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Kepler's Ellipse Generated by the Trigonometrically Organized Gravitons

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Abstract

Johannes Kepler made his great breakthrough when he discovered the elliptical path of the planet Mars around the Sun located in one focus of that ellipse (on the 11th October in 1605 in a letter to Fabricius). The first generation of researchers in the 17th century intensively discussed about the possible mechanism needed for the generation of that elliptical orbit and about the function of the empty focus of that ellipse. First generations of researchers proposed an interplay between attractive and repulsive forces that might guide the planet on its elliptical orbit. Isaac Newton made a giant mathematical progress in his Principia and introduced the concept of the attractive gravitational force between the Sun and planets. However, Newton did not propose a possible mechanism behind this attractive force. Albert Einstein in 1915 left the concept of attractive and repulsive forces and introduced his Theory based on the elastic spacetime. In his concept gravity itself became fictitious force and the attraction is explained via the elastic spacetime. In our proposed model we try to re-open the discussion of Old Masters on the existence of attractive and repulsive forces. The guiding principle for our trigonometrical model is the generation of the ellipse discovered by one of the last ancient Greek mathematicians – Anthemius of Tralles – who generated the ellipse by the so-called gardener's method (one string and two pins fixed to the foci of that ellipse). Frans van Schooten in 1657 invented a series of original simple mechanisms for generating ellipses, hyperbolas, and parabolas. Schooten's antiparallelogram might simulate the interplay of attractive and repulsive forces creating the elliptical path. We propose a model with trigonometrically organized Solar and planet gravitons. In this model the Solar and planet gravitons are reflected and refracted in predetermined directions so that their joint momentum transferred on the planet atoms guides the planet on an elliptical path around the Sun. At this stage we cannot directly measure the gravitons but we can use the analogy with behavior of photons. We propose to observe paths of photons emitted from one focus of the ellipse towards the QUARTER-silvered elliptical mirror. 1/4 of photons will be reflected towards to the second empty focus and the ¾ of photons might be reflected and refracted into the trigonometrically expected directions. (Until now we have experimental data only for the FULLY–silvered elliptical mirrors). The observed behavior of photons with the quarter-silvered elliptic mirror might support this concept or to exclude this model as a wrong model. The quantitative values of attractive and repulsive forces could be found from the well-known geometrical properties of the ellipse. The characteristic lengths of distances will be inserted into the great formula of Isaac Newton - the inverse square law. (In order to explain some orbit anomalies, we can use Paul Gerber's formula derived for the Pierre Fermat principle). We have found that the Kant's ellipse rotating on the Keppler's ellipse might express the co-operation of attractive and repulsive forces to guide the planet on its elliptic path. Finally, we have derived a new formula inspired by Bradwardine - Newton - Tan - Milgrom that might contribute to the MOND gravitational model. We have found that the Kepler ellipse is the very elegant curve that might still keep some hidden secrets waiting for our future research.

Keywords: Kepler's ellipse, attractive and repulsive forces, empty focus, Anthemius'generation of the ellipse, van Schooten's antiparallelogram generating the ellipse, geometrical properties of the ellipse, quarter-silvered elliptical mirror, reflection and refraction of photons on this partly-silvered mirror, Kant's ellipse rotating on the Kepler's ellipse, a new MOND formula

1. Introduction

The famous quote of Heraclitus "Nature loves to hide" was described in details by Pierre Hadot in 2008. Hadot in his valuable book give us many examples how Nature protects Her Secrets. In several situations the enormous research of many generations is strongly needed before the right "recipe" unlocking the true reality can be found.

From time to time some extraordinary events happen. The emperor Rudolph II. invited to Prague Tycho Brahe and Tadeáš Hájek z Hájku organized the invitation for Johannes Kepler in order to join the research group of Tycho Brahe on the 3rd February 1600.

It was a unique meeting of Tycho Brahe and Johannes Kepler in Benátky nad Jizerou (close to Prague) in 1600. Brahe collected the best experimental data on the motion of planets and Kepler was the best possible mathematician in those time with excellent knowledge of Ancient and medieval trigonometry. After the intensive five years of complicated work Kepler was able to refer his friends that he discovered the elliptical path of the planet Mars with the Sun in one focus of that ellipse.

Kepler was thinking for many years about two topics hidden in those elliptical paths: 1) what is the geometrical mechanism that guides the planet on the elliptical orbit? 2) what is the function of the empty focus? Kepler had assumed that that the elliptical orbit of the planets could be explained by the effect of two joined forces: an "anima force" emanating from the Sun and "vis insita" inherent in the planet itself. These two open topics passed into hands of other giants in the 17th century: Galileo, René Descartes, Christian Huygens, Gottfried Leibniz, Robert Hook and Isaac Newton. Isaac Newton in 1687 came with his superb mathematical development in his Principia. Newton quantitively described the attractive force between the Sun and planets. However, Newton did not propose any cause of the attractive gravitational force or any comment to the function of the empty focus of that ellipse.

