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Black hole temperature and Unruh effect from the extended uncertainty principle

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ABSTRACT

In this paper, we investigate the thermodynamic properties of the Schwarzschild black hole and modified Unruh effect by using the simplest form of the extended uncertainty principle (EUP). We obtain the mass-temperature relation and find that there should exist the lower bound for the EUP black hole temperature. Besides, we discuss the modified Unruh effect for EUP. We find that the modified Unruh temperature is larger than the Unruh temperature.

1. Introduction

Quantization of gravity is one of the oldest problems in physics. However, up to now, there does not exist a satisfactory solution. In this quantization, the concept of minimum measurable length is shown to be needed when considering the discreteness of the space-time which occurs beyond the Planck energy scale.

The ordinary Heisenberg uncertainty principle $\Delta x \Delta p \geq \hbar/2$ does not explain the existence of a minimum measurable length because Δx goes to zero in the high momentum limit. Thus, to incorporate the concept of minimum measurable length into quantum mechanics, one should deform the ordinary Heisenberg uncertainty principle which is called Generalized Uncertainty Principle (GUP). In the last decade, many papers have been appeared in the literature to address the effects of GUP on the quantum mechanical systems especially in high energy regime [1–26]. It should be noted that we have used this idea that from the discreteness of space-time, i.e. from the existence of a minimum length, then a modification of the ordinary Heisenberg principle is required leading to the generalized uncertainty principle (GUP). On the other hand, there is another point of view that a re-examination of the quantum measurement process leads to a modification of the uncertainty relation, and from there the existence of a minimum measurable length is inferred. This kind of a point of view can be found in these references [5,11,14,15]. The commutation relation of GUP is given by

$$[X, P] = i\hbar(1 + \beta^2 P^2). \tag{1}$$

This gives the uncertainty relation

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left[1 + \beta^2 (\Delta P)^2\right] \tag{2}$$

which suggests the existence of the fundamental minimal length $(\Delta X)_0 = \hbar\beta$ Here, $\beta^2 = \beta_0^2/(m_p c)^2$ where m_p is the Planck mass with $m_p c^2 \sim 10^{19}\mathrm{GeV}$ and β_0 is of order the unity. Thus we have

$$\frac{\beta^2}{c^2} \sim 10^{-38}\mathrm{GeV}^{-2} \tag{3}$$

Another possibility is to study the effects of gravity on quantum mechanical systems by using the assumption of minimal uncertainty in momentum. It is known that for large distances, where the curvature of space-time becomes important, there is no notion of a plane wave on a generally curved space-time [27,28]. This means that there appears a limit to the precision with which the corresponding momentum can be described. In order to incorporate the concept of minimum measurable momentum into quantum mechanics, one should deform the ordinary Heisenberg uncertainty principle which is called Extended Uncertainty Principle (EUP) [29–32]. The commutation relation of EUP is given by

$$[X, P] = i\hbar(1 + \alpha^2 X^2), \tag{4}$$

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where we know that this gives the minimal momentum $(\Delta P)_0 = \hbar\alpha$. Recently, it has been shown by Mignemi [31] that the eq. (4) can also be derived from the definition of quantum mechanics on an anti-de Sitter background, with a suitably chosen parametrization. He found that the relation (4) could be derived from the geometric properties of anti-de Sitter space-time. The one-dimensional quantum mechanics was discussed by Chung and Hassanabadi [32]. The Eq. (4) is also obtained when we consider quantum mechanics in an anti-de Sitter background where in which the expansion of the universe during a measurement is taken into account [30]. It is well known that in an anti-de Sitter background the Heisenberg uncertainty principle should be modified by introducing corrections proportional to the cosmological constant $\Lambda = -3/l_H^2$ with l_H the anti-de Sitter radius [29], where $l_H^2 > 0$:

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left[1 + \frac{(\Delta X)^2}{l_H^2}\right], \tag{5}$$

where we know

$$\alpha^2 = \frac{1}{l_H^2} = -\frac{\Lambda}{3} = \frac{|\Lambda|}{3} > 0 \tag{6}$$

The magnitude of the cosmological constant was known as

$$|\Lambda| \sim 10^{-52}\mathrm{m}^{-2} \tag{7}$$

In this paper, we shall study the thermodynamic properties of the Schwarzschild black hole and Unruh effect by using the simplest form of the EUP. This paper is organized as follows: In section 2 we discuss the mass-temperature relation for EUP black hole and minimal temperature of a black hole. In section 3 we discuss the EUP black hole entropy. In section 4 we discuss the modified Unruh effect for the EUP.

