

PRESENT STATUS AND PROSPECTS OF $(g_\mu - 2)/2^*$

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The current status of the muon anomalous magnetic moment is discussed. The leading order hadronic contribution is reevaluated based on the new data on e^+e^- annihilation. The experimental value is about 3.3 standard deviations higher than the Standard Model prediction.

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1. Anomalous magnetic moment

The muon anomalous magnetic moment, a_μ , is one of the most accurately known physical quantities recently measured by E821 [1] with a 5×10^{-7} relative accuracy:

$$a_\mu = (11659208.0 \pm 6.3) \times 10^{-10}. \quad (1)$$

Although for electron it is known much better (a_e is measured with a 4.9×10^{-10} accuracy [2]), a_μ is much more sensitive to new physics effects: the gain is usually $\sim (m_\mu/m_e)^2 \approx 4.3 \times 10^4$. Any significant difference of a_μ^{exp} from a_μ^{th} indicates new physics beyond the Standard Model (SM). It is conventional to write a_μ as

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}}. \quad (2)$$

Terms up to α^3 are known analytically [3]. Taking into account a recent more accurate numerical calculation of the α^4 terms and the leading $\log \alpha^5$ terms [4–6] one obtains

$$a_\mu^{\text{QED}} = (116584719.4 \pm 1.4) \times 10^{-11}. \quad (3)$$

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With the value of α from the latest result for $a_e \alpha^{-1} = 137.035999710(96)$ [7], one obtains [8]:

$$a_\mu^{\text{QED}} = (116584718.09 \pm 0.14 \pm 0.08) \times 10^{-11}. \quad (4)$$

Here the errors are due to the uncertainties of the $\mathcal{O}(\alpha^5)$ term and α . It is worth noting that the 4-loop term equals 38.1×10^{10} and is thus six times larger than the experimental uncertainty. Therefore, it is clear that its calculation as well as that of the 5-loop one is necessary.

The electroweak term is known rather accurately [9]:

$$a_\mu^{\text{EW}} = (15.4 \pm 0.1 \pm 0.2) \times 10^{-10}, \quad (5)$$

where the first uncertainty is due to hadronic loops while the second one is caused by the errors of M_H, M_t and 3-loop effects.

The hadronic contribution can also be written as a sum:

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}. \quad (6)$$

The dominant contribution comes from the leading order term

$$a_\mu^{\text{had,LO}} = \left(\frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{R(s) \hat{K}(s)}{s^2}, \quad (7)$$

where

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (8)$$

and the kernel $\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \rightarrow \infty$, $1/s^2$ emphasizes the role of low energies. Particularly important is the reaction $e^+e^- \rightarrow \pi^+\pi^-$ with a large cross section below 1 GeV.

Our new estimate takes into account the recent progress in the low energy e^+e^- annihilation and includes the data not yet available previously [10–12].

In addition to the previously published ρ meson data [13], CMD-2 reported their final results on the pion form factor F_π from 370 to 1380 MeV [14–16]. The new ρ meson sample has an order of magnitude larger statistics and a systematic error of 0.8%. SND measured F_π from 390 to 970 MeV with a systematic error of 1.3% [17]. KLOE studied F_π using the method of radiative return or ISR [18–20] at $590 < \sqrt{s} < 970$ MeV with a sample of 1.5×10^6 events and systematic error of 1.3% [21]. BaBar also used ISR and achieved impressive results on various final states with more than two hadrons [22–24].

2. New data

In Fig. 1 we show the pion form factor data from CMD-2, KLOE and SND. The $|F_\pi|$ values from CMD-2 and SND are in good agreement. The KLOE data are consistent with them near the ρ meson peak, but exhibit a somewhat different energy dependence: they are higher to the left and lower to the right of the ρ meson peak. However, the contributions to a_μ from all three experiments are consistent.

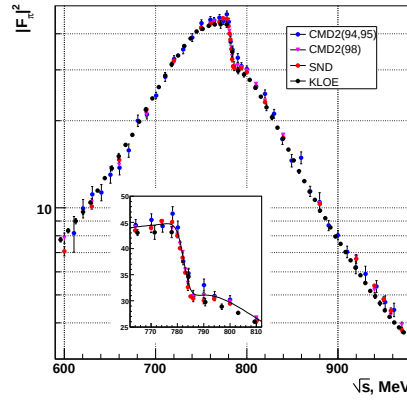


Fig. 1. $|F_\pi|$ from CMD-2, KLOE and SND.

3. Results

Using the new data below 1.8 GeV discussed above in addition to the whole data set of [10, 11] for old experiments, and assuming that for the hadronic continuum above 1.8 GeV one can already use the predictions of perturbative QCD [25], we can reevaluate the leading order hadronic contribution to a_μ . The results for different energy ranges are shown in Table I.