During next three centuries the subject changed - the repulsive forces became fictitious force as a pseudo-force artifact of rotating reference frames. The next change of this concept brought Albert Einstein in 1915 with his theory of the elastic spacetime. In this concept gravity force itself became a fictitious force and the attraction is explained via the elastic spacetime.

Recently, all the physical community celebrated centenary from the birth of Richard Feynman - one of the best physicists in the 20th century. Feynman openly and originally stimulated his readers to view some physical topics from a different angle. E.g., he was joking that angels do not have to fly tangentially in order to push the planet around the Sun but they have to fly at right angles toward to the Sun. This Feynman's joke might open a space for one question - how should be gravitons organized in order to generate the elliptical orbit?

In our attempt we want to return to the roots and re-open the concept of Johannes Kepler: what is the "planet mind" behind the elliptical path and what is the function of the empty focus? The guiding principle came from Anthemius of Tralles - one of the last ancient Greek mathematicians - who discovered the very well-known gardener's method for the generation of an ellipse: one string and two pins simulate the attractive forces. There existed a gap between the knowledge of geometrical properties of conic sections and their material generation in the 17th century. This gap was originally filled by the contribution of Frans van Schooten in 1657 who invented a series of simple mechanisms for generating ellipses, hyperbolas, parabolas, and straight lines. Van Schooten's antiparallelogram might simulate the interplay of attractive and repulsive forces creating the elliptical path. Immanuel Kant stressed that the elliptical path of planets around the Sun has to be guided by the co-operation of attractive and repulsive forces.

Old Masters discovered throughout ages many interesting properties of the ellipse (and parabola and hyperbola) that were very well-known till about the end of 19th century. During the last century some of those properties were forgotten and only several researchers used those old techniques in their physical concepts.

If we employ the lost know-how of Old Masters on the properties of conic sections we might easily deduce the quantitative description of attractive and repulsive forces. The trigonometrically determined reflection and refraction of the Solar and planet gravitons might transfer their momentum into the atoms of the rotating planet and thus guide that planet on the elliptical path.

At this stage of our experimental possibilities we cannot directly observe gravitons but we can study the behavior of photons and their reflection and refraction on the quarter-silvered elliptic mirror. E.g., it is very well known that silvered mirrors reflect about 25% of photons in the wavelength range from 200 - 230 nm. It could be interesting to analyze the paths of those 75% behind this quarter-silvered elliptic mirror. The new experimental data can be taken also for such parabolic and hyperbolic partly-silvered mirrors.

New experimental data will reveal if this proposed model is promising or just another wrong gravity model. The mechanism behind the gravity is very well hidden by Nature. Kepler was inspired in his study by muses depicted on his frontispiece of Tabulae Rudolphinae (Georg Celer in 1627) where the realm of Urania (the muse of astronomy) is represented by six visible muses: Physica lucis et umbranum, Optica, Logarithmica, Doctrina triangulorum, Stathmica, Magnetica, and six invisible muses: Geographica/Hydrographia, Computus, Chronologia, Mensoria altitudinum, Geometria and Harmonica. Our generation is very lucky because thanks to the internet connection the Muse Trigonometria opened for us the door to Her Realm - see the frontispiece on the book of Bonaventura Cavalieri “Trigonometria plana, et sphæрica, linearis and logarithmica (1643).

(We are aware of the famous quote of Richard Feynman from the year 1962: “There’s certain irrationality to any work in gravitation, so it is hard to explain why you do any of it.”)

2. Some Trigonometric Properties of the Ellipse

The discovery of ellipse, parabola, and hyperbola is attributed to Menaechmus. Apollonius of Perga - the Great Geometer - was the top Ancient Greek mathematician specialized on the conic sections. Pappus of Alexandria and Anthemius of Tralles were the last of great Ancient Greek mathematicians that contributed to this topic. After one thousand years this “geometric treasure” passed into the hands of Johannes Kepler and Isaac Newton.

Figure 1 and Figure 2 shows some trigonometrical properties of the ellipse that might be used for the description of motion of planets around the Sun.

