

Solar structure models

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Abstract. The Sun is a unique object in stellar evolution due to our unprecedented insight on its internal processes. We will illustrate in this review how the transition between a static vision to a more dynamical vision modifies the addressing questions on the solar radiative zone. Neutrinos and acoustic modes have first scrutinized the microscopic properties of the solar radiative plasma. Today, stimulated by the internal rotation profile determination, new questions emerge on the angular momentum transport by rotation, internal waves and on the role of magnetic fields to get access to the dynamical motions of this important region of the Sun. We will give some examples which demonstrate that the Sun is not yet under control.

1. Introduction

The Sun differs from other stars by its proximity and our capability of observation. We know for a long time its luminosity, radius and mass with a remarkable accuracy, and due to this fact, it has appeared very early as a reference for stellar evolution. Moreover since three decades, our star is scrutinized by two probes which help us to largely progress on our capability to check this plasma in great details. The knowledge coming from neutrinos has significantly increased since the detection by R. Davis of a mixture of neutrinos coming from different reaction rates. We are able today to look at two specific reaction rates (nearly three) and to sum the different flavours of neutrinos to really extract information on the central temperature of the Sun. On the helioseismic side, space measurements have stimulated a real insight on the thermodynamics of the radiative region and the dynamics of the convective zone from acoustic modes. Gravity modes appear now promising to reveal the last missing information, the dynamics of the core.

This review is organized in three parts: (1) the success and open questions on the classical solar models: standard (**SSM**) and seismic solar models (**SeSM**), (2) the new questions to solve in order to build a dynamical model (**DSM**), (3) the relevant future experimental or observational efforts for the determination of the real present internal solar structure.

2. The static vision of the Sun: the importance of the radiative zone

In the last century, when one was looked at the Sun, it seemed static. The so called solar constant seems not to vary more than 10^{-3} in mean value [1]. Present solar luminosity, radius and mass have guided the determination of the ingredients of the calibrated solar structure model. The temporal evolution of the Sun depends on the nuclear reaction reactions which produce the nuclear energy and on the way this energy is transported to the solar surface. It is not so easy to leave this rather "simple" way to determine the Solar Structure Model because the step beyond is difficult. Indeed we know that the four structural equations are not sufficient

to describe the phenomena that we observe in UV. High performance computers are not yet able to solve the MHD equations in 3D for an evolving Sun assuming simultaneously the internal dynamical motions and the whole detailed microphysics. So we need to progress step by step and verify that a specific progress maintains the previous success as a real success. At the dawn of the strong asteroseismology development, it is interesting to verify what we know or what we do not know from this static vision of the Sun.

2.1. The standard solar model: SSM

The ingredients of the classical structural equations are the nuclear reaction rates, the opacity coefficients, the equation of state and the initial composition. The radiative zone represents 98% of the total mass of the Sun and uses most of these ingredients. This shows its importance; the equilibrium between gravitational energy, nuclear energy production and the energy escaping by photon interaction is governed mainly by this region on long time-scales.

Nuclear cross sections have been measured in laboratory during at least three decades and the extrapolation towards the stellar plasma conditions has been largely studied and measured in one specific case. The sound speed in the core extracted from SoHO has put a real constraint on the fundamental reaction proton + proton. So one may consider that the reaction rates are reasonably under control now. For the two other ingredients (opacity coefficients and equation of state) the knowledge is purely theoretical. See review [2].

The conditions of temperature and density in the radiative zone ensure that the plasma is totally ionized for its main constituents: hydrogen and helium but heavier species such as iron and then silicon down to oxygen are considered as partially ionized. The bound-bound interaction of photons with matter is very efficient to evacuate the energy produced in the first radial quarter (practically half the solar mass). This kind of contribution is highly sensitive to the metal content ($\propto Z^4$), so it is necessary to calculate this interaction for *all* the elements present in the Sun (from hydrogen to iron). The small amount of iron (some 10^{-4} of hydrogen in fraction number) contributes to about one fifth of the opacity cross section in central conditions. This point shows the important role of the detailed knowledge of the internal composition (see section 4). Up to now the details of the ion interactions have never been verified except indirectly through acoustic pulsation eigenmodes which probe plasma properties throughout the Sun.

