

A Brief Review of Glueball Masses from Gauge/Gravity Duality

Elena Cáceres

CINVESTAV, Mexico City, Mexico
University of Texas at Austin, Austin, TX.

E-mail: elenac@zipy.ph.utexas.edu

Abstract. This is a brief review of the status of glueball mass calculations from Supergravity. After reviewing the basic concepts, we summarize results of glueball spectrum for different models and compare their assets as well as their shortcomings. We focus on AdS black-hole, Klebanov-Strassler and Maldacena-Nunñez backgrounds.

1. Introduction

String Theory was born as a theory of hadronic interactions. After the discovery of Quantum Chromodynamics (QCD), which is the currently accepted theory of strong interactions, String Theory was abandoned as a possible explanation of hadronic interactions and was hailed as a theory of Quantum Gravity. After thirty years, string theorists have come back to the study of problems related to the hadronic world. Guided by the Maldacena conjecture [1] and their refinements [2],[3] there have been many interesting achievements in the area. Indeed, the gravity duals of field theories with different amount of Supersymmetry and "similar" to Quantum Chromodynamics (QCD) are known. The important point is that many characteristic features of QCD, like confinement, chiral symmetry breaking, etc; have been understood based on dual String Theory backgrounds. In this review we study glueballs in the context of gauge/gravity duality. Even though the models reviewed here are dual to confining theories, it is not completely clear if they are in the same universality class of QCD. Nevertheless, they represent the best candidates we currently have for a study of a QCD dual.

From a modern QCD perspective, it is known that glueballs are composites made out of constituent glue, with no quark content. Of course, since we live in a world with quarks, one might think that the proposal of pure glue objects is impossible to study. When investigation higher order corrections to glueball operators, there will be quarks running around the loops rendering the object not-pure glue. But lattice theorist working in the quenched approximation overcame this limitation and have taught us many things about glueballs.

Results of lattice calculations show that there is a discrete spectrum of glueballs, the lightest glueball is a scalar, the next is a tensor, 1.6 times heavier and the mass of the lightest glueball should be around 1630 MeV . See [4] for a nice and clear review of these results.

With the discovery of the gauge/gravity duality we have a new tool to calculate glueball masses. This duality relates a strongly coupled gauge theory with a weakly coupled Supergravity. This allows us to calculate gauge theory observables like glueball masses directly from

Supergravity. The purpose of this article is to review the techniques to calculate glueball masses from Supergravity, collect the results obtained for different backgrounds and compare their virtues and shortcomings.

2. Glueballs from Supergravity, the Basics.

The glueball mass spectrum in a gauge theory can be obtained by computing correlation functions of gauge invariant glueball operators and looking for particle poles. That is, we compute a two-point correlation function of two glueball operators that should behave in a Wilson expansion as

$$\langle O(x)O(y) \rangle = \sum_j c_j e^{M_j|x-y|}$$

where M_j are the glueball masses

From the String Theory view point, using the Gauge/Gravity duality, correlation functions of local operators are related to tree level amplitudes in the dual Supergravity description. Thus, the study of glueballs proceeds by finding bound states for the fluctuations of the supergravity fields. Basically, the idea is to fluctuate all the fields in a given solution dual to a confining field theory and study the equations of motion to first order in the fluctuations. The system is usually reduced to a Schrödinger problem. The eigenfunctions are identified with the glueballs and the eigenvalues are identified with their masses. The quantum numbers of the glueballs J^{PC} are determined from the quantum numbers of the dual string theory field. We should point out that this procedure is not totally clear in many of the available confining-models and it should be important to understand it better.

This machinery has been applied to some confining models. Let us add that, since many of the existing Supergravity models are duals to confining field theories with only adjoint matter content, the objects under study are only glueballs (no hybrids) and since we work in the large N_c regime, the glueballs are stable.

There are four landmarks for backgrounds dual to confining theories in four dimensions: Black-Hole -or finite temperature- backgrounds [5], Klebanov-Strassler -or the deformed conifold -background [6], Maldacena-Nunez -or wrapped branes backgrounds [7] and Polchinski Strassler [8]. We will review the first three where much work on the glueball spectrum has been done.