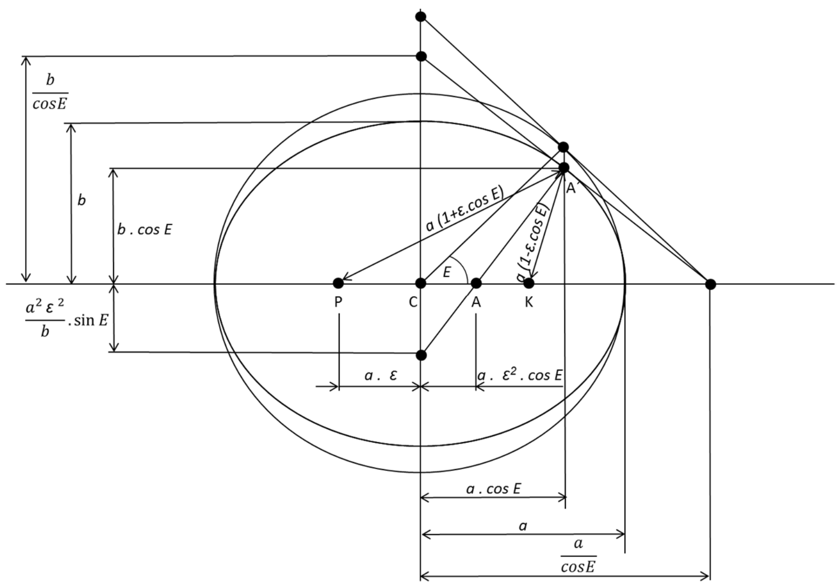

Figure 1. Some trigonometric relations derived for the eccentric anomaly

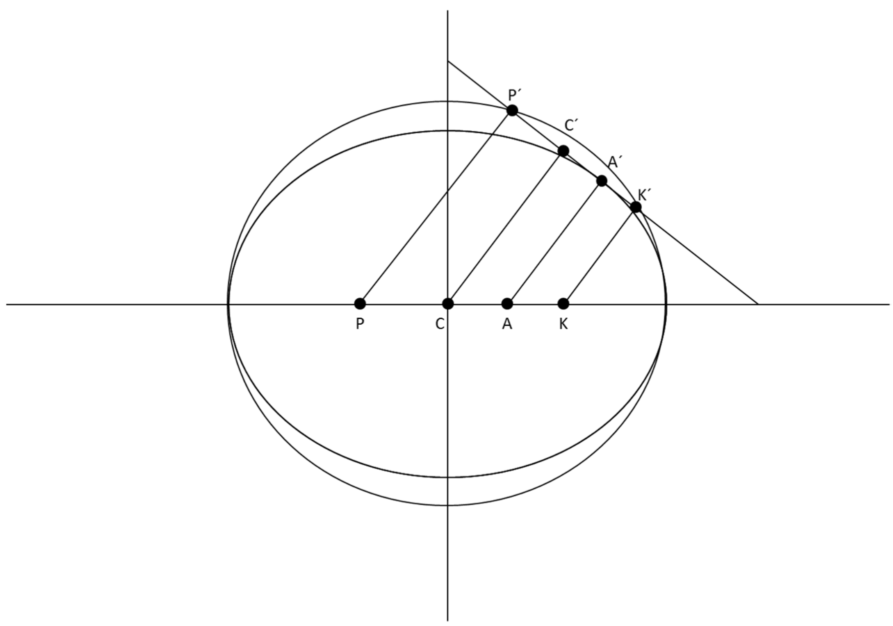

Figure 2. Distances from Ptolemy’s empty focus, Copernicus’ center of the auxiliary circle, Archimedean point, and Kepler’s occupied focus to the tangent

Table I summarizes some relations derived for the eccentric anomaly because of the complex behavior of the ellipse.

Table I. Some trigonometrical properties of the ellipse

Some trigonometrical properties of the ellipse
a … semi-major axis
b … semi-minor axis
E … eccentric anomaly
P … Ptolemy's empty focus
C … Copernicus' center of the auxiliary circle
A … Archimedean point
K … Kepler's occupied focus
A' … planet - Archimedean fulcrum
$PA' = a\left(1+\varepsilon\cos E\right)$
$KA' = a\left(1-\varepsilon\cos E\right)$
$\frac{PA}{KA} = \frac{PA'}{KA'} = \frac{\left(1+\varepsilon\cos E\right)}{\left(1-\varepsilon\cos E\right)}$
$CA' = a\sqrt{1-\varepsilon^2\sin^2 E}$
$PP' = b\frac{\sqrt{1+\varepsilon\cos E}}{\sqrt{1-\varepsilon\cos E}}$
$CC' = b\frac{1}{\sqrt{1-\varepsilon^2\cos^2 E}}$
$AA' = b\sqrt{1-\varepsilon^2\cos^2 E}$
$KK' = b\frac{\sqrt{1-\varepsilon\cos E}}{\sqrt{1+\varepsilon\cos E}}$

3. Reflecting Properties of Photons on the Fully-Silvered Elliptic Mirror

Since the Ancient times the reflecting properties of photons on the fully-silvered elliptical mirror are very well known both for the internal and external reflections - see Figure 3 and Figure 4.

