in DM detection experiments.

2.2.4 Resonant channel scattering

The Rein-Sehgal (RS) model describes all neutrino and antineutrino induce pion processes using one unified formalism. All non-strange resonant states below 2 GeV (18 resonances, usually the Δ exchange being the dominant mode) are combined, even interference terms, to produce the single pion channels. In addition, a small isospin 1/2 non-resonant background is generally added incoherently to improve the agreement with data. Thus, the differential cross-section is:

$$\frac{\partial\sigma}{\partial Q^2 \partial E_q} = \frac{1}{128\pi^2}\sum_{spins}|T(\nu N \to lN^*)|^2 \frac{\Gamma}{\left(W - M_{N^*}\right)^2 + \Gamma^2/4} \tag{2.45}$$

where M_{N^*} is the resonance mass, with width Γ and observed mass W. The hadronic matrix element can be written as

$$T(\nu N \to lN^*) = \frac{G_F}{\sqrt{2}} \left[\bar{u}_l \gamma^\alpha (1-\gamma_5) u_\nu\right] \langle N^*|J_\alpha|N\rangle \tag{2.46}$$

In the same way that CCQE, RS model parametrizes the form factors as dipoles separating outh the vector, axial-vector and pseudo-scalar components. The quarks are modeled as relativistic harmonic oscillators with the formalism develope by Feynman-Kislinger-Ravndal(FKR). In this framework, the following hamiltonian is used to calculate 75 transition amplitudes:

$$\mathcal{H} = 3(p_a^2 + p_b^2 + p_c^2) + \frac{\Omega^2}{36}((u_a - u_b)^2 + (u_b - u_c)^2 + (u_c - u_a)^2) + C \tag{2.47}$$

where p_a, p_b, p_c are quark 4-momenta, u_a, u_b, u_c are quark 4-positions and Ω, C are real constants. Using the 3 adjustable constants (energy level spacing per unit angular momentum, the pseudoscalar meson coupling to hadrons and a scaling factor as function of energy) and the particle masses, the transition amplitudes are calculated. Ravndal has improved further the model to calculate production cross-section for all nuclear resonances below 1.75 GeV using a separate vector and axial vector form factors, each one with its own free mass parameter

$$G^V(Q^2) = \left(1 + \frac{Q^2}{4m_N^2}\right)^{1/2-n} \left(\frac{1}{1+\frac{Q^2}{m_V^2}}\right)^2 \quad G^A(Q^2) = \left(1 + \frac{Q^2}{4m_N^2}\right)^{1/2-n} \left(\frac{1}{1+\frac{Q^2}{m_A^2}}\right)^2 \tag{2.48}$$

The RS model is not enough to provide the full kinematics in CCπ reactions. Indeed, experimental data disagree with the model in a very significative way, specially a low Q^2. There, the data show deficit of events with forward muons relative to predictions. There are several modifications proposed in order to solve the discrepancy. The RS model uses to neglect the muon mass, and the size of the correction varies depending on the method. Alternatively to consider the muon mass effect, there has been proposed different vector and axial form factors. A common popular model is based on a Rarita-Schwinger formalism to describe the Δ^{++} resonance. In this way, instead of implementing the FKR model, the vector and axial components for the Δ resonant transition amplitude are written more generally in terms of form factors $C_i^{V,A}$. See [99] and references therein for further information.

2.2.5 Deep Inelastic Scattering(DIS)

We first discuss CC DIS. It is common to define the four momentum transfer $q = p_\nu - p_l = p_X - p_N$. The kinematical variables relevant to the reaction are:

$$s = (p_\nu + p_N)^2 = m_N^2 + 2p_\nu \cdot p_N \tag{2.49}$$

$$Q^2 = -q^2 = 2p_\nu \cdot p_l \geq 0 \quad x = \frac{Q^2}{2p_N \cdot q} \quad y = \frac{p_N \cdot q}{p_\nu \cdot p_N} \tag{2.50}$$

These four Lorentz invariants are related

$$xy = \frac{Q^2}{s - m_N^2} \tag{2.51}$$

and where the variables x and y are confined to the ranges

$$0 < x \leq 1, \quad 0 < y \leq 1 \tag{2.52}$$

Therefore, the DIS region in terms of kinematical variables is given by:

$$Q^2 \gg m_N^2 \quad p_N \cdot q \gg m_N^2 \tag{2.53}$$

$$s \gg m_N^2 \quad xy \simeq Q^2/s \tag{2.54}$$

Thus, the neutrino (antineutrino) double CC DIS differential cross-section are given by the formulae:

$$\frac{d^2\sigma_{CC}^{\nu N,\bar{\nu}N}}{dxdy} = \sigma_{CC}^0 \left[xy^2 F_1^{W^\pm N} + (1-y) F_2^{W^\pm N} \pm xy \left(1 - \frac{y}{2}\right) F_3^{W^\pm N} \right] \tag{2.55}$$

where

$$\sigma_{CC}^0 = \frac{G_F^2}{2\pi} s \left(1 + \frac{Q^2}{m_W^2}\right)^{-2} \tag{2.56}$$

The plus and minus signs correspond to neutrino and antineutrino scattering witha a $W^\pm$ boson, respectively. The structure functions are real functions $F_i^{W^\pm N}$ that either are selected from previous experiments or they are the objects to be measured. Usually, they are chosen to satisfy a functional form $F_i = F_i(x, Q^2)$ and to have certain symmetry properties (see,e.g.,[20] and references therein). Generally, other simplification is also possible if we consider an isoscalar nucleon target. These nuclei are composed by an equal number of protons and neutrons, and the average (on nucleons) neutrino/antineutrino cross-section are given by:

$$\frac{d^2\sigma_{CC}^{\nu,\bar{\nu}}}{dxdy} = \frac{1}{2}\left[\frac{d^2\sigma_{CC}^{\nu p,\bar{\nu}p}}{dxdy} + \frac{d^2\sigma_{CC}^{\nu n,\bar{\nu}n}}{dxdy}\right] \text{ or equivalently}$$

$$\frac{d^2\sigma_{CC}^{\nu,\bar{\nu}}}{dxdy} = \sigma_{CC}^0 \left[xy^2 F_1^{W^\pm} + (1-y) F_2^{W^\pm} \pm xy \left(1 - \frac{y}{2}\right) F_3^{W^\pm} \right] \tag{2.57}$$

Note that in the laboratory frame

$$x = \frac{Q^2}{2m_N(E_\nu - E_l)}; \quad y = 1 - \frac{E_l}{E_\nu}; \quad Q^2 = 4E_\nu E_l \sin^2\frac{\theta}{2} \tag{2.58}$$

where θ is the scattering angle between of the outgoing lepton. In this LAB frame we also have that

$$\sigma_{CC}^0 \simeq \frac{G_F^2}{\pi} m_N E_\nu \simeq 1.58 \times 10^{-38} \left(\frac{E_\nu}{\text{GeV}}\right) \text{cm}^2 \tag{2.59}$$

It is interesting to note that, in the limit $y = 1$, the cross-section of neutrinos depends only on the quark PDFs and the cross-section of antineutrinos depends only on the antiquark PDFs, a fact that is caused by angular-moment conservation and the neglected quark/lepton masses.

By the other hand, NC DIS reactions are mediated by the exchange of a massive Z vector boson. In this case, the double differential cross-sections are:

$$\frac{d^2\sigma_{NC}^{\nu N,\bar{\nu}N}}{dxdy} = \sigma_{NC}^0 \left[xy^2 F_1^{ZN} + (1-y)\, F_2^{ZN} \pm xyF_3^{ZN}\right] \tag{2.60}$$

where

$$\sigma_{NC}^0 = \frac{G_F^2}{2\pi} s \left(1 + \frac{Q^2}{m_Z^2}\right)^{-2} \tag{2.61}$$

and where again the plus and minus signs refer to neutrino/antineutrino scattering. Note that if $Q^2 \ll m_N^2$ we have that $\sigma_{NC}^0 \simeq \sigma_{CC}^0 \sim G_F^2 s$. The quark/antiquark contributions to the structure functions can be seen, e.g., in [7, 20].

2.2.6 Coherent neutrino-nuclei scattering(COS)

In coherent neutrino production, a neutrino interacts with the entire complex nucleus rather than with its constituent particles. A modern short review is given by [54, 56, 63, 78]. The reaction for pure coherent scattering is written as $\nu N \to \nu N$. The reaction for coherent pion emission is written as $\nu A \to \nu A\pi$. Also, there could be coherent production of other hadrons. They are called coherent since every nucleon responds in phase and the global scattering amplitude is a sum of constructively interfering amplitudes from the inner nucleons, without any breakdown. We will only focus on two kinds of coherent modes: the elastic pure NC coherent mode (COH) and the single pion coherent mode(COHπ). There are some general condiditions on coherence:

- The transferred momentum to every nucleon is small enough that the nucleon remains bound in the nucleus.
- There is no transference of any quantum number, since it would spoil coherence otherwise.
- For scattering angles $\theta > 0$, processes are suppressed by $\sin^2\theta \leq (R\nu)^{-2}$, with $\nu = E - E'$ the difference energy before and after the coherent scattering.
- For convenience, a coherence length is introduced to be

$$l_c = \Delta t_c \simeq \frac{2\nu}{Q^2 + m^2} \tag{2.62}$$

  where m is the real hadron state mass. Note that if this coherence length is greater than the nucleus radius target, the weak current will behave like a real hadron current.

Purely Elastic NC coherent cross-section

The basic references are [57] and [58]. The "pure elastic" neutrino coherent scattering with the nucleus $\nu N \to \nu N$ is a fundamental neutrino interaction which has never been experimentally observed. It is a NC interaction and thus independent of the neutrino flavour. A neutrino scatters off a nucleus by exchanging a virtual Z boson, coherently in such a way that it leads to an enhanced cross section. For low transferred momenta, the wavelength of the Z is in the same order of magnitude as the radius of the nucleus. The SM cross-section for this process is given by:

$$\left.\left(\frac{d\sigma^{coh}}{dT}\right)\right|_{SM} = \frac{G_F^2}{4\pi} m_N \left[Z(1-4\sin^2\theta_w) - N\right]^2 \left[1 - \frac{m_N T_N}{2E_\nu^2}\right]$$
$$\sigma^{coh}_{SM,total} = \frac{G_F^2}{4\pi} E_\nu^2 \left[Z(1-4\sin^2\theta_w) - N\right]^2 \tag{2.63}$$

where m_N , N and Z are the mass, neutron number and atomic number of the nuclei, respectively, E_ν is the incident neutrino energy and T_N is the measurable (hard) recoil energy of the nucleus. Corrections due to nuclear form factors can be neglected at low energies: $Q^2 \ll m_N^2$. This formula is applicable for $E_\nu < 50$MeV where the momentum transfer Q^2 is small such that $Q^2R^2 < 1$, where R is the nuclear size. Usually, this cross-section is expressend in a more general and alternative way (nuclear form factor is included now):

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{8\pi}\left[Z(4\sin^2\theta_w - 1) + N\right]^2 E_\nu^2(1+\cos\theta)|f(q)|^2 \tag{2.64}$$
$$\frac{d\sigma}{dE_{recoil}} = \frac{G_F^2}{8\pi}\left[Z(4\sin^2\theta_w - 1) + N\right]^2 E_\nu^2 m_N \left(2 - \frac{E_{recoil} m_N}{E_\nu^2}\right)|f(q)|^2 \tag{2.65}$$
$$\sigma^{coh}_{total} = \frac{G_F^2 E_\nu^2}{4\pi}\left[Z(4\sin^2\theta_w - 1) + N\right]^2 |f(q)|^2 \tag{2.66}$$

with G_F the Fermi constant, Z the proton number of the target nucleus, N the neutron number of the target nucleus, θ_w the Weinberg angle, E_ν the neutrino energy, θ the scattering angle in the laboratory frame, m_N the mass of the target nucleus and E_{recoil} its recoil energy. For larger transferred momenta $q = \sqrt{2m_N E_{recoil}}$ the Z wavelength is smaller than the target nucleus. Thus, the cross-section is modified by a form factor $|f(q)|^2$. For this thesis and general purposes we used the so-called Helm form factor [58]:

$$|f(q)|^2 = \left(3\frac{j_1(qR_0)}{qR_0}\right)^2 e^{-q^2s^2} \tag{2.67}$$

where j_1 is the spherical Bessel function of the first kind and order 1, $R_0 = 1.14A^{1/3}$fm is the effective radius of the target nucleus, and $s = 0.9$ fm is the skin thickness of the nucleus. For the calculation of this form factor the distribution of the charge is assumed to be a "convolution" of a constant inner part and a gaussian distribution modeling the skin of the nucleus. Since $\sin^2\theta_w = 0.23$, the total cross-section is approximately

$$\sigma^{coh}_{total} \approx \frac{G_F^2 E_\nu^2}{4\pi} N^2 |f(q)|^2 = 4.2\cdot 10^{-45} N^2 \left(\frac{E_\nu}{1\text{MeV}}\right)^2 |f(q)|^2 \text{cm}^2 \tag{2.68}$$

Although the cross-section is relatively large due to the $\sim N^2$ enhancement by coherence, the cross-section coupling dependence $\sim G_F^2$ and the small kinetic energy from nuclear recoils poses severe experimental challenges both to the detector sensitivity and to background control. Measurement of the COS cross-section would provide a sensitive test to the SM, probing the weak nuclear charge and radiative corrections due to possible new physics above the weak scale. The coherent interaction plays important role in astrophysical processes where the neutrino-electron scatterings are suppressed due to Fermi gas degeneracy. It is significant to the neutrino dynamics and energy transport in supernovae and neutrons stars. Being a new detection channel for neutrinos, it may provide new approaches to study other aspects of neutrino physics and non-standard interactions. For instance, the detection of supernova neutrinos using this mode was recently discussed. Coherent scattering with the nuclei is also the detection mechanism adopted in the direct DM searches, such that its observations and measurements with the known particle neutrino is an important key to the next generation detectors and experiments as well. Furthermore, neutrino coherent scattering may be a promising avenue towards a compact and relatively transportable neutrino detector, an application of which can be for the real-time monitoring on the operation of nuclear reactors, a subject of paramount global importance in the non-proliferation of nuclear materials. Nuclear power reactors are intense source of electron anti-neutrinos ($\bar{\nu}_e$) at the MeV range, from which many important neutrino experiments are proposed[3]. The $\bar{\nu}_e$ spectra are well-modeled, while good experimental control is possible via the reactor ON/OFF comparisons.

The maximum nuclear recoil energy at momentum transfer much larger than neutrino masses is given by:

$$T_N^{max} = \frac{2E_\nu^2}{m_N + 2E_\nu} \tag{2.69}$$

The maximum neutrino energy for the typical reactor $\bar{\nu}_e$ spectra is about 8 MeV, such that $T_N =$ 1.9keV for Ge target ($A = 72.6$). The future scientific goals[4] in this case are straightforward: a) to develop advanced detectors with kg-size target mass, 100 eV-range threshold and low-background specifications for WIMP DM searches, and b) the studies of neutrino-nucleus COH scattering and neutrino magnetic moments. In the edge, "Ultra-Low-Energy" Germanium (ULEGe) detectors, developed originally for soft X-rays detection, are candidate technologies to meet these challenges of probing into the previously unexplored low-energy domain. These detectors typically will have modular (piece) mass of 5-10 grams while detector array of up to $N = 30$ elements will be necessary for a promising device for DM searches.

Coherent pion emission cross-section

Here, there are some different approaches to the cross-section. Although there are a relative large set of accepted theoretical models to describe COS, they agree on these coherence conditions. The predictions, however, vary widely in general. Using a vector meson dominance model, the various theoretical formulations agree up to the point that

$$\frac{d^2\sigma(\nu A \to \nu A)}{dxdy} = \frac{G_F^2 f_\pi^2}{2\pi^2} E_\nu (1-y) \left(\frac{m_A^2}{Q^2 + m_A^2} \right) \sigma(\pi A \to \pi A) \tag{2.70}$$

[3]Next generation experiments should not only fight for a reduction of the present energy thresholds but mainly focus on an increase of the target mass. Atmospheric neutrinos limit the achievable sensitivity for the background-free direct DM search to $\gtrsim 10^{-12} pb$.

