CONSTRUCTION OF AUTOMATIC CORRECTION SYSTEMS FOR THE RADIO FREQUENCY VOLTAGE AND THE RATE OF GROWTH OF THE MAGNETIC FIELD OF CYCLIC ACCELERATORS*)

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It is known that the dynamic parameters of the automatic correction system increase the order of the nonlinear differential equation which describes the motion of the accelerated particles. The conventional investigations of the acceleration process by the phase plane method permit to consider only the transfer coefficient of automatic systems [1]. The method of harmonic linearization proposed in this paper permits a study of the oscillation process with a consideration of the coefficient of transfer as well as of inertness and of other dynamic properties of the individual loops of the correction system.

The phase motion of particles can be described by the nonlinear differential equation

$$M(p)\psi + N(p)F(\psi) = H(p)[h(t) + g(t)], \quad p \equiv \frac{d}{dt}, \tag{1}$$

where M(p), N(p) and H_p are linear polynoms.

A very essential influence upon the oscillatory motion of charged particles is exerted by the nonlinearity

$$F(\psi) = \sin\psi_s - \sin\psi = \sin\psi_s - \psi + \frac{1}{6}\psi^3 - \frac{1}{120}\psi^5 + \ldots \tag{2}$$

of the accelerating voltage. By considering the great difference in the frequencies of the equilibrium ψ_s and the oscillatory ψ_α components

*) This report was not read.

of the phase motion

$$\psi = \psi_s + \psi_\alpha = \psi_s + a_\varphi \sin\theta, \quad \frac{da_\varphi}{dt} = -a_\varphi \gamma_\alpha, \quad \frac{d\theta}{dt} = \omega, \tag{3}$$

we divide the external effects also into adiabatic h(t) which determine the equilibrium phase ψ_s and into resonance g(t), vibratory synchrotron oscillations ψ_α.

1. Investigation of the oscillatory motion of particles

In the analysis of nonlinear processes the structural scheme of the system is divided according to equation (1) into a linear part

$$W_1(p) = \frac{N(p)}{M(p)} \tag{4}$$

and a nonlinear part $F(\psi)$ (fig. 1). The nonlinear properties of the

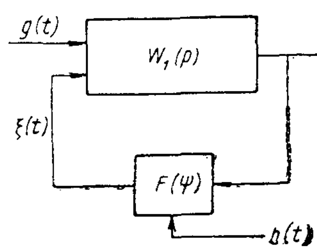

Fig. 1.

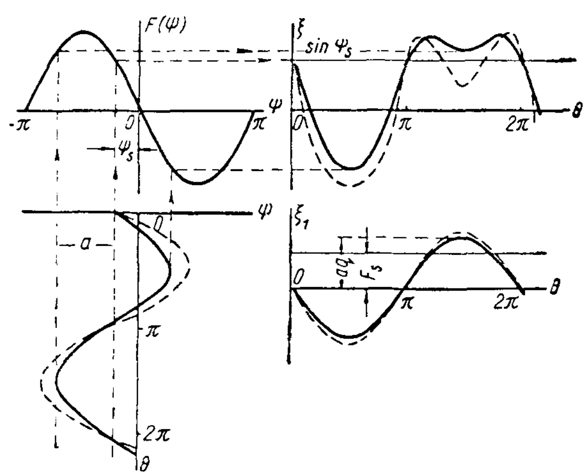

Fig. 2.

oscillation process represent the result from a successive nonlinear transformation (fig. 2) and a linear filtration. By assuming that the condition for an ideal filter

$$\left|\frac{N(j\omega)}{M(j\omega)}\right| \gg \left|\frac{N(kj\omega)}{M(kj\omega)}\right| \quad (k=2,\ 3,\ \ldots) \tag{5}$$

is fulfilled at the egress of the nonlinearity $F(\psi)$, we consider only the first harmonics of oscillations amplified q times as compared with the process ψ at the input, as well as the adiabatic component F_s (see fig. 2).