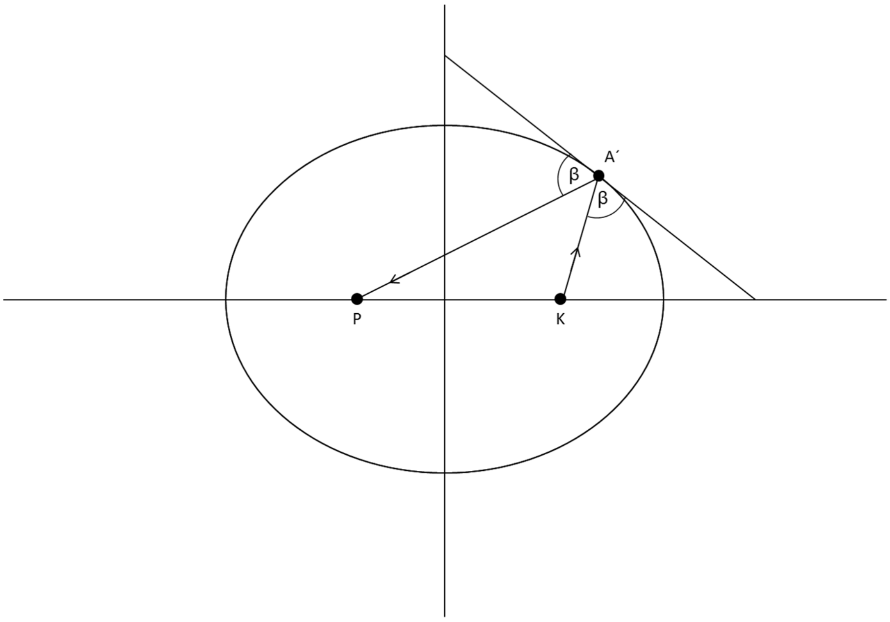

Figure 3. Internal reflection of photons on the fully-silvered elliptical mirror

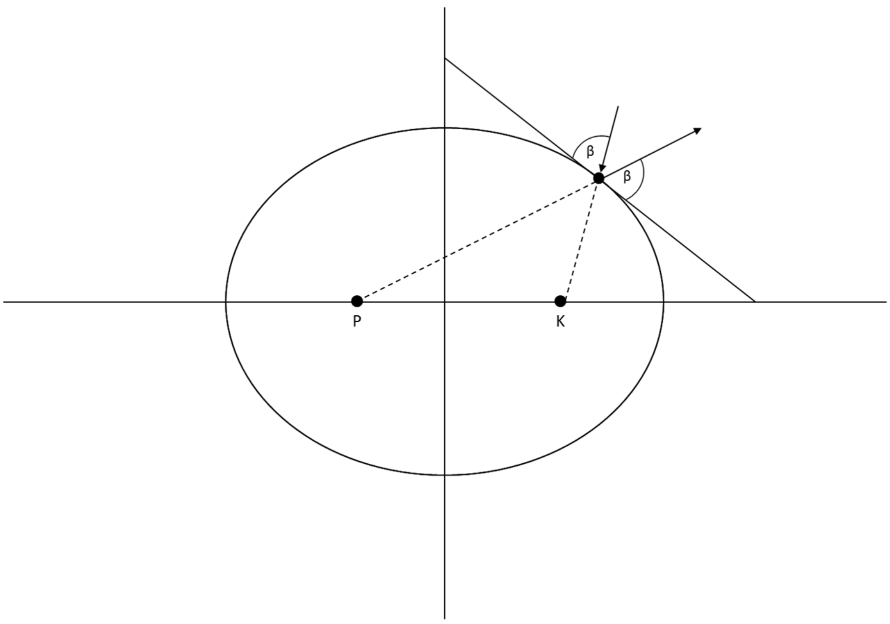

Figure 4. External reflection of photons on the fully-silvered elliptical mirror

We propose to collect experimental data for the photon reflections on a Quarter-silvered elliptical mirror in order to investigate the photon paths behind the partly-silvered elliptical mirror. It is very well-known that silvered mirrors reflect some 25% of photons with their wavelength in the range around 200 nm.

The newly obtained experimental data might support this proposed concept or to exclude the predicted and expected photon paths. Similar experiments might be done for partly-silvered parabolic and hyperbolic mirrors.

At this moment we are not able to describe graviton paths experimentally.

4. Proposed Reflecting and Refracting Properties of Solar and Planet Gravitons

In this section we will assume that Solar gravitons enter into the internal volume of planets and collide with planet gravitons in four possible scenarios. The planet is modelled as a quarter-silvered elliptic mirror. For this case we expect that 25% of Solar gravitons will be reflected towards the empty focus and 75% of Solar gravitons will be reflected and refracted on the tangent and the normal in directions depicted in Figures 5 - 8. The planet gravitons should be reflected into the expected directions. Both Solar and planet gravitons transfer their momentum into planet atoms. The resulting interplay of attractive and repulsive and tangential pushing and braking forces might generate that experimentally observed Kepler ellipse.