2. Mass-temperature relation for EUP black hole and minimal temperature of black hole

Now let us consider the one spatial dimensional case. The EUP (5) is related to the deformed Heisenberg algebra (4). Let us consider a Schwarzschild black hole of mass M. For any quantum particle (massless) near the horizon of a black hole, the momentum uncertainty characterizing its temperature can be written as [33,34]

$$T = \frac{c\Delta P}{k} \tag{8}$$

where k is the Boltzmann constant and c is the speed of light. Using this equation and the Eq. (5), we know that there exists the lower bound for the black hole temperature, indeed we get

$$T \geq T_{min} = \frac{\hbar c\alpha}{k} \sim 1.3221 \times 10^{-29}\mathrm{K} \tag{9}$$

For thermodynamic equilibrium, the temperature of the quantum particle is the same as the temperature of the black hole. In order to find the relation of this temperature with the black hole mass, we reconsider the EUP (5) in terms of T and M where EUP is saturated,

$$\Delta X \Delta P = \frac{\hbar}{2}\left[1 + \alpha^2(\Delta X)^2\right] \tag{10}$$

Near the horizon of the Schwarzschild black hole, the position uncertainty of a particle will be of the order of the Schwarzschild radius of the black hole [33,35,36],

$$\Delta X = \eta r_s, \quad r_s = \frac{2GM}{c^2} \tag{11}$$

where η is a scale factor, r_s is the Schwarzschild radius and G is the Newton's universal gravitational constant. Inserting the Eq. (8) and the Eq. (11) into the Eq. (10) we get

$$\frac{2\eta GMkT}{c^3} = \frac{\hbar}{2}\left(1 + \frac{4\alpha^2\eta^2G^2M^2}{c^4}\right) \tag{12}$$

This is the quadratic for M, whose solution is

$$M = \frac{(m_p c)^2 kT}{2\eta(\alpha\hbar c)^2}\left[1 - \sqrt{1 - \frac{(\alpha\hbar c)^2}{(kT)^2}}\right] \tag{13}$$

where m_p is the Planck mass obeying $(m_p c)^2 = \frac{\hbar c^3}{G}$. In the absence of correction due to EUP, Eq. (13) reduces to

$$M = \frac{(m_p c)^2}{4\eta kT} \tag{14}$$

Comparing this with the Hawking temperature $T_H = \frac{(m_p c)^2}{8\pi kM}$, we have $\eta = 2\pi$. This finally fixes the form of the mass-temperature relation (13) to be

$$M = \frac{(m_p c)^2 kT}{4\pi(\alpha\hbar c)^2}\left[1 - \sqrt{1 - \frac{(\alpha\hbar c)^2}{(kT)^2}}\right] \tag{15}$$

where

$$T \geq T_{min} = \frac{\hbar c\alpha}{k} \tag{16}$$

For a small value of α we have

$$M = \frac{\hbar c^3}{8\pi Gk}\left(\frac{1}{T} + \frac{(\alpha\hbar c)^2}{4k^2T^3} + \frac{(\alpha\hbar c)^4}{8k^4T^5} + \cdots\right) \tag{17}$$

Now the heat capacity of the EUP black hole can be defined as

$$C = c^2\frac{dM}{dT} = \frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\left[1 - \frac{1}{\sqrt{1 - \frac{(\alpha\hbar c)^2}{(kT)^2}}}\right] \tag{18}$$

