UDC 544.586.6

DYNAMICAL CP-VIOLATION IN QUASILOCAL QUARK MODELS AT NONZERO QUARK CHEMICAL POTENTIAL

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We consider the Quasilocal Quark Model of Nambu-Jona-Lasinio type as effective theory of non-perturbative QCD with scalar-pseudoscalar four-quark interaction with derivatives in fields at finite quark chemical potential. In the presence of a strong attraction in the scalar channel the chiral symmetry is spontaneously broken and as a consequence the composite meson states are generated. For special configurations of coupling constants, the dynamical CP-violation in the pseudoscalar sector can appear as a result of complexity of dynamical mass function generated at some value of quark density.

1. Introduction

Quark models with local four-quark interaction are widely applied for the description of low-energy phenomena in QCD in the hadronization regime (see, e.g., reviews [1] and references therein). The local four-fermion interaction is involved to induce the Dynamical Chiral Symmetry Breaking (DCSB) due to strong attraction in the scalar channel. As a consequence, the dynamical quark mass M_{dyn} is created, as well as an isospin multiplet of pions, massless in the chiral limit, and a massive scalar meson with mass $m_\sigma = 2M_{\text{dyn}}$ arise. However it is known from the experiment [2] that there are series of meson states with equal quantum numbers and heavier masses which traditionally are called "radial excitations" following an analogy with non-relativistic potential models. In order to describe the physics of those resonances at intermediate energies one can extend the quark model with local interaction of Nambu-Jona-Lasinio (NJL) type [3] taking into account higher-dimensional quark operators with derivatives, i.e. quasilocal quark interactions [4 - 12]. For sufficiently strong couplings the new operators promote the formation of additional meson states. Such a quasilocal approach (see also [13 - 16]) represents a systematic extension of the NJL model, where the low-energy gluon effects are hidden in the coupling constants. The alternative schemes including the condensates of low-energy gluons can be found in [11]. A distinguished feature of these models is that they naturally reproduce [8, 9, 14] the Chiral Symmetry Restoration (CSR) at high energies [17]. Due to this property such models can be successfully matched to QCD sum rules [8, 9, 14], i.e. they reproduce more dynamics typical for real QCD than usual NJL-type models.

It seemed to be the general feature for the quark models of NJL-type that, when being symmetry of the quark lagrangian, the CP-parity remains a good quantum number after DCSB. However, it happens that for particular combination of four-fermion coupling constants in the Quasilocal Quark Model (QQM), the CP-parity can be broken dynamically together with the chiral symmetry [18]. For simple two-channel pseudoscalar QQM this situation was considered in [18] at zero quark densities. On a particular plane in the coupling constant space the complex solution for the dynamical mass function was found, which yields the CP-parity breaking in meson sector. It means that in such a plane there exist heavy scalar states which can decay into two or three pions. On the other hand, as we know, the study of QCD and NJL-type models at nonzero temperature and quark chemical potential are of high importance to understand a wide range of different physical phenomena, from heavy-ion-collision experiments to neutron stars and cosmology. This has led to varios theoretical investigations of the phase diagram of QCD at finite quark-chemical potentials and temperatures (see, e.g., [19]). There are arguments for the possibility of CP-violation in the strong interactions at different external conditions in theories with local interactions [20].

In this paper we will focus on the problem of possible dynamical generation of CP-breaking in the strong interactions within QQM. The paper is organized as follows. In Section 2 we present at finite quark densities two-channel QQM with scalar-pseudoscalar self-interaction. The mass spectrum of meson states is derived in Section 3 in the large-log approximation. The physics behind the model is discussed in Section 4. We conclude in Section 5.

2. Quasilocal Quark Model with chemical potential

The quasilocal approach of [5] (see also[7, 21, 22]) represents a systematic extension of the NJL model [3] towards the complete effective action of QCD where many-fermion vertices with derivatives are incorporated with the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the perturbative QCD effective action. For sufficiently strong couplings, the new operators promote the formation of additional scalar and pseudoscalar states. These models allow an extension of the linear σ model provided by the NJL model, with the pion being a broken symmetry partner of the lightest scalar meson just as before, and with excited pions and scalar particles coming in pairs. In particular, when only scalar and pseudoscalar color-singlet channels are examined and dynamical quark mass is supposed to be sufficiently smaller than the DCSB cutoff one may derive the minimal two-channel lagrangian of the QQM [5, 7] where we have added the chemical potential μ

$$\mathcal{L}^{QQM} = \bar{q}(i\partial\!\!\!/ + \gamma_0\mu)q + \mathcal{L}^I ;$$

$$\mathcal{L}^I = \frac{1}{4N_f N_c\Lambda^2}\sum_{k,l=1}^{2} a_{kl}\left[\bar{q}f_k(s)q\,\bar{q}f_l(s)q - \bar{q}f_k(s)\tau^a\gamma_5 q\,\bar{q}f_l(s)\tau^a\gamma_5 q\right]. \quad (1)$$

Here a_{kl} represents a symmetric matrix of real coupling constants and $f_k(s)$, $s \equiv -\partial^2/\Lambda^2$ are the polynomial formfactors specifying the quasilocal (in momentum space) interaction. These formfactors are orthogonal on the unit interval

$$\int_0^1 f_k(s)\,f_l(s)ds = \delta_{kl}. \quad (2)$$

The results of calculations do not depend on a concrete choice of formfactors in the large-log approximation. Our choice is

$$f_1(s) = 2 - 3s, \qquad f_2(s) = -\sqrt{3}s. \quad (3)$$

As this model interpolates the low-energy QCD action it is supplied with the cutoff $\Lambda \sim 1$ GeV which bounds virtual quark momenta in quark loops. We restrict ourselves with consideration of two-flavor case, thus τ^a denote Pauli matrices.