[4]Those of TEXONO collaboration, at least.

We will present different models of COHπ due to its importance and implementation in Monte Carlo methods. We will discuss come of them.

The first model is due to ***Rein-Sehgal*** [22] and it uses the optical theorem in order to get the differential cross-section in terms of Lorentz kinematical invariants, a pion-nucleon total cross-section $\sigma_{tot}^{\pi N}$ that is an average measure of pion-deuteron scattering and the addition of the nuclear form factors, so:

$$\frac{d\sigma(\nu A \to \nu A)}{dt} = A^2 |F_A(t)|^2 \frac{d\sigma(\pi N \to \pi N)}{dt}\bigg|_{t=0} \tag{2.71}$$

with A the atomic mass of the nucleus, being

$$\frac{d\sigma(\pi N \to \pi N)}{dt}\bigg|_{t=0} = \frac{\left(\sigma_{tot}^{\pi N}\right)^2}{16\pi}\left(1 + r^2\right) \tag{2.72}$$

and the selected nuclear form factor

$$|F_A(t)|^2 = e^{-b|t|} F_{abs}; \quad F_{abs} = \exp\left(-\frac{9A^{1/3}}{16\pi R_0^2}\right) \tag{2.73}$$

Therefore, the final Rein-Sehgal COHπ cross-section is:

$$\frac{d^3\sigma(\nu A \to \nu A)}{dxdydt} = \frac{G_F^2}{2\pi^2} f_\pi^2 m_N E_\nu (1-y) A^2 \left(\frac{m_A^2}{Q^2 + m_A^2}\right) \frac{\left(\sigma_{tot}^{\pi N}\right)^2}{16\pi}(1 + r^2) e^{-b|t|} F_{abs} \tag{2.74}$$

One problem with this cross-section, firstly remarked by Belkov and Kopeliovich [60, 61] is that its crude approximation of absorption contradicts the coherent production. As absorption increases, the total pion-nucleus cross-section should increase too, but as it can be trivially seen from the above equation, the exponential function does not reach a maximum around $2\pi R^2$, as we would expect from coherence. Recent experiments have also shown that the RS model produces lower cross-section than the real ones. However, it works for energies about $\gtrsim$ 2GeV. Otherwise, it must be modified or substituted by a more accurate model of coherent scattering.

The ***model by Belkov and Kopeliovich*** of coherent pion scattering is more realistic and near of what we would expect from the physical viewpoint. It assumes that the scattering amplitude for hadron-nuclei interactions is one minus the product of amplitudes for the hadron not to interact with any of the targe nucleons. Moreover, unlikely the RS model, the momentum transfer is split into longitudinal and transverse components. A similar application of the optical theorem (following the same scheme than the RS model) gives:

$$\frac{d\sigma(hA)}{dt} = \frac{d\sigma(hA)}{dt}\bigg|_{t=0} \exp\left(B_L t_{min}\right) \exp\left(B_T\left(t - t_{min}\right)\right) \tag{2.75}$$

where t is related to the longitudinal and transverse components $-p_L^2 \approx t_{min}$ and $-p_T^2 \approx t - t_{min}$. The minimum possible momentum occurs when tha angle between the incoming and outgoing particle is zero, i.e., when the transverse component vanishes. The final Belkov-Kopeliovich(BK) cross-section is similar to the RS:

$$\frac{d^3\sigma(\nu A \to \nu A)}{dxdydt} = \frac{G_F^2}{2\pi^2} f_\pi^2 m_N E_\nu (1-y) \left(\frac{m_A^2}{Q^2+m_A^2}\right) \frac{\left(\sigma_{tot}^{\pi A}\right)^2}{16\pi}(1+r^2)e^{-B_T|t'|}e^{-B_L|t_{min}|} \tag{2.76}$$

Here, the values of B_L and B_T are calculated by the so-called Glauber method, with a Wood-Saxon model of nuclear charge density. Then, the result of using a more sophisticated nuclear form factor (hidden in the previous two parameters) is that the differential cross-section exhibits the correct behaviour as absorption increases. Finally, we mention the work of Paschos and Kartavtsev [59], who agree with the above results but remark the hard task of computing the coherent cross-section accurately. They perform calculations taking into account several Feynman diagrams but rely on data and numerical integration methods to estimate the COHπ cross-section at lower and higher energies.

There is also a model by Kelkar et. al. [62] that arises when coherent pion production is mediated by Δ resonances. It uses a even more detailed model of nuclear physics and the dramatic result is a suppresion of the cross-section at energy about 1 GeV in comparion with other models. We would like to end this beautiful section with one interesting remark. Neutral pions are more interesting particles in COS than the charged pions since for neutral pions there is no interference between pseudovector and vector currents. Hence, there are no substancial difference between coherent neutral pion for neutrinos and antineutrinos due to parity conservation. From the Adler relation and PCAC hypothesis, it is known that the hadronic current at low Q^2 is proportional to the pion field. The hadronic properties of the weak current in these kinematic regions have been investigated through the study of nuclear shadowing at low x and the coherent production of π, ρ, and a_1 mesons. Coherent scattering therefore allows investigation of the PCAC hypothesis and hadron dominance models of the weak current in detail. A number of calculations of coherent scattering, involving substantially different procedures and assumptions, have been made over the past forty years. These calculations factorize the problem in terms of the hadron-like component of the weak current and the scattering of this hadron with the nucleus. The calculations assume PCAC as a starting point but quickly diverge when it comes to the number of hadronic states required to describe the weak current and how the hadron–nucleus scattering should be treated. The Rein-Sehgal model, used by both the popular NUANCE and NEUGEN Monte Carlo generators, describes the weak current only in terms of the pion field; the Q^2 dependence of the cross-section is assumed to have a dipole form.

2.2.7 Other cross-section effects

It is also important to simulate the secondary interactions of mesons produced in neutrino interactions with nucleons inside the heavy nuclei. First, the initial pion production point in the nucleus, where neutrino-nucleon interactions occur, is determined by the Wood-Saxon density distribution

$$\rho(r) = \frac{Z}{A}\rho_0 \frac{1}{1+\exp\left(\dfrac{r-c}{a}\right)} \tag{2.77}$$

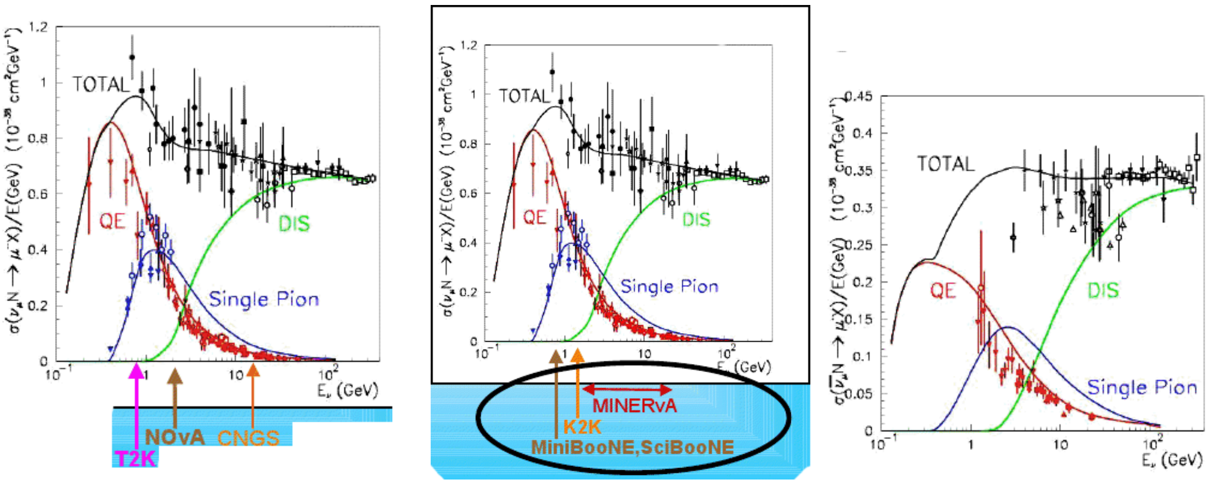

Figure 2.4: Cross sections(I). On the left, a plot of neutrino-nucleon cross-section(highlighting some current and future experiments). In the middle, other experiments relative to neutrino-nucleon scattering. On ther right, a plot of antineutrino-nucleon cross-section. Remark the lower number of points in the low-energy region, below 1 GeV.

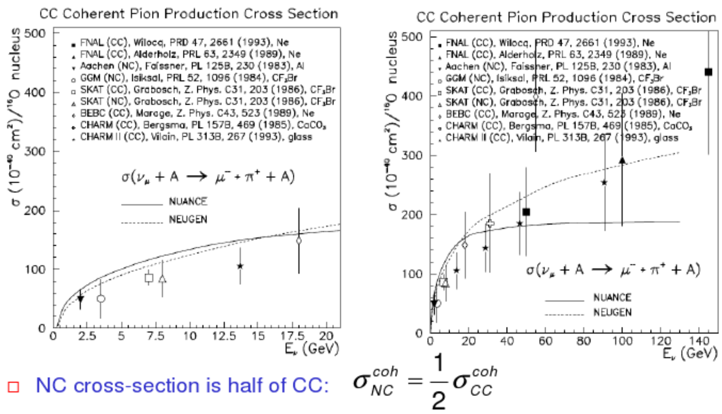

Figure 2.5: Cross-sections(II). CC Coherent one pion cross-sections data.

The mean free path of pions depends on their momenta and positions in the nucleus. Nuclei are fermions and their behaviour has to be implemented. Usually, it is done with the model of Smith and Moniz [23]. In the interactions, the Fermi motion of the nucleus and the Pauli blocking are considered, and the outgoing nucleon must have the energy above the Fermi surface momentum defined by :

$$p_F(r) = \left(\frac{3}{2}\pi^2\rho(r)\right)^{1/3} \tag{2.78}$$

Every nucleon below the Fermi level is suppressed. Of course, all these results can be modified in the presence of new physics. For instance, unparticles can change and modify the neutrino-nucleon cross-sections, see [102, 103, 104, 105, 106, 107, 108].

Chapter 3

Neutrino masses and mixing

As we have anticipated in the previous sections 1 and 2, the existence of the neutrino oscillations, that is, the neutrino mixing, provides a hint of Physics BSM. Neutrino masses are the central object of the theory of massive and mixed neutrinos. There a large set of lectures, talks and papers covering neutrino mixing. The introductory and fun [24, 25, 26] are interesting, and about neutrino mass games we highlight [32, 33, 34]. Between the lecture notes, we found useful [35, 36, 37, 38, 39, 40, 41, 42, 43] and the monographic-like big review papers [44, 45, 46, 47] and those by Dolgov [27, 28, 29, 30, 31] are specially good about the connections between neutrinos and Cosmology/Astrophysics.

Unlike any other particle in the SM, (electrically) neutral neutrinos allow the interesting theoretical possibility (noted by Majorana) of having two different classes of masses:

- Dirac mass terms.
- Majorana mass terms.

The second option forces to neutrinos to be *the same* particle than its own antiparticle, the antineutrinos. It is evident that this can be realized with neutrino fields and no other field in the SM.

By the other hand, there are the even most interesting possibility that there were other kind of *sterile* neutrinos. These neutrinos would have been unnoticed because they would not couple to the SM fields, or if so, they would couple very weakly. Sterile neutrinos could be a natural candidate for DM.

In modern theories, mass terms of fermions appear after the breaking (e.g.SSB) of underlying symmetries. They can also arise as radiative corrections in perturbation theory. In the SM, the neutrino mass term could occur with right-handed neutrino singlets, since they are not forbidden by the gauge symmetries of the lagrangian. LEP experiments established that only 3 light flavor species exist in Nature (i.e., there are only 3 light neutrinos feeling the SM electroweak interaction). These neutrinos are commonly referred as active neutrinos. These active neutrinos interact under the CC and NC

$$j_\alpha^{CC} = 2\sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_\alpha l_L(x) \quad j_\alpha^{NC} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_\alpha \nu_L(x) \tag{3.1}$$

The fields ν_{lL} *must* enter into the neutrino mass term. The structure of mass terms depends on: a) other fields that enter into the mass term, and b)the conservation of the total leptonic number: $L = L_e + L_\mu + L_\tau$. Remember that every fermion field $\Psi(x)$ allows a decomposition $\Psi = \Psi_L + \Psi_R$, where the left-handed and right-handed components, Ψ_L and Ψ_R, are defined by

$$\Psi_{L,R} = \frac{1 \mp \gamma_5}{2}\Psi, \quad \gamma_5 \Psi_{L,R} = \mp \Psi_{L,R} \tag{3.2}$$

3.1 Neutrino masses

3.1.1 Dirac masses

Let us suppose that in addition to the left-handed neutrinos ν_{lL}, there are 3 right-handed neutrinos ν_{lR} that enter into the mass term induced, for instance, after SSB. In this case, the most general neutrino mass term will be

$$\mathcal{L}_D(x) = -\sum_{l',l} \bar{\nu}_{l'L}(x) M^D_{l'l} \nu_{lR} + h.c. \tag{3.3}$$

where M^D is a 3x3 complex matrix, and the lepton index takes tha values $l = e, \mu, \tau$. Global gauge transformations

$$\nu'_{lL} = e^{i\Lambda}\nu_{lL}(x), \ \ \nu'_{lR} = e^{i\Lambda}\nu_{lR}(x) \ \ l'(x) = e^{i\Lambda} l(x) \ \ q'(x) = q(x) \tag{3.4}$$

where Λ is an arbitray constant phase leave invariant the lagrangian term $\mathcal{L}_D$, mass terms included. From this invariance, total lepton number is conserved and it is the same for all flavor neutrinos. We diagonalize the complex matrix M^D by the so-called biunitary transformation method:

$$M^D = U^\dagger m V \tag{3.5}$$

where U and V are unitary matrices, and $m_{ik} = m_i \delta_{ik} = diag(m_1, m_2, m_3)$, and the masses are (strictly) positive. Then, we find

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x) \qquad l = e, \mu, \tau \tag{3.6}$$

and

$$\nu_{lR}(x) = \sum_{i=1}^{3} V_{li} \nu_{iR}(x) \qquad l = e, \mu, \tau \tag{3.7}$$

After this process, the lagrangian takes the form

$$\mathcal{L}^D = -\sum_{i=1}^{3} m_i \bar{\nu}_i(x) \nu_i(x) \tag{3.8}$$

Consequences:

1. $\nu_i(x)$ are the neutrino fields, with masses m_i for $i = 1, 2, 3$.
2. The left-handed flavor neutrinos ν_{lL} that couple to the CC and NC in the SM are "mixed" fields. That is, the mass eigenstates are not the flavor eigenstates. Flavor (resp. mass) eigenstates are certain (usually complex) linear combinations of mass (resp. flavor) eigenstates.

The matrix U is usually called, as we mentioned before in section 2, the PMNS matrix. The total lagrangina *is* invariant under global gauge transformations leaving the total leptonic number unchanged. This invariance also means that the leptonic number of neutrinos and antineutrinos is the opposite to the another. That is, $L(\nu_i) = -L(\bar{\nu}_i) = 1$. The neutrino mass in (3.3) is called Dirac mass term (or Dirac mass lagrangian).