For a small value of α we have

$$C = \frac{\hbar c^5}{8\pi Gk}\left(-\frac{1}{T^2} - \frac{3(\alpha\hbar c)^2}{2k^2T^4} - \frac{5(\alpha\hbar c)^4}{8k^4T^6} - \cdots\right) \tag{19}$$

2.1. Comparison with the GUP black hole

In the GUP black hole [33–43] we know the mass-temperature relation as

$$M = \frac{\hbar c^3}{8\pi Gk}\left(\frac{1}{T} + \frac{k^2\beta^2}{c^2}T\right) \tag{20}$$

which has the minimum of mass coming from the minimal length,

$$M \geq M_0 = \frac{\hbar c^2\beta}{4\pi G} \tag{21}$$

Besides, the GUP black hole does not give the lower bound on the temperature while EUP black hole does. For quite massive BHs and small temperatures, we have Hawking mass-temperature formula that GUP can express it well but EUP cannot reproduce the well known and accepted semi-classical Hawking behavior for BH with large masses and low temperatures. We face with unphysical predictions for the Hawking temperature in the semi-classical

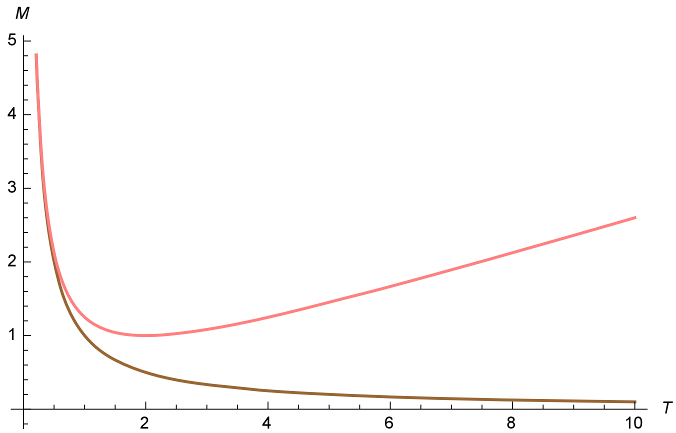

Fig. 1. Plot of the mass-temperature relation for the ordinary black hole (Brown) and the GUP black hole with $\beta = 0.5$ (Pink) where we set $\hbar = c = k = 1, 8\pi G = 1$.

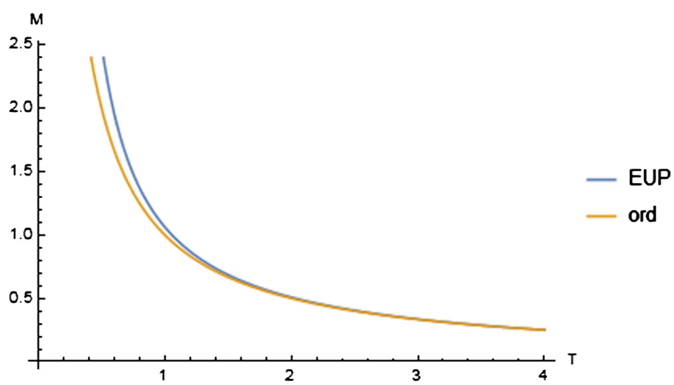

Fig. 2. Plot of the mass-temperature relation for the ordinary black hole (Brown) and the EUP black hole with $\alpha = 0.5$ (Purple) where we set $\hbar = c = k = 1, 8\pi G = 1$.

region of parameters. We state and emphasize that the results are suitable for as an analytic comparison between the prediction of GUP and those of EUP. But it should be noted that in EUP we have a minimum value for the temperature as we derived in Eq. (16) while the minimum temperature in GUP is zero. Fig. 1 shows the plot of the mass-temperature relation for the ordinary black hole (Brown) and the GUP black hole with $\beta = 0.5$ (Pink) where we set $\hbar = c = k = 1, 8\pi G = 1$. Fig. 2 shows the plot of the mass-temperature relation for the ordinary black hole (Brown) and the EUP black hole with $\alpha = 0.5$ (Purple) where we set $\hbar = c = k = 1, 8\pi G = 1$.