3. Black-Hole Backgrounds

The first calculation of glueball masses from supergravity were done in Black-Hole backgrounds. This study was pioneered by [9] and afterwards the spectrum was completed in [10]-[13]. As mentioned in the previous section, the computation of correlation functions amounts - using the gauge/gravity duality- to solving field equations for the corresponding fluctuation in the AdS background. For a given glueball one has to identify the corresponding glueball operator and compute its two point function. In order to do that we need to identify the supergravity field that couples to it at infinity. For example, the quantum numbers of the 0^{++} lead us to identify $Tr(F^2)$ as the glueball operator. And the string field that couples to it at infinity is the dilaton, Φ . Therefore the 0^{++} masses are obtained by solving equations of motion for the dilaton fluctuations,

$$\delta_\mu(\sqrt{g}g^{\mu\nu}\delta_\nu\delta\Phi) = 0. \quad (1)$$

Where the metric of a black-hole background is

$$dS^2 = r^2 dx_i dx^i + (r^2 - \frac{1}{r^4}) d\tau^2 + (r^2 - \frac{1}{r^4})^{-1} dr^2 + \frac{1}{4} d\Omega_4^2 \quad (2)$$

with worldvolume coordinates x_i , ($i = 1, 2, 3, 4$), τ is a compact direction, r the AdS radial direction and $d\Omega_4$ is the metric of a four-sphere. To find the lowest mass modes we consider

Table 1. Mass (squared) of the spin 0 glueball from a black-hole background.

State	(Mass) ²	Field
0 ⁺⁺	7.38	h_α^α
0 ^{++*}	22.07	$\delta\Phi$
0 ⁺⁺⁺	46.98	h_α^α
0 ⁺⁺⁺⁺	55.84	$\delta\Phi$
0 ^{++++*}	94.48	h_α^α
0 ^{++++**}	102.46	$\delta\Phi$

Table 2. Mass (squared) of the spin 1 glueball from a black-hole background.

State	(Mass) ²
1 ⁻⁻	83.04
1 ^{--*}	143.58
1 ^{--**}	217.39
1 ^{--***}	304.54

solutions of the form $\delta\Phi = f(u)e^{ikx}$. Equation (1) becomes,

$$\partial_r[r(r^6 - r_0^6)\partial_r f(r)] + M^2 r f(r) = 0, \quad -M^2 = k^2$$

The values of M^2 for which there are normalizable solutions are the glueball masses. The spectrum is found to be discrete and exhibits a mass gap. In [9],[10] the authors found that the mass of the 0⁺⁺ and its excited states is given by,

$$M^2 \sim \frac{0.74n(n+2)}{R_0^2}, \quad n = 1, 2, 3, \dots$$

Following a similar procedure Brower, Mathur and Tan in [11]-[13] carried out a detailed study of the complete spectrum in black-hole backgrounds. The 1⁻⁻ and 2⁺⁺ glueball masses are found by studying fluctuations of the one form A_μ and the metric $g_{\mu\nu}(x)$ respectively. Parts of their results are shown in Tables 1 and 2. Note that the dilaton fluctuation $\delta\Phi$ and the trace of the internal part of the metric h_α^α have the same quantum numbers as the scalar glueball and thus this two fluctuations will give the complete 0⁺⁺ spectrum. Brower, Mathur and Tan showed that the scalars coming from the dilaton fluctuation are degenerate with the 2⁺⁺ glueball and that the lightest scalar comes not from the dilaton but from the h_α^α fluctuation.

4. The deformed Conifold Background

The deformed conifold background [6] involves M D5 branes wrapped on an S^2 and N D3 branes. The solution is dual to a four dimensional $SU(N) \times SU(N+M)$ in the ultraviolet. The theory has $\mathcal{N} = 1$ supersymmetry. Going to lower energies, the theory cascades through chiral symmetry breaking $SU(N) \times SU(N+M) \rightarrow SU(N-M) \times SU(N) \rightarrow SU(N-2M) \times SU(N-M) \rightarrow \dots$. Originally it was thought that, if $N = kM$ with k and integer, deep in the infrared the gauge group was $SU(M)$, and that it was in the same universality class of Super Yang Mills. Later on

Table 3. Mass (squared) of the spin 2 glueball from a black-hole background.