For strong four-fermion coupling constants $a_{kl} \sim 8\pi^2\delta_{kl}$ Lagrangian (1) reveals the phenomenon of dynamical chiral symmetry breaking. This phenomenon can be described with the help of the effective potential for the attractive scalar channel where scalar mesons arise as composite states. Indeed its non-trivial minimum gives rise to a dynamical quark mass and the perturbative fluctuations around this minimum characterize the mass spectrum of meson states. To derive the required effective potential one should bosonize the quark action, i.e. incorporate auxiliary bosonic variables $\sigma_k \sim i\bar{q}f_k(s)q$, $\pi_k^a \sim \bar{q}f_k(s)\tau^a\gamma_5 q$, and integrate out fermionic degrees of freedom. At the first step we introduce the bosonic variables in two channels

$$L_I = \sum_{k=1}^{2} i\bar{q}\left(\sigma_k + i\gamma_5\pi_k^a\tau^a\right)f_k(s)q + N_f N_c\Lambda^2\sum_{k,l=1}^{2}\left(\sigma_k a_{kl}^{-1}\sigma_l + \pi_k^a a_{kl}^{-1}\pi_l^a\right). \quad (4)$$

Let us parametrize the matrix of coupling constants in a close vicinity of tricritical point

$$8\pi^2 a_{kl}^{-1} = \delta_{kl} - \frac{\Delta_{kl}}{\Lambda^2}, \qquad |\Delta_{kl}| \ll \Lambda^2. \quad (5)$$

The last inequality turns out to be equivalent to requirement for the dynamical mass to be essentially less than the cut-off.

The dynamical mass function (for the time being we denote $\bar{\sigma}_k \equiv \sigma_k$) is $M(s) = \bar{\sigma}_k f_k(s)$. Since the collective variables $\bar{\sigma}_k$ are complex functions, the dynamic mass is complex as well. However, by the global chiral rotation $M(s) \to M(s)e^{i\omega}$, $\omega = const$, we can ensure fulfilment of the condition $\mathrm{Im}\langle M_0\rangle_{\bar{\sigma}} = 0$ (here and further $M_0 \equiv M(0)$). In this case, the following parametrization of the solutions of the stationary-state equations is valid

$$\bar{\sigma}_1 = \sigma_1, \quad \bar{\sigma}_2 = \sigma_2 - i\frac{2\xi}{\sqrt{3}}, \quad \sigma_k \equiv \mathrm{Re}\,\bar{\sigma}_k. \tag{6}$$

In hermitian quark Lagrangian the imaginary part of the dynamical mass corresponds to the pseudoscalar mass. In the present analysis we identify the imaginary part of dynamical mass with v.e.v. of π^0-meson,

$$\frac{2\xi}{\sqrt{3}} = \left\langle \pi^0 \right\rangle. \tag{7}$$

As we will see the condensation of this meson can happen at some conditions.

The appearance of chemical potential results in the following modification of quark propagator[23]

$$\frac{i}{\not{p} - M + i\varepsilon} \to \frac{i}{\not{p} - M + i\varepsilon} - 2\pi\delta(p^0 - E_p)\Theta(p_F - |\,\mathrm{p}\,|)\frac{\not{p} + M}{2E_p}, \tag{8}$$

where $E_p = (\mathrm{p}^2 + M^2)^{1/2}$ is the energy and $p_F = (\mu^2 - M^2)^{1/2}$ is the Fermi momentum which is related to the baryon density ρ as follows

$$\rho = \frac{N_c N_f}{9\pi^2} p_F^3. \tag{9}$$

After performing the Wick rotation (further analysis we carry out in the Euclidean space) and integrating out the quark fields one comes to the bosonic effective action $S_{\mathrm{eff}}(\sigma_k, \pi_k^a)$ which is defined from the regularized vacuum functional

$$Z^\Lambda(\sigma_k, \pi_k^a) = \exp(-S_{\mathrm{eff}}) = \left\langle \exp(-\int d^4x\,\mathcal{L}) \right\rangle_{\bar{q}q} = \exp\left(-\int d^4x V_{\mathrm{eff}}\right), \tag{10}$$

and therefrom, for constant meson variables, to the effective potential

$$\begin{aligned} V_{\mathrm{eff}} = \frac{N_c N_f}{8\pi^2}\Bigg\{ & -\sum_{k,l=1}^{2} \sigma_k\sigma_l\Delta_{kl} - (\pi_2^a)^2\Delta_{22} - \frac{4}{3}\Delta_{22}\xi^2 + 8\sigma_1^4\left(\ln\frac{\Lambda^2}{4\sigma_1^2} + \frac{1}{2}\right) - \\ & -\frac{159}{8}\sigma_1^4 - \frac{5\sqrt{3}}{2}\sigma_1^3\sigma_2 + \frac{9}{4}\sigma_1^2\sigma_2^2 + \frac{\sqrt{3}}{2}\sigma_1\sigma_2^3 + \frac{9}{8}\sigma_2^4 + \\ & + \frac{3}{4}\bar{\sigma}(\pi_2^a)^2 + \frac{9}{8}(\pi_2^a)^4) + \bar{\sigma}\xi^2 + 2\xi^4 + \frac{2}{3}\mu^4 + \\ & + 4\Theta(\mu - 2\sigma_1)\left[\mu\sigma_1^2\sqrt{\mu^2 - 4\sigma_1^2} - \frac{\mu}{6}(\mu^2 - 4\sigma_1^2)^{3/2} - 4\sigma_1^4 \ln\frac{\mu + \sqrt{\mu^2 - 4\sigma_1^2}}{2\sigma_1}\right]\Bigg\} + O\left(\frac{\ln\Lambda}{\Lambda^2}\right). \end{aligned} \tag{11}$$

Here we have introduced the notation $\bar{\sigma}$ for the combination

$$\bar{\sigma} = \sigma_1^2 + \frac{2\sqrt{3}}{3}\sigma_1\sigma_2 + 3\sigma_2^2. \tag{12}$$