In the GUP black hole, the specific heat is

$$C = \frac{\hbar c^5}{8\pi G k}\left(-\frac{1}{T^2} + \frac{k^2\beta^2}{c^2}\right) \tag{22}$$

Comparing this with the eq. (18), we know that the heat capacity in EUP black hole is always negative, so the radiation process leads to a decrease in the mass of the EUP black hole which in turn leads to an increase in temperature. Thus, in EUP black hole, as the temperature increases, the heat capacity also increases, hence there is no point at which the heat capacity vanishes. On the contrary, for the GUP black hole, at $T = c/(k\beta)$ the specific heat vanishes. Fig. 3 shows the plot of specific heat versus temperature for the ordinary black hole (Brown) and the GUP black hole with $\beta = 0.5$ (Pink) where we set $\hbar = c = k = 1, 8\pi G = 1$. Fig. 4 shows the plot of specific heat versus temperature for the ordinary black hole (Brown) and the EUP black hole with $\alpha = 0.5$ (Purple) where we set $\hbar = c = k = 1, 8\pi G = 1$.

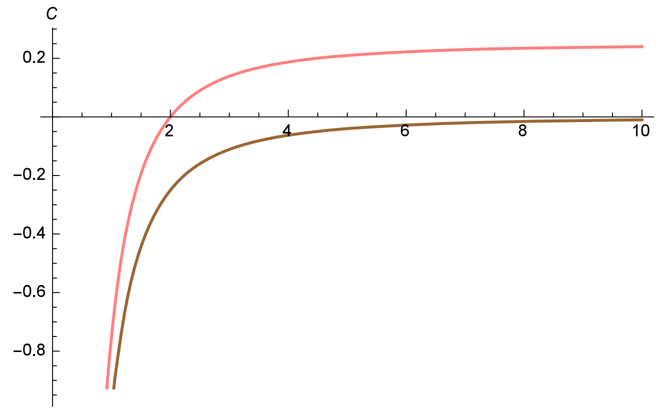

Fig. 3. Plot of specific heat versus temperature for the ordinary black hole (Brown) and the GUP black hole with $\beta = 0.5$ (Pink) where we set $\hbar = c = k = 1, 8\pi G = 1$.

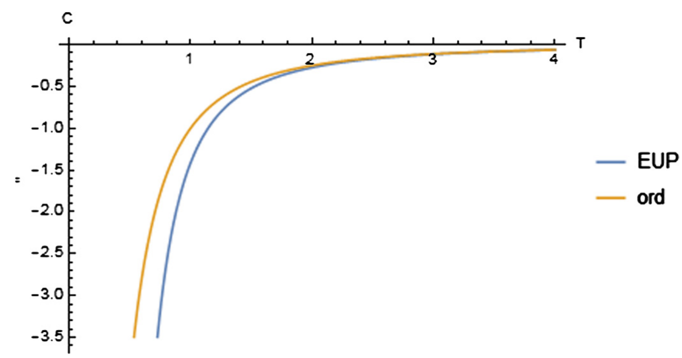

Fig. 4. Plot of specific heat versus temperature for the ordinary black hole (Brown) and the EUP black hole with $\alpha = 0.5$ (Purple) where we set $\hbar = c = k = 1, 8\pi G = 1$.

3. EUP black hole entropy

One can also determine the EUP black hole entropy from the first law of black hole thermodynamics given by

$$S = c^2 \int_{T_{min}}^{T} \frac{dM}{T} = \int_{T_{min}}^{T} C(T)\frac{dT}{T} \tag{23}$$

Performing the above integration leads to

$$S = -\frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\ln\left[1 + \sqrt{1 - \frac{(\alpha\hbar c)^2}{(kT)^2}}\right] + K \tag{24}$$

where K is the integration constant. Expanding the above expression for small α, we get

$$S = K - \frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\ln 2 + \frac{k(m_p c^2)^2}{16\pi(kT)^2} + \frac{3k(\alpha\hbar c)^2(m_p c^2)^2}{128\pi(kT)^4} + \cdots \tag{25}$$

which fixes the value of K as

$$K = \frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\ln 2 \tag{26}$$

Thus, we have

$$S = -\frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\ln\left[1 + \sqrt{1 - \frac{(\alpha\hbar c)^2}{(kT)^2}}\right] + \frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\ln 2 \tag{27}$$