The harmonic amplification coefficient q and the translocation function F_s can be written in the shape

$$q=\frac{1}{\pi a_\varphi}\int_0^{2\pi}\left(\sin\psi_s-\psi+\frac{1}{6}\psi^3-\frac{1}{120}\psi^5+\ldots\right)\times$$
$$\times\sin\theta\,d\theta=1-\frac{1}{2}\psi_s^2-\frac{1}{8}a_\varphi^2+\frac{1}{24}\psi_s^4+$$
$$+\frac{1}{16}a_\varphi^2\psi_s^2+\frac{1}{192}a_\varphi^4-\ldots\approx 1-\frac{1}{2}\psi_s^2-\frac{1}{8}a_\varphi^2; \tag{6}$$

$$F_s=\frac{1}{2\pi}\int_0^{2\pi}\left(\sin\psi_s-\psi+\frac{1}{6}\psi^3-\frac{1}{120}\psi^5+\ldots\right)d\theta=$$
$$=\sin\psi_s-\psi_s\left(1-\frac{1}{6}\psi_s^2-\frac{1}{12}a_\varphi^2+\frac{1}{120}\psi_s^4+\right.$$
$$\left.+\frac{1}{24}\psi_s^2a_\varphi^2+\frac{1}{64}a_\varphi^4-\ldots\right)\approx$$
$$\approx\sin\psi_s-\psi_s\left(1-\frac{1}{6}\psi_s^2-\frac{1}{12}a_\varphi^2\right). \tag{7}$$

At the nonlinearity egress we have the harmonically linearized process

$$\xi_1=F(\psi_1)=F_s+q\psi_\alpha. \tag{8}$$

A translocation of the oscillation process is energetically impossible without the adiabatic component of the external effect h(t). The differential equation (1) can be presented in the harmonically linearized shape

$$M(p)(\psi_s+\psi_\alpha)+N(p)(F_s+q\psi_\alpha)=H(p)\,h(t) \tag{9}$$

or as interconnected through F_s and q with a system of equations for the adiabatic and oscillatory process

$$M(p)\,\psi_s+N(p)\,F_s=H(p)\,h(t), \tag{10}$$
$$M(p)\,\psi_\alpha+N(p)\,q\psi_\alpha=0. \tag{11}$$

For the equation (1) we can write

$$eU_0(t)\sin\psi_s(t) - J(t) = b_{a3}\frac{dH_M}{dt}, \qquad (12)$$

where $U_0(t)$ is the amplitude of the accelerating voltage, J(t) the intensity of the magnetic field. $\psi(t)$ depends also on the frequency of the accelerating voltage and on the velocity of particles.

The stability and quality of the acceleration process can be investigated according to the linear equation (11). The acceleration process proceeds normally when the particles delayed from the equilibrium position with respect to the phase pass through the resonators at a higher voltage than in the equilibrium phase ψ_s. This requires that the roots of the characteristic equation

$$M(p) + N(p)q = 0 \qquad (13)$$

should not possess positive real parts. Equation (13) can be examined according to the Hurwitz criterion and by frequency and other criteria used in the analysis of linear systems. Random low-frequency variations of the parameters of the accelerating voltage and of the magnetic field lead to a gradual change of the values h(t) and $\psi_s(t)$.

In the investigation of the correction system of synchrotron oscillations by the position of the beam the analysis of the influence of external effects g(t) whose spectrum is located in the vicinity of the frequency $\Omega_c(t)$ of synchrotron oscillations is of a great importance. Resonance interferences affect the slowly changing phase as well as the oscillatory component of the process:

$$\psi_p = \psi_s + \psi_s^{\text{rand}} + a_\varphi \sin\theta, \qquad (14)$$

where $\psi_s^{\text{random}} + \alpha\sin\theta$ is treated as a total random process. The translocation function is given by the equation

$$F_{sp} = \frac{1}{2\pi}\int_0^{2\pi} d\theta \int_{-\infty}^{\infty} F(\psi_p)\, w_c(\psi_p^{\text{rand.}})\, d\psi_p, \qquad (15)$$

and the harmonic amplification coefficient is substituted by the equivalent [2]

$$q^{\text{equil.}} = \frac{q^{\text{st}} + \frac{2\sigma_p^2}{a_\varphi^2}\, q^{\text{transloc. equil. ph.}}}{1 + \frac{2\sigma_p^2}{a_\varphi^2}} \qquad (16)$$

which is determined from the condition of the minimum of the mean square deflection of the substituting function from the initial one

$$M'[(F(\psi) - F_{sp} - q^{\text{экв}}\psi_\alpha)^2] = \min,$$

where

$$w_c(\psi_p^{\text{rand}}) = \frac{1}{2\pi}\int_0^{2\pi} w(\psi_s^{\text{rand}} + a_\varphi \sin\theta)\, d\theta \text{ —}$$

is the differential law of the distribution of the process; $q^{\text{st.}}$ considers the variation of the amplitude at ψ_s = const, and $q^{\text{transloc. equil. ph.}}$ the variation of the amplitude by a random translocation of the equilibrium phase; M' [ ] - symbol of the mathematical expectancy.