The conditions on extremum of the effective potential, the mass-gap equations,

$$\Delta_{11}\sigma_1+\Delta_{12}\sigma_2=16\sigma_1^3\ln\frac{\Lambda^2}{4\sigma_1^2}-\frac{159}{4}\sigma_1^3-\frac{15\sqrt{3}}{4}\sigma_1^2\sigma_2+\frac{9}{4}\sigma_1\sigma_2^2+\frac{\sqrt{3}}{4}\sigma_2^3+$$

$$+\xi^2\left(\sigma_1+\frac{\sigma_2}{\sqrt{3}}\right)+8\sigma_1\Theta(\mu-2\sigma_1)\left[\mu\sqrt{\mu^2-4\sigma_1^2}-4\sigma_1^2\ln\left(\frac{\mu}{2\sigma_1}+\sqrt{\frac{\mu^2}{4\sigma_1^2}-1}\right)\right],$$

$$\Delta_{12}\sigma_1+\Delta_{22}\sigma_2=-\frac{1}{4}\left(\sigma_1-\sqrt{3}\,\sigma_2\right)\left(5\sqrt{3}\,\sigma_1^2+6\sigma_1\sigma_2+3\sqrt{3}\,\sigma_2^2\right)+\xi^2\left(\frac{\sigma_1}{\sqrt{3}}+3\sigma_2\right), \tag{13}$$

$$\Delta_{22}\xi=3\xi\left(\frac{\bar{\sigma}}{4}+\xi^2\right).$$

The critical values of the coupling constants correspond to the cancellation of contributions quadratic in the momentum cutoff Λ, i.e. $a_{kl}=\delta_{kl}$. Equations (13) allow to find certain relations between the components of dynamic mass function and (reduced) coupling constants Δ_{kl}. In practice, one uses the v.e.v.'s of scalar fields as input parameters, in particular, $2\sigma_1=M_{\text{dyn}}=250\div 400$ MeV, and determines the required Δ_{kl}. But in order to classify the solutions we will follow the inverse procedure, i.e. we will keep the parameters Δ_{kl} as inputs. We suppose a scale (the reason will be explained below) $\Delta_{22}=\mathcal{O}(1)$, then the condition of selfconsistency of Eqs. (13) is $\Delta_{12}=\mathcal{O}(1)$, $\Delta_{11}=\mathcal{O}(\ln\frac{\Lambda^2}{M_0^2})$.

Consider $\mu=0$.

For the real solutions ($\xi=0$) several phases are possible.

1) Gross-Neveu (GN) phase

$$\sigma_1^2=\frac{\Delta_{11}}{16\ln\frac{\Lambda^2}{M_0^2}}\left[1+\mathcal{O}\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right)\right]. \tag{14}$$

The expression for σ_2 is very lengthy, but it is evident that there is a solution which behaves as $\sigma_1/\sigma_2=const$ at $\sigma_1\to 0$. This solution delivers minimum to the effective potential for $\Delta_{22}<0$, $\det\Delta<0$.

2) Anomalous phase

$$\sigma_1^2=\frac{\Delta_{22}^{1/3}(\Delta_{22}-3\sqrt{3}\Delta_{12})^{2/3}}{12\left(3\ln\frac{\Lambda^2}{M_0^2}\right)^{2/3}}\left[1+\mathcal{O}\left(\frac{1}{\ln^{1/3}\frac{\Lambda^2}{M_0^2}}\right)\right],$$

$$\sigma_2^2=\frac{4}{9}\Delta_{22}\left[1+\mathcal{O}\left(\frac{1}{\ln^{1/3}\frac{\Lambda^2}{M_0^2}}\right)\right]. \tag{15}$$

This solution corresponds to minimum of potential for - $\Delta_{22}>0$, $\Delta_{22}-3\sqrt{3}\Delta_{12}\neq 0$.

3) Solutions (14) and (15) are not valid when $\Delta_{22}-3\sqrt{3}\Delta_{12}=0$ or $\Delta_{22}=0$ or $\Delta_{12}=\Delta_{22}=0$. In these cases one has the so-called transitional, singular, and special solutions correspondingly. They were analyzed (for a different choice of formfactors) in [7]. We will consider further only the special phase where one has

$$\sigma_1^2=\frac{\Delta_{11}}{16\left(\ln\frac{\Lambda^2}{M_0^2}-\frac{1039}{384}\right)},\qquad \sigma_2=\frac{\sigma_1}{\sqrt{3}}. \tag{16}$$

The complex solutions to Eqs. (13) with $\xi\neq 0$ are as follows.

The axial part of the mass function

$$\xi^2 = \frac{\Delta_{22}}{3} - \frac{\bar{\sigma}}{4}. \tag{17}$$

The second v.e.v.

$$\sigma_2 = \sqrt{3}\sigma_1 + \frac{\sqrt{3}(3\sqrt{3}\Delta_{12} - \Delta_{22})}{12\sigma_1}, \tag{18}$$

and the solution for σ_1 is

$$\sigma_1^2 = \frac{\Delta_{11} + 2\sqrt{3}\Delta_{12} - \Delta_{22}}{16\left(\ln\frac{\Lambda^2}{M_0^2} - 3\right)}. \tag{19}$$

Consider $\mu \neq 0$ and the normal (GN) phase. The exact expressions for v.e.v.'s as functions of chemical potential are very cumbersome. We will be interested in the behaviour near the point of phase transition only. In the vicinity of this point the asymptotics are as follows

$$\sigma_1^2 \simeq \frac{\mu_{\text{cr}}^2 - \mu^2}{4\ln\frac{\Lambda}{2\mu}}\left[1 + \mathcal{O}\left(\frac{1}{\ln\frac{\Lambda}{2\mu}}\right)\right], \qquad \sigma_2 \simeq -\frac{\Delta_{12}}{\Delta_{22}}\sigma_1, \tag{20}$$

where the critical value of chemical potential is

$$\mu_{\text{cr}}^2 = \frac{\det\Delta}{8\Delta_{22}}. \tag{21}$$

In the special phase the qualitative behavior is the same up to the factor in the second relation of (20), $\sigma_2 = \sigma_1/\sqrt{3}$.