Now let us rewrite the eq. (15) as

$$M = m_0 T\left[1-\sqrt{1-\left(\frac{T_0}{T}\right)^2}\right] \tag{28}$$

where we set

$$m_0 = \frac{(m_p c)^2 k}{4\pi(\alpha\hbar c)^2}, \quad T_0 = T_{min} = \frac{\hbar c\alpha}{k} \tag{29}$$

The inverse relation of the eq. (28) is

$$T = \frac{M}{2m_0} + \frac{m_0 T_0^2}{2M} \tag{30}$$

Thus, we always have the real temperature for all possible values of M, which differs from the case of GUP black hole where there exists the critical mass below which the temperature becomes a complex quantity.

The specific heat is given by

$$C = m_0 c^2\left[1-\frac{1}{\sqrt{1-\left(\frac{T_0}{T}\right)^2}}\right] = m_0 c^2\left[1-\left(1-\left(\frac{M}{2m_0 T_0}+\frac{m_0 T_0}{2M}\right)^{-2}\right)^{-1/2}\right] \tag{31}$$

The entropy is given by

$$S = -\frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\ln\left[1+\sqrt{1-\frac{(\alpha\hbar c)^2}{k^2\left(\frac{M}{2m_0}+\frac{m_0 T_0^2}{2M}\right)^2}}\right] + \frac{(m_p c)^2 k}{4\pi(\alpha\hbar)^2}\ln 2 \tag{32}$$

For a small α, we have

$$\frac{S}{k} \approx \frac{S_{BH}}{k} - \frac{2\pi(\hbar\alpha)^2}{(km_p c)^2}S_{BH}^2 \tag{33}$$

where the semi-classical Bekenstein-Hawking entropy for the Schwarzschild black hole is

$$S_{BH} = 4\pi\left(\frac{M}{m_p}\right)^2 \tag{34}$$

In terms of the area of the horizon $A = 4\pi r_s^2 = 4L_p^2\left(\frac{S_{BH}}{k}\right)$ where L_p is the Planck length, the eq. (33) can be written as

$$\frac{S}{k} \approx \frac{A}{4L_p^2} - \frac{\pi(\hbar\alpha)^2 A^2}{4(m_p c)^2 L_p^4} \tag{35}$$

3.1. Comparison with GUP black hole

Now let us compare the EUP black hole and GUP black hole. For EUP, the Eq. (32) gives the relation between the EUP black hole entropy and the area as

$$\frac{S^{EUP}}{k} = \frac{A}{4L_p^2} + \sum_{n=1}^{\infty} c_n^{EUP}\left(\frac{A}{4L_p^2}\right)^n \tag{36}$$

For the GUP black hole we have [33–43]

$$\frac{S^{GUP}}{k} = \frac{A}{4L_p^2} + c_0^{GUP}\ln\left(\frac{A}{4L_p^2}\right) + \sum_{n=1}^{\infty} c_n^{GUP}\left(\frac{A}{4L_p^2}\right)^{-n} \tag{37}$$

where the coefficients c_n^{GUP} can be regarded as model dependent parameters. Many researchers have expressed a vested interest in fixing c_0^{GUP} (the coefficient of the subleading logarithmic term). Recent rigorous calculations of loop quantum gravity predicts the value of c_0^{GUP} to be $-1/2$ [44]. But, for the EUP black hole, we do not have the logarithmic term and we have the correction terms with the positive power for $\left(\frac{A}{4L_p^2}\right)$ while for the GUP black hole we have negative power as well as the logarithmic term.