3.1.2 Majorana mass term

A mass term is a sum of Lorentz-invariant products of left-handed (LH) and right-handed (RH) components of the fields. Let us introduce the charge conjugated fields:

$$(\nu_{lL})^c = C\bar{\nu}_{lL}^T \qquad (\nu_{lR})^c = C\bar{\nu}_{lR}^T \tag{3.9}$$

where C is the unitary matrix of *charge conjugation*. It satisfies

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha \quad C^T = -C \tag{3.10}$$

We obtain, therefore

$$\gamma_5\nu_{lL} = -\nu_{lL} \quad \gamma_5\nu_{lR} = \nu_{lR} \tag{3.11}$$

From these relations, after hermitian conjugation and multiplication by γ^0 from the right side, we deduce that

$$\bar{\nu}_{lL}\gamma_5 = \nu_{lL}, \quad \bar{\nu}_{lR}\gamma_5 = -\nu_{lR} \tag{3.12}$$

Further, from these last equations by transposition and multiplication from the left side by the matrix C, we get

$$\gamma_5 (\nu_{lL})^c = (\nu_{lL})^c \quad \gamma_5 (\nu_{lR})^c = - (\nu_{lR})^c \tag{3.13}$$

where we have used that $C\gamma_5^T C^{-1} = \gamma_5$. Then, it follows that $(\nu_{lL})^c$ is the RH and $(\nu_{lR})^c$ is the LH component of the neutrino field with these definitions. Moreover, we have

$$\overline{(\nu_{lL})^c} = -\nu_{lL}^T C^{-1} \quad \overline{(\nu_{lR})^c} = -\nu_{lR}^T C^{-1} \tag{3.14}$$

Therefore, if we *impose* that $(\nu_{lL})^c = \nu_{lR}$ and $(\nu_{lR})^c = \nu_{lL}$, we get a new kind of mass term, called Majorana mass term (or Majorana mass lagrangian):

$$\mathcal{L}^M = -\frac{1}{2}\sum_{l',l=e,\mu,\tau} \bar{\nu}_{l'L} M^M_{l'l} (\nu_{lL})^c + h.c. = -\frac{1}{2}\sum_{l',l=e,\mu,\tau} \bar{\nu}_{l'L} M^M_{l'l} C \nu^T_{lL} + h.c. \tag{3.15}$$

and where M^D is a 3×3 complex non-diagonal mass matrix, called Majorana matrix. The extra prefactor 1/2 is conventional. Thus, the (Majorana) mass term (lagrangian) can be written as:

$$\mathcal{L}^M = -\frac{1}{2}\bar{\nu}_L M^M (\nu_L)^c + h.c. \quad \nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \tag{3.16}$$

An important feature of M^M is that it is a symmetric (complex) matrix, since

$$\bar{\nu}_L M^M (\nu_L)^c = \bar{\nu}_L M^M C \bar{\nu}_L^T = -\bar{\nu}_L \left(M^M\right)^T C^T \bar{\nu}_L = \bar{\nu}_L \left(M^M\right)^T (\nu_L)^c \tag{3.17}$$

so $\left(M^M\right)^T = M^M$. We can diagonalize the symmetric complex matrix with unitary transformations given by a matrix U. Thus, $M^M = UmU^T$, where we write the diagonal matrix $m = m_{ik} = diag(m_1, m_2, m_3) = m_i\delta_{ik}$ for positive mass values. With these definitions and transformations, we can write

$$\mathcal{L}^M = -\frac{1}{2}\bar{\nu}_L U m U^T C \bar{\nu}_L^T + h.c. = -\frac{1}{2}\overline{U^\dagger \nu_L} m \left(U^\dagger \nu_L\right)^c - \frac{1}{2}\overline{(U^\dagger \nu_L)^c} m U^\dagger \nu_L \tag{3.18}$$

and in this way, the Majorana mass term acquires the compact expression:

$$\mathcal{L}^M = -\frac{1}{2}\bar{\nu}^M \nu^M \tag{3.19}$$

provided we define the Majorana neutrino field as

$$\nu^M = U^\dagger \nu_L + \left(U^\dagger \nu_L\right)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{3.20}$$

In this fashion, the Majorana mass lagrangian after diagonalization can be written as

$$\mathcal{L}^M = -\frac{1}{2}\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i \tag{3.21}$$

where we have dropped the Majorana superindices, and so, the Majorana (reality) condicion holds on neutrino spinors:

$$\left(\nu^M(x)\right)^c = \nu^M(x) \tag{3.22}$$

This last relationship has dramatic consequences for the quantum theory. If the neutrino field satisfies the Majorana condition, then the neutrino is equal to its antineutrino, i.e., they are the same particle. This neutrino field can also suffer a quiral decomposition:

$$\nu^M(x) = \nu_L^M(x) + \nu_R^M(x) \tag{3.23}$$

and therefore we can write

$$\nu_L^M(x) = U^\dagger \nu_L(x), \quad \nu_R^M(x) = \left(U^\dagger \nu_L(x)\right)^c \tag{3.24}$$

We conclude that the RH and the LH components of Majorana fields are related by the equation

$$\nu_R^M(x) = \left(\nu_L^M(x)\right)^c \tag{3.25}$$

and also the next relations hold

$$\nu_{iR}(x) = (\nu_{iL})^c \tag{3.26}$$

because of the calculation

$$(\nu_{iL})^c = \left(\frac{1-\gamma_5}{2}\nu_i\right)^c = C\frac{1+\gamma_5^T}{2}\bar{\nu}_i^T = C\frac{1+\gamma_5}{2}\nu_i^c = \frac{1+\gamma_5}{2}\nu_i = \nu_{iR} \tag{3.27}$$

The Majorana mass term (3.15) *is not* invariant under global gauge transformation leaving the total leptonic number conserved. The absence of total leptonic number conservation is the main phenomenological difference between Dirac and Majorana neutrinos. We can similarly write the neutrino mixing matrix for Majorana neutrinos

$$\nu_L(x) = U\nu_L^M(x) \tag{3.28}$$

or

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x) \tag{3.29}$$

where U is the unitary PMNS-like mixing matrix. In the case of Majorana mass terms, the LH flavor fields ν_{lL}, through their insertion into the CC and NC of the modified SM, are connected with the LH components of the Majorana fields ν_{iL} by the above relation.

The kinetic term for the Majorana mass is given by

$$\mathcal{L}_0^M = \sum_l \bar{\nu}_{lL} i\gamma^\alpha \partial_\alpha \nu_{lL} = \bar{\nu}_L i\gamma^\alpha \partial_\alpha \nu_L \tag{3.30}$$

The PMNS matrix allows us to write this as

$$\mathcal{L}_0^M = \bar{\nu}_L^M i\gamma^\alpha \partial_\alpha \nu_L^M = \sum_i \bar{\nu}_{iL} i\gamma^\alpha \partial_\alpha \nu_{iL} \tag{3.31}$$

It is trivial to check that

$$\bar{\nu}_{iL} i\gamma_{\alpha}\partial^{\alpha}\nu_{iL} = -\partial^{\alpha}\nu_{iL}^{T} i\gamma_{\alpha}^{T}\bar{\nu}_{iL}^{T} = -\partial^{\alpha}\overline{(\nu_{iL})^{c}} i\gamma_{\alpha}(\nu_{iL})^{c} \tag{3.32}$$

and simple calculus provides

$$-\partial^{\alpha}\overline{(\nu_{iL})^{c}} i\gamma_{\alpha}(\nu_{iL})^{c} = -\partial\left(\overline{(\nu_{iL})^{c}} i\gamma_{\alpha}(\nu_{iL})^{c}\right) + \overline{(\nu_{iL})^{c}} i\gamma_{\alpha}\partial^{\alpha}(\nu_{iL})^{c} \tag{3.33}$$

The final ultimate expression for the kinetic Majorana term reads

$$\mathcal{L}_{0}^{M} = \frac{1}{2}\sum_{i}\bar{\nu}_{iL} i\gamma^{\alpha}\partial_{\alpha}\nu_{iL} + \frac{1}{2}\sum_{i}\overline{(\nu_{iL})^{c}} i\gamma_{\alpha}\partial^{\alpha}(\nu_{iL})^{c} = \frac{1}{2}\bar{\nu}_{i} i\gamma^{\alpha}\partial_{\alpha}\nu_{i} \tag{3.34}$$

where we have written $\nu_i = \nu_{iL} + (\nu_{iL})^c$. In the flavor base, for the Majorana case, we have derived the kinetic and mass terms that together can be assembled into

$$\mathcal{L}_{0,m}^{M} = \frac{1}{2}\sum_{i=1}^{3}\bar{\nu}_{i}\left(i\gamma^{\alpha}\partial_{\alpha} - m_{i}\right)\nu_{i} \tag{3.35}$$

Important remark: The Majorana mass term has only active LH neutrino fields in the total lagrangian. There are no RH components in the mass terms.

3.1.3 Dirac-Majorana mass term

It can be easily understood that the most general mass term for a neutrino field is a combination of Dirac and Majorana terms. This is called Dirac-Majorana mass term (lagrangian). Let us write it:

$$\mathcal{L}^{DM} = -\frac{1}{2}\bar{\nu}_{L} M_{L}^{M}(\nu_{L})^{c} - \frac{1}{2}\overline{(\nu_{R})^{c}} M_{R}^{M}\nu_{R} - \bar{\nu}_{L} M^{D}\nu_{R} + h.c. \tag{3.36}$$

Here, the matrices M_L^M and M_R^M are complex non-diagonal symmetrical 3×3 matrices, while M^D is a complex non-diagonal 3×3 matrix and

$$\nu_L = \begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\end{pmatrix} \quad \nu_R = \begin{pmatrix}\nu_{eR}\\ \nu_{\mu R}\\ \nu_{\tau R}\end{pmatrix} \tag{3.37}$$

Remark: The presence of a Majorana left-handed mass term spoils the renormalizability of the theory, so it is usually set to zero to ensure it. Similarly to the Majorana case, this Dirac-Majorana mass term also violates the total lepton number conservation law. Thus, we expect that the fields of neutrinos with definite masses are Majorana fields. We have to diagonalize the mass matrix in the Dirac-Majorana case. Writing the lagrangian as

$$\mathcal{L}^{DM} = -\frac{1}{2}\bar{N}_{L} M^{DM}(N_{L})^{c} + h.c. \tag{3.38}$$

with

$$N_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} \tag{3.39}$$

and

$$M^{DM} = \begin{pmatrix} M_L^M & M^D \\ \left(M^D\right)^T & M_R^M \end{pmatrix} \tag{3.40}$$

This matrix is a 6×6 matrix that can be diagonalized with a unitary transformation U, in such a way $M^{DM} = UmU^T$, where U is also a 6×6 matrix and $m = m_i \delta_{ik}$. The DM lagrangian reads

$$\mathcal{L}^{DM} = -\frac{1}{2}\overline{U^\dagger N_L} m \left(U^{\dagger N_L}\right)^c + h.c. = -\frac{1}{2}\bar{\nu}^M m \nu^M = -\frac{1}{2}\sum_{i=1}^{6} m_i \bar{\nu}_i \nu_i \tag{3.41}$$

wiht the Majorana neutrino field

$$\nu^M = \nu_L^M + \left(\nu_L^M\right)^c = \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_6 \end{pmatrix} \qquad \nu_L^M = U^\dagger N_L \tag{3.42}$$

and where the Majorana conditions hold for $\left(\nu^M\right)^c = \nu^M$ and $\nu_i^c = \nu_i$. It is obvious too that we get the mixing terms

$$\nu_{lL}(x) = \sum_{i=1}^{6} U_{li}\nu_{iL}, \quad (\nu_{lR}(x))^c = \sum_{i=1}^{6} U_{li}\nu_{iL}(x) \tag{3.43}$$

Therefore, for a DM mass term, the flavor field ν_{lL} is a "mixture" or complex linear combination of the six LH fields of Majorana particles with mass m_i. The sterile neutrino field, a RH neutrino field $(\nu_{lR})^c$, is also a "mixture" of the same components[1].

In summary, whenever we have only a Dirac or Majorana mass term, we get the mixing scheme

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL} \tag{3.44}$$

and if we have a DM mass term we get the mixing

$$\nu_{lL}(x) = \sum_{i=1}^{6} U_{li}\nu_{iL}, \quad (\nu_{lR}(x))^c = \sum_{i=1}^{6} U_{li}\nu_{iL}(x) \tag{3.45}$$

[1] The most popular seesaw mechanism of neutrino mass generation allows to explain the tiny (LH) neutrino masses using a DM term.

The minimal number of massive neutrinos is equal to the number of the flavor neutrinos that we have observed in the SM tests of the electroweak theory. However, we can add some extra sterile neutrinos that *cannot* be produced in weak processes and so detected by the SM interactions. The most general case would be then

$$\nu_{lL} = \sum_{i=1}^{3+n_s} U_{li}\nu_{iL} \quad \nu_{sL} = \sum_{i=1}^{3+n_s} U_{si}\nu_{sL} \tag{3.46}$$

Here, the PMNS matrix is a $(3+n_s)\times(3+n_s)$ unitary mixing matrix, where $l = e, \mu, \tau$ and $s = s_1 + s_2 + \ldots + s_{n_s}$, being n_s the number of sterile neutrinos.

3.2 The PMNS Matrix

In this section we will study the general features of the PMNS matrix. What have we learned? What are the PMNS matrix elements? As we have conclude from previous sections, flavor eigenstates are linear combinations of mass eigenstates, and the matrix that provides the coefficients *is* precisely the PMNS mixing matrix:

$$\nu_{lL}(x) = \sum_{i} U_{li}\nu_{iL}(x) \tag{3.47}$$

The unitary matrix U is the PMNS matrix and it provides a way to understand the neutrino mixing, well established right now, and it is a mixing matrix irrespectively the Dirac, Majorana or Dirac-Majorana Nature of neutrino fields. However, the concrete mechanism of neutrino mass generation is yet uncovered, and, as we mentioned before there exists some good candidates for such a mechanism (complementary to the SM Higgs mechanism).

3.2.1 Angles and phases

Let us consider a unitary mixing matrix U in the general $n\times n$ case. The unitary matrix can be written $U = e^{iH}$, where H in a hermitian $n\times n$ matrix. A unitary matrix owns the following number of *real* parameters:

$$N_U^{dof} = n + 2\frac{n(n-1)}{2} = n^2 \tag{3.48}$$

The number of angles which characterizes U is the same as the number of parameters of a real orthogonal $n\times n$ matrix O, and since $O = \exp(A)$ with $A^T = -A$, the diagonal elements of the matrix A are equal to zero, and as it is well known, orthogonal matrices have the following number of angles

$$N_\theta = \frac{n(n-1)}{2} \tag{3.49}$$

The other parameter of the unitary matrix U are complex phases. The number of complex phases is equal to

$$N_\phi = n^2 - \frac{n(n-1)}{2} = \frac{n(n+1)}{2} \qquad (3.50)$$

The number of *physical phases* of the mixing matrix *is* generally *smaller* than N_ϕ. The reason is that the PMNS matrix enters into the CC together with fields of charged leptons and neutrinos. The CC reads

$$j_\alpha^{\dagger CC} = 2\sum_{l,i} \bar{l}'_L(x)\gamma_\alpha U'_{li}\nu'_{iL}(x) \qquad (3.51)$$

We can write $U' = S^\dagger(\beta)US(\alpha)$, and thus

$$U = S(\beta)U'S^\dagger(\alpha) \quad S_{ll'}(\beta) = e^{i\beta_l}\delta_{ll'}S_{ik}(\alpha) = e^{i\alpha_i}\delta_{ik} \qquad (3.52)$$

The unitary matrix S can be rearranged:

$$S(\alpha) = e^{i\alpha_n}\begin{pmatrix} e^{i(\alpha_1-\alpha_n)} \\ \vdots \\ 1 \end{pmatrix} \qquad (3.53)$$

and there are $n + n - 1 = 2n - 1$ free parameters (phase differences). These parameters can be chosen in such a way to make $2n - 1$ phases in the matrix U equal to zero. Therefore, the number of physical phases in the PMNS matrix supposing neutrinos are Dirac fields is:

$$N_\phi^D = \frac{n(n+1)}{2} - (2n-1) = \frac{(n-1)(n-2)}{2} \qquad (3.54)$$

For instance, the mixing of two Dirac neutrinos, the mixing matrix is real and there is only a mixing angle and there is no phase. For the interesting case with 3 Dirac neutrinos, we apply the above formulae and deduce that there are 3 mixing angles and one phase.