In the anomalous phase the asymptotics at large μ are completely different

$$\sigma_2^2 \simeq \frac{4}{9}\Delta_{22}, \qquad \sigma_1 \simeq \frac{\sigma_2(3\sqrt{3}\,\Delta_{12} - \Delta_{22})}{24\sqrt{3}\,\mu^2}. \tag{22}$$

In order to guarantee the negative sign of quark condensate in the model,

$$\langle \bar{q}q \rangle \simeq -\frac{N_c\Lambda^2}{8\pi^2}(\sigma_1 - \sqrt{3}\,\sigma_2), \tag{23}$$

the v.e.v. σ_2 must be negative. Since σ_2 tends to a constant value in this scenario, the phase transition cannot be reached in the anomalous phase.

In the CP-breaking phase the relation for σ_2 is fixed by Eq. (18). Out of the hyperplane $3\sqrt{3}\Delta_{12} - \Delta_{22} = 0$ we can not reach the phase transition as σ_2 becomes infinite in this point. However, σ_1 degreases in response to increasing the chemical potential according to the law

$$\sigma_1^2 = \frac{\Delta_{11} + 2\sqrt{3}\Delta_{12} - \Delta_{22} - \frac{1}{2}\mu\sqrt{\mu^2 - 4\sigma_1^2}}{16\left(2\ln\frac{\Lambda}{\mu + \sqrt{\mu^2 - 4\sigma_1^2}} - 3\right)}. \tag{24}$$

It must be noticed that if we are in the CP-breaking phase than, as it follows from Eqs. (17) and (18), at small enough value of σ_1 we go out this phase as $\bar{\sigma} > 0$. Only at the hyperplane $3\sqrt{3}\Delta_{12} - \Delta_{22} = 0$ it is possible to reach the phase transition being in the CP-breaking phase. More precisely, the CP-breaking phase exists between the values of v.e.v. σ_1

$$(\sigma_1^2)_{1,2} = \frac{-5(\sqrt{3}\Delta_{12} - \Delta_{22}) \mp \sqrt{(4\sqrt{3}\Delta_{12} - 3\Delta_{22})(\sqrt{3}\Delta_{12} - 2\Delta_{22})}}{24}. \tag{25}$$

The CP-breaking phase appears either at $\Delta_{12} < 0$, $\Delta_{22} > 0$ or at $\Delta_{12} > 0$, $\Delta_{22} > \frac{4\sqrt{3}}{3}\Delta_{12}$. At the hyperplane $3\sqrt{3}\Delta_{12} - \Delta_{22} = 0$ (the transitional phase) the point of phase transition, $\sigma_1 = 0$, and way out of CP-breaking phase coincide.

In each of the considered regions, four collective states arise: two scalar and two pseudoscalar for real solutions, and one pseudoscalar and three with mixed P-parity, generally speaking, for a complex solutions. Their spectra are analyzed in the next section.

3. Mass spectrum

The spectrum of the excitations is determined by the matrix of the second variation of the effective action. Let us divide that matrix into a constant part $\hat{B}$ (independent of the momentum p) and a kinetic part $\hat{A}p^2$ which appears to be quadratic in momentum in the large-log approximation

$$\frac{\delta^2 S_{\text{eff}}}{\delta\sigma_k(p)\delta\sigma_l(p')} = \frac{N_c}{8\pi^2}(A_{kl}^{\sigma\sigma} p^2 + B_{kl}^{\sigma\sigma})\delta^{(4)}(p+p'),$$

$$\frac{\delta^2 S_{\text{eff}}}{\delta\pi_k(p)\delta\pi_l(p')} = \frac{N_c}{8\pi^2}(A_{kl}^{\pi\pi} p^2 + B_{kl}^{\pi\pi})\delta^{(4)}(p+p'), \tag{26}$$

$$\frac{\delta^2 S_{\text{eff}}}{\delta\sigma_k(p)\delta\pi_l(p')} = \frac{N_c}{8\pi^2}(A_{kl}^{\sigma\pi} p^2 + B_{kl}^{\sigma\pi})\delta^{(4)}(p+p').$$

For the two-channel model discussed in the previous section, the kinetic-energy matrix $\hat{A}p^2$ looks as follows

$$A_{kl}^{\sigma\pi} = 0, \qquad A_{kl}^{\sigma\sigma(\pi\pi)} = \begin{pmatrix} 8\ln\frac{\Lambda}{\mu + \sqrt{\mu^2 - 4\sigma_1^2}} - \frac{15}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{pmatrix}. \tag{27}$$

