GAUGE THEORIES: ANOMALIES, REGULARIZATION, AND RENORMALIZATION

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I. Introduction

Quantum electrodynamics is accurate to a few parts in a million. Its remarkable success is due to the smallness of the coupling constant and to the possibility of renormalizing the perturbation-theory divergences. The usual theory of weak interactions, on the other hand, is not renormalizable. Therefore, in spite of the smallness of the coupling constant ($Gm_p^2 \approx 10^{-5}$) higher-order calculations are meaningless. The quadratic divergences present a problem not only because the theory is not finite, but because it is not clear how the higher-order corrections can be smaller than the lowest order. For instance, if a process goes like G in lowest order, the next order will go like $G^2\Lambda^2$, where Λ is a cutoff, and it will not be smaller unless $G\Lambda^2$ is appreciably smaller than unity. For some processes, this kind of interpretation requires rather small values of Λ, an admission that the theory fails at uncomfortably small energies.

One may hope that a unified theory of weak and electromagnetic interactions based on a gauge group will be renormalizable because of the cancellations due to the Yang-Mills relations among the couplings. Furthermore, because of the properties of the nonabelian gauge group, it could explain the universality of both interactions. Since the intermediate boson must be taken to be massive and the theory of Yang-Mills fields with mass is not renormalizable, a special procedure had to be found--the Higgs mechanism; as described below, a renormalizable theory seems possible. Universality, on the other hand, has been only partially achieved. In the presently known models either the electromagnetic or the weak universality has to be put in by hand.[1]

II. Spontaneously Broken Gauge Groups

The Higgs[2] Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu}{}^2 - |(\partial_\mu - igA_\mu)\phi|^2 - \mu^2|\phi|^2 - h|\phi|^4$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

is invariant under the gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \Lambda, \ \phi \to \phi e^{ig\Lambda}.$$

Treating it at first classically (tree diagrams) we distinguish the two situations $\mu^2 > 0$ and $\mu^2 < 0$ (in both cases $h > 0$). In the second case the potential

$$V(\phi) = \mu^2|\phi|^2 + h|\phi|^4$$

has a minimum for $|\phi| = \lambda = (-\mu^2/h)^{1/2}$ and the complex field ϕ has a nonvanishing vacuum expectation value. Choosing it to be real, $\langle\phi\rangle = \lambda$, we see that the solution no longer has the symmetry of the equations. Rather, the group transforms the particular solution chosen into an infinity of equivalent solutions, none of which exhibits the gauge symmetry. The symmetry is spontaneously broken. In the absence of gauge fields the spontaneous breaking of a symmetry implies the existence of massless bosons (Goldstone bosons), as many as the number of generators of the original group which are no longer conserved. In the presence of gauge fields some or all of these massless bosons diappear from the physical spectrum and provide instead the additional degrees of freedom necessary to give a mass to a corresponding number of gauge fields. In the Higgs model this can be seen by introducing the fields χ and Θ through

$$\phi = \frac{1}{\sqrt{2}} (\lambda + \chi) e^{i\Theta/\lambda} .$$

Under the gauge transformation, the field Θ transforms as

$$\Theta \rightarrow \Theta + g\lambda\Lambda ,$$

while the field χ and the vector field

$$B_\mu = A_\mu - \frac{1}{g\lambda} \partial_\mu \Theta$$

are invariant. The Lagrangian can be expressed completely in terms of these gauge-invariant fields

$$L = -\frac{1}{4} B_{\mu\nu}^{\ 2} - \frac{1}{2}(g\lambda)^2 B_\mu^{\ 2} - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}(3h\lambda^2 + \mu^2)\chi^2$$

$$- \frac{h}{4} \chi^4 - \frac{1}{2} g^2 B_\mu^{\ 2} (2\lambda\chi + \chi^2).$$

We see that the phase Θ has disappeared and at the same time the vector field has acquired a mass $m = g\lambda$. The gauge group is no longer visible, although the particular relations among the coupling constants are a reminder of the original gauge invariance.