The theoretical error consists of 1.9×10^{-10} due to uncertainties of radiative corrections in old measurements and 0.7×10^{-10} related to the mentioned above use of perturbative QCD. It can be seen that due to a higher accuracy of e^+e^- data the uncertainty of $a_\mu^{\text{had,LO}}$ is now 4.4 (0.63%) compared to 15.3 of Ref. [10] and 7.2 of Ref. [12].

We move now to the higher order hadronic contributions. Their most recent estimate performed in [26] gives $(-9.8 \pm 0.1) \times 10^{-10}$ and has a negligible error compared to that of the leading order one.

The most difficult situation is with the light-by-light hadronic contribution, which is estimated only theoretically. Even the correct sign of this term was established quite recently [27]. The older predictions based on

TABLE I

Updated $a_\mu^{\text{had,LO}}$	
$\sqrt{s}$, GeV	$a_\mu^{\text{had,LO}}, 10^{-10}$
2π	$504.6 \pm 3.1 \pm 1.0$
ω	$38.0 \pm 1.0 \pm 0.3$
ϕ	$35.7 \pm 0.8 \pm 0.2$
0.6–1.8	$54.2 \pm 1.9 \pm 0.4$
1.8–5.0	$41.1 \pm 0.6 \pm 0.0$
$J/\psi, \psi'$	$7.4 \pm 0.4 \pm 0.0$
> 5.0	$9.9 \pm 0.2 \pm 0.0$
Total	$690.9 \pm 3.9_{\text{exp}} \pm 2.0_{\text{th}}$

the chiral model and vector dominance [28, 29] were compatible and much lower than that using short-distance QCD constraints [30] (see also [31]). Their approximate averaging in [32] gives $(120 \pm 35) \times 10^{-11}$. Even higher uncertainty is listed in Ref. [33] who added some terms not taken into account in Ref. [30] to obtain $(110 \pm 40) \times 10^{-11}$. It is very tempting to be able to find an approach to estimate the light-by-light hadronic contribution from the data, like, *e.g.*, it was done in Ref. [34], where CLEO single-tag measurements [35] of $\gamma\gamma^* \rightarrow \pi^0, \eta, \eta'$ were used to estimate the contribution from the pseudoscalar resonances.

Adding all hadronic contributions we obtain $a_\mu^{\text{had}} = (693.1 \pm 5.6) \times 10^{-10}$ [36]. This result agrees with other recent estimations, *e.g.*, [12, 26, 37–39] and has better accuracy due to the new e^+e^- data. All separate contributions are collected in Table II. The improved precision of the leading order hadronic contribution allows to confirm previously observed excess of the experimental value over the SM prediction with a higher than before significance of more

TABLE II

Experiment <i>vs.</i> theory.	
Contribution	$a_\mu, 10^{-10}$
Experiment	11659208.0 ± 6.3
QED	11658471.94 ± 0.14
Electroweak	$15.4 \pm 0.1 \pm 0.2$
Hadronic	693.1 ± 5.6
Theory	11659180.5 ± 5.6
Exp.–Theory	$27.5 \pm 8.4 \ (3.3\sigma)$

than three standard deviations. For the first time during last years the accuracy of the SM prediction is slightly better than the experimental one.

How real is a very high accuracy of the leading order hadronic contribution obtained above? We believe that we understand well the radiative corrections due to initial state radiation and vacuum polarization, but should not forget that they are numerically rather large and may reach $\sim 20\%$, so their critical reanalysis and tests of the existing Monte Carlo generators are needed. The situation with the radiative corrections due to final state radiation is not so well established, so we have to rely on the model of scalar electrodynamics and confront it with the data. There is also a question of double counting of the hadronic final states in the leading and higher order hadronic terms [40].

One of the most serious experimental questions is that of the missing states. An obvious candidate is final states with neutral particles only, which were badly measured previously. Recent experiments in Novosibirsk in which the $\pi^0\gamma$, $\eta\gamma$, $\pi^0\pi^0\gamma$, $\eta\pi^0\gamma$ final states were studied in the energy range from threshold to 1.4 GeV by CMD-2 and SND (see Refs. [41, 42] and references therein) showed that the cross sections are dominated by the ρ^- , ω^- , ϕ^- -mesons and thus the corresponding contributions are properly taken into account. From the upper limits on nonresonant cross sections obtained in these papers we can estimate that a possible not yet accounted for contribution is $a_\mu^{\text{had,LO}} < 0.7 \times 10^{-10}$. However, one should remember that there are no measurements at all of such channels above 1.4 GeV although they are expected to be small.