Helioseismology was already a mature discipline when SoHO has been launched. The theoretical framework was developed and ground single sites or networks (GONG, IRIS, BiSON) were operational. Two very important results have appeared just before the launch: the determination of the photospheric helium content [3], [4] and the determination of the depth of the convective zone [5]. In fact, before helioseismology, the solar *helium content* (the second element in mass fraction) was only deduced from theoretical solar models. Its practically cosmological estimate (0.25 in mass fraction) showed the limit of one basic hypothesis of stellar models (the initial composition is equal to the present photospheric composition) and confirmed the need to introduce extra phenomena such as the slow atomic diffusion introduced first by [6] in 1989. This process leads today to a reduction of practically 10-15% of the He mass fraction at the solar surface [7], [8], [9].

Then, SoHO has played a dominant role for the investigation of the radiative zone. The very long and stable mission of SoHO has been crucial to progress on the properties of these layers down to the core because one needs very precise frequencies to scrutinize the whole radiative zone. Global acoustic modes of high frequencies (easier to observe) have a resonant cavity that includes the outer layers, largely perturbed by the turbulence and the varying magnetic field component along the 11 year cycle. Furthermore, these modes have a reduced lifetime leading to broad peaks dominated by the stochastic excitation of the modes. One success of SoHO, obtained by measuring the Doppler velocity shifts through two instruments (GOLF: Global Oscillations at Low Frequency especially designed for this purpose and MDI: Michelson Doppler

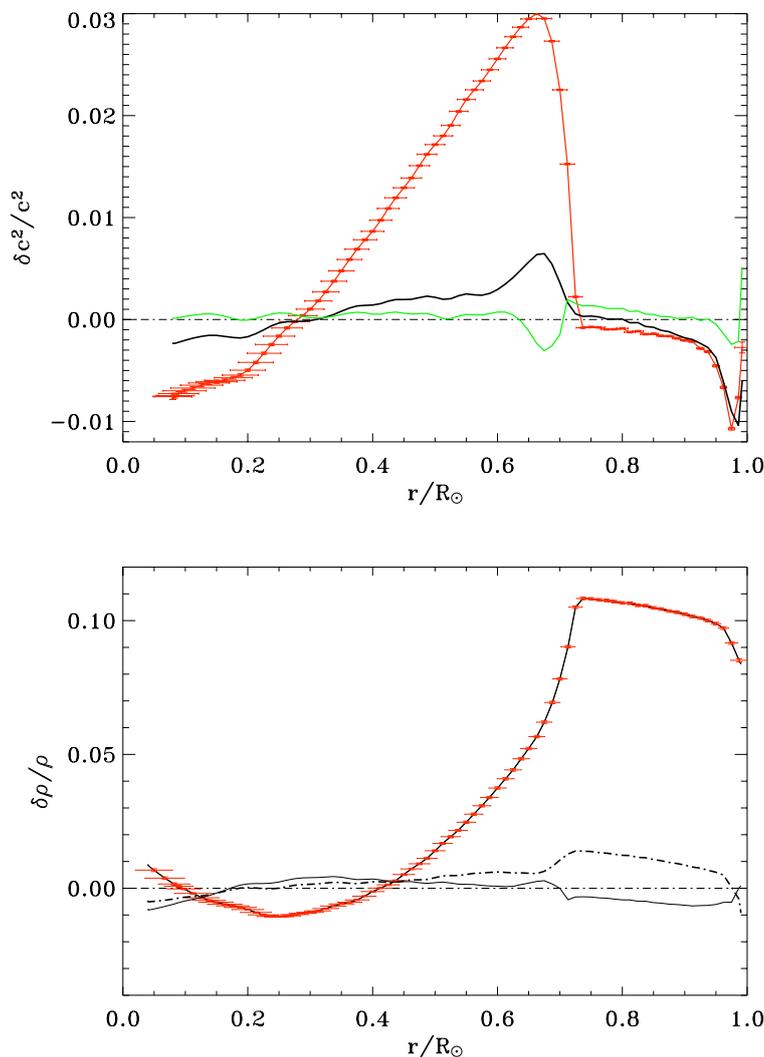


Figure 1. Squared sound speed and density differences between the seismic inversions obtained with the GOLF+MDI/SOHO acoustic modes and the standard model (using the Grevesse & Noels composition[14], black solid line), the seismic model (green or dashed lines) and the standard model (with the Asplund composition [15], red solid line with seismic error bars). The vertical error bars are so small that they are not visible on the figures, the horizontal error bars are rather large in the nuclear core (below $0.3 R_\odot$ which contains more than half the mass of the Sun), they will be reduced in measuring precisely several gravity mode frequencies. Deduced from [13, 19].