The pervading idea of the work described here is that if a theory is renormalizable in the symmetric case ($\mu^2 < 0$), it will also be renormalizable in the case of spontaneous symmetry breaking. Since the original Higgs Lagrangian is renormalizable in the symmetric case, we expect the new form to correspond to a renormalizable theory. In the new form the vector meson propagator

$$\underbrace{B_\mu B_\nu} \sim \left(\delta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2} \right) \frac{1}{k^2 + m^2}$$

will give rise to highly divergent Feynman diagrams. Cancellations will hopefully take place which will render the theory renormalizable. This kind of quantization which uses the conventional vector meson propagator and no unphysical fields gives rise to a manifestly unitary S-matrix and is therefore called the U-formalism. In this formulation renormalizability is the hard thing to show.

An alternative quantization scheme leaves the unphysical degrees of freedom and makes explicit use of the gauge group. Here renormalizability is obvious by simple power counting and therefore one speaks about the R-formalism. Unitarity, however, has to be proved. This can be done either by showing that the unphysical singularities cancel as a consequence of the Ward identities of the group, or by showing that the R-formalism is equivalent to the unitary U-formalism. Let us write

$$\phi = \frac{1}{\sqrt{2}} (\lambda + \phi_1 + i\phi_2).$$

The real fields ϕ_1 and ϕ_2 have vanishing vacuum expectation value and transform into each other under the gauge group. One can fix the gauge by taking as Lagrangian

$$L - \frac{1}{2} \xi (\partial_\mu A_\mu + \frac{m}{\xi} \phi_2)^2,$$

where L is the Higgs Lagrangian expressed in terms of ϕ_1 and ϕ_2. Other gauge conditions are possible but the above is especially convenient because it cancels the bilinear $A_\mu \partial_\mu \phi$ term in L generated by the shift in the scalar field. Now the vector meson has a propagator with good high-momentum behavior

$$\underbrace{A_\mu A_\nu} \sim \left[\delta_{\mu\nu} + \frac{k_\mu k_\nu (1-\xi)}{\xi k^2 + m^2} \right] \frac{1}{k^2+m^2},$$

while the would-be Goldstone boson ϕ_2 has the propagator

$$\underbrace{\phi_2 \, \phi_2} \sim \frac{1}{k^2+m^2/\xi} .$$

Observe that the vector meson propagator differs from its unitary counterpart given earlier by

$$\frac{k_\mu k_\nu}{m^2 (k^2+m^2/\xi)} .$$

This scalar ghost must cancel in all amplitudes against the Goldstone ghost to give a unitary S-matrix. Notice that for $\xi \to 0$, the vector propagator tends to the unitary form while the Goldstone ghost acquires infinite mass and drops out of the theory. Therefore, there exists a one-parameter set of renormalizable gauges, starting from $\xi = \infty$ (Landau gauge), which tends in the limit of $\xi \to 0$ to the unitary gauge. The on-mass-shell S-matrix elements must be independent of the particular gauge chosen. These considerations provide the basis for a proof of equivalence between the R- and the U-formalism.

Let us already mention here that in the nonabelian case the proof is complicated by the occurrence, in the correct Feynman rules, of the Feynman-Faddeyev-Popov ghosts.[3] It turns out that the interaction between these ghosts and the residual physical scalar (the analogue of ϕ_1 above) is proportional to $1/\xi$. Therefore, the ghost loops with external physical scalar lines give a contribution which diverges in the limit as $\xi \to 0$. In this case one must combine the ghost loops with all other diagrams contributing to the same amplitude. The S-matrix element itself is well behaved as $\xi \to 0$. Finally, let us remark that for a satisfactory proof one must first regularize and subtract the divergences. These points will be discussed later.