In the Majorana case, since neutrinos are identical to antineutrinos, but the CC has the same universal expresion than that for Dirac neutrinos:

$$j_\alpha^{\dagger CC} = 2\sum_{l,i} \bar{l}'_L(x)\gamma_\alpha U'_{li}\nu_{iL}(x) \qquad (3.55)$$

The difference comes from the phase counting, since $\nu_i^c = \nu_i$. Now, the n phases entering $e^{i\alpha_n}S(\beta)$ can be absorbed into the fermion fields. Thus, as we know

$$j_\alpha^{CC} = 2\sum_{l,i} \bar{l}_L(x)\gamma_\alpha U_{li}^M\nu_{iL}(x) \quad U^M = US^M(\bar{\alpha}) \qquad (3.56)$$

The non-diagonal elements of S^M are not null. For diagonal elements, the counting of phases changes with respect to the Dirac neutrino case. Now, there are

$$N_\phi^M = \frac{(n-1)(n-2)}{2} + (n-1) = \frac{n(n-1)}{2} \tag{3.57}$$

For 2 Majorana neutrinos would have one mixing angle and one phase (note the difference with 2 Dirac neutrinos). If we have 3 Majorana neutrinos, we would have 3 mixing angles and 3 different physical phases.

3.2.2 Parametrization

Let us consider a unitary 3×3 mixing matrix for Dirac neutrinos. The standard PDG parametrization[2] is given by

$$U^D = U_1 U_2 U_3 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \tag{3.58}$$

where we have defined $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ for $ij = 12, 13, 23$ and where δ is the $\not{CP}$-phase. We also note that, apart from the CP violating phase, this parametrization is identical to that of the Euler angles introduced to describe the orientation of a rigid body. If neutrinos have a Majorana mass term, then we must include the additional Majorana phase matrix in order to obtain the most general parametrization. That is, if neutrino are Majorana particles, the PMNS matrix will be[3]:

$$U^M = U^D S^M(\alpha) \quad \text{with} \quad S^M(\alpha) = \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix} \tag{3.59}$$

Let us stress that if there is CP invariance in the neutrino sector, the above matrix has to be real and, therefore, the complex CP-violating phase has to be zero ($\delta_{\not{CP}} \neq 0$).

In summary, the PMNS matrix[4] is (for active neutrinos):

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{3.60}$$

and it can be rewritten[5] as $U = U^D S^M$.

[2] There are some other possible useful parametrizations, like the so-called FX in [1]

[3] It is also popular the election $S^M = diag(e^{i\alpha_1}, e^{i\alpha_2}, 1)$ since only phase differences are observable.

[4] The object that can be measured with neutrino oscillations.

[5] The Majorana phases, i.e., the Majorana character of neutrino fields *can not* be measured with neutrino oscillation experiments. It requires an extra kind of experiment, such as double-β decay.

3.3 Oscillations in vacuum

In this section, we review the general theory of neutrino oscillations in vacuum. The most general mixing with both active and sterile neutrinos is driven by the mixing matrix equation:

$$\nu_{lL} = \sum_{i=1}^{3+n_s} U_{li}\nu_{iL} \text{ with } l = e, \mu, \tau \qquad \nu_{sL} = \sum_{i=1}^{3+n_s} U_{si}\nu_{iL} \text{ with } s = s_1, \ldots, s_{n_s} \tag{3.61}$$

The neutrino fields ν_i can be Dirac or Majorana with mass m_i. When all the neutrino masses are smaller than the neutrino energy, the neutrino masses can be neglected in the matrix elements of the processes in which neutrinos are produced and for the LH states, we get for active neutrinos with mass m_i and momentum p_i

$$|\nu_l\rangle = \sum_{i=1}^{3+n_s} U_{li}^* |\nu_i\rangle \tag{3.62}$$

and for the sterile neutrino fields

$$|\nu_s\rangle = \sum_{i=1}^{3+n_s} U_{si}^* |\nu_i\rangle \tag{3.63}$$

Remember that sterile neutrinos do not couple or enter into the SM lagrangian and respective currents of the weak interaction. From unitarity, we obtain

$$\langle\nu_s|\nu_l\rangle = 0, \quad \forall s, l \tag{3.64}$$

Thus, the total system of active plus sterile LH states is

$$|\nu_\alpha\rangle = \sum_{i=1}^{3+n_s} U_{\alpha i}^* |\nu_i\rangle \quad \text{with} \quad l = e, \mu, \tau, s_1, \ldots, s_{n_s} \tag{3.65}$$

and it verifies

$$\langle\nu_{\alpha'}|\nu_\alpha\rangle = \sum_{i=1}^{3+n_s} U_{\alpha' i}U_{\alpha i}^* = \delta_{\alpha'\alpha} \tag{3.66}$$

If the neutrino initial state at $t =$ is prepared in the *flavor* state $|\nu_\alpha\rangle$, then, at time $t \geq 0$, Quantum Mechanics says that the state evolves accordingly to a hamiltonian (unitary) transformation:

$$|\nu_\alpha(t)\rangle = e^{-iHt} |\nu_\alpha\rangle = \sum_{\alpha'} |\nu_{\alpha'}\rangle \langle\nu_{\alpha'}| e^{-iHt} |\nu_\alpha\rangle \tag{3.67}$$

from which we can read the amplitude as a function of time t

$$A(\nu_\alpha \to \nu_{\alpha'})(t) = \langle \nu_{\alpha'} | \, e^{-iHt} \, | \nu_\alpha \rangle = \sum_{i=1}^{3+n_s} U_{\alpha' i} e^{-iE_i t} U^*_{\alpha i} \tag{3.68}$$

with E_i being the neutrino eigenenergy for the i-th mass state. In an analogue way, for antineutrinos, the same procedure yields

$$A(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'})(t) = \langle \bar{\nu}_{\alpha'} | \, e^{-iHt} \, | \bar{\nu}_\alpha \rangle = \sum_{i=1}^{3+n_s} U^*_{\alpha' i} e^{-iE_i t} U_{\alpha i} \tag{3.69}$$

As a consequence, the probabilities of the transition between neutrinos and antineutrinos can be calculated to be:

$$P\left(\nu_\alpha \to \nu_{\alpha'}\right) = \left| \sum_{i=1}^{3+n_s} U_{\alpha' i} e^{-iE_i t} U^*_{\alpha i} \right|^2 \quad P\left(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}\right) = \left| \sum_{i=1}^{3+n_s} U^*_{\alpha' i} e^{-iE_i t} U_{\alpha i} \right|^2 \tag{3.70}$$

These transition probabilities are normalized with our present conventions, since

$$\sum_{\alpha'} P\left(\nu_\alpha \to \nu_{\alpha'}\right) = \sum_{\alpha', i, k} U_{\alpha' i} U^*_{\alpha' k} e^{-i(E_i - E_k)t} U^*_{\alpha i} U_{\alpha k} = \sum_i |U_{\alpha i}|^2 = 1 \tag{3.71}$$

Sterile neutrinos can not be detected in SM weak processes, so if we want to obtain information about transitions into sterile states, we have to measure the following quantity:

$$\sum_{l'=e,\mu,\tau} P(\nu_l \to \nu_{l'}) = 1 - \sum_{s=s_1,\ldots,s_n} P(\nu_l \to \nu_s) \tag{3.72}$$

If this observable is different from the unit at some distance from the source, by the observation of a NC event, since the LH side is the total probability of the transition of a light flavor neutrino into every possible flavor active neutrinos (ν_e, ν_μ, ν_τ), it would be an experimental proof of the sterile neutrino existence. The different energies can be written in the case of neutrinos (ultrarelativistic particles) as

$$E_k = \sqrt{p^2 + m_i^2} \simeq E_k + \frac{m_k^2}{2p} \tag{3.73}$$

and thus, the difference between energies, as $E \simeq p$ is the neutrino energy, will read

$$\Delta E = E_k - E_i \simeq \frac{\Delta m^2_{ki}}{2E} \tag{3.74}$$

where the squared mass difference between (mass) states are

$$\Delta m^2_{ki} = m_i^2 - m_k^2 \tag{3.75}$$

The same ultrarelativist approximation provides $t \simeq x = L$, where L is the distance between the neutrino source and the detection point. So, gathering these last results together, we find that

$$(E_k - E_i)\, t \simeq \frac{\Delta m^2_{ki}}{2}\frac{L}{E} = \frac{\Delta m^2_{ki}}{2E}L \tag{3.76}$$

This important relationship allows us to understand better the neutrino oscillation experiments and their types. Different kinds of neutrino mixing experiments arise from the different ratios L/E between the travelled distance and the neutrino energy. Using all this stuff, we can derive the following relationships from the PMNS matrix unitarity:

$$P(\nu_\alpha \to \nu_{\alpha'}) = \left|\delta_{\alpha\alpha'} + \sum_{i\neq j} U_{\alpha' i}\left(e^{-i\frac{\Delta m^2_{ji}}{2E}} - 1\right)U^*_{\alpha i}\right|^2 \tag{3.77}$$

$$P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) = \left|\delta_{\alpha\alpha'} + \sum_{i\neq j} U^*_{\alpha' i}\left(e^{-i\frac{\Delta m^2_{ji}}{2E}} - 1\right)U_{\alpha i}\right|^2 \tag{3.78}$$

or equivalently, using againg unitarity and the identity $Re(ab) = (Rea)\,(Reb) - (Ima)\,(Imb)$, we would get

$$\begin{aligned} P(\nu_\alpha \to \nu_{\alpha'}) = \delta_{\alpha'\alpha} - 4\sum_{i>k}\Re\left(U_{\alpha' i}U^*_{\alpha' k}U^*_{\alpha i}U_{\alpha k}\right)\left(\sin^2\frac{\Delta m^2_{ji}}{4E}L\right) \\ +2\sum_{i>k}\Im\left(U_{\alpha' i}U^*_{\alpha' k}U^*_{\alpha i}U_{\alpha k}\right)\left(\sin\frac{\Delta m^2_{ji}}{2E}L\right) \end{aligned} \tag{3.79}$$

$$\begin{aligned} P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) = \delta_{\alpha'\alpha} - 4\sum_{i>k}\Re\left(U_{\alpha' i}U^*_{\alpha' k}U^*_{\alpha i}U_{\alpha k}\right)\left(\sin^2\frac{\Delta m^2_{ji}}{4E}L\right) \\ -2\sum_{i>k}\Im\left(U_{\alpha' i}U^*_{\alpha' k}U^*_{\alpha i}U_{\alpha k}\right)\left(\sin\frac{\Delta m^2_{ji}}{2E}L\right) \end{aligned} \tag{3.80}$$

We must mention the commonly used numerical relationships for the trigonometric argument in the probability formulae for neutrino oscillations:

$$\frac{\Delta m^2_{ji}}{2E}L = \frac{c^4}{\hbar c}\frac{\Delta m^2_{ji}}{2E}L = 1.267\frac{\Delta m^2_{ji}}{1\text{eV}^2}\frac{L}{1\text{km}}\frac{1\text{GeV}}{E} = 1.267\frac{\Delta m^2_{ji}}{1\text{eV}^2}\frac{L}{1\text{m}}\frac{1\text{MeV}}{E} \tag{3.81}$$

and the so-called oscillation length for neutrinos is defined as

$$L_{osc} = \lambda_{osc} = 4\pi\frac{E}{\Delta m^2} = 4\pi\frac{E\hbar c}{c^4\Delta m^2} = 2.47\frac{E}{\Delta m^2}\ \text{m} \tag{3.82}$$

Like decays, oscillations are suppressed at large energy by "time-dilation" Lorentz factors m/E from special relativity. In order to observer neutrino mixing, one requires neutrinos of low enough energy with small cross-sections. Furthermore, some reactions can be kinematically allowed only at high enough energies. For instance, ν_τ scattered by the process $\nu_\tau e \to \nu_e \tau$ needs $E_{\nu_\tau} > m^2_\tau/2m_e \approx 3$TeV while $\nu_\tau n \to p\tau$ only requires $E_{\nu_\tau} > m_\tau + m^2_\tau/2m_n \approx 3.5$GeV.

The general oscillation neutrino formulae (3.70) satisfy certain relationships provided unitarity and/or CPT invariance hold (and these are very satisfactory properties in the local QFT framework):

- PMNS unitarity implies:

$$\sum_{l'} P\left(\nu_l \to \nu_{l'}\right) = 1 \quad \sum_{l'} P\left(\bar{\nu}_l \to \bar{\nu}_{l'}\right) = 1 \quad \sum_{l} P\left(\nu_l \to \nu_{l'}\right) = 1 \quad \sum_{l} P\left(\bar{\nu}_l \to \bar{\nu}_{l'}\right) = 1 \tag{3.83}$$

- PMNS matrix,with CPT invariance, implies for survival probabilities:

$$P\left(\nu_l \to \nu_l\right) = P\left(\bar{\nu}_l \to \bar{\nu}_l\right) \tag{3.84}$$

- CP invariance for PMNS matrix means:

$$P\left(\nu_l \to \nu_{l'}\right) = P\left(\bar{\nu}_l \to \bar{\nu}_{l'}\right) \tag{3.85}$$

In this last case, we would get a real PMNS matrix (up to a sign in the case of Majorana neutrinos), i.e., $U_{li} = U^*_{li}$ for Dirac neutrinos or $U_{li} = \pm U^*_{li}$ for Majorana.

3.3.1 The 2 flavor case

The above formalism is specially simple in the two flavor case, the leading term in many experiments. Due to its intrinsic importance, we study this case in some detail here. The mixing matrix reads:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{3.86}$$

Then the probability of a neutrino changing its flavor is

$$P_{\alpha\to\beta,\alpha\neq\beta} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)\ (\text{natural units}) \tag{3.87}$$

Or, using SI units and the convention introduced in this section

$$P_{\alpha\to\beta,\alpha\neq\beta} = \sin^2(2\theta)\,\sin^2\left(1.267\frac{\Delta m^2 L}{E}\frac{\text{GeV}}{\text{eV}^2\,\text{km}}\right) \tag{3.88}$$

This formula is often appropriate for discussing the transition $\nu_\mu \to \nu_\tau$ in atmospheric mixing , since the electron neutrino plays almost no role in this case (or its influence is neglected). It is also appropriate for the solar case of $\nu_e \to \nu_X$, where ν_X is a superposition of ν_μ and ν_τ. These approximations are possible because the mixing angle θ_{13} is very small and because two of the mass states are very close in mass compared to the third.

3.3.2 3 flavor formulae without $\not{CP}$

In the 3 flavor case, neutrino oscillation formulae exist but they are very messy. In order to mantain simple this example, we will consider the CP conserving case. The PMNS matrix is real. Thus, we can derive the following equation:

$$P_{\nu_l \nu_{l'}}(L,E) = \delta_{ll'} - 4 \sum_{i>j} U_{li} U_{l'i} U_{lj} U_{l'j} \sin^2 \left(\frac{\Delta m^2_{ij}}{4E} L \right) \tag{3.89}$$

This formula is yet general in the sense it can be applied to *any* number of generations with CP conservation. For three families, we can obtain some useful limits.

- **One mass difference is very small**. For instance,

$$L \frac{\Delta m^2_{12}}{2E} << 1 \tag{3.90}$$

  In this particular case, that mass difference is neglected and, without sake of generality, we can assume that $\Delta m^2_{32} = \Delta m^2_{31}$, and the probability simplifies to

$$P_{\nu_l \nu_{l'}}(L,E) = \delta_{ll'} - 4 U_{l3} U_{l'3} \left(\delta_{ll'} - U_{l3} U_{l'3} \right) \sin^2 \left(\frac{\Delta m^2_{31}}{4E} L \right) \tag{3.91}$$

  This equation is very similar to the 2 flavor example when there are different flavors, i.e., when $l \neq l'$. Using the standard PMSN parametrization, for electron neutrinos we would get the survival probability

$$P_{\nu_e \nu_e}(L,E) = 1 - \sin^2 2\theta_{13} \, \sin^2 \left(\frac{\Delta m^2_{32} L}{4E} \right) \tag{3.92}$$

- **Two very large mass differences**. That is,

$$L \frac{\Delta m^2_{32}}{2E} >> 1, \quad L \frac{\Delta m^2_{31}}{2E} >> 1 \tag{3.93}$$

  In this special case, the oscillatory terms with these arguments have to be averaged out when one integrates over the energy spectrum of the initial state (or beam). We get,

$$P_{\nu_l \nu_{l'}} = \delta_{ll'} - 2 U_{l3} U_{l'3} \left(\delta_{ll'} - U_{l3} U_{l'3} \right) - 4 U_{l1} U_{l'1} U_{l2} U_{l'2} \sin^2 \left(\frac{\Delta m^2_{21} L}{4E} \right) \tag{3.94}$$

  and the survival probability for electron neutrinos is

$$P_{\nu_e \nu_e}(L,E) = \cos^4 \theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{21} L}{4E} \right] + \sin^4 \theta_{13} \tag{3.95}$$

  If $\theta_{13} \to 0$, we recover again the two flavor formula.