The matrices of constant parts take the following form

$$B_{11}^{\sigma\sigma} = -2\Delta_{11} + 32\sigma_1^2 \ln\frac{\Lambda^2}{4\sigma_1^2} + 128\sigma_1^2 \ln\frac{\Lambda}{\mu + \sqrt{\mu^2 - 4\sigma_1^2}}$$
$$-\frac{605}{2}\sigma_1^2 - 15\sqrt{3}\sigma_1\sigma_2 + \frac{9}{2}\sigma_2^2 + 2\xi^2, \tag{28}$$

$$B_{12}^{\sigma\sigma} = -2\Delta_{12} - \frac{15\sqrt{3}}{2}\sigma_1^2 + 9\sigma_1\sigma_2 + \frac{3\sqrt{3}}{2}\sigma_2^2 + \frac{2\sqrt{3}}{3}\xi^2, \tag{29}$$

$$B_{22}^{\sigma\sigma} = -2\Delta_{22} + \frac{9}{2}\bar{\sigma}, \tag{30}$$

$$B_{11}^{\pi\pi} = -2\Delta_{11} + 32\sigma_1^2 \ln\frac{\Lambda^2}{4\sigma_1^2} - \frac{159}{2}\sigma_1^2 - 5\sqrt{3}\sigma_1\sigma_2 + \frac{3}{2}\sigma_2^2 + 6\xi^2, \tag{31}$$

$$B_{12}^{\pi\pi} = -2\Delta_{12} - \frac{5\sqrt{3}}{2}\sigma_1^2 + 3\sigma_1\sigma_2 + \frac{\sqrt{3}}{2}\sigma_2^2 + 2\sqrt{3}\xi^2, \tag{32}$$

$$B_{22}^{\pi\pi} = -2\Delta_{22} + \frac{3}{2}\bar{\sigma}, \tag{33}$$

$$B_{kl}^{\sigma\pi} = -2\xi \begin{pmatrix} -5\sigma_1 + \sqrt{3}\sigma_2 & \sqrt{3}\sigma_1 + \sigma_2 \\ \sqrt{3}\sigma_1 + \sigma_2 & \sigma_1 + 3\sqrt{3}\sigma_2 \end{pmatrix}. \tag{34}$$

The spectrum of the collective excitations is determined from the equation

$$\det(\hat{A}p^2 + \hat{B}) = 0, \tag{35}$$

which represents the condition of simultaneous diagonalization of the matrices $\hat{A}$ and $\hat{B}$. The minimum condition (positiveness of the second variation) for the Euclidean momenta ($p^2 > 0$) leads to the existence of solutions in the region $p^2 \le 0$, i.e., for physical values of the particle masses. Below we present the mass spectra for different regions in the vicinity of the tricritical point. Usually the corresponding expressions are very cumbersome. For the sake of simplicity we present many of them in the large-log approximation.

First of all, everywhere one has the Goldstone boson

$$m_\pi = 0. \tag{36}$$

Consider the case $\xi = 0$. Due to existence of exact solution (36), one can obtain a compact expression for the mass of π'-meson

$$m_{\pi'}^2 = -\left(4 - \frac{(\sigma_1 - \sqrt{3}\,\sigma_2)^2}{4\sigma_1^2\left(\ln\dfrac{\Lambda}{\mu + \sqrt{\mu^2 - 4\sigma_1^2}} - 1\right)}\right)\xi^2, \tag{37}$$

where quantity ξ^2 is given by Eq. (17). Nevertheless, it is often convenient to present an approximate relation for $m_{\pi'}$ in order to compare with other masses.

In the GN phase one has

$$m_\sigma^2 = 16\sigma_1^2 + \mathcal{O}\left(\frac{\sigma_k\sigma_l}{\ln\dfrac{\Lambda^2}{4\mu^2}}\right), \tag{38}$$

$$m_{\pi'}^2 = -\frac{4}{3}\Delta_{22} + \bar{\sigma} + \mathcal{O}\left(\frac{\sigma_k\sigma_l}{\ln\dfrac{\Lambda^2}{4\mu^2}}\right) + \mathcal{O}\left(\frac{\Delta_{22}}{\ln\ln\dfrac{\Lambda^2}{4\mu^2}}\right), \tag{39}$$

$$m_{\sigma'}^2 = -\frac{4}{3}\Delta_{22} + 3\bar{\sigma} + \mathcal{O}\left(\frac{\sigma_k\sigma_l}{\ln\ln\dfrac{\Lambda^2}{4\mu^2}}\right) + \mathcal{O}\left(\frac{\Delta_{22}}{\ln\ln\dfrac{\Lambda^2}{4\mu^2}}\right), \tag{40}$$

where $\sigma_k\sigma_l$ denotes different combinations of v.e.v.'s for $k,l = 1,2$.

Spectrum in the anomalous phase

$$m_{\sigma}^{2} = 16\sigma_1^2 + \mathcal{O}\left(\frac{\sigma_k\sigma_l}{\ln\ln\frac{\Lambda^2}{4\mu^2}}\right) + \mathcal{O}\left(\frac{\Delta_{22}}{\ln\ln\frac{\Lambda^2}{4\mu^2}}\right), \tag{41}$$

$$m_{\sigma'}^{2} = \frac{8}{3}\Delta_{22} + \mathcal{O}\left(\frac{\sigma_k\sigma_l}{\ln\frac{\Lambda^2}{4\mu^2}}\right) + \mathcal{O}\left(\frac{\Delta_{22}}{\ln\frac{\Lambda^2}{4\mu^2}}\right), \tag{42}$$

and $m_{\pi'}$ is given by Eq. (37).

In the special phase one has

$$m_{\sigma}^{2} = 8\sigma_1^2 + \mathcal{O}\left(\frac{\sigma_1^2}{\ln\frac{\Lambda^2}{4\mu^2}}\right), \tag{43}$$

$$m_{\pi'}^{2} = \frac{8}{3}\sigma_1^2 + \mathcal{O}\left(\frac{\sigma_1^2}{\ln\frac{\Lambda^2}{4\mu^2}}\right), \tag{44}$$

$$m_{\sigma'}^{2} = 16\sigma_1^2 + \mathcal{O}\left(\frac{\sigma_1^2}{\ln\frac{\Lambda^2}{4\mu^2}}\right). \tag{45}$$

The case $\xi \neq 0$.