If a stationary random disturbance is characterized by the two first probable moments (by the mathematical expectancy and dispersion), then because of the nonlinear structure of the accelerator the determination of the random component phases and of the oscillation process is connected with the highest probable moments. Because of the closed state of the structural scheme of the accelerator the linear part smooths the highest probable moments. A consideration of the amplification of only the two first probable moments in the nonlinearity $F(\psi)$ means a statistical linearization.

2. Analysis of the system of automatic correction of the position of the center of the beam

A variation of the amplitude of accelerating voltage leads to a change of the tolerable amplitude of synchrotron oscillations (fig. 3).

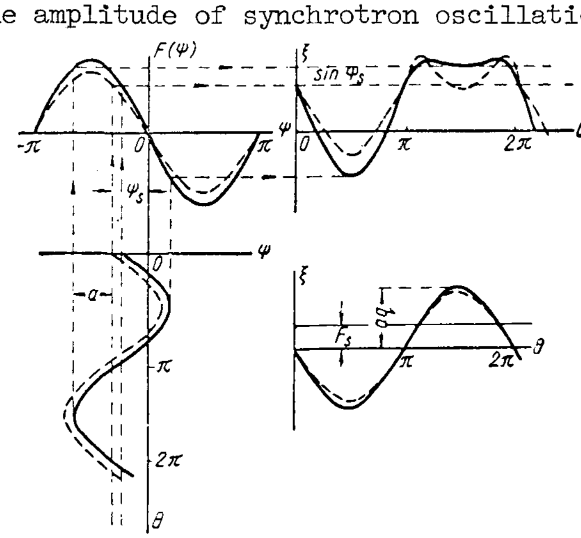

Fig. 3.

An analogous result is obtained also from a variation of the frequency of the accelerating voltage and also of the magnetic field intensity [3].

If the external perturbations cause a discrepancy of the voltage pulse with respect to the value α_1^0, then in the uncorrected accelerator the center of the beam starts to perform synchrotron oscillations with the initial amplitude $\psi_s''' - \psi'$ (fig. 4). By considering the opera-

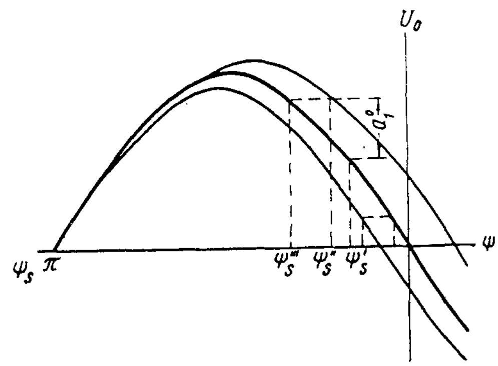

Fig. 4.

tional mechanism as a modulator of the frequency of the accelerating voltage, we find that the amplitude of the voltage varies with the variation of the frequency according to the characteristics of a real resonator (fig. 5). For the attainment of favorable conditions for the

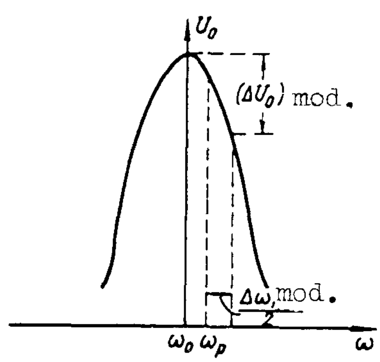

Fig. 5.

automatic correction of the voltage in the phase of the center of the beam it is required that the mean value of the frequency ω_p of the accelerating voltage should be slightly higher than the resonance frequency of the resonator (see fig. 5). Then the lagging of the center of the beam behind the equilibrium phase is compensated by an increase of the voltage pulse not only because of the increase of the period of the accelerating voltage, but also because of a simultaneous increase of the voltage amplitude (see fig. 4). In the corrected accelerator the center of the beam performs synchrotron oscillations with an amplitude of not $\psi_s''' - \psi_s'$ but $\psi_s'' - \psi_s'$ and transits into the new equilibrium phase ψ_s''' after several periods of synchrotron oscillations.

For the oscillatory motion of the center of the beam in the corrected accelerator we have [3]

$$[M(p) + H(p) W_c(p)] \psi_\alpha + N(p) q \psi_\alpha = 0. \tag{17}$$

According to the requirement of the process stability the maximum quick operation can be determined at a given transfer coefficient of the system from equation (17).