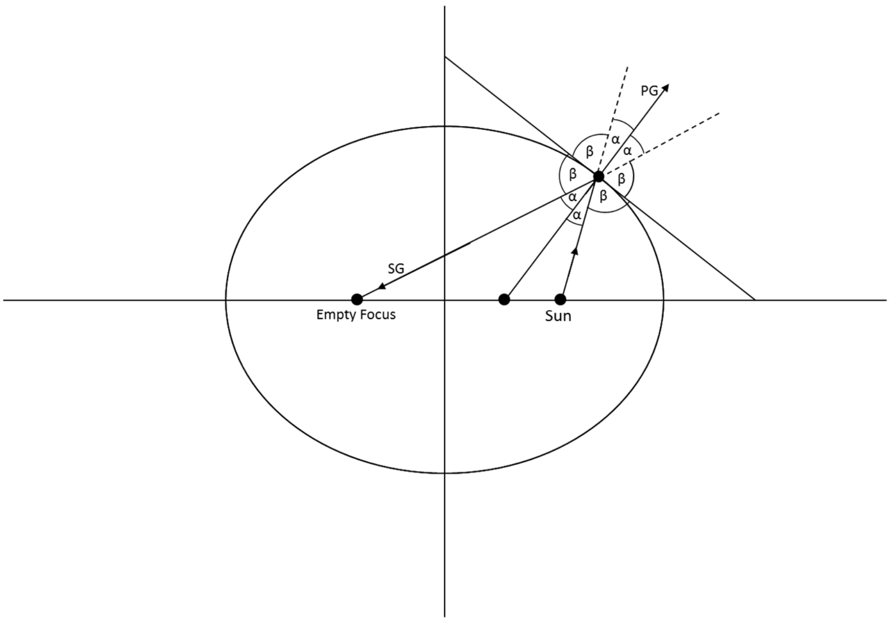

Figure 5. Proposed reflection of the solar graviton (SG) and the planet graviton (PG) on the tangent

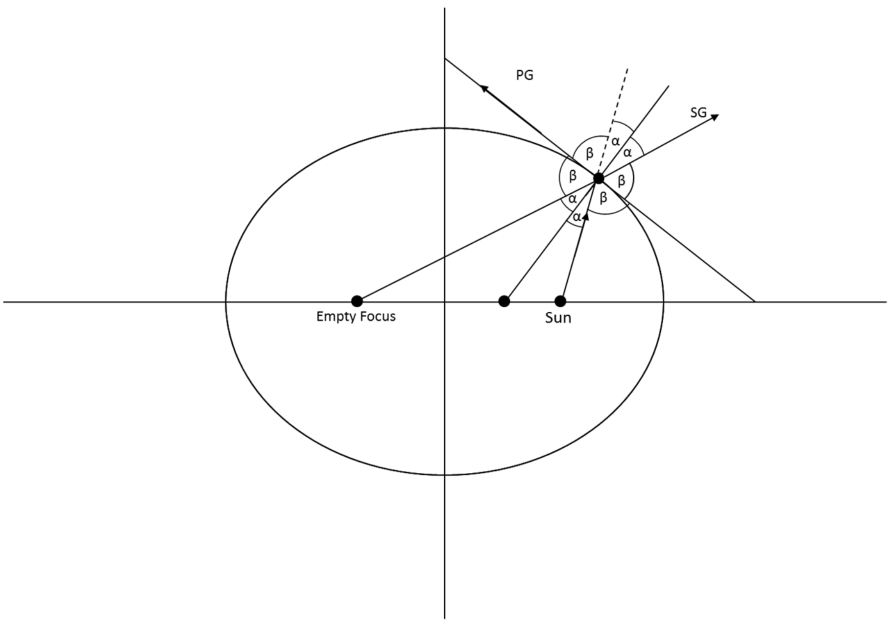

Figure 6. Proposed reflection of the solar graviton (SG) and the planet graviton (PG) on the normal

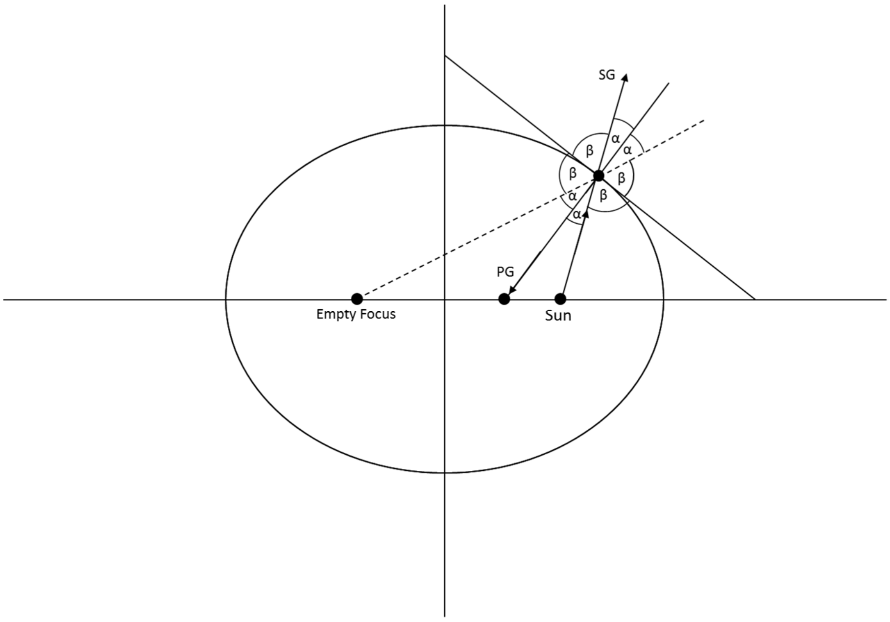

Figure 7. Proposed refraction of the solar graviton (SG) on the tangent and the reflection of the planet graviton (PG) on the tangent