We can't compare EUP and GUP in a figure, because they do not have the same or close value or the same numerical order, but just we can compare them checking at the critical temperature. From the Eq. (17), we only consider the first two terms for the mass-temperature relation for EUP black hole

$$M_{EUP} = \frac{\hbar c^3}{8\pi Gk}\left(\frac{1}{T} + \frac{(\alpha\hbar c)^2}{4k^2 T^3}\right) \tag{38}$$

Then, the mass-temperature relation for GUP black hole from the Eq. (20) is

$$M_{GUP} = \frac{\hbar c^3}{8\pi Gk}\left(\frac{1}{T} + \frac{k^2\beta^2}{c^2}T\right) \tag{39}$$

and for the ordinary mass-temperature from the Eq. (14) we have

$$M_{ord} = \frac{\hbar c^3}{8\pi GkT} \tag{40}$$

Now, we calculate ΔM_{EUP} and ΔM_{GUP}

$$\Delta M_{EUP} = M_{EUP} - M_{ord} = \frac{\hbar^3 c^5\alpha^2}{32\pi Gk^3}\frac{1}{T^3} \tag{41}$$

$$\Delta M_{GUP} = M_{GUP} - M_{ord} = \frac{\hbar ck\beta^2}{8\pi G}T \tag{42}$$

Eqs. (41) and (42) show the difference between and having different numerical order such that they cannot be plotted in the same figure. If we calculate the numerical value of temperature coefficient in the Eqs. (41) and (42) we have

$$\frac{\hbar^3 c^5\alpha^2}{32\pi Gk^3} \simeq 1.6113\times 10^{-87} \tag{43}$$

$$\frac{\hbar ck\beta^2}{8\pi G} \simeq 2.34133\times 10^{-83} \tag{44}$$

It is seen that $\Delta M_{GUP} \ll \Delta M_{EUP}$ about thousands times.

To obtain the critical temperature, we should equal the mass difference together then we have

$$\Delta M_{EUP} = \Delta M_{GUP} \tag{45}$$

$$T_c = \frac{c}{k}\sqrt{\frac{\hbar\alpha}{2\beta}} \tag{46}$$

Now, in the SI system we have

$$T_c = 3.94176\times 10^{11}K \tag{47}$$

Now, inserting T_c in the Eq. (1) and Eq. (2) we have

$$M_{EUP}(T_c) = M_{GUP}(T_c) \propto \frac{\hbar c^2}{8\pi G}\sqrt{\frac{2\beta}{\hbar\alpha}}\left(1+\frac{\beta\alpha\hbar}{2}\right) \tag{48}$$

We see that at the critical temperature for EUP and GUP we have the same mass. Also, we obtain this relation for the heat capacity by inserting T_c in the heat capacity equation

$$C_{EUP} = C_{GUP} \propto \frac{1}{8\pi G}(\hbar k\beta^2 c^3 - \frac{\beta k c^3}{\alpha}) \tag{49}$$

4. Unruh effect

In this section, we derive the Unruh temperature [45] starting directly from the EUP. Let us consider some elementary particles kept at rest in a uniformly accelerated frame. The kinetic energy acquired by each of these particles while the accelerated frame moves a distance ΔX will be given by [46]

$$E = ma\Delta X \tag{50}$$

where m is the mass of the particle and a the acceleration of the frame. Now, suppose this energy is sufficient to create N pairs of the same kind of particles from the quantum vacuum. Namely, we set

$$E \approx 2Nmc^2 \tag{51}$$

The distance along which each particle must be accelerated in order to create N pairs is given by [46]

$$\Delta X \approx \frac{2Nc^2}{a} \tag{52}$$

First, consider the case of absence of EUP effect. The original particles and the pairs created in this way are localized inside a spatial region of width ΔX, therefore the fluctuation in energy of every single particle is given by

$$\Delta E \approx \frac{\hbar c}{2\Delta X} \approx \frac{\hbar a}{4Nc} \tag{53}$$

If we interpret this fluctuation as a classical thermal one, we can write

$$\frac{3}{2}kT \approx \Delta E \tag{54}$$

or

$$T = \frac{\hbar a}{6Nck} \tag{55}$$

Comparing this with the Unruh temperature $T_U = \frac{\hbar a}{2\pi ck}$, we know that $N = \frac{\pi}{3} \approx 1$.