$$m_{\sigma}^{2} = 16\sigma_1^2 + \mathcal{O}\left(\frac{\Delta_{22}}{\ln\frac{\Lambda^2}{4\mu^2}}\right) + \mathcal{O}\left(\frac{\sigma_1^2}{\ln\frac{\Lambda^2}{4\mu^2}}\right), \tag{46}$$

$$m_{\pi'}^{2} = \frac{4}{3}\Delta_{22} - \frac{4}{3}\sqrt{\Delta_{22}^2 - 8\sigma_1^2\xi^2} + \mathcal{O}\left(\frac{\sigma_1^2\xi^2}{\Delta_{22}\ln\frac{\Lambda^2}{4\mu^2}}\right), \tag{47}$$

$$m_{\sigma'}^{2} = \frac{4}{3}\Delta_{22} + \frac{4}{3}\sqrt{\Delta_{22}^2 - 8\sigma_1^2\xi^2} + \mathcal{O}\left(\frac{\sigma_1^2\xi^2}{\Delta_{22}\ln\frac{\Lambda^2}{4\mu^2}}\right) + \mathcal{O}\left(\frac{\Delta_{22}}{\ln\frac{\Lambda^2}{4\mu^2}}\right). \tag{48}$$

In the expressions above v.e.v. σ_1 can be fixed by a value of dynamical mass, $M_0 = 2\sigma_1$. Other parameters are free but subject to mass-gap equations (13). The properties of obtained mass spectra are discuseed in the next Section.

4. Dynamical CP-violation

Now we will relate our model to the real physics. As the value of dynamical mass M_0 is about 0.3 GeV, we should have $\sigma_1 = M_0/2 \approx 0.15$ GeV. Typical masses of the first scalar and pseudoscalar radial excitations are about 1.3 GeV. As follows from Eqs. (39), (40), and (12) we should then have a large (compared to σ_1)

negative Δ_{22} or/and large σ_2. In [14] the scaling of reduced couplings was chosen to be $\Delta_{ik} = \mathcal{O}(\ln\frac{\Lambda^2}{M_0^2})$. Then the first term in the r.h.s. of Eqs. (39) and (40) is dominant in the large-log approximation. As it directly seen, this scenario naturally reproduces CSR in radial excitations: (masses)^2 are equal in the first logarithmic approximation, next-to-leading corrections appear due to DCSB. As $\Delta_{22} < 0$ this scenario is realized in the GN phase. When one reaches the phase transition, the masses of excited states do not change drastically in comparison with their values at $\mu = 0$ since the reduced couplings Δ_{ik} do not depend on μ.

Let us consider another possibility. Suppose that at $\mu = 0$ the sign of r.h.s. of Eq. (17) is negative. Switching on the chemical potential it may happen that at certain point $\xi^2(\mu_{\mathrm{CP},1}) = 0$. Needless to say that to make this case possible one has to assume the scaling $\Delta_{22} = \mathcal{O}(1)$. That is why such a scaling was chosen in this paper. After this point ξ^2 becomes positive and we can enter the CP-breaking phase with mass spectrum (46) - (48)! As was shown above (see Eqs. (24) and (25)) at some other point $\mu_{\mathrm{CP},2} > \mu_{\mathrm{CP},1}$ we inevitably leave this phase. The solutions smoothly conform to each other in these points,

$$m_\sigma^2 \simeq 16\sigma_1^2, \quad m_{\pi'} = 0, \quad m_{\sigma'}^2 \simeq \frac{8}{3}\Delta_{22}. \tag{49}$$

Thus, the π'-meson mass is the order parameter for the beginning and the end of CP-breaking phase. As $\bar{\sigma}$ is a positive quantity (see its definition (12)) it is evident from Eq. (17) that such a scenario may be realized only when $\Delta_{22} > 0$. As was discussed above, in this case the minimum of effective potential is given by anomalous solution. Therefore, one can enter the CP-breaking phase only from anomalous one. In addition, from Eqs. (15), (17), and (22) follows that in this case $\xi^2 \simeq 0$ in the first log approximation, i.e. the change of phase turns out to be a $\mathcal{O}(1/\log)$ effect. For the same reason the π'-meson is logarithmically lighter than its chiral partner σ'.

The last and quite unusual scenario, which we would like to mention, is given by the special phase. In this case the point of phase transition and that of CP-breaking coincide if the latter happens. All states become massless in that point.

5. Conclusions

The present analysis is rather qualitative, we do not expect to describe quantitatively such higly complicated phenomenon as possible dynamical CP-breaking within the present simple toy-model. Indeed, the effect might appear only in the next-to-leading approximation. In this order there are contributions from some higher dimensional vertices in effective Lagrangian which were not taken into account. Moreover, we have not included the vector isoscalar interaction. As is known [24] the zero-component of the corresponding vector field acquires a nonzero v.e.v. which shifts the chemical potential. The mass-gap equations are then supplemented by a relation between the bare chemical potential and the renormalized one, with all other quantities being functions of only latter one (we briefly sketch the procedure in Appendix). This can result in the change of character of phase transition (it can become discontinuous) and in a shift of absolute value of critical chemical potential. These topics, however, are not the subject of our analysis. What we pretend to demonstrate are some general conditions which could lead to the phenomenon of dynamical CP-breaking. The omitted vertices in the effective Lagrangian seem not affecting our conclusions. First of all, according to our analysis, the CP-breaking phase cannot be reached starting from the Gross-Neveu phase. In our opinion, this is the reason why such an effect cannot appear within usual NJL-type models. The considered model has several additional phases with unusual scalings of masses. We have shown that in some of these phases the dynamical CP-breaking, in principle, could appear. A common feature of the spectrum in these phases is that one should have a rather light radial excitations, especially in the pseudoscalar channel (say, in the special phase $m_{\pi'} \simeq m_{\sigma'}/\sqrt{6}$). A possible existence of such light pseudoscalar meson is still excluded neither by experiment nor by relativistic potential quark models (see, e.g., [25]. An interesting consequence of our analysis is that the existence of this state and the CP-violation in QCD could be tightly related. A further development of this subject will be presented elsewhere.