3.3.3 Some frequently used neutrino oscillation formulae

Here we review 4 very used formulae in 4 main types of neutrino oscillation experiments. In the standard parametrization, the $\nu_\mu \to \nu_\tau$ oscillation probability, which describes atmospheric neutrino [48, 49, 50, 51, 52, 53] disappearance, can be expressed with the survival probability:

$$P(\nu_\mu \to \nu_\mu) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{23}\sin^2\left(\frac{\Delta m^2_{23}}{4E}L\right) \tag{3.96}$$

The solar (electron) neutrino disappearance, which has been further confirmed by the KamLand reactor (electron anti-)neutrino experiment (with average baseline 100 km), can be expressed as:

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{12}\sin^2\left(\frac{\Delta m^2_{12}}{4E}L\right) \tag{3.97}$$

The measurements in the solar and atmospheric sectors have shown that the mixing angles θ_{12} and θ_{23} are large, but there remains one mixing angle which has not been determined yet (although hints have been unveiled this year), θ_{13} . This angle would be manifest by electron neutrino disappearance a few kilometers from a reactor, or electron neutrino appearance in a few-GeV ν_μ beam a several hundred kilometers from an accelerator. In the latter case the oscillation probability in vacuum is

$$P(\nu_\mu \to \nu_e) = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{32}}{4E}L\right) + O\left(\frac{\Delta m^2_{12}}{\Delta m^2_{23}}\right) \tag{3.98}$$

and in the former (reactor) case, in the leading approximation too,

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{31}}{4E}L\right) \tag{3.99}$$

3.3.4 3 flavor $\not{CP}$ term

CP symmetry interchanges neutrinos with negative helicity and antineutrinos with positive helicity, transforming the $\nu_\alpha \to \nu_{\alpha'}$ channel into the $\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}$ channel. In the case of three-neutrino mixing, the mixing matrix is, in general, complex and leads to violations of the CP symmetry. Such violations can be revealed in neutrino oscillation experiments by measuring the CP asymmetry $A^{CP} = P(\nu_\alpha \to \nu_{\alpha'}) - P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'})$. CPT symmetry implies that the CP asymmetry is antisymmetric in the flavor indices $A^{CP}_{\alpha\alpha'} = -A^{CP}_{\alpha'\alpha}$, because the CPT relation. Hence, it is clear that a CP asymmetry can be measured only in transitions between *different* flavors. The oscillation probabilities of neutrinos and antineutrinos are related by complex conjugation of the elements of the mixing matrix. One realizes that they differ only in the sign of the terms depending on the imaginary parts of the quartic products of the elements of the mixing matrix. Thus, only these terms contribute to the CP asymmetry, leading to the celebrated equation:

$$A^{CP}_{\alpha\alpha'} = 4\sum_{k>i}\Im\left(U^*_{\alpha k}U_{\alpha' k}U_{\alpha i}U^*_{\alpha' i}\right)\sin\left(\frac{\Delta m^2_{ki}}{2E}L\right) \tag{3.100}$$

This asymmetry, like the correponding T violating one, is proportional to the famous Jarlskog invariant

$$J = \sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2\sin\delta \tag{3.101}$$

3.3.5 Decay effects

If there are some unstable neutrinos (active and/or sterile) that could decay or annihilate with rate $\Gamma_j = cn_j\sigma_j$, the oscillation formulae are modified with damping terms (exponentials). A convenient formalism of this kind of flavor mixing is given by the density matrix. At the initial time we have

$$\rho(t=0) = \rho_0 = \rho(0) = \sum_\beta w_\beta \left|\nu_\beta\right\rangle \left\langle\nu_\beta\right| \tag{3.102}$$

where the weights w_β represent the proportion in which the different flavors are combined, with $\sum_\beta w_\beta = 1$. ρ_0 is normalized according to the normal condition for density matrices $\mathrm{Tr}(\rho_0) = 1$ and describes the ensemble (averaged) neutrino compositeness. The main result of unstable neutrinos or neutrinos that can annihilate at certain rate is to introduce some decoherence (i.e. destroyed cohererence) and writing the density matrix in the eigenmass basis allows to include the effect of resonant absorption or decays. Let us write $\left|\nu_\beta\right\rangle = U_{\beta j}\left|\nu_j\right\rangle$, i.e., $U_{\beta j} = \langle\nu_j|\nu_\beta\rangle = \langle\nu_\beta|\nu_j\rangle^*$. Then, after a density matrix time evolution, we would get

$$\rho(t) = \sum_\beta w_\beta \sum_j \left[e^{-i\frac{m_j^2}{2E}}e^{-\frac{\Gamma_j}{2}t}U_{\beta j}\left|\nu_j\right\rangle\right]\sum_k\left[\left\langle\nu_k\right| U^*_{\beta k}e^{+i\frac{m_k^2}{2E}}e^{-\frac{\Gamma_k}{2}t}\right] \tag{3.103}$$

or

$$\rho(t) = \sum_\beta w_\beta \sum_{j,k} U_{\beta j}U^*_{\beta k}e^{+i\frac{\Delta m^2_{kj}}{2E}t}e^{-\frac{\Gamma_j+\Gamma_k}{2}}\left|\nu_j\right\rangle\left\langle\nu_k\right| \tag{3.104}$$

This decay or damping effect is specially useful considering models with annihilation or neutrino absorption in flight, and it is also useful in asphophysics or cosmology [6]. When this neutrino mixture arrives to Earth, it can be detected. The probability to detect some particular flavor α will be:

$$P^{det}_{\nu_\alpha} = \langle\nu_\alpha|\rho(t)|\nu_\alpha\rangle = \sum_\beta w_\beta \sum_{j,k} U_{\beta j}U^*_{\beta k}U_{\alpha k}U^*_{\alpha j}e^{+i\frac{\Delta m^2_{kj}}{2E}t}e^{\frac{-\Gamma_j+\Gamma_k}{2}} \tag{3.105}$$

This equation can be separated into two pieces, one giving the diagonal probabilities and the second giving the interference terms, that is:

$$P^{det}_{\nu_\alpha} = P^{diag}_{\nu_\alpha} + P^{int}_{\nu_\alpha} \tag{3.106}$$

[6] It is straightforward to include the effect of redshift in the final formulae for cosmological applications.

with

$$P_{\nu_\alpha}^{diag} = \sum_\beta w_\beta \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2 e^{-\Gamma_j t} \qquad P_{\nu_\alpha}^{int} = 2 \sum_\beta w_\beta \sum_{j<k} e^{-\frac{\Gamma_j + \Gamma_k}{2}} A_{\beta jk} \tag{3.107}$$

where

$$A_{\beta jk} = \left[\Re\left(U_{\beta j} U_{\beta k}^* U_{\alpha k} U_{\alpha j}^*\right) \cos\left(\frac{\Delta m_{kj}^2}{2E} t\right) - \Im\left(U_{\beta j} U_{\beta k}^* U_{\alpha k} U_{\alpha j}^*\right) \sin\left(\frac{\Delta m_{kj}^2}{2E} t\right)\right] \tag{3.108}$$

The averaging procedure over time of this equation produces:

$$P_{\nu_\alpha}^{det} = \sum_j |U_{\alpha j}|^2 e^{-\Gamma_j t} \sum_\beta w_\beta |U_{\beta j}|^2 \to P_{\nu_\alpha}^{det} = \frac{1}{3} \sum_j |U_{\alpha j}|^2 e^{-\Gamma_j t} \to P_{\nu_\alpha}^{det} = \frac{1}{3} \sum_j e^{-\Gamma_j t} \tag{3.109}$$

3.3.6 Coherence and wave packet effect

A complete treatment of the beautiful and subtle issue of coherence and localization of neutrinos, and the wave-packet formalism is beyond the scope of this thesis. However, due to its intrinsic importance we write the effect of coherence through the formula (see,e.g., [20]):

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_{k,j} U_{\alpha k}^* U_{\alpha j} U_{\beta k} U_{\beta j}^* \exp\left(-2\pi i \frac{L}{L_{kj}^{osc}} - \left(\frac{L^{osc}}{L_{kj}^{coh}}\right)^2\right) \tag{3.110}$$

where the oscillation lengths are the known expressions, and the (new) coherence lengths are defined as

$$L_{kj}^{coh} = \frac{4\sqrt{2}E^2}{|\Delta m_{kj}^2|} \sigma_x \tag{3.111}$$

This means that in order to measure the interference of the massive neutrino components ν_k and ν_j the production and detection processes must be localized in space-time regions much smaller than the oscillation length L^{osc}. For all practical purposes, this requirement is satisfied in all neutrino oscillation experiments.

Other interesting effects on the basic neutrin oscillation formulae are those caused by new physics. For instance extra dimensions are suggested as a method to explain the reactor anomaly instead of sterile neutrinos in the framework of [114].

3.4 Oscillations in matter

The previous section, oscillation in vaccum, formalism applies to neutrinos passing through vacuum. In the presence of matter the neutrinos acquire effective masses from coherent scattering processes. This effect was anticipated by Mikheyev-Smirnov-Wolfenstein and it is called MSW-effect. In particular, coherent forward scattering $\nu_e e^- \to \nu_e e^-$ via the charged current amplitude differentiates electron neutrinos from the other neutrinos. The coherent forward scattering is analogous to the electromagnetic process leading to the refractive index of light in a medium.

In many important cases (e.g. the Earth mantle, the sun), neutrinos travel through matter with certain density. This density can be approximately constant, slowly time-varying (i.e.adiabatic), non-uniform (non-adiabatic),... Matter density is usually dependent on space and time. Some common cases include the neutrinos crossing through the mantle (density is almost aproximately constant there) or the solar matter (position dependent in general).

3.4.1 Oscillations in normal matter

Neutrinos with ordinary energies cross the Earth or the sun without being absorbed in general. However, matter affects the neutrino oscillation pattern. Forward scattered neutrinos interfere with free neutrino propagation, allowing refraction (similar to that of light in water or air). At low energy, this scattering can be described by the effective hamiltonian

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} \left(\bar{\nu}_e \gamma_\mu P_L \nu_e\right) \left(\bar{e}\gamma^\mu P_L e\right) \tag{3.112}$$

If the background is composed by non-relativistic and non-polarized electrons and no positrons, one can easily derive

$$\langle \mathcal{H}_{eff} \rangle = \sqrt{2} G_F N_e \left(\bar{\nu}_e \gamma_0 P_L \nu_e\right) \tag{3.113}$$

where N_e is the electron number density in the matter. Including the Z-contribution, the hamiltoninan density in ordinary matter reads

$$\langle \mathcal{H}_{eff} \rangle = \bar{\nu}_l V_m \gamma_0 P_L \nu_l \tag{3.114}$$

with the "matter potential"

$$V_m = \sqrt{2} G_F \left[N_e diag(1,0,0) - \frac{N_n}{2} diag(1,1,1) \right] \tag{3.115}$$

This matter potential is a 3x3 flavour matrix (if extra sterile neutrinos exist, it increases its diagonal size). Adding the matter correction to the Hamiltonian density at vacuum, in the ultrarelativistic approximation, one gets a modified dispersion relation between energy and momentum. Moreover, this effect is also independent of the Dirac or Majorana character of neutrinos [45].There, it is obtained that oscillations in matter of ultrarelativistic neutrinos are described by the differential equation

$$i\frac{d}{dx}\nu = H\nu \quad H = \frac{m \cdot m^\dagger}{2E} + V_m, \quad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \tag{3.116}$$

This equation can be solved starting from the production point what flavor/s is/are there produced. Moreover, $m \cdot m^\dagger = U^* diag(m_1^2, m_2^2, m_3^2) U^T$, where U is the PMNS matrix and the m_i the neutrino mass eigenvalues. For antineutrinos, we shift $m \to m^*$ and $V_m \to -V_m$, loosing CP invariance (this $\not{CP}$ is matter induced and it is different from fundamental $\not{CP}$ in vaccum). If we face Majorana neutrinos, the m matrix will be real.

We can focus on the case with two flavors and constant density. We define effective-dependent neutrino mass eigenvalues m_m^2, eigenvectors in matter ν_m and mixing angle θ_m if we diagonalize the matter hamiltonian $H = H_0 + V_m$. In this simple example, with only ν_e and ν_μ flavors with mixing θ_m, we write

$$mm^\dagger = \frac{m_1^2 + m_2^2}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{\Delta m^2}{2} \begin{pmatrix} -\cos 2\theta & \sin \theta 2 \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \tag{3.117}$$

where $\Delta m^2 = m_1^2 + m_2^2$ so the oscillation parameters in matter are defined as follows

$$\mathbf{S} = \Delta m^2 \sin 2\theta; \quad \mathbf{C} = \Delta m^2 \cos 2\theta \mp 2\sqrt{2} G_F N_e E \tag{3.118}$$

$$\Delta m_m^2 = \sqrt{S^2 + C^2}; \quad \tan 2\theta_m = \frac{S}{C} \tag{3.119}$$

where the $\mp$ sign refers to ν(resp.$\bar{\nu}$). This simple example allows us to point out the main differences with respect to oscillation in vaccum. Firstly, **Oscillations in matter** *distinguish* **between** θ **and** $\pi/2 - \theta$. Therefore, $\sin^2 2\theta$ is not a well-posed experimental variable. The common practice is to represent $\tan^2 \theta$ in a logarithmic scale plot and $\sin^2 \theta$ in a linear scale plot. The most useful variables are then

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\lambda^2} \qquad \Delta m_m^2 = \lambda \cdot \Delta m^2 \tag{3.120}$$

$$\lambda = \sqrt{\sin^2 2\theta + \left(\cos^2 2\theta \mp \frac{2\sqrt{2} G_F N_e E}{\Delta m^2} \right)} \tag{3.121}$$

Secondly, we have a **resonance condition**. The oscillation amplitude *can be maximal* irrespectively of the vacuum mixing angle value θ. This happens whenever

$$\sqrt{2} G_F N_e \mp \frac{\Delta m^2}{2E} \cos 2\theta = 0 \Longrightarrow \sin^2 \theta_m = 0 \to \Delta m_m^2 = \Delta m^2 \sin 2\theta \tag{3.122}$$

We can estimate the resonance energy

$$E_\nu \sim \frac{\Delta m^2}{\sqrt{2} G_F N_e} = 3\text{GeV} \frac{\Delta m^2}{10^{-3}\text{eV}^2} \frac{1.5\text{g/cm}^3}{\rho Y_e} \tag{3.123}$$

Finally, in some cases, **oscillations are completely dominated by matter**. If neutrinos have high enough energy, the matter term dominates over the oscillation in vaccum since it is flavor diagonal, and hence, it erases (vacuum) oscillations. In this approximation, neglecting oscillation in vacuum effects, neutrino mixing in matter has an energy-independent wavelength $\lambda = \pi / \sqrt{2} G_F N_e$. For the Earth mantle we have $\lambda \sim 3000\text{km} \simeq R_\oplus/2$, i.e.,about one-half of the Earth radius. In the more interesting case of astrophysical bodies (the sun, supernovae,...), neutrinos are produced and they arrive as an incoherent (in general) mixtures of mass eigenstates. These neutrinos are detected crossing the Earth through the mantle of the atmosphere and the probability functions are peaked in the resonance energy. It is important to note that, in the case of atmospheric neutrinos, their effect are important only if $\theta_{13} > 0$.

