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Gas Flow Model for the MDT Chambers.

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We present a model of the gas flow in the MDT Chambers. As a specific example we will use the US MDT prototype. Detailed derivation and explanations of the formulas and tables presented here can be found in many books. The one we used is Vacuum Technology by Alexander Roth (North-Holland Publishing Company).

The Basics. (Skip this section if you are familiar with this subject.)

Descriptions of gas flow are valid only in given ranges of pressure. At very low pressure where the mean free path of gas molecules exceed the distance between walls and collisions between gas molecules rarely occur and collisions with the container walls determine the properties of gas flow. At intermediate pressures molecular collisions dominate and the mean free path of molecules in the gas determines the behavior of the gas. At very high pressure the distance between molecules is sufficiently small that intermolecular forces become important.

The pressure regime that we are working in is the intermediate pressure where flow is generally described as *viscous* or *molecular* flow. In addition, we will consider only isothermal, steady state flow situations.

For molecular flow the mean free path of a molecule is determined by the molecular cross section, the velocity distribution, and the pressure of the gas. The mean free path is given by the equation:

$$\lambda = \frac{\Gamma}{P\left(1 + \frac{c}{T}\right)} \qquad \begin{array}{l} P\ (Torr) \\ \Gamma\ (Torr \bullet cm) \\ c\ (^\circ K) \\ \lambda\ (cm) \end{array}$$

where **c** is Southerland's constant which is a measure of the strength of the attractive forces between molecules. For the operating point of the MDT's the mean free path is much less than a micron. Typical values are given in the table below.

Table 1

Gas	Reduced Mean Free Path (Γ) (Torr•cm) @ ∞ temperature and 1 Torr.	Southerland's Constant (c) (°K)	Viscosity (η) (micropoise) @ 0°C and 760 Torr
Argon	7.×10-3	169	208.8
Helium	1.6×10-3	79	186.9
Hydrogen	10.56×10-3	76	84.7
Neon	11.2×10-3	56	312.4
Nitrogen	6.1×10-3	112	166.6
Oxygen	6.87×10-3	132	191.0
Water Vapor	5.9×10-3	142	–

A gas streaming through a narrow-bore tube experiences a resistance to flow so the velocity decreases smoothly from the tube axis until it reaches zero at the wall. In this situation each layer of gas (parallel to the wall of the tube) exerts a tangential force on the neighboring layers. This force is proportional to the velocity gradient. The coefficient of viscosity is defined as the tangential force per unit area for unit rate of decrease of velocity with distance. This is a measure of the rate at which momentum is being transported between adjacent layers of a gas. The coefficient of the viscosity η has units of $ML^{-1}T^{-1}$ and in cgs units:

$$1\ poise\ =\ 1\ \frac{gm}{cm\ \bullet\ sec}$$

and in the MKS system:

$$1 \;\; Pascal \;\; sec \;=\; 1 \;\frac{kg}{m \,\bullet\, sec} \;=\; 10 \;\; poise.$$

The variation of η with temperature is given by:

$$\eta \;=\; \eta_0 \left(\frac{T}{T_0}\right)^{\frac{1}{2}} \left(\frac{1 \;+\; \frac{c}{T_0}}{1 \;+\; \frac{c}{T}}\right)$$

where **c** is Southerland's constant. We see that the viscosity of a gas increases with increasing temperature unlike a liquid in which the viscosity decreases with increasing temperature.

In the molecular flow regime the flow can be either turbulent or laminar. At lower velocities the flow is laminar e.g. in a pipe the layers of gas with constant velocity are parallel with their velocity increasing from the walls toward the axis of the pipe. When the velocity of the gas exceeds certain values the flow is turbulent. The gas layers are not parallel. In the region between layers, spaces of lower pressure appear. The limit between turbulent and laminar is empirically observed to be defined by the value of the Reynold's number, a dimensionless quantity expressed by:

$$R \;=\; \frac{\rho \;\; v \;\; D}{\eta} \qquad \begin{array}{l} \rho \;\; density \;\; of \;\; the \;\; gas \\ v \;\; velocity \;\; of \;\; the \;\; gas \\ \eta \;\; viscosity \;\; of \;\; the \;\; gas \\ D \;\; diameter \;\; of \;\; the \;\; tube \end{array}$$

If the Reynold's number is larger than 2100 the flow is entirely turbulent while for a Reynold's number less than 1100 the flow is entirely laminar. In all cases that we will be dealing with for the MDT's, the gas flow will be laminar.