Acknowledgements

We are very grateful to the organisers of the NPAE 2006 for hospitality. The work was supported by RFBR, grant 05-02-17477 and grant RNP 2.1.1.1112, and also Grant President of Russia SS-5538.2006.2. The work of A.S. was also supported by CYT FPA, grant 2004-04582-C02-01, and CIRIT GC, grant 2001SGR-00065.

Appendix: inclusion of vector isoscalar interaction

Let us include vector isoscalar interaction in the minimal way (i.e. without derivatives in the vertices). Interaction part of Lagrangian will be

$$\mathcal{L}^I = \frac{1}{4N_f N_c \Lambda^2} \sum_{k,l=1}^{2} a_{kl} \left[\bar{q} f_k(s) q \bar{q} f_l(s) q - \bar{q} f_k(s) \tau^a \gamma_5 q \bar{q} f_l(s) \tau^a \gamma_5 q \right] + \\ + \frac{G_v}{N_f N_c \Lambda^2} \bar{q} i \gamma_\nu q \bar{q} i \gamma_\nu q. \tag{50}$$

After introduction of auxiliary bosonic variables, $\sigma_k \sim i\bar{q} f_k(s) q$, $\pi_k^a \sim \bar{q} f_k(s) \tau^a \gamma_5 q$, and $v_\nu \sim \bar{q}\gamma_\nu q$, one has

$$\mathcal{L}_I = \sum_{k=1}^{2} i\bar{q} \left(\sigma_k + i\gamma_5 \pi_k^a \tau^a \right) f_k(s) q + N_f N_c \Lambda^2 \sum_{k,l=1}^{2} \left(\sigma_k a_{kl}^{-1} \sigma_l + \pi_k^a a_{kl}^{-1} \pi_l^a \right) + \\ + i\bar{q} i \gamma_\nu v_\nu q + \frac{N_f N_c \Lambda^2}{4 G_v} v_\nu^2. \tag{51}$$

Fine-tuning of couplings is extended to

$$8\pi^2 a_{kl}^{-1} = \delta_{kl} - \frac{\Delta_{kl}}{\Lambda^2}, \qquad \frac{4\pi^2}{G_v} = 1 - \frac{4\Delta_v}{3\Lambda^2}, \qquad |\Delta_{kl}, \Delta_v| \ll \Lambda^2. \tag{52}$$

At $\mu > 2\sigma_1$ the zero-component of vector field, v_0 , acquires a non-zero v.e.v. due to the second term in modified propagator (8). The effect can be taken into account by shifting the chemical potential. Its renormalized value μ_r is then $\mu_r = \mu - v_0$, which will enter all formulas instead of μ . Performing the Wick rotation and calculating v.e.v. v_0 one has finally

$$\mu_r = \mu + \frac{2}{\Delta_v} \left(\mu_r^2 - 4\sigma_1^2 \right)^{3/2}. \tag{53}$$

This relation has to be added to mass-gap equations (13). The parameter Δ_v can be fixed from the mass of vector particle in our model. If we identify this particle with ω-meson, one obtains

$$m_\omega^2 = -\frac{2\Delta_v}{\ln \frac{\Lambda^2}{M_0^2}}. \tag{54}$$

REFERENCES

1. *Vogl U., Weise W.* // Prog. Part. and Nucl. - 1991. - Vol. 27. - P. 195; *Klevansky S.P.* // Rev. Mod. Phys. - 1992. - Vol. 64. - P. 649; *Volkov M.K.* // Phys. Part. Nucl. - 1993. - Vol. 24. - P. 35; *Hatsuda T., Kunihiro T.* // Phys. Rept. - 1994. - Vol. 247. - P. 338; *Hufner J., Klevansky S.P., Zhuang P., Voss H.* // Ann. Phys. - 1994. - Vol. 234. - P. 225; *Zhuang P., Hufner J., Klevansky S.P.* // Nucl. Phys. - 1994. - Vol. A576. - P. 525.
2. *Eidelman S. et al.* [Particle Data Group Collaboration]. // Phys. Lett. - 2004. - Vol. B592. - P. 1.
3. *Nambu Y., Jona-Lasinio G.* // Phys. Rev. - 1961. - Vol. 122. - P. 345.
4. *Andrianov A.A., Andrianov V.A.* // Z.Phys. - 1992. - Vol. C55. - P. 435; Theor. Math. Phys. - 1992. - Vol. 93. - P. 1126; J. Math. Sci. (N.Y.) - 1995. - Vol. 77. - P. 3021; [Zap. Nauch. Sem. POMI. - 1992. - Vol. 199. - P. 3].
5. *Andrianov A.A., Andrianov V.A.* // Int. J. Mod. Phys. - 1993. - Vol. A8. - P. 1981; Theor. Math. Phys. - 1993. - Vol. 94. - P. 3; Proc. School-Sem. "Hadrons and nuclei from QCD", Tsuruga/Vladivostok/Sapporo 1993 / Ed. by