Imager), has been the capability to reach the low frequency range of the acoustic spectrum. The corresponding modes have higher lifetime but smaller intensities [10], [11]. From these modes, we have extracted a very clean sound speed profile down to $0.06 R_\odot$ [12], [13] and a reasonable density profile (Figure 1).

Helioseismology was the key for validating the various ingredients used in the construction of the standard solar model. It is in fact interesting to notice that each phenomenon (specific nuclear rate, specific opacity coefficient, screening or Maxwellian tail distribution) has a specific influence on the sound speed profile [16]. We have indeed shown that the present sound speed

does not favour any tiny variation of the Maxwellian distribution nor strong screening or large mixing in the core [17]. This is of particular importance to check the validity of the involved nuclear processes. It has also been possible from the sound speed profile in the core and due to the signature of each specific reaction rate [16] to put an observational constraint on the value of the p-p reaction rate which was known only theoretically because of the weak character of the interaction. Its influence on the sound speed profile in the core is strong and the cross section is well constrained nowadays within 1%.

2.2. The seismic solar model: *SeSM* for neutrino and gravity mode predictions

The quality of these seismic observations has allowed to build a *seismic model* which reproduces the measured solar sound speed [12] in the context of classical stellar evolution. The interest of such a model comes from the idea that the framework of the standard model could be too crude for reproducing all the existing observables. From such a model, we predict observables like neutrinos (see below) or gravity modes deduced not uniquely from the classical assumptions of the standard model but also from the observed sound speed [13, 18, 19, 20]. This model allows us to avoid any conflict with new updates such as the recent reestimate of the heavy element mass fraction contrary to predictions of the SSM [15]. The new solar chemical abundances are often considered as the origin of a crisis in helioseismology these last years and encourage a lot of studies. However, there is no reason to consider the present situation better or worse than previously as far as we know that the standard model is not the final representation of the real Sun (see sections 3 and 4).

Another important fundamental solar model value is $P_0 = 2\pi^2 \left(\int_0^{r_c} \frac{N}{r} dr \right)^{-1}$ where N is the Brunt-Väisälä frequency, r_c is the internal limit radius of the convective zone. P_0 characterizes the asymptotic behaviour of the gravity modes nearly equally spaced in period for frequencies below $\sim 100 \mu\text{Hz}$. Before the launch of SoHO, there was a great dispersion on the theoretical predicted values of P_0 . Following Hill [21], its value was varying between 29 mn to 63 mn depending on the models. Today, the values for standard and seismic models agree within 1 mn.

2.3. Neutrino properties and the solar central region

Two decades ago, the neutrino predictions coming from the astrophysical community were in disagreement with the measured neutrino fluxes [23], [24]. This is why we have used helioseismic constraints to improve the neutrino detections, see the review [27]. Progressively, we have injected the progress done on the characteristics of the plasma or thanks to helioseismic observations.

Table 1 illustrates the time evolution of the predicted ^8B neutrino flux which depends strongly on the central temperature and consequently, on the details of the plasma properties. At each step, the sound speed profile has evolved and the discrepancy with the observed one has increased or decreased. The seismic model prediction agrees remarkably well with the measured value obtained with the SNO detector (filled with heavy water), this detector is sensitive to all the neutrino flavours. For the gallium or water detector predictions, one needs to inject the energy dependent reduction factor due to the fact that the electronic neutrinos are partially transformed into muon or tau neutrinos (confirmed this year by the Borexino results [28]). Doing so, the agreement between the predictions of the seismic model and all the detectors is extremely good [29]. A larger deviation in the sound speed profile between the model and the Sun corresponds to a greater discrepancy between predictions and measured neutrino fluxes. Therefore, there is a remarkable agreement (see Figure 1 and Table 1) between the two probes of the central region of the Sun (neutrinos and helioseismology), but there are discrepancies with the actual SSM predictions. The central temperature of the Sun is now determined with an accuracy of 0.005.

Table 1. Time evolution of the boron neutrino flux prediction associated with the reaction ${}^7\text{Be}(p, \gamma) {}^8\text{B} \longrightarrow {}^8\text{Be} * + e^+ + \nu_e \longrightarrow {}^4\text{He}$, for solar structural models. Also mentioned are the corresponding central temperature T_c , initial helium content Y_0 and the origin of the improvements introduced in the corresponding solar model. These results must be compared to the recent results of the SNO detector of 5.27 ± 0.27 (stat) ± 0.38 (syst) [22].