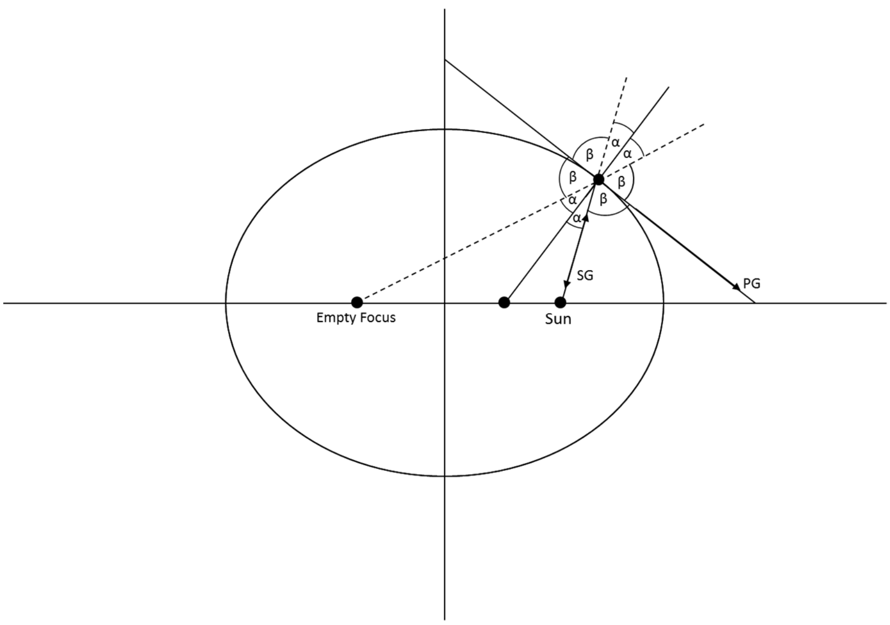

Figure 8. Proposed reflection of the solar graviton (SG) on the tangent and the reflection of the planet graviton (PG) on the normal

5. Rotation of the Kant's Ellipse on the Kepler's Ellipse

These four proposed scenarios for the reflection and refraction of Solar and planet gravitons via momentum transfer into the planet atoms create a combination of forces that leads to the generation of the elliptical path of that planet. For the quantitative determination of those forces we have to find trigonometrically characteristic lengths. These characteristic lengths we will insert into the Newton's gravitational law. These characteristic lengths could be determined with the help of directrix circles around both foci with the radius 2a - see Figure 9.