The throughput **Q** is the quantity of gas flowing through a pipe:

$$Q \;=\; P \;\frac{dV}{dt} \qquad \left(units\!: \;\; \frac{pressure \times volume}{time} \;=\; \frac{Torr \times liter}{sec} \right).$$

This is calculated using:

$$Q \;=\; C \;(P_1 - P_2)$$

where **C** is the conductance and $\mathbf{P_1 - P_2}$ is the pressure differential across the pipe. Often an analogy is made between the flow and electrical conductance, however, the flow conductance depends on the pressure so in general it cannot be considered a constant. For two pipes with parallel flow the conductance is given by:

$$C_P = C_a + C_b + \bullet\bullet\bullet$$

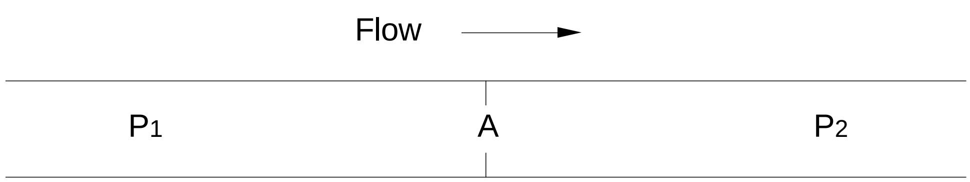
and for pipes where the flow is in series the conductance is given by:

$$\frac{1}{C_S} = \frac{1}{C_a} + \frac{1}{C_b} + \bullet\bullet\bullet \,.$$

The conductance for some common situations:

I) Conductance of an Aperture

For an aperture in which the mean free path is small compared to the dimensions of the aperture:



The conductance is given by:

$$C = \frac{9.13 \; A}{1 - \left(\frac{P_2}{P_1}\right)} \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} \left\{ \frac{2 \; \gamma}{\gamma - 1} \left(\frac{T}{M}\right) \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$

where **C** is the conductance of the aperture in liter/sec, **A** is the area of the aperture in cm^2, **P** is the pressure in the pipe in Torr, **T** is the temperature of the gas in °K, **M** is the molecular weight of the gas in grams, and $\gamma = \mathbf{c_p/c_v}$ is the ratio of specific heat at constant pressure to the specific heat at constant volume. (γ = 1.67 for monatomic gases, 1.40 for diatomic gases, and 1.30 for triatomic gases). The conductance limit for large and small pressure differentials in Argon at 300 °K are:

$$C = \frac{55.9\ A}{\left(1 - \frac{P_2}{P_1}\right)^{\frac{4}{5}}} \left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} \qquad \frac{P_2}{P_1} < .7$$

$$C = \frac{2.7 \times 10^4\ A}{\left(P_1 - P_2\right)^{\frac{4}{5}}} \qquad P_2 \approx P_1 .$$

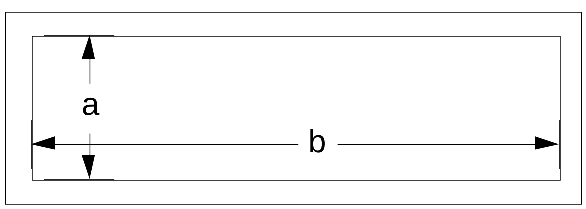
The regime in which we are interested in is $\mathbf{P_2} \approx \mathbf{P_1} \approx$ 2280 Torr (= 3 bar). As we discuss later, typical pressure differences in the MDT's are ~.001 Torr. We can see that under these conditions the conductance is very large so the aperture size has little or no effect on gas flow. It is entirely determined by the conductance of the tubes. The commonly heard assertion that **C**/**A** for an aperture is a constant independent of flow, and thus can be used to control flow, is only true in high flow (large $\Delta\mathbf{P}$) situations. For the operating regime of the MDT's this is very far from an accurate statement.

II) Conductance of a cylindrical pipe of diameter **D** and length **L**.

$$C = 3.27 \times 10^{-2} \left(\frac{D^4}{\eta\ L}\right) \overline{P}$$

$$\overline{P} = \frac{P_1 + P_2}{2}\ in\ Torr$$
$$D,\ L\ in\ centimeters$$
$$\eta\ viscosity\ in\ poise$$
$$C\ in\ \frac{liters}{sec}$$

III) Conductance of a rectangular pipe of area **A = a•b** and length **L**.



$$C = 4.17 \times 10^{-2}\ Y \left(\frac{A^2}{\eta\ L}\right) \overline{P}$$

$$\overline{P} = \frac{P_1 + P_2}{2}\ in\ Torr$$
$$L\ in\ centimeters$$
$$A = a \bullet b\ in\ cm^2$$
$$\eta\ viscosity\ in\ poise$$
$$C\ in\ \frac{liters}{sec}$$

Where Y is a correction factor for the asymmetry of the pipe.