Now let us repeat the same argument using the EUP. We have the Eq. (5) as position-momentum relation for extended uncertainty principle. Now, we have time-energy relation for extended uncertainty principle:

$$\Delta t \Delta E \geq \frac{\hbar}{2}\left[1 + \frac{(\Delta t)^2}{l_H^2}\right]. \tag{56}$$

On the other hand, $\Delta X = c\Delta t$ and $\frac{|\Lambda|}{3} = \frac{1}{l_H^2}$ so, we obtain:

$$\Delta X \Delta E \geq \frac{\hbar c}{2}\left[1 + |\Lambda|\frac{(\Delta X)^2}{3c^2}\right]. \tag{57}$$

Then, solve the quadratic equation as follows:

$$\frac{\hbar|\Lambda|(\Delta X)^2}{6c} - (\Delta E)(\Delta X) + \frac{\hbar c}{2} = 0 \tag{58}$$

or

$$\Delta X = \frac{3c(\Delta E)(1 - \sqrt{(1 - |\Lambda|\hbar^2/3(\Delta E)^2)}}{|\Lambda|\hbar} \tag{59}$$

where we get $T \geq T_{min} = \frac{2\hbar}{3k}\sqrt{\frac{|\Lambda|}{3}}$. Then, we expand the Eq. (59) as below:

$$\Delta X = \frac{\hbar c}{2\Delta E}(1 + \frac{\hbar^2|\Lambda|}{12(\Delta E)^2}) \tag{60}$$

And we have the equations (52), (54), (55) with substituting them in Eq. (60) we obtain:

$$T = \frac{\hbar a}{2\pi kc}(1 + \frac{\hbar^2|\Lambda|}{27k^2T^2}) \tag{61}$$

And we know that $T_U = \frac{\hbar a}{2\pi kc}$ so, we obtain:

$$T = T_U(1 + \frac{\hbar^2|\Lambda|}{27k^2T^2}) \tag{62}$$

Here we substituting the T formula in Eq. (62) Therefore, we have:

$$T = T_U(1 + \frac{\hbar^2|\Lambda|}{27k^2T_U^2(1 + \frac{\hbar^2|\Lambda|}{27k^2T^2})^2}) \tag{63}$$

Then

$$T = T_U(1 + \frac{\hbar^2|\Lambda|}{27k^2T_U^2}(1 + \frac{\hbar^2|\Lambda|}{27k^2T^2})^{-2}) \tag{64}$$

Now, we expand the second terms as below:

$$T = T_U(1 + \frac{\hbar^2|\Lambda|}{27k^2T_U^2}(1 - 2\frac{\hbar^2|\Lambda|}{27k^2T^2})) \tag{65}$$

We neglect from the term of order $|\Lambda|^2$ so, we have:

$$T = T_U(1 + \frac{\hbar^2|\Lambda|}{27k^2T_U^2}) \tag{66}$$

EUP correction to the Unruh temperature becomes larger than the ordinary Unruh effect. Since $T_U = \hbar a/(2\pi kc)$, and $|\Lambda| > 0$, eq. (66) predicts an unphysical divergent temperature T when the acceleration a goes to zero. Namely, if $a \to 0$, then $T_U \to 0$, finally $T \to \infty$. Therefore eq. (66) satisfies $a > 0$.

5. Conclusion

In this paper, we investigated the thermodynamic properties of the Schwarzschild black hole and Unruh effect by using the simplest form of the EUP. From the EUP we obtained the mass-temperature relation for EUP black hole. From the minimal momentum of EUP, we found that there should exist the lower bound for the EUP black hole temperature. Using the mass-temperature relation for EUP black hole, we computed the specific heat and entropy. We found that the specific heat is always negative and increasing with the temperature. We also reexpressed the specific heat and entropy of the EUP-corrected Schwarzschild black hole in terms of the temperature instead of the mass. We found the relationship between the entropy and the area of the horizon for the EUP-corrected Schwarzschild black hole. Here we found that the expansion of the entropy in α does not contain the $\ln A$ which appears in the GUP-corrected Schwarzschild black hole case.

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