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The Klein-Nishina Formula & Quantum Electrodynamics

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One of the greatest scientific revolutions in the history of mankind was the development of Quantum Mechanics. Its birth was a very difficult process, extending from Planck's paper of 1900 to the papers of Einstein, Bohr, Heisenberg, Schrödinger, Dirac and many others. After 1925-1927, a successful theory was in place, explaining many complicated phenomena in atomic spectra. Then attention moved to higher energy phenomena. It was in this period, 1928-1932, full of great new ideas and equally great confusions, that the Klein-Nishina Formula played a crucial role. The formula was published in 1929, in the journals *Nature* and *Z. Physik*. It dealt with the famous classical problem of the scattering of light waves by a charged particle. This classical problem had been studied by J. J. Thomson. Conceptually in classical theory, the scattered wave's frequency must be the same as the incoming frequency, resulting in a total cross-section:

$$\sigma = \frac{8\pi}{3}\frac{e^4}{m^2c^4}.$$

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But in 1923 in an epoch making experiment, Compton found that the scattered waves had a lower frequency than the incoming waves. He further showed that if one adopts Einstein's ideas about the light quanta, then conservation laws of energy and of momenta in fact led quantitatively to the lower frequency of the scattered waves.

Compton also tried to guesstimate the scattering cross-section, using a half-baked classical picture with ad hoc ideas about the frequency change, obtaining:

$$\sigma = \frac{8\pi}{3}\frac{e^4}{m^2c^4}\frac{1}{1+2\alpha},$$
$$\alpha = h\nu/mc^2.$$

Now, when $h\nu$ is very small compared to mc^2, this formula reduces to Thomson's.

This Compton theory was one of those magic guess works so typical of the 1920's, He knew his theory cannot be entirely correct, so he made the best guess possible.

Later on, in 1926, Dirac and Gordon used Quantum Mechanics in different ways, but obtained the same formula:

$$\sigma = \frac{2\pi e^4}{m^2c^4}\frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha}\ln(1+2\alpha)\right].$$

This formula is very much like the Compton formula, only more complicated. It is also not entirely correct, because it doesn't have the electron spin.

Then came Dirac's relativistic equation of 1928 which led to great success but greater confusion. Some forty years later, Oppenheimer in his interview by T. S. Kuhn, used the metaphor:

Magic and Sickness

to describe Dirac's equation. Why Magic? Because

1. Before Dirac's equation, the spin was a hypothesis, but with Dirac's equation the spin was natural.
2. It had the correct spin-orbit coupling,
3. It had the correct magnetic momenta for the electron.

Yet there is also Sickness because of the "negative energy states", which led to great contradictions. Sickness led to confusion, sometimes even to madness. To give one example: Eddington entered the picture, saying that Dirac's equation is 4×4, and 4 is 2×2. But $(8\times8+2\times2)\times2 = 136$, so he claimed the fine structure constant should be 1/136. One year later he modified the theory, and said "no, you should add one to it", so it should be $1/137$.

A few months before he died, Eddington said

I am continually trying to find out why people find the procedure obscure. But I would point out that even Einstein was considered obscure, and hundreds of people have thought it necessary to explain him. I cannot seriously believe that I ever attain the obscurity that Dirac does. But in the case of Einstein and Dirac people have thought it worthwhile to penetrate the obscurity. I believe they will understand me all right when they realize they have got to do so ? and when it becomes the fashion "to explain Eddington."

Thus Dirac's magic and sickness did in a way influence Eddington.

The dominant question in physics in 1928-1930 was:

Was Dirac's equation correct?

In this atmosphere Klein and Nishina arrived at their formula in September 1928:

$$\sigma = \frac{2\pi e^4}{m^2c^4}\frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha}\ln(1+2\alpha)\right] + \frac{2\pi e^4}{m^2c^4}\left[\frac{1}{2\alpha}\ln(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2}\right].$$

This was a remarkable formula for two reasons: (a) It turned out to be correct, and (b) it however was based on a wrong theory, i.e. Dirac's 1928 paper which had the sickly negative energy states.