Year	Boron neutrino flux	T_c	Y_0	Problem solved
1988 [24]	3.8 ± 1.1	15.6	0.276	CNO opacity, ${}^7\text{Be}(p, \gamma)$
1993 [25]	4.4 ± 1.1	15.43	0.271	Fe opacity, screening
1998 [9]	4.82	15.67	0.273	Microscopic diffusion
1999 [26]	4.82	15.71	0.272	Turbulence in tachocline
2001 [12]	4.98 ± 0.73	15.74	0.276	Seismic model
2003 [18]	5.07 ± 0.76	15.75	0.277	Seismic model +magnetic field
2004 [19]	3.98 ± 1.1	15.54	0.262	- 30% in CNO composition
2004 [19]	5.31 ± 0.6	15.75	0.277	Seismic model, ${}^7\text{Be}(p, \gamma)$, ${}^{14}\text{N}(p, \gamma)$

The neutrino masses are not yet determined, so the comparison with neutrino fluxes continues to identify new properties of neutrinos beyond the supersymmetry framework.

3. The Dynamical solar model: DSM

SoHO results allow the seismic community to test physics beyond the standard model. Helioseismic inversions of the rotation splittings leads to a rather flat rotation profile [30], [31] between 0.2 and 0.65 R_\odot with a potential increase in the nuclear core which remains to be confirmed [32], [33], [34]. In fact, all the recent seismic results, including those dedicated to the 11 year evolution, call for a revolution in stellar modeling. Dynamical processes have been first introduced to describe massive stars and then been applied to the case of the Sun.

3.1. The rotation profile obtained by previous works

Pinsonneault and collaborators [35], [36] have first treated the rotation effect via a diffusion equation and predicted a large amount of differential rotation in the solar interior. Later studies, using a refined version of rotational mixing in which the advective nature of the Eddington-Sweet meridional circulation is taken into account, reached the same conclusion [37], some example is shown on the first figure 2.

Other authors computed the effect of a static fossil dipolar magnetic field on the solar rotation. They showed that this magnetic field indeed spins down the radiative zone if it is disconnected from the convection zone [38], [39]. Eggenberger et al. [40] showed that they seem to reproduce the flat solar radiative rotation rate by introducing the magnetic instability of the Tayler-Spruit dynamo but they do not produce the core increase and this kind of instability they use is still in debate [41]. In parallel, 3-D MHD calculations of portions of the Sun have been undertaken in order to reproduce the seismic observations. The first 3-D MHD simulations of the radiative zone have been performed, including the differential rotation of the tachocline. They show that the fossil dipolar field diffuses outward during the main sequence and connects to the surface convection zone, imprinting its differential rotation to the radiative core [42].

Another approach to solve the flat solar rotation profile, observed in a large part of the radiative zone, involves internal gravity waves (IGWs). The low frequency traveling waves are excited at the base of the convection zone and may be a source of angular momentum