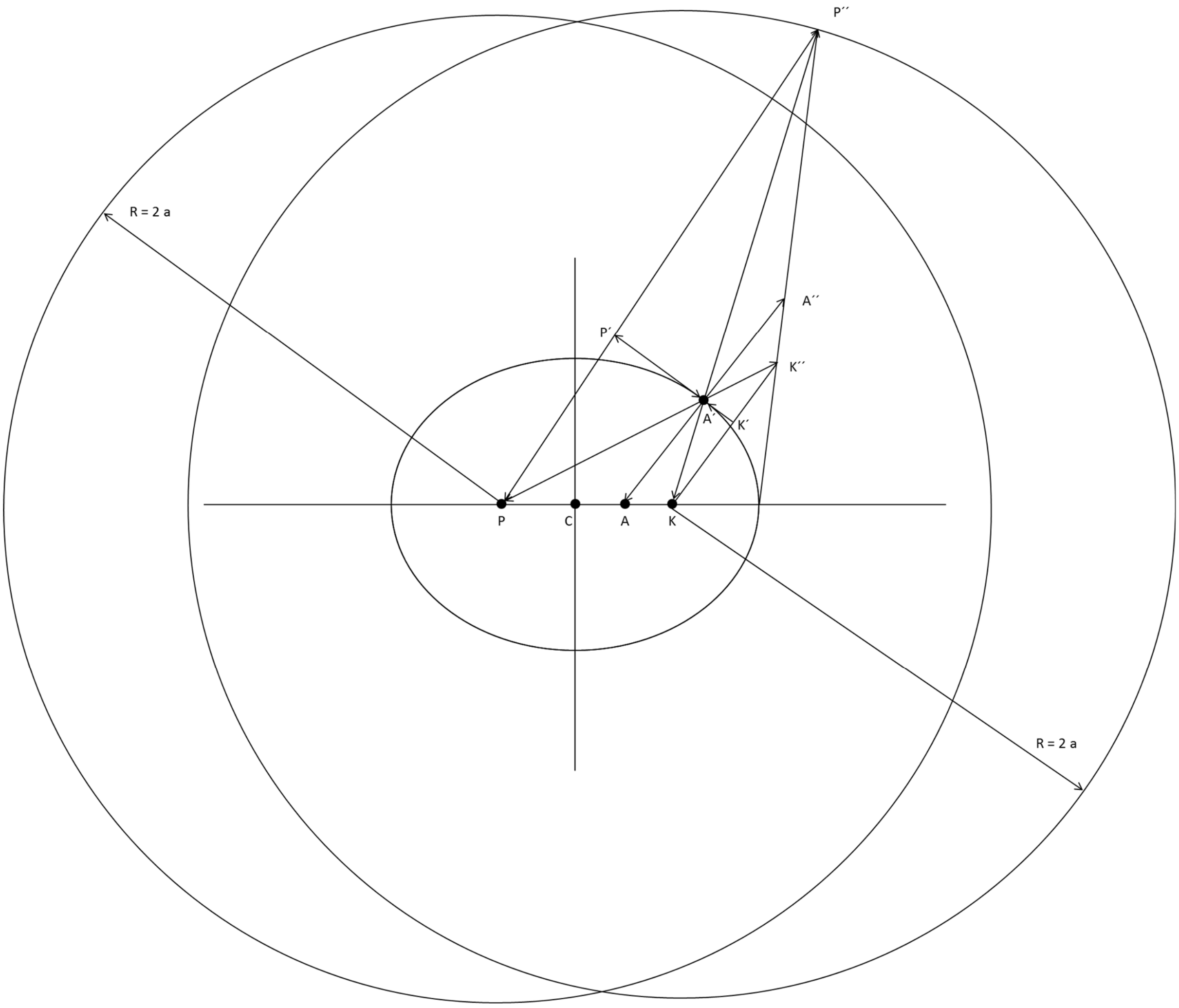

Figure 9. Construction of the characteristic lengths of repulsive forces using two directrix circles with R = 2a and centers in both foci of the Kepler's ellipse

Table 2. The combination of attractive, repulsive and tangential forces.

Characteristic lengths for the determination of forces
F_1: A'K - Newton's attractive force, F_5: A'P'' - Leibniz's repulsive force
F_2: A'A - Kepler's attractive force, F_6: A'A'' - Huygens' repulsive force
F_3: A'P - Anthemius' attractive force, F_7: A'K'' - Kant's repulsive force
F_4: K'A' - Descartes' pushing force, F_8: P'A' - Galileo's braking force

By this trigonometric approach we came back to Frans van Schooten and his antiparallelogram from 1657 that simulates the interplay of attractive and repulsive forces creating the elliptical path.

We can draw the Kant's ellipse (describing the repulsive forces) rotating without slipping on the Kepler ellipse (describing the attractive forces). The tangent to both ellipses characterizes the pushing and braking forces needed for the orbital motion around the Sun in the occupied focus. See Figure 10.