State	(Mass) ²
2 ⁺⁺	22.09
2 ^{++*}	55.58
2 ^{+++*}	102.46

[18] it was showed that the cascade stops before getting to $SU(M)$ and the theory at the bottom of the cascade is $SU(2M) \times SU(M)$. Furthermore, there is a massless scalar corresponding to a $U(1)$ Goldstone boson. Thus, there is no mass gap and the deformed conifold theory is not in the same class as Super Yang Mills. More precisely, the theory is in a baryonic branch where certain baryon operators acquire vacuum expectation value. Nevertheless, the fact that the theory exhibits chiral symmetry breaking and confinement makes it interesting in its own right. The Klebanov-Strassler metric is,

$$ds_{10}^2 = h^{-1/2}(\tau)dx_n dx_n + h^{1/2}(\tau)ds_6^2, \quad (3)$$

with ds_6^2 the metric of the deformed conifold. There is a basis, $\{\tau, g^{i=1,\dots,5}(\psi, \theta_1, \theta_2, \phi_1, \phi_2)\}$ where this metric becomes diagonal,

$$ds_6^2 = \frac{1}{2}\epsilon^{4/3}K(\tau) \left[\frac{1}{3K^3(\tau)}[d\tau^2 + (g^5)^2] + \cosh^2\left(\frac{\tau}{2}\right) [(g^3)^2 + (g^4)^2] \right] \quad (4)$$

$$+ \sinh^2\left(\frac{\tau}{2}\right) [(g^1)^2 + (g^2)^2], \quad (5)$$

where

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh(\tau)}. \quad (6)$$

The harmonic function in (3) is given by the integral expression

$$h(\tau) = \alpha \frac{2^{2/3}}{4} \int_{\tau}^{\infty} dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3}, \quad (7)$$

which cannot be evaluated in terms of elementary or special functions. The constant α is $\alpha \sim (g_s M)^2$.

The solution contains also a self-dual five-form and a three-form flux. $F_5 = \mathcal{F}_5 + \star \mathcal{F}_5$, $G_3 = F_3 + iH_3$. The explicit forms of these function can be found in [6]. For our purposes suffices to note that they only depend on the radial variable τ . the procedure for finding the 0^{++} and 1^{--} masses is very similar to the one outlined in the previous section. The 2^{++} case involves fluctuations of the metric and is therefore, more technically involved. The calculation of the spectrum in this background can be found in [14]-[16]. we summarize the results in the following table.

All the masses are measured in units related to $\epsilon^{2/3}$ which is a parameter that controls the deformation of the conifold. It is interesting to note that if we plot the lowest lying 0^{++} , 1^{--} and 2^{++} in a Spin vs. Mass² plot, these states lie on a straight line. Strictly speaking this cannot be called a Regge trajectory since this calculation is done in an infinite tension regime. Nevertheless, the fact that the states align themselves in a line in a J vs. M^2 plot is definitely reminiscent of a Regge trajectory and is quite remarkable.

Table 4. Mass (squared) of the spin 0,1 and 2 glueballs from supergravity in the Klebanov-Strassler background. The masses are measured in units of $\epsilon^{2/3}/g_s M \alpha'$