The Higgs model has been shown by B. W. Lee[4] to be renormalizable to all orders. The generalization of the Higgs mechanism to the nonabelian case is due to Kibble.[5]

III. The Weinberg-Salam Model[6]

If one tries to construct a minimal gauge theory containing the known currents and only the presently known leptons, one is led almost unavoidably to the group $SU(2) \times U(1)$. Consider, for instance, the electromagnetic current $\bar{e}\gamma_\mu e$ of the electron and the charged weak current $\bar{e}\gamma_\mu(1+\gamma_5)/2\ \nu_e$ with its hermihan conjugate. In terms of the left-handed doublet

$$L = \frac{1+\gamma_5}{2}\begin{pmatrix}\nu_e \\ e\end{pmatrix}$$

the charged current is

$$\bar{e}\gamma_\mu \frac{1+\gamma_5}{2}\nu_e = \bar{L}\gamma_\mu \tau^- L .$$

Together with its hermihan conjugate, it closes to an SU(3) structure with

$$\bar{L}\gamma_\mu \tau_3 L = \bar{\nu}_e\gamma_\mu \frac{1+\gamma_5}{2}\nu_e - \bar{e}\gamma_\mu \frac{1+\gamma_5}{2} e.$$

Since this neutral current differs from the electromagnetic current, one must introduce at least one more neutral current. Using the right-handed singlet

$$R = \frac{1-\gamma_5}{2} e,$$

one notices that

$$-\bar{e}\gamma_\mu e = \tfrac{1}{2}\bar{L}\gamma_\mu\tau_3 L - \tfrac{1}{2}\bar{L}\gamma_\mu L - \bar{R}\gamma_\mu R$$

and that the current $\frac{1}{2}\bar{L}\gamma_\mu L + \bar{R}\gamma_\mu R$ commutes with the above SU(2) group. The Yang-Mills interaction is written in the form

$$\frac{g}{2}\bar{L}\gamma_\mu \vec{\tau} L\cdot \vec{A}_\mu - g'(\tfrac{1}{2}\bar{L}\gamma_\mu L + \bar{R}\gamma_\mu R)B_\mu,$$

where B_μ is the vector field of the U(1) group. The charged intermediate boson field is identified as

$$W_\mu = \frac{1}{\sqrt{2}}(A_{\mu 1} + iA_{\mu 2}),$$

while the photon field is fixed by the fact that it couples to the electromagnetic current. It is given by

$$A_\mu = (g'A_{\mu 3} + gB_\mu)/(g^2 + g'^2)^{\frac{1}{2}}$$

while the orthogonal combination

$$Z_\mu = (gA_{\mu 3} - g'B_\mu)/(g^2 + g'^2)^{\frac{1}{2}}$$

describes a neutral intermediate boson. One finds easily that the electron charge satisfies

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$

from which it follows that $g > e$, $g' > e$. The muon is treated in a perfectly analogous way.

In this model neutral currents are very important. In particular, $\nu_\mu e$ scattering and νp scattering are consequences of the theory. To make the model realistic one must find a way to generalize it to hadrons so as to explain why $\Delta S = 1$ neutral currents are suppressed experimentally by a factor of $10^{-4} \sim 10^{-5}$ in the amplitude with respect to charged-current processes. The same applies to $\Delta S = 2$ transitions. This can be done by using an idea due to Glashow, Iliopoulos, and Maiani[7] who make use of cancellations between intermediate states with ordinary hadrons and states with new "charmed" hadrons (or quarks).

The vector mesons W_μ and Z_μ must be given masses by means of the Higgs mechanism. One takes a scalar doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

and writes its Lagrangian

$$-\left|(\partial_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{A}_\mu - i\frac{g'}{2}B_\mu)\phi\right|^2 .$$

When ϕ, due to its self-interaction, acquires a vacuum expectation value

$$<\phi> = \begin{pmatrix} 0 \\ \lambda/\sqrt{2} \end{pmatrix},$$

the vectors acquire masses $m_W = \frac{1}{2}\lambda g$, $m_Z = \frac{1}{2}\lambda(g^2 + g'^2)^{\frac{1}{2}}$. Identifying the Fermi constant from

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8\, m_W^2} = \frac{1}{2\lambda^2},$$

one finds $m_Z > m_W > 37.2$ GeV. The photon is, of course, massless. Observe that

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2+g'^2} .$$

This is an example of a zeroth-order relation among observable parameters which may be corrected but should remain finite when loops are included (see later).