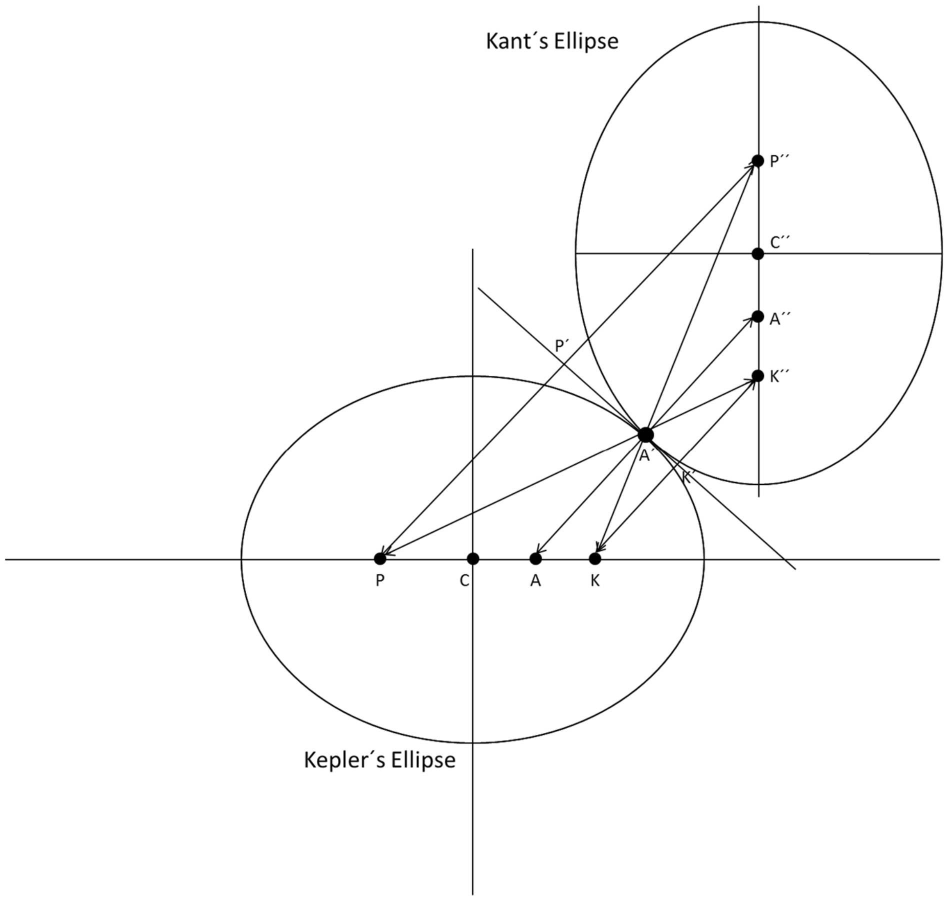

Figure 10. Kant's ellipse describing the repulsive forces rotates without sliding on the Kepler's ellipse describing the attractive forces (See the impressive applet created by Graeme McRae in 2017)

6. Bradwardine - Newton - Tan - Milgrom Formula - The MOND Formula Derived Trigonometrically

In order to introduce a possible application of this trigonometrical model we want to present a new MOND formula that was inspired by four great researchers:

1. Thomas Bradwardine in 1328 formulated a concept for the change of speeds based on the ratio of pushing force/braking force. Details in the book of W.R. Laird and S. Roux (2008).
2. Isaac Newton in 1687 published his Principia with the inverse square law for the attraction forces.
3. Arjun Tan in 1979 developed several new relationships for planet orbital speeds based on the properties of the ellipse. (See his valuable book 2008).
4. Mordehai Milgrom in 1983 proposed his Modified Newtonian dynamics (MOND) with a very interesting relationship $v^4 = GMa_0$. Details in his and other researcher contributions. We made a trigonometric modification for v_E tangential orbital speed at eccentric anomaly E, G gravitational constant, M mass of the Sun, and a_E inward acceleration at eccentric anomaly (if $E = \pi/2$ then we write v_0 for the orbital speed at the end of the minor axis and a_0 for the inward acceleration at the end of the minor axis):

$$v_E^4 = GMa_E \tag{1}$$

Arjun Tan discovered in 1979 (book from the year 2008, page 18) a very impressive speed formula given his Theorem 1.6: "The speeds at the ends of a diameter are inversely proportional to the distances between the focus and the points where the tangents to the ellipse meet the major axis extended." We have expressed Tan's formula trigonometrically as:

$$\frac{v_E}{v_0} = \sqrt{\frac{\left(1 + \varepsilon \cos E\right)}{\left(1 - \varepsilon \cos E\right)}} \tag{2}$$

We have used the Newton's gravitation law in order to express the ratio of Newtonian attractive force F_1 and the Anthemius' attractive force F_3, the ratio of Kant's repulsive force F_7 and the Leibniz's repulsive force F_5, and the ratio of the Descartes pushing force F_4 and the Galileo's braking force F_8 (see Table 2).

The combined Bradwardine - Newton - Tan - Milgrom formula is written as:

$$\frac{F_1}{F_3} = \frac{F_7}{F_5} = \frac{F_4}{F_8} = \frac{a^2\left(1 + \varepsilon \cos E\right)^2}{a^2\left(1 - \varepsilon \cos E\right)^2} = \frac{v_E^4}{v_0^4} = \frac{a_E}{a_0} \tag{3}$$

It could be interesting to study in details this Bradwardine - Newton - Tan - Milgrom formula for the Solar system as well as for galaxy systems in order to check if the existence of the so-called dark matter is just a mathematical artefact of some gravitational models.

We have found that the Kepler ellipse is the very elegant curve that might still keep some hidden secrets waiting for our future research. The Muse Trigonometria has been inviting Readers of this Journal to Her Trigonometric Realm.

7. Conclusions

1. We proposed to apply the antiparallelogram of Frans van Schooten (1657) as a model for attractive and repulsive forces generating the ellipse.

2. We proposed the trigonometric model to find characteristic lengths for attractive, repulsive, and tangential forces to generate the ellipse.

3. The characteristic lengths might be inserted into the Newton's inverse square formula to get values for those forces.

4. We have combined the great philosophical school represented by Immanuel Kant and the great physical school represented by Johannes Kepler and derived the Kant's ellipse rotating on the Kepler's ellipse.

5. We proposed to get experimental data for photon reflection and refraction of partly-silvered elliptic, hyperbolic, and parabolic mirrors.

6. We have derived trigonometrically one MOND formula based on the joint stimulating ideas of Bradwardine - Newton - Tan - Milgrom.

Acknowledgments

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