Table 2

$\frac{a}{b}$	Y
1	1
.9	.99
.8	.98
.7	.95
.6	.90
.5	.82
.4	.71
.3	.58
.2	.42
.1	.23

A Model for Gas Flow in the MDT Chambers.

We have constructed a model for gas flow in the MDT chambers. The particular parameters chosen correspond to the tube organization proposed by the BMC but the general principles apply to all MDT chambers. The flow in and out of the chamber is arranged in manifolds ("service bars") along each edge of the chamber. Small tubes called jumpers connect the drift tubes to the manifolds and each other. The gas flow is organized so that there are three tubes in series between manifolds. There are 56 such "strings" in each multilayer. The input and exhaust flow for the chamber is arranged to be diagonally opposite corners. The case where the exhaust is on the same side of the chamber as the input is also considered. The layout and tube lengths and inside diameters are shown in figure 1.

The drift tube vary from 1.7 to 2.2 meter with an ID of 29.2 mm. The manifolds have an ID of 10 mm and are consider to be short (30.0 mm) segments between the tube strings. The jumpers have an ID of 1.8 mm and the manifold to tube length is 63 mm and the tube to tube jumpers have a length of 54 mm. We have calculated the conductance for the string of 4 jumpers and 3 tubes. Typical tube conductance's are 10^5 liters/sec while each jumper conductance is around 60 liters/sec. The conductance is totally dominated by the jumpers with the tubes making essentially no contribution.