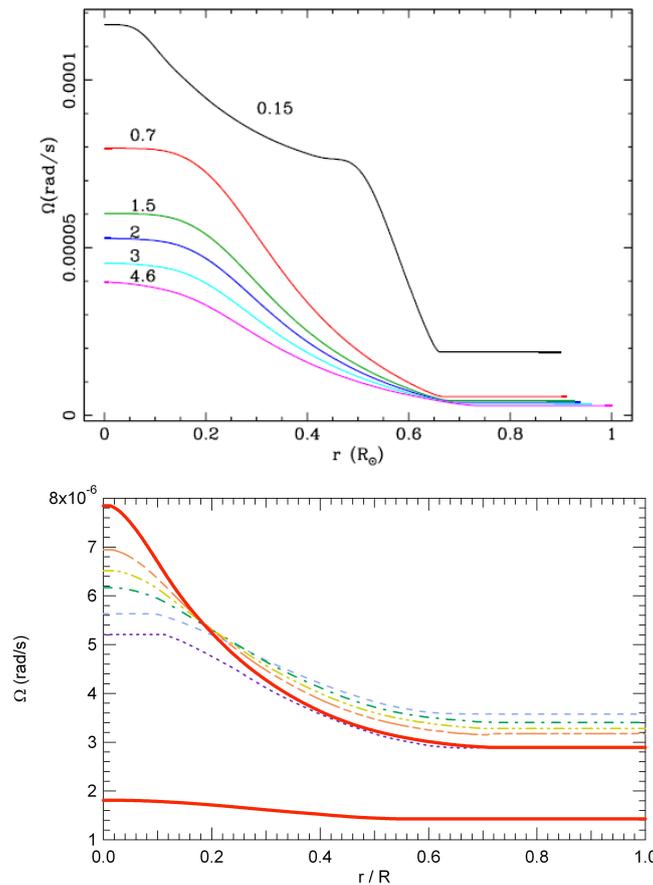


Figure 2. Evolution of the internal solar rotation profile with different initial hypotheses. On the first figure, the profile is chosen uniform at the arrival on the main sequence and the sun is then quickly slowed down to reach 2 km/s at the present time. The lines correspond to age from 0.15 to 4.6 Gyrs. From [43]. On the following figure, we present an extreme model where the solar internal rotation is shown for ages 0.010, 0.025, 0.050, 1, 2, 3, 4.6 Gyrs (from bottom to the top) beginning the computation from the PMS (initial and final profiles in red full lines), but ignoring any braking. The initial external rotation (red full line between 1.8 to $1.2 \cdot 10^{-6}$ rad/s) is chosen to get the superficial rotation of 2 km/s at the present time. From [52].

redistribution, since they take momentum from the region where they are excited and deposit it where they are damped. When both prograde and retrograde waves are excited, in the presence of shear turbulence, this produces a rapidly oscillating shear layer similar to the quasi-biennial oscillation of the Earth's stratosphere. If the surface convection zone is rotating more slowly than the core as expected from surface magnetic braking, the shear layer oscillation (SLO) becomes asymmetrical, and produces differential filtering that favors the penetration of low-degree, low-frequency retrograde wave [44]. These waves may then deposit their negative angular momentum in the deep interior, causing the spin-down of the solar core on evolutionary time-scales. A complete formalism has been developed [45, 46] to include such an effect in stellar evolution codes. In the absence of differential rotation, wave transport becomes negligible away from the thin SLO. Such formalism has been applied to an evolving solar mass model in which the surface convection zone is slowly spun-down with time [47].

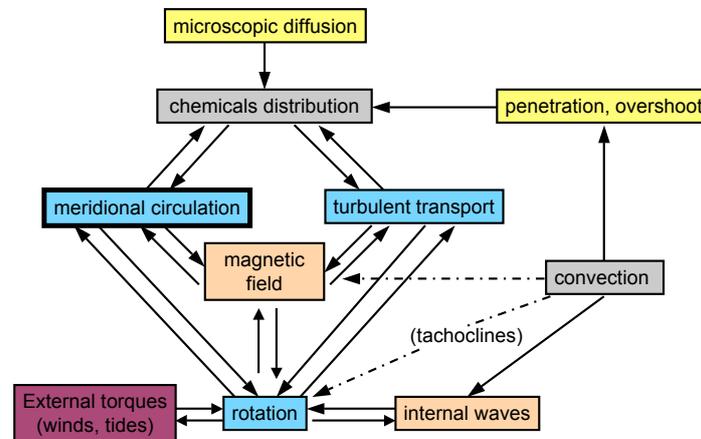


Figure 3. Description of the different processes that one needs to incorporate in the stellar equations in order to take into account the internal dynamics of stars. From [48].

3.2. Building the DSM step by step

Despite all these efforts, the observed solar rotation profile from the core to the surface remains to be explained in details. All the dynamical processes, convection, rotation and magnetic fields and their interconnection need to be included simultaneously in solar (stellar) models as shown in Figure 3 together with the role of the low frequency gravity waves. It will result a dynamical solar structure model which must reproduce all the present observations. This new objective involves the introduction of the various terms which contribute to the angular momentum transport along the evolution. A complete formalism has been established recently which takes into account the different aspects of the transport of momentum in stellar equations [49, 50, 51]. This complex system of 16 equations is under implementation in different stellar evolution codes (STAREVOL, CESAM) to build step by step this solar dynamical model using also the observational constraints coming from the young stars and those coming from asteroseismology.

In reality the solar magnetic torquing arrives very early. So we have computed with the CESAM code [52] an extreme model beginning in the premain sequence stage to estimate the role of the different phases of the solar evolution. In these conditions the star is not totally contracted at the beginning and our objective was to reach the present superficial rotation at the present age without imposing any high initial rotation. This objective is satisfied with an early flat and relatively low rotation profile. One notices on Figure 2 bottom, that such model leads also to a radial differential profile in the radiative zone. In this case, the slope of such a profile is due to the advection of angular momentum by the meridional circulation; this work shows that that the theoretical present central rotation is clearly due to its whole past story but does not reflect evidently its birth profile. The resulting solar sound speed profile is examined in parallel, one has already noticed that introducing rotational mixing could slightly increase the difference with the observational profile due to a change in the chemical profile [43]. A quantitative estimate of this change depends largely on the initial conditions of the rotation profile and its story. The case showing on Figure 2 top has more impact on the sound speed profile than the second case illustrated on the same figure [53] but the contrast between central and superficial rotation in the second case is in better agreement with the one deduced from observations [34] even the flat

profile between 0.3 and 0.7 R_{\odot} is up to now never properly reproduced.