IV. Anomalies

We have mentioned that the proof of unitarity in the R-formalism makes use of the Ward identities to prove that the unphysical ghost singularities do not appear in the S-matrix. In theories like the Weinberg-Salam model in which the gauge group involves chiral transformations, the Ward identities may develop anomalies,[8] and it is easy to see that the presence of anomalies spoils the proof of unitarity. Fortunately it is possible to arrange things so that the anomalies due to the various fundamental spinor fields cancel. This has been shown for one-loop anomalies by Bouchiat, Iliopoulos and Meyer,[9] by Gross and Jackiw,[10] and by Wess and the author.[11] The more difficult question of cancellation in higher orders has been considered by Bardeen,[12] who reached the conclusion that the anomalies will not cause difficulties in higher order if they cancel in the one-loop case.

The one-loop anomaly is relatively easy to treat. Its general form in the nonabelian case has been given by Bardeen.[13] The relevant property here is that it is proportional to the symbol

$$d_{abc} \sim \mathrm{Tr}\,\{\lambda_a, \lambda_b\}\,\lambda_c$$

of the gauge group. In the case of the Weinberg-Salam model the interaction can be written in general (for leptons as for quarks) in the form

$$\bar{L}\,\gamma_\mu\left(\frac{g}{2}\,\vec{C}\cdot\vec{A}_\mu + \frac{g'}{2}\,C_0 B_\mu\right) L - \bar{R}\,\gamma_\mu g' Q B_\mu R,$$

where Q is the (diagonal) charge matrix, $C_0 = C_3 - 2Q$ and L and R are the left-handed and right-handed parts of the spinor fields which we imagine arranged in a single column Ψ containing all leptons and quarks. The matrices C_1, C_2, and C_3 of the SU(2) algebra will therefore have, in general, a highly reduced form. Separating the vector and the axial vector part in the above interaction, one can write it as

$$\bar{\Psi}\gamma_\mu V_\mu \Psi + \bar{\Psi}\gamma_\mu \gamma_5 A_\mu \Psi,$$

where

$$V_\mu = \frac{g}{4}\vec{C}\cdot\vec{A}_\mu - \frac{g'}{4}\,C_3 B_\mu + g'\left(-Q + \tfrac{1}{2}C_3\right)B_\mu$$

$$A_\mu = \frac{g}{4}\vec{C}\cdot\vec{A}_\mu + \frac{g'}{4}\,C_3 B_\mu .$$

Now we use the fact that the anomaly can always be eliminated from the vector Ward identities by a suitable definition of the one-loop amplitudes (which always allow redefinition by contact terms). The axial vector anomaly will cancel if the corresponding d_{abc} symbol vanishes. This gives rise here to the condition $\mathrm{Tr}\, C_3^2\, Q = 0$. In all models based on SU(2) × U(1) which have been proposed, the eigenvalues of the matrix C_3 are such that this equation simplifies to $\mathrm{Tr}\, Q = 0$. So, if the sum of the charges of all elementary spinor fields vanishes, the one-loop anomaly cancels. This condition can be satisfied by arranging the quark charges in a suitable way. However, it is not possible, e.g., to cancel the electron against the muon since they have the same charge.

A further restriction to be considered in model building is that the purely hadronic part of the anomaly should give the right $\pi^0 \to 2\gamma$ decay amplitude in both magnitude and sign. This requires that $2\,\mathrm{Tr}\, T_3 Q^2 \approx 1$, where T_3 is the hadronic isospin matrix and only the hadronic part of the charge matrix is taken in the trace.

Georgi and Glashow[14] have considered the general question of anomaly-free gauge group. The anomaly of a gauge group is proportional to the d_{abc} symbol of the representation to which one assigns the fundamental spinor fields. Now, pseudoreal representations (equivalent to their conjugates) have vanishing d_{abc}. Groups which have only pseudoreal representations cannot have anomalies. This leads to a classification of gauge groups which are safe for model building. On the other hand "vector-like" models in which the interaction can be transformed into a purely vector interaction by redefining the spinor fields have no anomaly since the one-loop amplitudes can always be defined so as to satisfy anomaly-free vector Ward identities.