K. Fuji et al. - Singapore: World Scientific, 1994. - P. 341 [hep-ph/9309297].
6. *Andrianov A.A., Andrianov V.A.* // Nucl. Phys. Proc. Suppl. - 1995. - Vol. 39BC. - P. 256.
7. *Andrianov A.A., Andrianov V.A., Yudichev V.L.* // Theor. Math. Phys. - 1996. - Vol. 108. - P. 1069.
8. *Andrianov V.A., Afonin S.S.* // Eur. Phys. J. - 2003. - Vol. A17. - P. 111; J. Math. Sci. (N.Y.) - 2005. - Vol. 129. - P. 99: [Zap. Nauch. Sem. POMI. - 2002. - Vol. 291. - P. 5] [hep-ph/0304140].
9. *Andrianov A.A., Andrianov V.A., Afonin S.S.* // hep-ph/0101245; hep-ph/0209260.
10. *Espriu D., de Rafael E., Taron J.* // Nucl. Phys. - 1990. - Vol. B345. - P. 22; erratum ibid. - 1991. - Vol. B355. - P. 278.
11. *Andrianov A.A., Andrianov V.A.* // Zap. Nauch. Sem. POMI. - 1996. - 245. - P. 5 [hep-ph/9705364].
12. *Andrianov A.A., Espriu D., Tarrach R.* // Nucl. Phys. - 1998. - Vol. B533. - P. 429.
13. *Andrianov A.A., Espriu D.* // JHEP. - 1999. - Vol. 10. - P. 022.
14. *Andrianov A.A., Andrianov V.A.* // Coimbra 1999: Proc. of the Int. Workshop on Hadron Physics / Ed. by A.H. Blin et al. - N.Y.: AIP, 2000. - P. 328 [hep-ph/9911383].
15. *Andrianov A.A., Andrianov V.A., Rodenberg R.* // JHEP. - 1999. - Vol. 06. - P. 3.
16. *Andrianov A.A., Andrianov V.A., Yudichev V.L., Rodenberg R.* // Int. J. Mod. Phys. - 1999. - Vol. A14. - P. 323 [hep-ph/9709475].
17. *Beane S.R.* // Phys. Rev. - 2001. - Vol. D64. - P. 116010; *Afonin S.S., Andrianov A.A., Andrianov V.A., Espriu D.* // JHEP. - 2004. - Vol. 04. - P. 039; Int. J. Mod. Phys. - 2006. - Vol. A21. - P. 885; *Shifman M.* // hep-ph/0507246; *Afonin S.S., Espriu D.* // hep-ph/0602219; *Afonin S.S.* // hep-ph/0603166.
18. *Andrianov A.A., Andrianov V.A., Yudichev V.L.* Proc. 10th Int. Conf. QFTHEP, 1995 // [hep-ph/9512256].
19. *Quark*-Gluon Plasma / Ed. by R. C. Hwa. - Singapore: WSPC, 1990; *Alford M., Rajagopal K., Wilczek F.* // Phys. Lett. - 1998. - Vol. B422. - P. 247; *Rapp R., Schafer T., Shuryak E.V., Velkovsky M.* // Phys. Rev. Lett. - 1998. - Vol. 81. - P. 53; *Rajagopal K., Wilczek F.* // hep-ph/0011333; *Alford M.G.* // Ann. Rev. Nucl. Part. - 2001. - Vol. 51. - P. 131; hep-ph/0102047; nucl-th/0312007; *Schafer T.* // hep-ph/0304281; *Rischke D.H.* // nucl-th/0305030; *Hong D.K.* // Acta Phys. Polon. - 2001. - Vol. B32. - P. 1253; hep-ph/0101025; hep-ph/0401057; *Muroya S., Nakamura A., Nonaka C., Takaishi T.* // Progr. Theor. Phys. - 2003. - Vol 110. - P. 615; hep-lat/0306031; *Buballa M.* // Phys. Rept. - 2005. - Vol. 407. - P. 205.
20. *Kharzeev D.E., Pisarski R.D., Tytgat M.H.G.* // Phys. Rev. Lett. - 1998. - Vol. 81. - P. 512; hep-ph/0012012; hep-ph/0406125.
21. *Andrianov A.A., Andrianov V.A.* // Nucl. Phys. Proc. Suppl. - 1995. - Vol. 39BC. - P. 257.
22. *Pallante E., Petronzio R.* // Z. Phys. - 1995. - Vol. C65. - P. 487.
23. *Landsman N.P., Van Weert Ch.G.* // Phys. Rept. - 1987. - Vol. 145. - P. 141.
24. *Asakawa M., Yazaki K.* // Nucl. Phys. - 1989. - Vol. A504. - P. 668.
25. *Ivanshin Yu.I., Skachkov N.B.* // Nuovo Cim. - 1995. - Vol. A108. - P. 911.

ДИНАМИЧЕСКОЕ CP-НАРУШЕНИЕ В КВАЗИЛОКАЛЬНЫХ КВАРКОВЫХ МОДЕЛЯХ ПРИ НЕНУЛЕВОМ КВАРКОВОМ ХИМИЧЕСКОМ ПОТЕНЦИАЛЕ

А. А. Андрианов, В. А. Андрианов, С. С. Афонин

Рассмотрена квазилокальная кварковая модель типа Намбу-Йона-Лазиньо в качестве эффективной теории непертурбативной КХД со скалярным-псевдоскалярным четырехкварковым взаимодействием, включающем производные и при конечном химическом потенциале. При достаточно сильном взаимодействии в скалярном канале происходит спонтанное нарушение киральной симметрии и, как следствие, образуется составное скалярное состояние. При специальном выборе констант связи и некотором значении кварковой плотности, в псевдоскалярном секторе может появится фаза с CP-нарушением, как результат комплексности динамической массовой функции.

ДИНАМІЧНЕ CP-ПОРУШЕННЯ В КВАЗІЛОКАЛЬНИХ КВАРКОВИХ МОДЕЛЯХ ПРИ НЕНУЛЬОВОМУ КВАРКОВОМУ ХІМІЧНОМУ ПОТЕНЦІАЛІ

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Розглянуто квазілокальну кваркову модель типу Намбу-Йона-Лазіньо як ефективну теорію непертурбативної КХД із скалярною-псевдоскалярною чотирикварковою взаємодією, що включає похідні і при скінченому хімічному потенціалі. При достатньо сильній взаємодії в скалярному каналі відбувається спонтанне порушення кіральної симетрії і, як наслідок, утворюється складений скалярний стан. При спеціальному виборі констант зв’язку і певному значенні кваркової густини в псевдоскалярному секторі може з’явитися фаза із CP-порушенням, як результат комплексності динамічної масової функції.