Soon after the K-N paper, people found the Klein-Nishina formula to be in rough agreement with the data about the absorption of x-rays by matter, which was taken as additional support for Dirac's equation. But still the negative energy states remained a fundamental sickness and caused great agonies. Oppenheimer later remembered that Pauli's opinions then was "Any theory that had such a sickness must agree with experience only by accident." That was the typical attitude of the theoretical physicists around that time.

Then came the hole theory of Dirac. Dirac said: Okay, there are many negative states, but this sea of negative states is usually fully occupied. Once in a while, there is a hole in the sea and that would appeared as a positively charged particle. That was the hole theory. He first proposed the idea in a letter to Niels Bohr dated Nov. 26, 1929, and later published it in 1930.

This revolutionary idea of Dirac's introduced the subtle modern view of the complexity of the vacuum. Thus began the modern quantum field theory of electromagnetism: which today we call QED. In a speech of 1959 at Bryn Mawr, I had likened

Dirac's bold proposal of the hole theory to the first introduction of negative numbers.

How about the K-N formula in the new hole theory? It was then shown by Dirac, and by Waller independently, that the Klein-Nishina formula, derived without the infinite sea of holes, was nevertheless magically correct. Thus the Klein-Nishina formula became the first correct formula of QED discovered by physicists. Its agreement with experiments was, e.g. reported by Rutherford in his Presidential address to the Royal Society.

Despite this, most theorists still refused to believe the hole theory. It was deemed too revolutionary. Pauli, Bohr, Landau and Peierls all argued against it. The negative energy states were dubbed "donkey electrons" and ridiculed. Why "donkey electrons"? Because with a negative energy state, if you apply force on it, the more force you apply, the more it resists, like the behavior of a donkey. Adding to the confusion was Dirac's original proposal to have protons as the holes. His 1930 paper had the title "A Theory of Elections and Protons".

This particular confusion was later resolved through the theoretical papers of Oppenheimer, Tamm and Dirac, all in 1930. In these papers they proved that a hole cannot be a proton, because if the hole were a proton, then an electron would jump into it, in approximately 10^{-10} seconds, making the hydrogen atom unstable. The conclusion was: there has to be 2 seas, one for the electron and one for the proton.
Thus by the end of 1930 the main theoretical framework for QED was complete. But a new experimental confusion arose, also in 1930, delaying the general acceptance of QED for another 2 years. The experiment was the absorption of γ rays by heavy elements. Earlier work with lower energy photons had produced agreement with the K-N formula. Now higher energy γ-rays became available at about 2.6 MeV, from thorium C. People began to check with the new γ-ray the validity of the Klein-Nishina formula. In a 1983 article by C. D. Anderson, reminiscing about 1930-1932:

At that time it was generally believed that the absorption of 'high energy' γ-rays was almost wholly by Compton scattering, as governed by the Klein-Nishina formula.

Anderson was a graduate student of Millikan, who had assigned another graduate student, C. Y. Chao, to study with counters this absorption process, to see whether for γ-rays it also agrees with the K-N formula.

In a beautifully simple experiment, Chao found that for heavy targets at 2.6 MeV there is more absorption than predicted by the K-N formula. He called these "anomalous absorption". Furthermore, in a second experiment, also in 1930, Chao found what he called "additional scattered rays" in the scattering.

These discoveries were not understood theoretically, and were thought to be nuclear phenomena, unrelated to QED. Adding to the confusion was the unfortunate situation that two other experimental groups did not agree with Chao's findings. As we shall mention below, Chao's findings were in fact correct, and "anomalous absorption" and "additional scattered rays" were essential QED phenomena required by the hole theory.

But that understanding came only after Anderson's discovery in 1932 of the positron with his cloud chamber pictures, vividly showing the correctness of the hole theory. People then looked back at the Chao experiment and found that in fact

"Anomalous absorption" was pair creation, and
"Additional scattered rays" was pair annihilation.

Both essential phenomena in QED! Thus by 1932 the theoretical framework of QED was complete, and was in agreement with all experiments.

The next chapter in the history of QED opened with the theoretical discovery in the mid-1930s of higher-order divergences, which led eventually to the renormalization program of 1947-1949. But that is not the subject of my talk today.