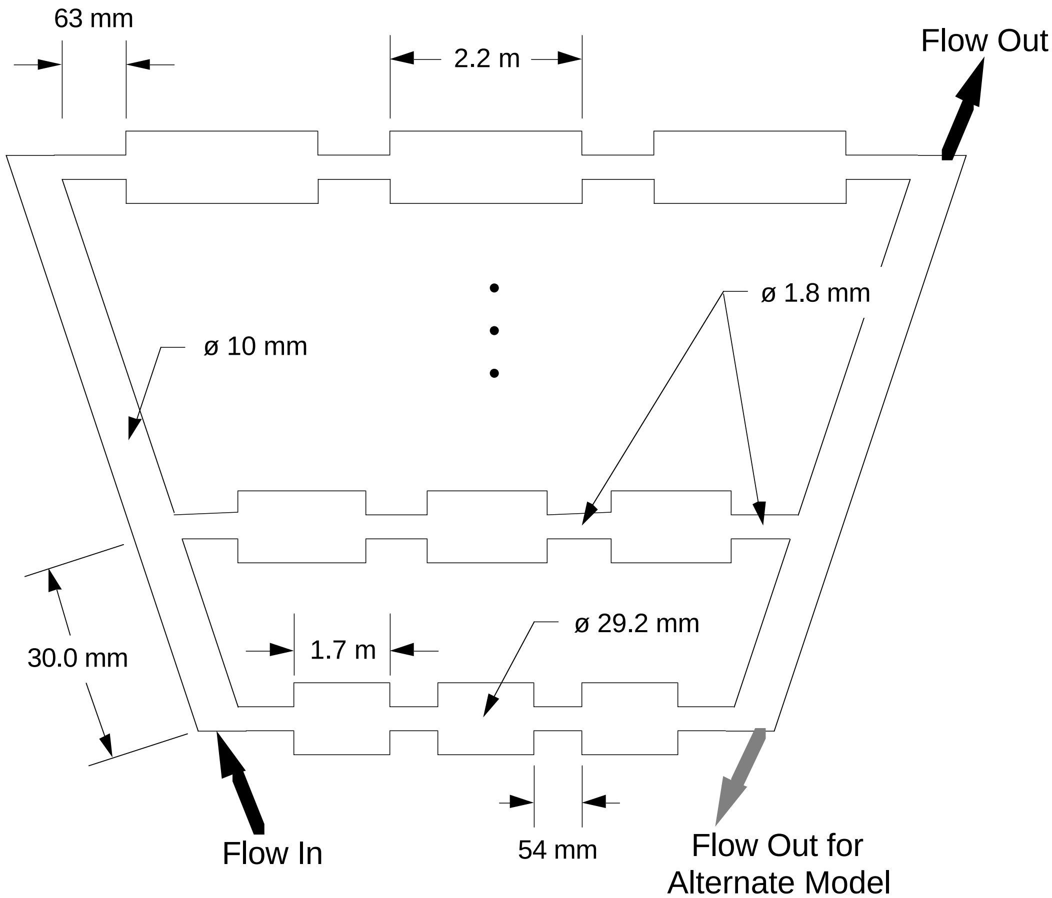


Figure 1 - Model of US prototype for gas flow calculation. The gas input and exhaust are in manifold along the edges of the chambers (service bars). The tubes are connected to the manifolds and each other by small tubes called jumpers. For the gas flow the chamber is organized into 56 strings of three tubes each. The lengths and diameters (ø) are shown.

A simple network model for flow in the tube strings was constructed and is shown graphically in figure 2. **Q** represents the total flow in and out of the chamber. The symbols $\mathbf{q_{in}}$, $\mathbf{q_n}$, and $\mathbf{q_{en}}$ represent the flow in the nth element of the input, tube string, and exhaust elements respectively. The total flow was constrained to be 22 liters (at 3 atmospheres) per day, a flow corresponding to one volume change every 10 days. The input pressure was fixed at 2280 Torr. The node equations were setup and the pressure at each node was determine.

As a simple example the equations for a system of three tube strings is shown in figure 2. The node equations are shown following the figure.