Absence or cancellation of the anomalies is necessary to obtain a finite unitary S-matrix. On the other hand, the anomalies would begin to cause problems only in a relatively high order (sixth order), and one may ask oneself if the constraint on model building is not being taken too seriously since the essential physical requirement is perhaps not renormalizability but rather the smallness of the second order with respect to the first. The constraint given by the $\pi^0 \to 2\gamma$ amplitude may also possibly be relaxed if different explanations of the process[15] turn out to be correct.

V. Regularization

An important step in the proof of finiteness and unitarity is that of regularizing the theory in a gauge-invariant way. Most known regularization methods violate the Yang-Mills gauge invariance. Two regularization methods which preserve gauge invariance will be described here.

The first is the use of higher-order covariant derivatives due to Slavnov[16] and Lee and Zinn-Justin.[17] As an example, in the Yang-Mills theory one can take the Lagrangian

$$-\frac{1}{4}\vec{F}_{\mu\nu}^{\,2} - \frac{\alpha}{4\Lambda^2}\left(D_\sigma \vec{F}_{\mu\nu}\right)\cdot\left(D_\sigma \vec{F}_{\mu\nu}\right) - \frac{\beta}{4\Lambda^4}\left(D_\sigma^2 \vec{F}_{\mu\nu}\right)\cdot\left(D_\rho^2 \vec{F}_{\mu\nu}\right) .$$

The first term is the usual Yang-Mills Lagrangian. The second and third term contain derivatives up to sixth order and give rise to a regularized propagator

$$\underbrace{A_\mu A_\nu} \sim \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2}\,\frac{1}{\left[1+\alpha\frac{k^2}{\Lambda^2}+\beta\left(\frac{k^2}{\Lambda^2}\right)^2\right]} +$$

+ gauge-dependent terms.

At the same time, the need to use covariant derivatives in the second and third term of the Lagrangian introduces additional interactions of maximum dimension eight. The occurrence of additional interactions limits the order of derivatives since interactions of too high dimension offset the advantage due to the higher derivatives. With the above choice one finds that only one-loop diagrams with two, three, and four external lines are primitively divergent (like Λ^2, Λ, and $\log\Lambda$ respectively). Although not all divergences are regularized, the divergence of the theory is sufficiently reduced and the divergences identified and isolated so as to become amenable to treatment. Lee and Zinn-Justin use this regularization in combination with the use of regulator fields for scalars and spinors.

The second regularization method is the n-dimensional regularization of 't Hooft and Veltman,[18] which had been used earlier in a different context by Bollini and Giambiagi[19] and others. A Feynman integral can be transformed by the standard parameter method. For a one-loop diagram, for instance, one obtains an integral like

$$\int \frac{d^n k}{(2\pi)^n} \frac{1}{\left[k^2 - L + i\epsilon\right]^m} = \frac{i^{1-m}}{(2\sqrt{\pi}\,)^n} L^{\frac{n}{2}-m} \frac{\Gamma(m-\frac{n}{2})}{\Gamma(m)} ,$$

where n is the dimension of space-time. The divergence of such an integral for n = 4 appears in the right-hand side as a pole in the gamma function at n = 4. For instance, if the original integral was logarithmically divergent, one would find a pole at n = 4; if it was quadratically divergent, one would find poles at n = 2 and n = 4. With n away from the poles, the integral is regularized and one can evaluate the Feynman graph by using formulas such as $g_{\mu\nu}g^{\mu\nu} = n$, $k_\mu k_\nu \to \frac{1}{n} k^2 g_{\mu\nu}$ etc. The important point is that 't Hooft and Veltman were able to prove that this regularization method respects the Ward identities coming from gauge invariance. It also provides a way of defining a subtraction procedure by subtracting the poles at n = 4 with appropriate residues so that the amplitude becomes finite for n = 4.

The n-dimensional regularization method can be used in connection with either the U- or the R-formalism. It seems to be the best available at this time. In the case of theories with spinors and chiral gauge groups, it cannot be applied to the spinor loops because of the special properties of the matrix γ_5 in four dimensions. It must then be supplemented by another regularization method, such as the Pauli-Villars regularization, for the spinor loops.