State	M^2
0 ⁺⁺	9.78
0 ^{++*}	33.17
1 ⁻⁻	14.05
1 ^{--*}	42.90
2 ⁺⁺	18.33

5. Wrapped Branes or the Maldacena-Nunez Background

The Maldacena-Nuñez background [7] involves D5 branes wrapped in an S^2 . This model exhibits some very distinctive features. It is dual to four dimensional Super-Yang-Mills in the infrared but the ultraviolet completion is a five dimensional theory. Thus, as we go to high energies one dimension opens up and the model stops being dual to SYM. This fact alone suggests that some kind of regularization will be needed. Another distinctive feature is the presence of a varying dilaton. In Black-Hole backgrounds as well as in the Klebanov-Strassler models reviewed in the previous section the background dilaton was constant. The procedure for calculating glueball masses is essentially the same as the one outlined in the previous sections but the existence of a varying dilaton in the Maldacena-Nuñez background makes it technically much more involved. We will not present all the equations here, for details see [7]. The background has the topology of $R^{1,3} \times R \times S^2 \times S^3$ and there is a fibration between the two spheres that allows the supersymmetry preservation. The topology of the metric, near $r = 0$ is $R^{1,6} \times S^3$. The metric in Einstein frame reads,

$$ds_{10}^2 = \alpha' g_s N e^{\frac{\phi}{2}} \left[\frac{1}{\alpha' g_s N} dx_{1,3}^2 + e^{2h} (d\theta^2 + \sin^2 \theta d\varphi^2) + dr^2 + \frac{1}{4} (w^i - A^i)^2 \right], \quad (8)$$

where ϕ is the dilaton. The angles $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$ parametrize a two-sphere. This sphere is fibered in the ten dimensional metric by the one-forms A^i ($i = 1, 2, 3$). The A^i 's can be written as a function of r and the angles (θ, φ) , for explicit expressions see (ccc). The dilaton is given by,

$$e^{-2\phi} = \frac{2e^{-2\phi_0}}{\sinh 2r} \sqrt{\coth 2r - \frac{r^2}{\sinh^2 2r} - \frac{1}{4}} \quad (9)$$

The glueball spectrum of this model was investigated in [19], [20] and the meson spectrum in [21]. The authors of [20] found that unlike the backgrounds previously studied, in Maldacena-Nuñez background not even the simplest scalar mode decouples from the rest of the fluctuations. Indeed, assuming only fluctuations of the dilaton leads to inconsistent equations. Therefore, the glueball 0⁺⁺ in the Maldacena-Nuñez background is not dual to the dilaton, but to a mixture of dilaton and trace of the internal part of the metric. This mixing might persist for higher spin modes. The presence of a non-constant dilaton background seems to be the reason for the mixing of the fluctuations.

The Maldacena-Nuñez background produces a discrete spectrum. Due to the particular nature of the potential, the eigenvalues are bounded from above and below. The spectrum is not normalizable. In [20] the authors proposed a regularization procedure that amounts to subtracting the unphysical region where the theory is no longer dual to a four dimensional gauge theory.

Table 5. Mass (squared) of the spin 0 glueball, measured in units of $\frac{1}{Ng_s\alpha'}$, in the Maldacena-Nuñez background

State	(Mass) ²
0 ⁺⁺	18.41

6. Comparison and Conclusions

The three backgrounds reviewed here present very distinctive characteristics. The black-hole backgrounds exhibit a degeneracy of the 2⁺⁺ and 0⁺⁺ glueballs. Also, the lightest 0⁺⁺ glueball does not come from the dilaton but from the trace of the internal part of the metric. The Klebanov-Strassler background does not present these degeneracy; the lowest states lie on a line in a Spin vs Mass² plot. This linear trajectory is not, strictly speaking, a Regge trajectory since the supergravity calculation is done in an infinite coupling regime. Nevertheless, it is quite suggestive. The Klebanov-Strassler spectrum does not exhibit a mass gap, there is a massless scalar state. The Maldacena-Nuñez solution, which, unlike the other solutions presented here, involves a varying dilaton in the background, presents also some new features. The spectrum is discrete but the modes are non-normalizable and a regularization procedure is needed. The scalar glueball is in this case a mixture of the dilaton and the trace of the internal part of the metric.

The interest of studying glueballs goes beyond the simple fact of getting a discrete spectrum—that is by itself of enough interest. Indeed, glueballs play an important role in some advances that happened recently regarding the study of Deep Inelastic Scattering using AdS/CFT techniques [22]. The knowledge of glueballs masses and profiles in different models might help to extend the results in papers like [22] to other ‘more realistic’ models.

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