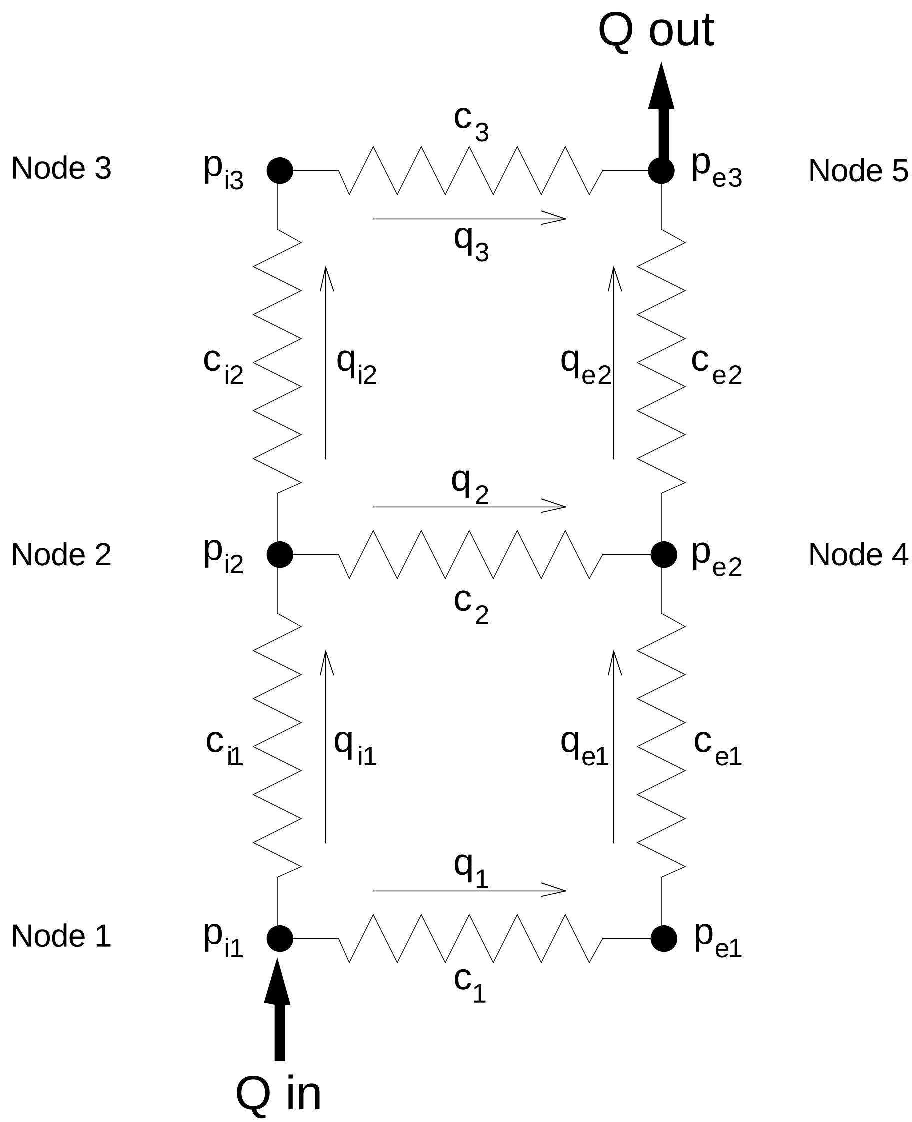


Figure 2 - Conductance network for MDT chambers assuming three strings of tubes. C_n is the conductance of the drift tubes and jumpers connected in series. C_{in} and C_{en} are the conductance's for the short sections of manifold connecting the tube strings. Q is the total flow into and out of the chamber. The P's are the pressures at each node. Q and p_{i1} are assumed to be given, all the other P's are determined by solving the node equations.

The node equations shown in figure 2 are given by;

$$\begin{bmatrix} Q - c_{i1}P_{i1} - c_1 P_{i1} \\ -c_{i1}P_{i1} \\ 0 \\ 0 \\ Q \end{bmatrix} = \begin{bmatrix} -c_{i1} & 0 & -c_1 & 0 & 0 \\ -c_{i1} - c_{i2} - c_2 & c_{i2} & 0 & c_2 & 0 \\ c_{i2} & -c_{i2} - c_3 & 0 & 0 & c_3 \\ c_2 & 0 & c_{e1} & -c_{e1} - c_{e2} - c_2 & c_{e2} \\ 0 & c_3 & 0 & c_{e2} & -c_3 - c_{e3} \end{bmatrix} \begin{bmatrix} P_{i2} \\ P_{i3} \\ P_{e1} \\ P_{e2} \\ P_{e3} \end{bmatrix}$$

which can be solved by matrix inversion. These equations can be easily generalized to any number of tube strings. The flow in each tube string for the US prototype (as shown in figure 1) are given in figure 3. The flow assumes that there is one volume change every 10 days and that the input pressure is held constant. The flow through out the chamber is nearly constant. The maximum variation of flow between the first and middle tube strings is 9.6%. The maximum pressure difference between any two tubes is .5 milli-Torr. Since the flow is linear, a X10 flow (one volume change per day) would result in a 5 milliTorr pressure differential, although the variation in flow pattern doesn't change.