These studies will be pursued during the next years in introducing all the processes and their interconnection and in using 3D simulations to guide us in the understanding of some instabilities or some estimate of the real energy budget. The corresponding Dynamical Solar Model will be confronted to all the seismic and neutrino observations.

4. Emerging questions pushing new observational and experimental constraints

SoHO has delivered a lot of information on the solar variability and a 3D vision of the Sun slowly appears. New questions emerge today which justify complementary observations or some experiments. It is certainly important to understand the solar cycles in duration and amplitude since Galilee observation of the first sunspots. This step is necessary before being able to predict such phenomena for the next century, SoHO has demonstrated the important role of the inner solar behavior, so the simulation of the inner variability on century time-scale is one objective of the Dynamical Solar Model.

4.1. *The next generation of observations*

After 10 years of SoHO observation, one still needs to establish properly what is the present central rotation and if the core turns obliquely in comparison with the rest of the radiative zone. We must also better understand the story of the radiative magnetic field and if this magnetic field or the gravity waves contribute to generate some other kind of variability than the 11 year dynamo attributed now to the whole convective zone including the tachocline. To answer to these questions, a new generation of instruments is required. Among them the development of the GOLF-NG instrument is specifically dedicated to the dynamics of the core, the PICARD microsatellite and the SDO satellite (launch 2009) will observe the impact of the variability on the shape evolution and will follow the internal dynamics at least down to the limit of the convective zone. A large international community considers that a permanent and simultaneous observation of the different crucial regions of the Sun: the core, the transition photosphere-chromosphere and the corona plus the associated solar wind and mass ejections continues to be a fundamental need after SDO. A large formation flying DynaMICCS/HIRISE proposal around the L1 Lagrangian point has been retained in the Cosmic Vision 2015-2025 perspective of ESA but not selected for the first mission of this program due to the schedule of Solar Orbiter which will be launched only in 2015 probably (see the corresponding presentation in this conference and the internal references).

4.2. *The solar composition and the associated experiments*

One important question today is to know if we understand the inner composition of the Sun for the heavy elements. Recent studies of the solar atmosphere both through the line analyses and through the 3D turbulent character of these layers have led to a reduction of 30 % of the CNO composition (after a similar reduction of iron 10 years before for different reason) [15]. Such information is crucial in the context of the Galactic evolution both for the evolution of helium and for the formation of the Sun in a well known neighbourhood [19, 54]. After 30 years of helioseismic investigation, we ask the following questions: do we know properly the internal composition of the Sun? could we deduce it from the hypotheses of the standard model for the solar radiative interior and the convective region or do we need to add some dynamical effects with a redistribution of the chemical species?

The results we get for the SSM are shown on Figure 1. The sound speed and density profiles of the model including the updated composition are very similar than those corresponding to the case where we omitted to take into account the slow gravitational settling of the elements [9, 20]. This point helps us to quantify the effect of the change of composition: a 30% variation of CNO is practically equivalent to 10-12 % variation of all the heavy elements or by a change

of opacity coefficients of the same order for these elements. Of course it is interesting to look for the responsibility coming from some other element which may be badly determined like the neon. Nevertheless thinking of only an error on this element to compensate the large effect produced is excluded by the large number of measurements already obtained on this specific element in different thermodynamical conditions [55] and by recent new determination which confirms the old ones. Another way to try to disentangle the different effects is to extract the CNO composition from helioseismic investigation of the heavy elements [56].

In order to progress on the composition in the radiative zone, we will develop the Dynamical Solar Model following the equations quickly mentioned above and we will also begin a campaign of opacity measurements. The high intensity lasers which are developing in France and United States will offer conditions corresponding to the basis of the convective zone and we prepare these experiments in the continuity of what we have done previously for other conditions [57]. Laboratory opacity and equation of state experiments might justify in the future an equivalent effort as the one dedicated to nuclear reaction rate measurements in the last century for the benefit of stellar evolution. We need such an effort for a proper knowledge of the radiative zones, it will contribute to understand the real composition of the Sun in the deep interior. These measurement will be also useful for a good understanding of pulsation in stars.

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