3.4.2 Oscilations in varying density

In order to study neutrinos coming from the sun, other stars or more interestingly from supernovae/hypernovae, it is necessary to develop the approximation for the core neutrinos(those produced in the core of the star where matter dominates) and then they scape into vacuum(where matter effects are neglected), taking into account that, in some intermediate points, we can have resonant (MSW) effects. We will focus on the solar neutrinos for space reasons. Solar neutrinos have the following properties:

- ν_e are produced in the sun core, $r \approx 0$. In the core, $P(\nu_{1m}) = \cos^2\theta_m$ and $P(\nu_{2m}) = \sin^2\theta_m$ are valid. When matter dominates, we get $\nu_e \simeq \nu_{2m}$ due to $\sin^2\theta_m = 1$.
- Solar radius satisfies $R_\odot \gg \lambda_{osc}$, the oscillation wavelength is tiny in comparison with solar size. Then, neutrinos travel for many oscillation wavelengths and the phase averages out combining probabilities, instead of amplitudes. Moreover, the so-called adiabatic approximation is often used. It applies when the density changes very slowly in such a way that each neutrino mass eigenstate will remain the same along its path. Otherwise, the neutrinos will flip to other mass eigenstate with some probability, called "level-crossing" probability P_C. Thus:

$$\nu_{2m}(r\approx 0)\ \boxed{\text{adiabatically flips to}} \begin{cases} \nu_{2m} = \nu_2 \ \text{ with } \ P_2 = 1 - P_C \\ \nu_{2m} = \nu_2 \ \text{ with } \ P_1 = P_C \end{cases} \tag{3.124}$$

- Neutrinos from the sun to the Earth can cross the earth mantle before reaching the detector point. We will consider these effects neglectable.
- ν_1 and ν_2 are respectively detected with probabilites $\cos^2\theta$ and $\sin^2\theta$.

Assemblying all these results, we form the survival probability for electron neutrinos

$$P(\nu_e \to \nu_e) = \frac{1}{2} - \left(\frac{1}{2} - P_C\right)\cos 2\theta \cos 2\theta_m \tag{3.125}$$

Some limits of this formular are popular:

- **Averaged vacuum oscillations.** Matter effects are negligible for $\theta_m = 0, P_C = 0$. This is the case when solar neutrinos have low enough energy. $P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$.
- **Dominant matter effects** with electron neutrino being the heaviest mass eigenstate. It happens if $\cos 2\theta_m = -1$ and $\theta \ll 1$.
- **Adiabatic propagation** (valid for solar neutrinos at high enough energies). $P_C = 0$ and $\cos 2\theta_m = -1$.
- **Non-adiabatic limit.** $P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$ when $\cos\theta_m = -1$ and $P_C = \cos^2\theta$.

In order to calculate the crossing probability, a more careful treatment of adiabaticity has to be done. It shows that the resonance width and the density profile approximation are essential to get some adiabaticity conditions. See [45] for additional approximations and details.

3.4.3 Density matrix and matter oscillations

In the general and more complex 3 flavor (or even with extra generations) neutrino species, the density matrix formalism is a very simple tool that simplifies hard calculations. The "averaged" oscillation case can be handled then in this framework. To extend the previous results to 3 active flavors, in the solar neutrino case, we take the density matrix at the sum to be:

$$\rho_S^m = diag \begin{pmatrix} 1-P_C & P_C & 0 \\ P_C & 1-P_C & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |V_{e1}^m|^2 \\ |V_{e2}^m|^2 \\ |V_{e3}^m|^2 \end{pmatrix} \tag{3.126}$$

so

$$\rho_S^m = diag \left(\cos^2\theta_{13} \left[\frac{1}{2} - \left(\frac{1}{2} - P_C \right) \cos 2\theta_{12}^m \right], \cos^2\theta_{13} \left[\frac{1}{2} - \left(\frac{1}{2} - P_C \right) \cos 2\theta_{12}^m \right], \sin^2\theta_{13} \right) \tag{3.127}$$

This equation holds in vacuum where matter eigenstates are equal to vacuum eigenstates. In the last equation above, we could write $\theta_{13}^m \simeq \theta_{13}$ since matter effects are not important for the "atmospheric" mixing effect of θ_{13}. To obtain a closed equation for probabilities, we consider that solar neutrinos detected during the day *do not* cross the Earth planet, and thus

$$P(\nu_e \to \nu_e)_{day} = Tr(\Pi_e \rho_S) = \rho_S^{ee} = \sin^4\theta_{13} + \cos^4\theta_{13} \left[\frac{1}{2} + \left(\frac{1}{2} - P_C \right) \cos 2\theta_{12} \cos 2\theta_{12}^m \right] \tag{3.128}$$

During the night, the density matrix can also be calculated if we consider that the density matrix at the detector of a mixed neutrino beam becomes $\rho_E = U\rho_S U^\dagger$. However, the evolution is computed commonly numerically performing an average procedure (see [45, 20]). Taking solar neutrino energis below 10 MeV, the following formula is deduced:

$$P(\nu_e \to \nu_e)_{night} = P(\nu_e \to \nu_e)_{day} + \frac{P_{2e} - c_{13}^2 s_{12}^2}{c_{13}^2 \cos 2\theta_{12}} \left(1 - 2P(\nu_e \to \nu_e)_{day} - 2s_{13}^2 + 3s_{13}^4 \right) \tag{3.129}$$

This equation is called the *earth regeneration formula* for solar neutrinos. Note that we need to compute the function $P_{2e}(E)$ taking into account the neutrino path inside our planet. Neglecting matter effects we would recover the "day" formula since $P_{2e} = c_{13}^2 s_{12}^2$ in that case.

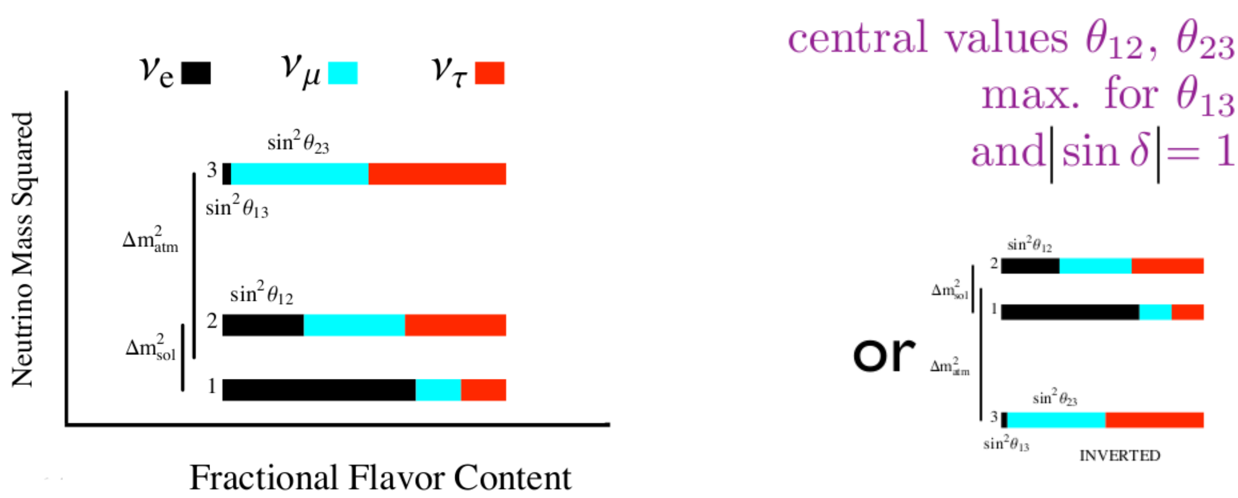

Figure 3.1: Mass-squared vs.fractional flavor content plot(I). It shows the fractional flavor content of neutrino mass eigenstates and the neutrino spectrum type (hierarchical here). The picture supposes a maximal mixing between two species and a tiny, almost null, mixing with the remaining neutrino flavor according to the neutrino atmospheric experiments.

3.5 The Neutrino Spectrum

The neutrino spectrum ordering and absolute scale is yet completely unknown, unlikely that of quarks and charged leptons in the SM. However, neutrino mixing implies that at least one neutrino is massive. Neutrino spectrum can be classified intro hierarchical or degenerated (this sometimes called quasi-detenerated). Furthermore, it can be “normal” or “inverted”. Usually, though, only 3 main different classes are exposed:

1. Normal hierarchy. $m_1 < m_2 \ll m_3$
2. Inverted hierarchy. $m_1 \ll m_2 < m_3$
3. Quasidegeneracy. $m_1 \simeq m_2 \simeq m_3$, with $m_1(m_3) \gg \sqrt{\Delta m^2_{23}}\left(\sqrt{\Delta m^2_{13}}\right)$

A common technique used to present results from neutrino mixing and thus, the effects that these experiments have, is the mass-squared vs. (fractional)flavor content plot. It consists on a frame representing:

- The mass-splittings obtained in the different experiments in the vertical axis, usually in eV.
- The fractional flavor content of the different mass eigenstates. In general, in a two mixing scheme, the mass eigenstate $|\nu_i\rangle = \sum_i U_{ij}\,|j\rangle = \cos\theta\,|\nu_\alpha\rangle + \sin\theta\,|\nu_\beta\rangle$, and hence the fractional flavor content is proportional to the $\sin^2\theta$ and $\cos^2\theta$.

In other words, this frame representation is a very visual way to represent a complex linear combination between three dimensional flavor and mass eigenstates, and their respective change of base (the PMNS matrix), including the relative mass scale.

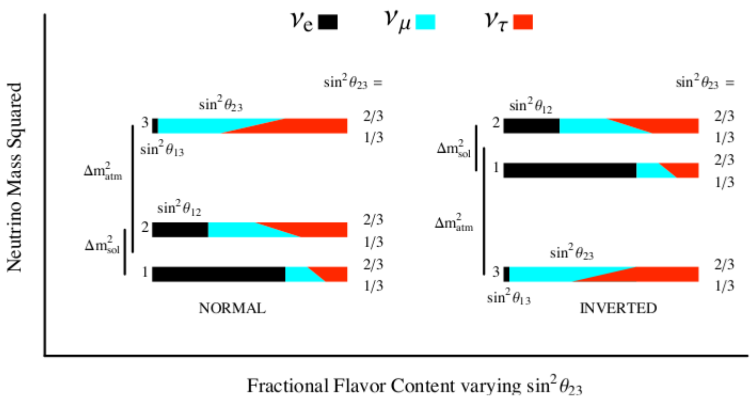

Figure 3.2: Mass-squared vs.fractional flavor content plot (II). The picture shows a non-maximal scenario not excluded by the data.

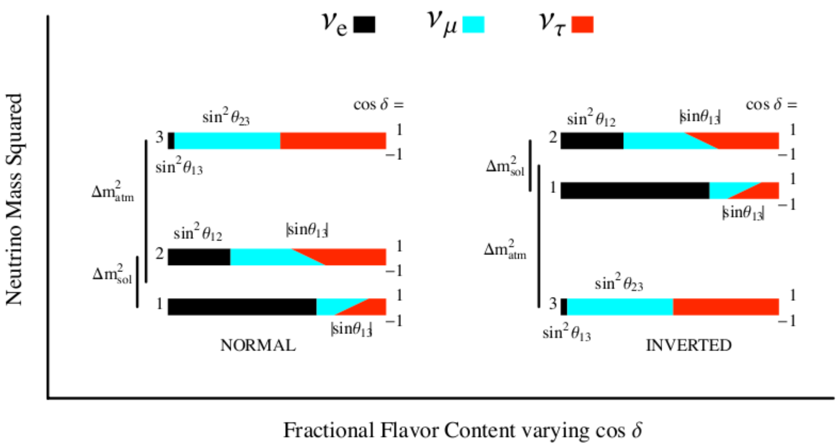

Figure 3.3: Mass-squared vs.fractional flavor content plot(III). The picture represents the effect of the CP violating phase on the spectrum.

Chapter 4

Neutrino Experiments

The usual classification of the different types of neutrino experiments. As we have anticipated before, there are three main categories: *neutrino oscillation experiments* and *double beta decay experiments* $\beta_{0\nu ee}$ are the main two. Some good references focused in the experimental results from neutrino experiments are [85, 86, 87, 88, 89]

4.1 Oscillation Experiments

Neutrino oscillation experiments can be [90, 91, 92, 93, 95, 96, 97, 98, 99]are divided into:

- **Appearance experiments.** These experiments *measure* **transitions** between *different neutrino flavors.* If the final flavor to be searched for in the detector is not present in the initial beam, the background can be very small. In this case, an experiment can be sensitive to rather small values of the mixing angle.
- **Disappearance experiments.** These experiments *measure* **the survival probability** of a *neutrino flavor by counting* the number of interactions in the detector and comparing it with the expected one. Since, even in the absence of oscillations, the number of detected events has statistical fluctuations, it is very difficult to reveal a small disappearance. Therefore, in this type of experiment, it is hard to measure small values of the mixing angle.

Focusing on two neutrino mixing, an important characteristic of neutrino oscillations is, as we can understand from the oscillation wavelenth, that the transitions to different flavors cannot be measured if

$$\frac{L\Delta m^2}{2E} \ll 1 \tag{4.1}$$

On the other hand, whether

$$\frac{L\Delta m^2}{2E} \gg 1 \tag{4.2}$$

only the average transition probability is observable, yielding information only on $\sin^2 2\theta$. Since the value of Δm^2 is fixed by nature, different experiments can be designed in order to be sensitive to its different values, by choosing appropriate values of the ratio L/E. The so-called sensitivity to

Δm^2 of an experiment is the value of Δm^2 for which

$$\frac{L\Delta m^2}{2E} \sim 1 \tag{4.3}$$

Different types of neutrino mixing experiments are traditionally classified depending on the average value of the ratio L/E for an experiment, which determines its sensitivity:

Type	L(km)	E(GeV)	Δm^2(eV^2) sensitivity
Reactor SBL	$\sim 10^{-2}$	$\sim 10^{-3}$	~ 0.1
Accelerator SBL (Pion DIF)	~ 1	$\gtrsim 10^{-3}$	$\gtrsim 1$
Accelerator SBL (Muon DAR)	$\sim 10^{-2}$	$\sim 10^{-2}$	~ 1
Accelerator SBL (Beam Dump)	~ 1	$\sim 10^{2}$	$\sim 10^{2}$
Reactor LBL	~ 1	$\sim 10^{-3}$	$\sim 10^{-3}$
Accelerator LBL	$\sim 10^{3}$	$\gtrsim 1$	$\gtrsim 10^{-3}$
ATM	$\sim 20-10^{4}$	$\sim 0.5-10^{2}$	$\sim 10^{-4}$
Reactor VLB	$\sim 10^{2}$	$\sim 10^{-3}$	$\sim 10^{-5}$
Accelerator VLB	$\sim 10^{4}$	$\gtrsim 1$	$\gtrsim 10^{-4}$
SOL	$\sim 10^{11}$	$0.2-15\cdot 10^{-3}$	$\sim 10^{-12}$

Table 4.1: Types of neutrino oscillation experiments. Based on reference [20].

4.1.1 Short baseline experiments (SBL)

- **Reactor SBL.** These are experiments that utilize large isotropic fluxes of electron antineutrinos produced in nuclear reactors by β^- decays of heavy nuclei (mainly fission fragments of uranium or plutonium isotopes). Since the antineutrino energy is too low to produce muons or taus, only the survival probability of electron antineutrinos can be measured by detecting, in a liquid scintillator,the inverse β-decay reaction $\bar{\nu}_e + p \to n + e^+$ with a threshold $E_{th} = 1.8$MeV. Examples: ILL, Bugey, RENO(future experiment).
- **Accelerator SBL.** These are experiments with beams of neutrinos produced by decay of pions, kaons, and muons created by a proton beam hitting a target. They can be divided in three subtypes:
  - *Pion Decay In Flight (DIF).* Examples: BEBC, CCFR,CHARM, BNL-E776, CHORUS, NOMAD ($\nu_\mu \to \nu_\tau, \nu_e \to \nu_\tau, \nu_\mu \to \nu_e$), LSND $\nu_\mu \to \nu_e$, NuTeV ($\nu_\mu \to \nu_e, \bar{\nu}_\mu \to \bar{\nu}_e$).
  - *Muon Decay At Rest (DAR).* Examples: KARMEN, LSND, both in the channel $\bar{\nu}_\mu \to \bar{\nu}_e$.
  - *Beam Dump.* Examples: old experiments BEBC,CHARM, CDHSW.