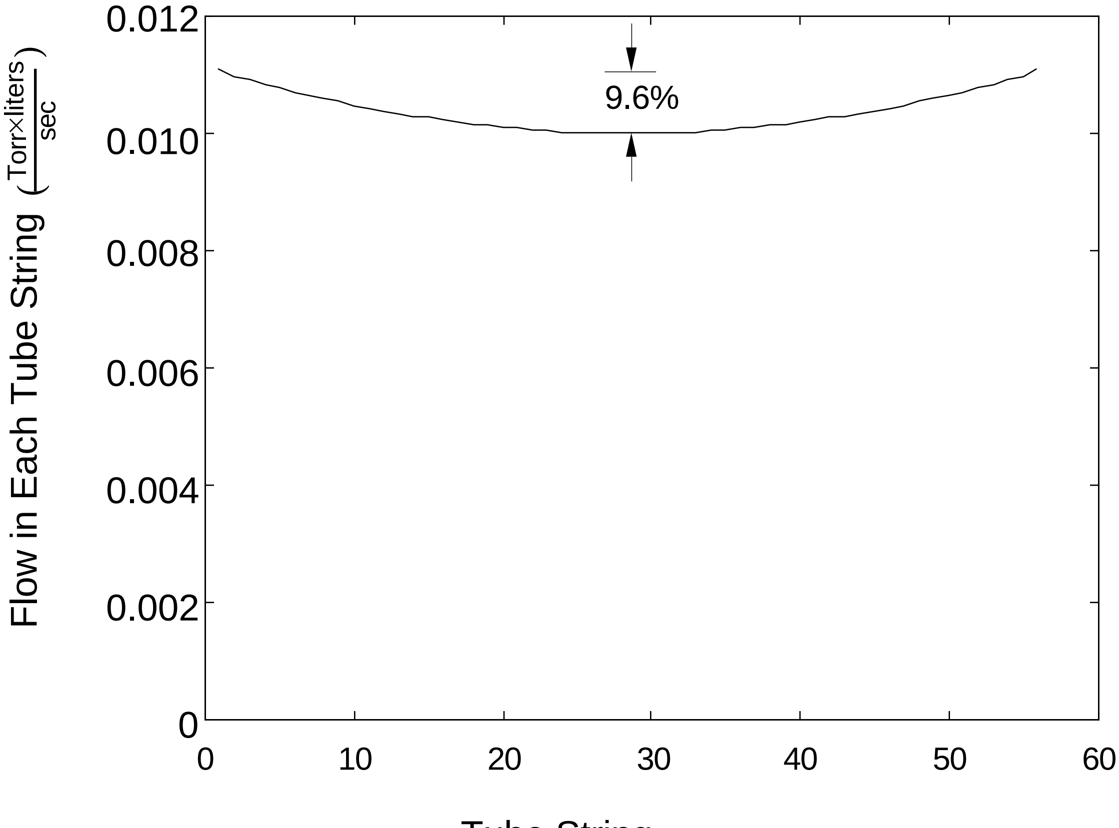


Figure 3 - Flow in the tube string for the US prototype for diagonal flow. See figure 1 for the parameters used.

An alternate flow arrangement, where the input and exhaust are along one edge of the chambers, is possible. This model is also indicated in figure 1. The flow pattern for this model is shown in figure 4. Here the maximum variation in flow from the first to the last strings is about 31%.

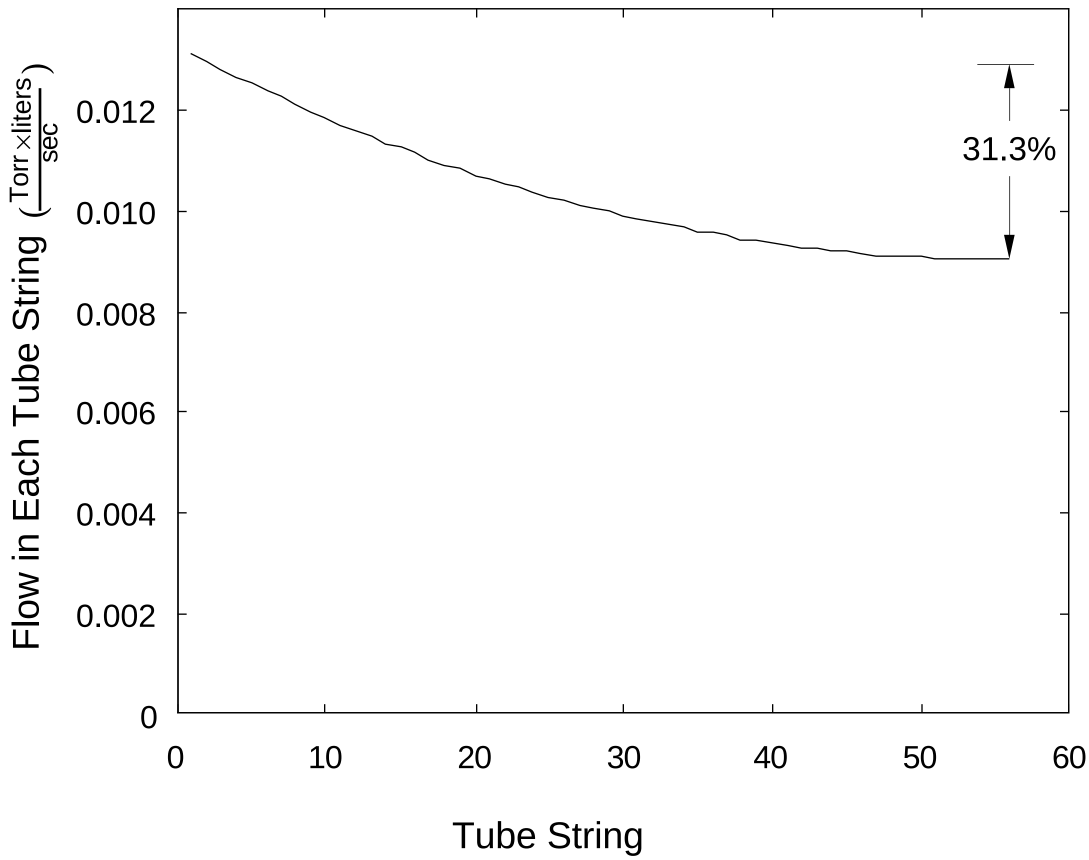


Figure 4 - Flow in the tube string for the US prototype for same side flow for input and exhaust for alternate model. See figure 1 for the parameters used.