Recently there has been serious progress with ISR studies from BaBar. The discussion of their effect on the $a_\mu^{\text{had,LO}}$ estimation can be subdivided into two parts: new results on already measured states and studies of various new final states. In the first part there are processes which cross sections are consistent with the older measurements and more precise, *e.g.*, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$, $\dots$. There are also final states for which the cross sections strongly differ from the older, less accurate measurements, *e.g.*, $\pi^+\pi^-\pi^0$, 6π , $\dots$. In the second part there are final states, which have never been measured before, *e.g.*, $K^+K^-\pi^0\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $4\pi^\pm\eta$, $K^+K^-\eta$. Obviously, one should calculate what contribution to a_μ^{LO} comes from them and add it to the previous estimate. While doing that one should be very thorough since the final states observed may be only a subset of more general processes. For example, the $K^+K^-\pi^+\pi^-\pi^0$ final state may come from the process $\phi\eta$, so that our estimate of the contribution to a_μ^{LO} should be correspondingly divided by the relevant branching fractions, in this case $\mathcal{B}(\phi \rightarrow K^+K^-)\mathcal{B}(\eta \rightarrow \pi^+\pi^-\pi^0) = 0.1118$, effectively increasing our estimate by a factor of 8.94! Fortunately, we are interested in exclusive cross sections only below 2 GeV and the new processes above usually have a

rather small cross section in this energy range. The first estimate shows that these new contributions may increase $a_\mu^{\text{had,LO}}$ by $(1-3) \times 10^{-10}$, only slightly decreasing the discrepancy between the theoretical expectation and the experimental result.

In view of the new measurements of the cross sections of the processes with K^+K^- and pions in the final state one should carefully reconsider the contribution from $K\bar{K}n\pi$, which was previously estimated using isospin relations [11]. Anyway, it is clear that we have to process new information thoroughly and understand the size and accuracy of the continuum contribution below 2 GeV (now $(62.4 \pm 2.0 \pm 0.5) \times 10^{-10}$) compared to that from the $\pi\pi$ ($504.6 \pm 3.1 \pm 1.0 \times 10^{-10}$).

What is the future of this SM test? From the experimental side there are suggestions to improve the accuracy by a factor of 2.5 at E969 (BNL) or even by an order of magnitude at J-PARC. It is clear that it will be extremely difficult to improve the accuracy of the SM prediction significantly. One can optimistically expect that by 2008 new high-statistics ISR measurements at KLOE, BaBar and Belle together with the more precise R below 4.3 GeV from CLEO-c will decrease the error of $a_\mu^{\text{had,LO}}$ from 4.4 to 2.8×10^{-10} . Experiments are planned at the new machine VEPP-2000 now commissioning, which is a VEPP-2M upgrade with two detectors (CMD-3 and SND) up to $\sqrt{s}=2$ GeV with $L_{\text{max}} = 10^{32} \text{ cm}^{-2}\text{s}^{-1}$. A similar machine (DAΦNE-II) is discussed in Frascati. New R measurements below 5 GeV will be done at the $\tau - c$ factory now under construction in Beijing. We can estimate that by 2010 the accuracy of $a_\mu^{\text{had,LO}}$ will be improved from 2.8 to 2.2×10^{-10} and the total error of 4.1 will be limited by the LBL term (3.5) and still higher than the expected 2.5 in E969.

There is still no explanation for the observed discrepancy between the predictions based on τ lepton and e^+e^- data [12]. For this reason we are not using τ data in this update. One expected that more light on the problem would be shed by the high-statistics measurement of the two-pion spectral function by Belle whose preliminary results indicated better agreement with e^+e^- data than before [44]. However, it turns out that while in a relatively small range of masses from 0.8 to 1.2 GeV the $\pi\pi$ spectral function measured at Belle is below the ALEPH one, this effect is compensated by the spectral function behavior at low and high masses, so that the resulting contribution to the hadronic part of the muon anomaly is about the same as before.

Let us hope that progress of theory will allow a calculation of a_μ^{had} from first principles (QCD, Lattice). One can mention here a new approach in the QCD instanton model [45] or calculations on the lattice, where there are encouraging estimates of $a_\mu^{\text{had,LO}}$, e.g., [46] $(667 \pm 20) \times 10^{-10}$ or attempts to estimate $a_\mu^{\text{had,LBL}}$ [47].

In conclusion, I would like to emphasize once again that BNL success stimulated significant progress of e^+e^- experiments and related theory. Improvement of e^+e^- data (BaBar, BES, CMD-2, KLOE and SND) led to a substantial decrease of an error of $a_\mu^{\text{had,LO}}$. For the first time the accuracy of the theoretical prediction is better than that of the experimental measurement. Future experiments as well as development of theory should clarify whether the observed difference between a_μ^{exp} and a_μ^{th} is real and what consequences for the Standard Model it implies.

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