4.1.2 Long baseline experiments (LBL)

These are experiments which have sources similar to SBL experiments, but the source–detector distance is larger, about two or three orders of magnitude larger. LBL experiments are classified as follows:

- **Reactor LBL.** These are reactor neutrino experiments in which the source– detector distance is of the order of 1 km. Examples: CHOOZ, Palo Verde, Double CHOOZ, Daya Bay.
- **Accelerator LBL.** These are neutrino experiments with a muon neutrino or antineutrino beam produced by the decay in flight of pions and kaons created by shooting a proton beam to a target. The source-detector distance is about $10^2 - 10^3$ km. Examples: K2K($\nu_\mu \to \nu_\mu, \nu_\mu \to \nu_e$), MINOS experiment ($\nu_\mu \to \nu_\mu, \nu_\mu \to \nu_e$), ICARUS ($\nu_\mu \to \tau_\mu, \nu_\mu \to \nu_e$), OPERA ($\nu_\mu \to \nu_\tau$), T2K [633] ($\nu_\mu \to \nu_\mu, \nu_\mu \to \nu_e$).
- **ATMospheric neutrino experiments (ATM).** Primary cosmic rays interact with the upper layers of the atmosphere producing a large flux of pions and kaons which decay in the atmosphere into muons and neutrinos. Many muons further decay into electrons and neutrinos before hitting the ground. Atmospheric neutrino experiments detect these neutrinos. The energy of detectable atmospheric neutrinos cover a very wide range, from about 500 MeV to about 100 GeV. The source-detector distance ranges from about 20 km for neutrinos coming from above, to about 1.3×10^4 km for neutrinos coming from below, initially produced on the other side of the Earth. Examples: Kamiokande, IMB, Super-Kamiokande, MACRO, Soudan-2, MINOS.

4.1.3 Very Long baseline experiments (VLBL)

These are experiments with a source-detector distance even larger than LBL experiments by one or two orders of magnitude. They are:

- **Reactor VLB.** These experiments measure the combined neutrino flux of many reactors at a distance of the order of 100 km. Example: KamLAND.
- **Accelerator VLB.** These are accelerator neutrino experiments with a source-detector distance of the order of several thousands of km, comparable with the diameter of the Earth. These experiments are presently under study: new and more intense neutrino beams are needed in order to observe a sufficient number of events at such large distances. Candidate types of beam are: Super-Beam , Beta-Beam , and Neutrino Factory.
- **SOLar neutrino experiments (SOL).** These are experiments which detect the neutrinos generated in the core of the Sun by the thermonuclear reactions that power the Sun. The Sun–Earth distance is about $1.5 \times 10^{11} m$. Examples: Homestake, Kamiokande , GALLEX, SAGE, GNO , SNO(upgraded to SNO+), KamLAND.

Finally, we could mention that there is an extra class of neutrino oscillation experiments, those trying to detect neutrinos or neutrino mixing profiles from cosmic (astrophysical and cosmological) sources such as supernovae, ultra high energy and extreme high energy cosmic rays, or relic neutrinos. The detectors in this category are commonly referred as **neutrino telescopes**. Examples: ICECUBE, Pierre Auger observatory, ANTARES,...

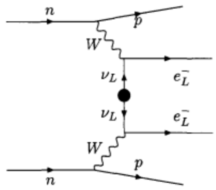

Figure 4.1: Double beta decay(I). If the neutrino is a Majorana particle, then neutrinoless double beta decay becomes possible.

4.2 Double Beta Decay Experiments

In the weak hamiltonian, the following process can arise:

$$n + n \to p^+ + p^+ + e^- + e^- + \nu_e + \nu_e \tag{4.4}$$

On a nucleus with Z protons and $N = Z - A$ neutrons, this reaction can be written as

$$(Z, A) \to (A, Z + 2) + e^- + e^- + \nu_e + \nu_e \tag{4.5}$$

This reaction is called double beta decay, or $\beta\beta$-decay since two β-rays (i.e.electrons) are produced in the final state. This concrete example of $\beta\beta$-decay has also two (anti)neutrinos that can not be detected, and thus, the notation $\beta\beta_{2\nu}$ is commonly seen. The amplitude for this process is very small, since the effective coupling is proportional to G_F^2 and it is very rare, and hard, to find it. Moreover, if the $\beta\beta$-decay happens naturally, the nuclei has to be arranged so that the single β-decay

$$(Z, A) \to (A, Z + 1) + e^- + \nu_e \tag{4.6}$$

is energetically forbidden. In this way, some $\beta\beta_{2\nu}$-decay have been observed in some nuclei. The $\beta\beta_{2\nu}$-decay conserves global lepton number and so, is a test of SM physics. There is other reaction that could also arise in the context of BSM: neutrinoless $\beta\beta$-decay ($\beta\beta_{0\nu}$ or $\beta_{0\nu ee}$-decay). This new process, yet more fantastic and rarer than $\beta\beta_{2\nu}$-decay, can appear only with Majorana neutrinos in BSM models. There are mainly 3 effective reactions driving a $\beta\beta_{0\nu}$:

$$(A, Z) \to (A, Z + 2) + 2e^- \tag{4.7}$$
$$(A, Z) \to (A, Z + 2) + 2e^- + \mathcal{M} \tag{4.8}$$
$$(A, Z) \to (A, Z + 2) + 2e^- + 2\mathcal{M} \tag{4.9}$$

where $\mathcal{M}$ is a massless Goldstone boson called the Majoron arising from SSB of the $B-L$-symmetry. Note that these kind of reactions violate global lepton number by two units, i.e. $\Delta L = \pm 2$. Other neutrinoless double beta decay reactions are:

$$K^+ \to \pi^- + \mu^+ + \mu^+ \ \ , K^+ \to \pi^- + e^+ + e^+ \ \ , K^+ \to \pi^- + \mu^+ + e^+ \tag{4.10}$$
$$\mu^- + (A, Z) \to (A, Z - 2) + e^+ \tag{4.11}$$
$$\tau^- \to e^+ + \pi^- + \pi^-, \tau^- \to \mu^+ + \pi^- + \pi^-, \tau^- \to e^+ + \pi^- + K^- \tag{4.12}$$

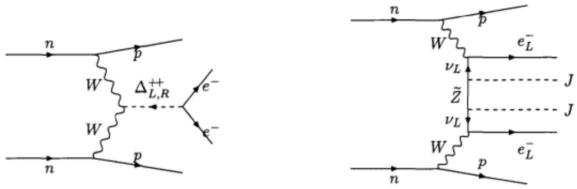

Figure 4.2: Double beta decay(II). Other examples of double beta decay. A doubly charged Higgs model (left) and a double Majoron emission in a SUSY model with a Zino(right).

Searches for $\beta\beta0\nu$ hints have been pursued without conclusive results, so neutrinoless double beta decay lifetime have arised[1]. Present results suggest that $\tau^{1/2}_{\beta\beta0\nu} \gtrsim 10^4 \tau^{1/2}_{\beta\beta2\nu}$. Furthermore, in the framework of a minimally Majorana extended SM, a crude estimate allows us to get the effective coupling[2]:

$$G_{\beta\beta0\nu} \simeq G_F^2 m_\nu \left\langle \frac{1}{q^2} \right\rangle_{ij} \tag{4.13}$$

where the bracket expression is the nuclear matrix element that has to be taken in the final and initial nuclear state (the hard part of nuclear experiments is to compute this "hadronic" matrix element). We can also understand that the process is highly suppressed since neutrino masses are of the order 1eV and typical neutrino energies are of the order MeV, thus the reaction is cancelled by about $m^2/E^2 \sim 10^{-12}$ or less. A more careful estimate taking into account the neutrino mixing is the next calculation:

$$\sum_i U_{li} U_{l'k} \frac{1-\gamma_5}{2} \frac{(\gamma \cdot q + m_i)}{q^2 - m_i^2} \frac{1-\gamma_5}{2} C \simeq m_{ll'} \frac{1}{q^2} C \tag{4.14}$$

with

$$m_{ll'} = \sum_i U_{li} U_{l'i} m_i \tag{4.15}$$

As the matrix element in which l^+ and l'^+ are produced is proportional to $\sum_i U^*_{li} U^*_{l'i} = m^*_{ll'}$. Noting that we are interested in real observable quantities, we can compute $|m_{ll'}|$ and taking into account the familiar Cauchy-Schwarz inequality,

$$|m_{ll'}| \leq \sqrt{\sum_i |U_{li}|^2 m_i^2} \sqrt{\sum_i |U_{l'i}|^2} \leq m_{max} \tag{4.16}$$

beint m_{max} the mass of the heaviest neutrino. For instance, from tritium β-spectrum is found that the bound is about 2.2eV. In summary, the exceptional and more promising experiment sensible

[1]Only a group has claimed the detection of this decay, but their results are not accepted and are in strong tension with other data.

[2]This holds for SM neutrinos with light masses. Heavy neutrinos would satisfy $G_{\beta\beta0\nu} \simeq G_F^2/m_\nu \gtrsim 10^7$GeV. Other models or theories with doubly charged Higgs like those in SUSY models can also modify dramatically the effective coupling for $\beta\beta0\nu$ decays.

to Majorana neutrino masses is nuclear $\beta\beta0\nu$-decay of even-even nuclei, where there are many possibilities to use large targets to reach small background and high energy resolution. The total decay rate for $\beta\beta0\nu$-decay depends on three main parameters: the so-called effective Majorana mass, the square of nuclear matrix element and the effective matrix coupling:

$$\Gamma^{\beta\beta0\nu} = \frac{1}{T_{1/2}^{\beta\beta0\nu}} = |m_{\beta\beta}|^2 |M^{\beta\beta0\nu}|^2 G^{\beta\beta0\nu}(Q, Z) \tag{4.17}$$

with the effective coupling (see,e.g.,[2])

$$G^{\beta\beta0\nu}(Q,Z) = \frac{G_F^4 g_A^4}{2(2\pi)^5 R^2} \int_0^Q dT_1 \int_0^\pi d\theta \sin\theta \,(E_1E_2 - p_1p_2\cos\theta) \cdot \\ \cdot\, p_1p_2\,(F(E_1, Z+2)F(E_2, Z+2)) \tag{4.18}$$

where $T_1 = E_1 - m_e$,$Q = M_i - M_f - 2m_e$, $F(Z) \simeq \frac{2\pi\eta}{1-e^{2\pi\eta}}$,$\eta = Z\alpha m_e/E$.

The effective Majorana mass is given by the equation:

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \tag{4.19}$$

and it provides information about the Majorana character of neutrinos and its mass scale. Note that neutrino oscillation angles and mass squared differences can introduce interference so the real value can be low. In fact, this is important since it allows to determine the kind of neutrino spectrum[3]. The effective Majorana mass is determined not only by the lightest neutrino mass and neutrino mass-squared differences but also by the character of the neutrino mass spectrum. We will analyse the three main spectrum classes (in the case of 3 flavors):

- **Normal (hierarchical) spectrum.** This corresponds to

$$m_1 \ll \sqrt{\Delta m_{12}^2} \;\; m_2 \simeq \sqrt{\Delta m_{12}^2} \;\; m_3 \simeq \sqrt{\Delta m_{23}^2} \tag{4.20}$$

The neutrino masses determine two mass-squared differences $\Delta m_{23}^2, \Delta m_{13}^2$. The lightest neutrino has a very small mass. The effective Majorana mass reads:

$$|m_{\beta\beta}| = m_1 c_{12}^2 c_{13}^2 + \sqrt{m_1^2 + \Delta m_{21}^2}\, s_{12}^2 c_{13}^2 e^{2i\alpha} + \sqrt{m_1^2 + \Delta m_{31}^2}\, s_{13}^2 e^{2i(\beta - \delta_{CP})} \tag{4.21}$$

Neglecting the lightest neutrino mass in the effective Majorana mass, we get

$$|m_{\beta\beta}| \simeq \left| \sin^2 \sqrt{\Delta m_{12}^2} + e^{2i\alpha} \sin^2\theta_{13} \sqrt{\Delta m_{23}^2} \right| \tag{4.22}$$

The first term is small due to the smallness of the first mass-square splitting. The other mass-square splitting is proportional to the $\sin^2\theta_{13}$ so it will be suppressed if it is small. We can estimate the value using the CHOOZ bound $\sin^2\theta_{13} < 5 \cdot 10^{-2}$ realizing that in that case the effective Majorana mass tends to zero if $\alpha \simeq \pi/2$ and it would not be observable. However,

[3]Let us stress that the kind of neutrino mass spectrum is related to some details of the theory or model.

this is not necessary in order to be unobservable in forthcoming experiments. Without a Majorana phase the bound is:

$$|m_{\beta\beta}| \simeq \left| \sin^2 \sqrt{\Delta m_{12}^2} + \sin^2 \theta_{13} \sqrt{\Delta m_{23}^2} \right| \lesssim 5.3 \cdot 10^{-3} \text{eV} \tag{4.23}$$

and it is too small for the next experiments (like KATRIN, with sensitivity about 0.2 eV).

- **Inverted (hierarchical) spectrum.** In this case

$$m_3 \ll \sqrt{|\Delta m_{13}^2|} \;\; m_1 \simeq \sqrt{|\Delta m_{13}^2|} \;\; m_2 \simeq \sqrt{\Delta m_{13}^2} \left(1 + \frac{\Delta m_{12}^2}{2|\Delta m_{13}^2|}\right) \tag{4.24}$$

Now, the lightest neutrino mass is m_3. Therefore,

$$|m_{\beta\beta}| = \sqrt{m_3^2 + |\Delta m_{31}^2|} c_{12}^2 c_{13}^2 + \sqrt{m_3^2 + \Delta m_{12}^2 + |\Delta m_{31}^2|} s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3^2 s_{13}^2 e^{2i(\beta - \delta_{CP})} \tag{4.25}$$

Taking into account the spectrum structure, we obtain

$$|m_{\beta\beta}| \simeq \sqrt{|\Delta m_{13}^2|} \left(1 - \sin^2 2\theta_{12} \sin\alpha\right)^{1/2} \tag{4.26}$$

where α is the difference of Majorana phases between U_{e1} and U_{e2}. Hence, we can derive the bound

$$\cos 2\theta_{12} \sqrt{|\Delta m_{13}^2|} \leq |m_{\beta\beta}| \leq \sqrt{|\Delta m_{13}^2|} \tag{4.27}$$

corresponding to the $\alpha = 0, \pi$ (upper bound) and $\alpha = \pm\pi/2$ (lower bound). Using the known values of the boundary limits, we get

$$1.8 \cdot 10^{-2} \leq |m_{\beta\beta}| \leq 4.9 \cdot 10^{-2} \tag{4.28}$$

The future $\beta\beta 0\nu$ experiment sensitivities lay in this range, and then, future experiments are sensitive to inverted spectrum.