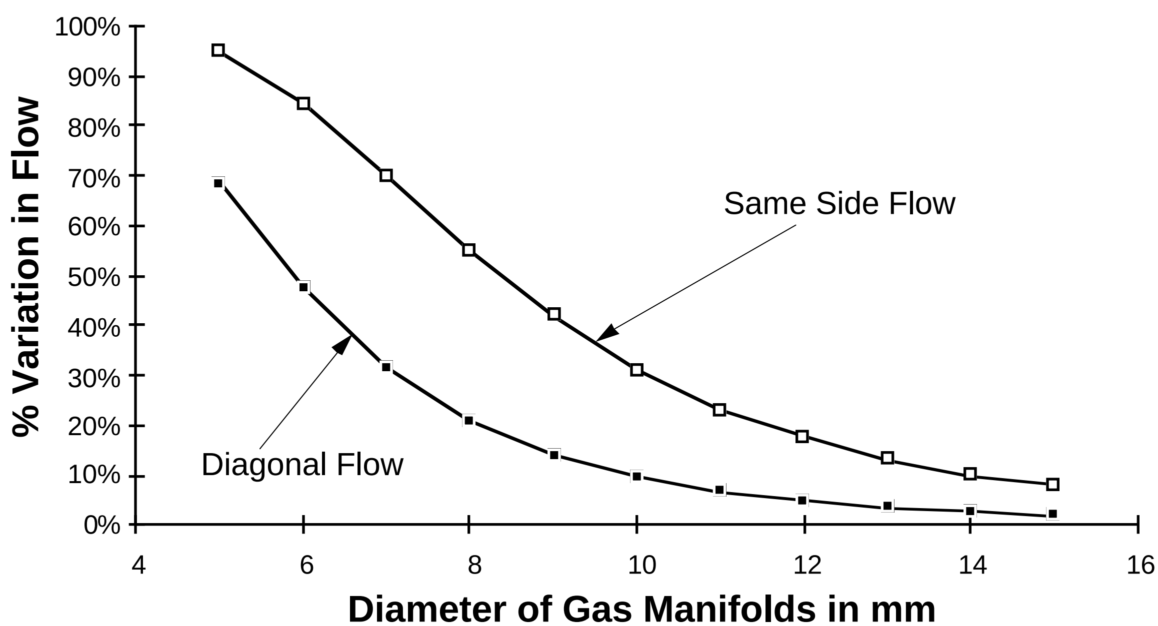


Figure 5 - Percentage variation between the minimum and maximum flow in tube strings for both diagonal and same side flow models.

We also investigated the dependence of the gas flow on the various parameters of the MDT chambers. As an example the dependence on the inner diameter of the input and exhaust manifolds is shown on the maximum variation in flow for both the diagonal flow and alternate flow models are shown in figure 5.

Another interesting issue to investigate is the inner diameter of the jumpers. If they are too small, then the MDT chambers will have significant pressure differences between neighboring tubes. If the ID is too large, they will no longer control the flow and the flow in different tube strings will have a large variation. The flow variation for jumpers of ID = 3.0, 2.0, 1.0, and .5 mm are shown in figure 6. The largest pressure differences between any two tubes in a chamber for these ID's are shown in table 3. From this data one can conclude that the ID of the jumpers should be between .5 and 2 mm. Larger than 2 mm would cause too great a variation in flow between strings. Less than .5 mm ID would give rise to pressure variations between tubes exceeding 2×10^{-4} in $\Delta P/P$. [NB: Temperature variation of .1C° will result in density variations of 3×10^{-4} in $\Delta n/n$.]