VI. Renormalization

Different aspects of the renormalization program for a theory with a spontaneously broken gauge group have been studied among others by 't Hooft,[20] 't Hooft and Veltman,[21] B. W. Lee and Zinn-Justin,[22] Ross and J. C. Taylor.[23] From the work of these authors emerges the possibility of a proof of renormalizability and unitarity to all orders in the R-formalism. No attempt has been made to give a proof in the U-formalism although a number of concrete calculations have been performed in the U-formalism, in the one-loop approximation, with finite results after renormalization.[24]

The proof in the R-formalism is too involved to be reported here. The main idea, which is simple, is to make use of an invariant regularization procedure, subtract the infinities, and then prove the unitarity of the renormalized on-mass-shell amplitudes either by using the Ward identities in the particular renormalizable gauge being used or by proving gauge independence and the equivalence to the U-formalism. Because of the length of its expression, we renounce writing here a Lagrangian with all necessary counterterms. Let us only observe that, contrary to the familiar situation in electrodynamics, the simplest choice for the finite parameters entering in the Lagrangian (coupling constants) does not correspond to a simple and direct physical meaning in terms of observable processes. Furthermore, the simplest choice of renormalized fields does not correspond to normalized fields, so that additional multiplicative renormalizations will be needed in the S-matrix elements. Finally, let us observe that the field and coupling constant renormalization are not inversely proportional

$$\vec{A}_\mu^{\,\text{unr.}} = Z_3^{\frac{1}{2}} \vec{A}_\mu^{\,\text{ren.}}$$

$$g^{unr.} = Z_3^{-\frac{1}{2}} Z' g^{ren.},$$

and the additional renormalization constant Z' is infinite in general. This means that the renormalized field transforms under the gauge group by a transformation with an infinite coefficient and the Ward identities for the renormalized quantities are correspondingly complicated.

VII. Mass Relations

In a renormalizable model, such as the Weinberg-Salam model, all renormalized masses are finite since there must exist a sufficient number of counterterms to obtain this result. However, the masses and mass differences are arbitrary. Are there situations in which finite calculable mass relations arise? Weinberg[25] and Georgi and Glashow[26] have discussed this possibility. Suppose that in a particular model a relation among masses is satisfied to zeroth order (tree graphs and tadpoles). Suppose further that the mass relation is valid to zeroth-order for all renormalizable Lagrangians which are invariant under the given gauge group and which are constructed with a given set of fields. In general, the original gauge group will be spontaneously broken down to a subgroup, but let us consider the case in which the mass relation is not a consequence of the invariance under the subgroup. Then the inclusion of loops will be expected to modify the mass relation. Nevertheless, if the theory is renormalizable, the corrections must be automatically finite since one has already taken into account all counterterms allowed by the original gauge group. Zeroth-order mass relations of the kind described here can arise because of the special representation content of the scalar multiplet giving rise to the Higgs phenomenon (which may imply for instance that certain spinor fields do not couple to a scalar which has a nonvanishing vacuum expectation value and therefore have no zeroth-order mass) or because of the particular dynamics of the scalar multiplet (which may imply that a particular scalar does not acquire a nonvanishing vacuum expectation value although this is not a consequence of group theoretic arguments).

The ideas outlined here open the possibility of understanding, within the framework of spontaneously broken gauge theories, mass relations such as the Gell-Mann-Okubo relation. They generate the hope that one might find models in which, e.g., the neutron-proton mass difference or the electron-mass are calculable. For instance, if in a model the electron-mass vanishes to zeroth-order, it could come out proportional to αm_μ in the one-loop approximation. Unfortunately, so far these are only possibilities in principle since no realistic models have been found.

Finally, let us point out that the above considerations apply equally well to relations involving not only masses but also other observable parameters such as coupling constants.

VIII. Conclusion

The renormalization program appears to work. However, it seems fair to say that none of the models suggested for a unified theory of weak and electromagnetic interactions is physically satisfactory. The experience gathered poses very strong constraints on model building. If a model exists which satisfies them all, it has a good chance of being the right theory. We must continue to look for it.

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