- **Quasidegenerated spectrum.** We define in this case the minimum neutrino mass as m_{min}. Then,

$$|m_{\beta\beta}| \simeq m_{min} \left(1 - \sin^2 2\theta_{12} \sin\alpha\right)^{1/2} \tag{4.29}$$

and thus

$$\cos 2\theta_{12} m_{min} \leq |m_{\beta\beta}| \leq m_{min} \tag{4.30}$$

If it is the case with neutrino masses, $\beta\beta 0\nu$-decay will be observed and the effective Majorana mass will be relatively large $|m_{\beta\beta}| \gg |\sqrt{|\Delta m_{23}^2|}$. So, neutrinos would be Majorana particles and its spectrum would be quasidegenerated, with

$$|m_{\beta\beta}| \leq m_{min} \leq 2.8 |m_{\beta\beta}| \tag{4.31}$$

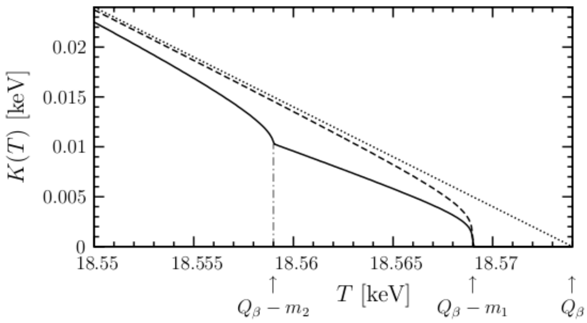

Figure 4.3: Kurie plot(I). We also show how the neutrino mixing deforms the massless case, provided $m_1 = 5$eV and $m_2 = 15$eV, with $\theta = \pi/4$.

We note and remember that the Mainz and Troitsk tritium experiments yield $m_\beta < 2.2$eV at 95% C.L. This stringent bound is obtained from the measurement of the high-energy part of the β-spectrum of tritium, and where the effective mass is defined as

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2} \tag{4.32}$$

Tritium is the most promising candidate for absolute determination of the electron anti-neutrino mass. A non-zero mass modifies the shape of the energy spectrum of the emitted β-electrons near the high-energy endpoint according to

$$\frac{dN_e}{dE} \sim p\left(E + m_e\right)\left(E_0 - E\right)\sqrt{\left(E_0 - E\right)^2 - m_\beta^2} \tag{4.33}$$

For the measurement of the electron neutrino mass it is convenient to define the so-called **Kurie function**(see Figure 4.3)

$$K(T) = \sqrt{\frac{2\pi^3 dN/dT}{G_F^2 m_e^5 \cos^2\theta_C |M|^2 F(Z, E_e) E_e p_e}} \tag{4.34}$$

or

$$K(T) = \left[\left(Q_\beta - T\right)\sqrt{\left(Q_\beta - T\right)^2 - m_{\nu_e}^2}\right]^{1/2} \tag{4.35}$$

If the electron neutrino mass is zero, the Kurie function is a linear function of the kinetic energy T of the electron $K(T)|_{m_\nu=0} = Q_\beta - T$ This is also represented in tritium end-point experiments, like Figure 4.4.

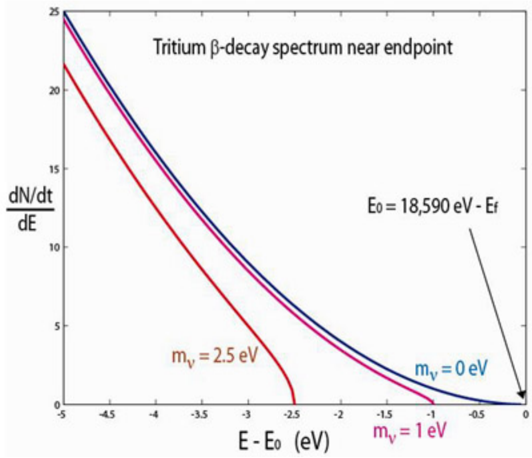

Figure 4.4: Kurie plot(II): The *tritium end-point spectrum.* For a massless neutrino, the spectrum would be a straight line. The effect of massive neutrinos is shown as a parabolic deformation of the end-point for the beta decay. Some crude values or neutrino mass are also shown as orientative explanation of what happens.

4.3 Data

In this section we will briefly review some best values and fits coming from some neutrino experiments. The experimental data in the neutrino area is very intense, and it requires quite dedication to be continuously updating results, but it deserves the prize in this unknown chart of the SM. As scientists, we love experiments, data and theories.

4.3.1 Neutrino mixing data

According to the PDG, oscillation parameters, **angles and mass differences**, read:

- From **KAMLAND and a solar neutrino** global fit, we get: $\sin^2(2\theta_{12}) = 0.861^{+0.026}_{-0.022}$, $\Delta m^2_{12} = \Delta m^2_{solar} = 7.59^{+0.20}_{-0.21} \cdot 10^{-5}\text{eV}^2$
- **Atmospheric neutrino** yields (sign of Δm^2_{23} is unknown): $\sin^2(2\theta_{23}) > 0.92, CL = 90\%$ $\Delta m^2_{23} = \Delta m^2_{atm} = 2.43 \pm 0.13 \cdot 10^{-3}\text{eV}^2 \;\; CL = 68\%$
- **Reactor neutrino** provides[4]: $\sin^2(2\theta_{13}) < 0.15,\ CL = 90\%$

The absolute scale of neutrino masses or their Majorana character are also unknown from neutrino oscillation results.

4.3.2 $\beta\beta0\nu$ and absolute mass bounds

The most known bound for the electron neutrino mass is the one from the Mainz and Troitsk data. It yields: $m_e < 2.2\text{eV}$. The double beta decay $\beta\beta0\nu$ measurement is very hard and challenging. It

[4]Hints of a non-zero θ_{13} have appeared in T2K and MINOS, this year 2011.

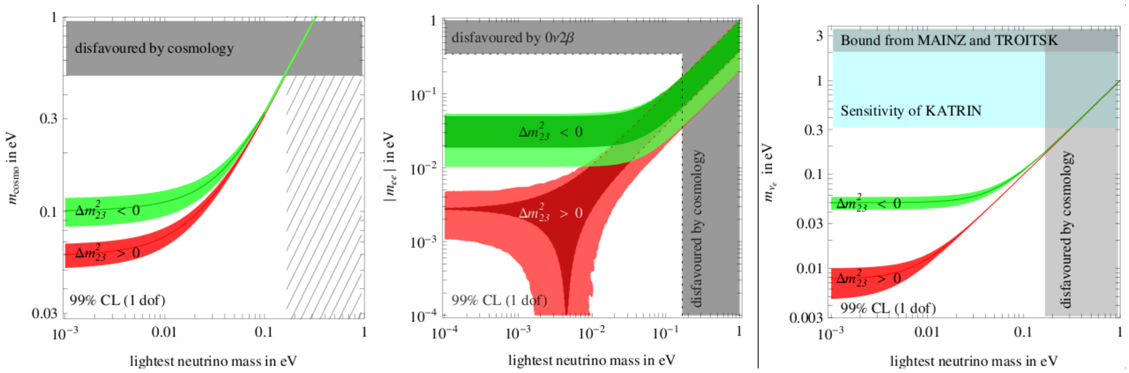

Figure 4.5: Double beta decay and bounds on the absolute neutrino masses. The third picture shows the future parameter range of $\beta\beta0\nu$ experiments.

is also highly dependent from the chosen method and isotope.
From IGEX (^{76}Ge):$|m_{\beta\beta}| < 0.3 - 1.2\text{eV}\ CL = 90\%$.
From CUORICINO(^{130}Te):$|m_{\beta\beta}| < 0.19 - 0.68\text{eV}\ CL = 90\%$
From Heidelberg-Moscow(^{76}Ge):$|m_{\beta\beta}| < 0.3 - 1.3\text{eV}\ CL = 90\%$.
From NEMO-3 (^{96}Zr) we get the 2010 bound: $|m_{\beta\beta}| < 7.2 - 19.5\text{eV}\ CL = 90\%$.
The present bounds on double beta decay provide the bounds on the absolute neutrino masses given by the above Figure 4.5.

4.4 Controversial anomalies

Beyond the accepted results from the previous sections, there are also intriguing experimental results not fully understood with the present data. They are simply called *anomalies* since it has not been either confirmed or refuted with the current paradigms (SM, 2 or 3 mixing within neutrino oscillations,. . .) or they have not yet reached the 5σ-threshold usually marked for a solid claim/test of "discovery".

- The **NuTeV anomaly** [109] arose when the neutrinos at the Tevatron (NuTeV) collaboration at Fermilab were measuring the ratio of NC to CC processes in the collisions of high-energy neutrinos (and antineutrinos) with a large steel target. The measurements gave a value for $\sin^2\theta_w$ that was three standard deviations higher than predicted by the Standard Model.

- **LSND anomaly.** The LSND project was created to look for evidence of neutrino oscillation, and its results conflict with the SM expectation of only three neutrino flavors, when considered in the context of other solar and atmospheric neutrino oscillation experiments. It implies *other different* mass squared splitting $m^2_{LSND} \sim 1\text{eV}^2$. Cosmological data bound the mass of the sterile neutrino to $m_s < 0.26\text{eV}(0.44\text{eV}), \text{CL}= 95\%\ (99.9\%)$, excluding at high significance the sterile neutrino hypothesis as an explanation of the LSND anomaly.

- **Reactor anomaly**. The forthcoming and promising Double Chooz reactor experiment forced a re-evalutation of the antineutrino reactor flux. The new calculation provided a small flux

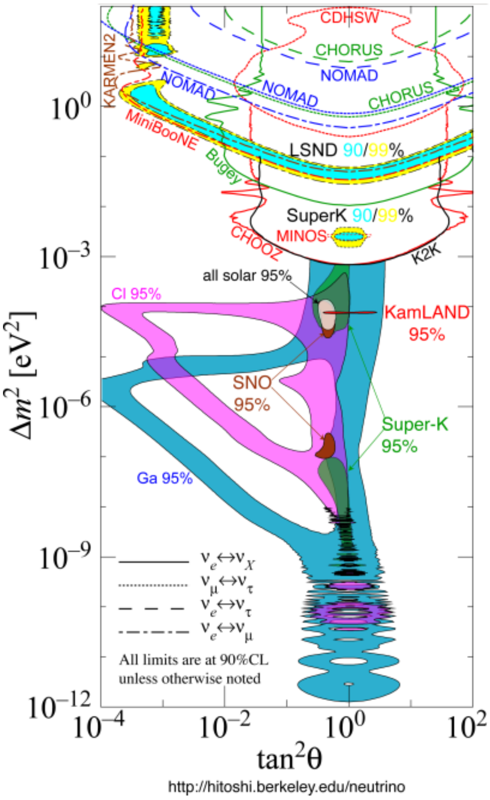

Figure 4.6: Bounds on neutrino oscillation parameters from different experiments (2010 data).

increase $\sim 3.5\%$. Although this increase has no significant effect on KamLAND data, when it is combined with the previously reported small deficits at nearer distances, it implies in a larger average deficit of 5.7%, at 0.943 ± 0.023. This is the *reactor antineutrino anomaly*[111, 110, 112, 113], significant at the level of 98.6% C.L. This requires a neutrino with a mass squared splitting $\sim m^2_{LSND}$ or higher.

The last MiniBooNE data are indeed consistent with $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations at mass splitting order around eV^2 and consistent with the evidence for antineutrino oscillations from LSND. The MiniBooNE $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation allowed region appears to be different from the $\nu_\mu \to \nu_e$ oscillation allowed region. Can nuclear effects possibly explain this difference? We do not know. MINOS was also seeing different pattern between oscillation in the neutrino and antineturino modes, although last results are decreasing the initial difference. Increasing evidence from various experimets at $L/E \sim 1$ that there is more than three active neutrinos. No conclusive evidence yet of new physics at $L/E \sim 1$, but hints are arising quickly. However, a new set of experiments under construction or proposed could measure neutrino oscillations with high significance (more than the critical 5 sigmas) and potentially prove that there is new physics at the $m^2 \sim$ eV^2 scale. As we can observe, neutrinos have a long story providing anomalous results in every kind of experiment in which they are involved. Neutrinos are beautiful specially due to these experiments for both, theorists and phenomenologists.

Chapter 5

Conclusions

Neutrino physics has a promising present and future. However, although the main goal of neutrino oscillation experiments is to study neutrino properties, we have learnt that the targets are complex nuclei and nuclear effects must be under control in order to interpret the data, due to the non-elementary character of nucleons. A new insight on nucleonic physics will be gained by probing the nucleus with neutrinos:

- CC events are sensitive to the nucleon axial mass M_A. Its value is presently poorly known and and better accuracy is necessary in order to face neutrino mixing and other kind of experiments (like those related with proton decay)with exceptional precision.
- NC events can probe the strangeness content of the nucleon. An improvement in the understaning of this type of process is essential since it is one of the most important kind of events forming the background in DM experiments, and it can provide information about what properties of nucleons are badly modeled by present models.
- Nuclear effects (FSI, correlations, two-body currents,...) must be well understood before drawing conclusions on the nucleonic physics, and it is a highly non trivial task.

The neutrinos (as weak probes) can give informations on the nuclear structure and dynamics complementary to electrons (electromagnetic probes) thanks to their tiny cross-sections and masses. Neutrino interactions have provided valuable information (often in the form of anomalies, surprising results) into the theory of weak interactions. The maximal parity violation,the V-A theory and finally the Glashow-Weinberg Salam electroweak theory were developed in part from information on neutrino interactions. In this sense, neutrino interaction data has been used to test the electroweak theory, such as in the measurements of s_w^2(NuTeV). Neutrino interactions have provided information on the deeper structure of nucleons. MINERνA experiment, for instance, seeks to measure low energy neutrino interactions both in support of neutrino mixing and also to study the strong dynamics of the nucleon and nucleus that affect these interactions. By the other hand, neutrino oscillations can allow us to probe the GUT scale(maybe the Planck scale?), but it is *crucial* that we understand further the 1 GeV (low energy) region to be able to exploit the full neutrino oscillation phenomenon information. A new generation of experiments is commencing to lead the way towards a new precision era in neutrino interaction physics. We remark and remember that there are some problems in the detection of some modes of neutrino mixing(such as $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ from pion decays): huge underground detectors are required, background events coming from kaon and muon decays,...Some present and future double beta decay experiments are: CUORE, GERDA, MAJORANA, EXO and superNEMO or KATRIN. Reactor experiments like Daya-Bay, Double

CHOOZ, and RENO will be searching θ_{13} and possibly confirm or refute the recently found reactor anomaly. Even before a new experiment, neutrinos are producing interesting results. The future neutrino experiments will be likely made in experimental accelerator facilities (neutrino factories, superbeams, beta beams,...) and also in neutrino telescopes as Baikal, ANTARES, KM3NET (proposed), NESTOR Project (under development since 1998), or ICECUBE.

In the LHC, and although it is not covered in this thesis, the neutrino mass generation mechanism has also to be searched within the collider experiments[100]. A speculative emergent electroweak gravity theory was proposed in [101]. Moreover, there are interesting speculative theories in the context of neutrino physics such as decoherence models (triggered by quantum gravity and space-time foam?), CPT violation in the neutrino sector, neutrino tests of the equivalence principle, unitarity violations, neutrino decays, sterile neutrinos, extra dimensions, and many others that we expect we can test in the near future either with UHECR detectors such as Pierre AUGER or future space telescopes/detectors.

The plum pudding model was the prevailing theory on the structure of the atom almost one century ago until it was disproved by Ernest Rutherford in 1911. The gold foil experiment, conducted under the supervision of Rutherford at the University of Manchester in 1909 by scientist Hans Geiger and undergraduate student Ernest Marsden, drove us towards a more complex picture of the atom and it showed us how complex nuclei are. Today, we have increasing evidence that the most precise theory ever invented for subatomic particles, the SM, can not be the full story. A fundamental issue in the SM is the origin of mass and the neutrino mass spectrum. One century after Rutherford's pioneer work, we are also wondering about the same topics and questions about the (sub)atomic deepest structure of our Universe using neutrino probes.

Everywhere and everytime the neutrinos are providing to us beautiful neutrino oscillations, a fully quantum phenomenon that is opening a new experimental window to the Universe (beyond the one of photons, for instance). Their oscillation pattern is something we can not completely understand yet. Moreover, future experiments on $\beta\beta 0\nu$ and the determination of the kind of spinorial structure behind the neutrino would provide us extra information in this not completely understood part of the SM: the neutrino sector. Many neutrino experimental challenges are yet ahead of us.

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