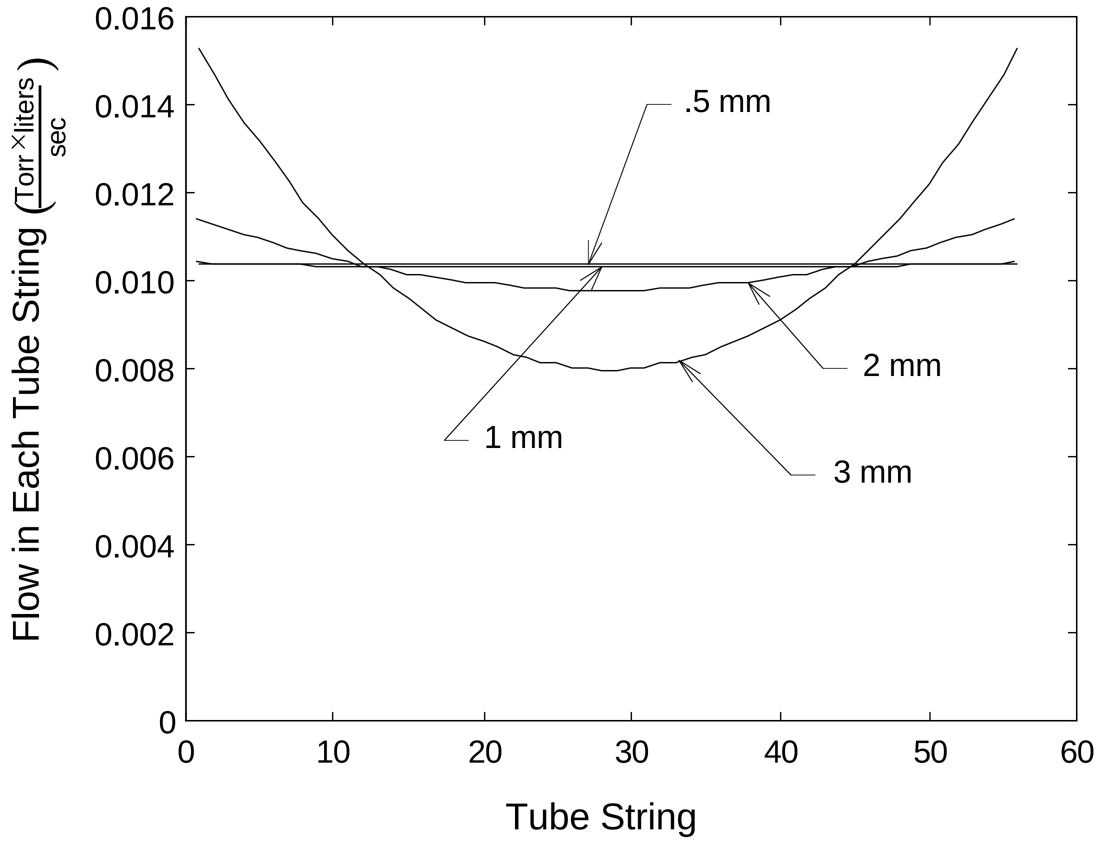


Figure 6 - Flow variation between the tube strings for different inner diameters of the jumpers. Curves for jumper ID's of 3, 2, 1, and .5 mm are shown. Other parameters used are standard for the US Prototype as shown in fig. 1.

Table 3

Jumper Inner Diameter (mm)	Maximum Pressure Difference between Tubes in a Chamber. (milliTorr)
3	.21
2	.38
1	3.6
.5	57.

Protection from Tube Failure

One potential problem with the MDT chambers is that some failure modes are catastrophic. For instance, a puncture of a tube or other major leak would quickly exhaust all the gas in tubes between the nearest cut-off valves in the input and exhaust line. The requirement that the cut-off valves be closed to protect the rest of the muon system means that all tubes between these cut-off valves would become non-functional.

One possible protection against this eventuality is to place a high impedance (low conductance) capillary at the beginning and end of each tube string. Should a puncture occur in one of the tubes in that string, then the capillaries would limit the flow from that string and prevent the entire chamber (and perhaps surrounding chambers) from being lost. The exact placement of the capillary is somewhat arbitrary. It could be in the manifold as part of the design of the manifold, or it could be incorporated in the jumpers from the manifold to the drift tubes.

If we have capillary tubes one possible design strategy is to limit the gas loss from the puncture to be an amount of gas equal to the flow in a typical chamber. This will allow the rest of the MDT system to keep functioning until repairs can be effected. This could be accomplished by installing a capillary 1 cm long and 100 microns in diameter at each end of the tube string. This would result in a 9 Torr pressure difference across each capillary during normal operation of the MDT's. All MDT tubes, however, would be at the same pressure. In the event of a puncture to a drift tube, the capillaries would limit the loss to an amount of gas roughly to the normal flow in a chamber. In addition the remainder of the chamber would be largely unaffected by the loss of gas.

A note of caution is necessary here, the diameter of the capillary is about that of a human hair. It could be easily clogged with a small dust particle. In practice the reduced flow could be difficult to detect. This could be a case of the cure being as bad as the disease. Substantial care would have to be taken to insure that the gas system is free of debris.