3. Principle of the construction of extreme acceleration systems for charged particles

An accelerator is discussed with an extreme regulation of the rate of growth of the magnetic field which decreases the energy losses caused by radiation without losses of particles in the beam. At the maximum rate of growth of the magnetic field the time of acceleration diminishes and the energy losses by radiation which are proportional to the radiation time decrease. The information concerning the magnitude of the equilibrium phase and the beam dimensions leaves the pick-ups of the regulator and enters the analyzing assembly where the prompt values of the equilibrium phase are compared with the ones tolerable at certain dimensions of the beam and the signal is generated for an adjustment of the magnetic control field.

If in the simplest case the characteristic equation obtained from the equations (11) or (17) possesses the shape

$$p^2 + 2\gamma_\alpha p + \Omega_b^2 q = 0, \tag{18}$$

then the program of the analyzing assembly is determined from the condition

$$q = 1 - \frac{1}{2}\psi_s^2 - \frac{1}{8}a_\varphi^2 + \frac{1}{24}\psi_s^4 + \frac{1}{16}\psi_s^2 a_\varphi^2 + \dots \geqslant \frac{\gamma_\alpha^2}{\Omega_b^2}, \tag{19}$$

where $\alpha_2 = 2\alpha$ are the dimensions of the beam along the phase.

Refined calculations obtained from a consideration of the highest harmonics of the oscillatory motion or by means of a simulator, show that the maximum values of the amplitude α of synchrotron oscillations in dependence on the magnitude of the equilibrium phase ψ_s (fig. 6) are determined from the equation

$$1-\frac{2}{\pi}|\psi_s|-\frac{1}{\pi}\alpha_\varphi=0. \qquad (20)$$

We believe that it is the easiest to materialize an extreme regulator for an accelerator where the rate of growth of the magnetic field at the end of the acceleration cycle has been increased previously. In this case a commutator can be used as the performing mechanism which

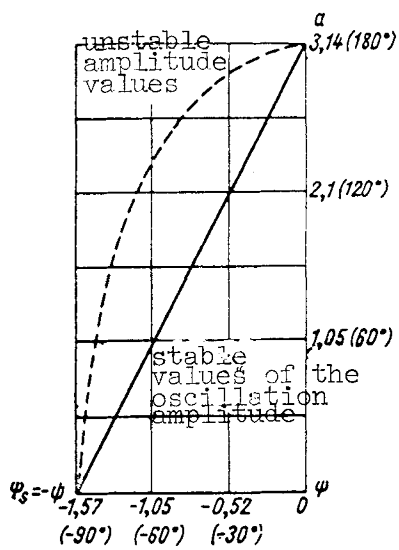

Fig. 6

transfers the corresponding part of the energy from the source of the feeding of the electromagnet to other objects by the signal of the analyzing assembly.

4. Refinement of the calculation results on a simulator

The method of harmonic linearization is approximate and does not yield exact quantitative data, in particular for such processes which are close to the limit of stability. The calculating units of the electronic simulator are connected according to the same structural scheme (see fig. 1) which is the basis for the calculation by the method of harmonic linearization.

In the performance of calculations on the electronic simulator the condition (5) of the ideal filter is not indispensable, since at the egress of linear blocks we have not only a translocated first harmonics, but also a real process which corresponds to all parameters of the differential equation. This explains the good accuracy of calculation by means of a simulator. A loss of the stability of motion of particles in the chamber corresponds in the simulating device to a periodically increasing process. In fig. 2 and 3 (dash curves) the intersection of the "dip" at the second half-period of the curve $\xi(\theta)$ with the line $\xi = \sin \psi_s$ characterizes the loss of stability.

The perturbing effect is transmitted to the oscillation process in the simulating device by means of low-frequency generators. The circuits RC imitate the parameters of automatic correction systems.

The method of harmonic linearization and the calculation procedure on a simulating device supplement each other and provide a comparatively simple and descriptive picture of the acceleration process for the choice of the optimum parameters of the accelerator.

Literature cited

1. A. A. Kolomenskii and A. N. Lebedev. Theory of cyclic accelerators. Moscow, Fizmatgiz, 1962.
2. A. A. Pervozvanskii. Random processes in nonlinear automatic systems. Moscow, Fizmatgiz, 1962.
3. V. A. Bertulis. "Izv. vusov. Radiofizika" 6, 4 (1963).