

# RF Stacking Without Emittance Dilution

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## Abstract

A long-established technique for accumulating high current in a storage ring is to inject additional beam off-momentum and move it into the main beam stack by rf acceleration. This procedure necessarily dilutes the longitudinal phase space density, because the rf perturbs the stack as the new beam approaches closely in momentum. For accumulators that use beam cooling to obtain a high phase space density, this perturbation may result in an unacceptable performance limitation. Using broadband rf to establish and manipulate azimuthal barriers to the motion of the stack and injected beam permits practically dilution-free longitudinal phase space stacking. The concept is described and illustrated with a detailed example which pertains to the Fermilab Recycler ring.

## 1 General Considerations

Stacking is the slow accumulation of beam current in a storage ring by collecting many injected batches closely in phase space. In practice the accumulation has been made in the longitudinal (momentum-azimuth) space. Two schemes have been used separately or in combination, *viz.*, off-momentum injection with rf acceleration to the edge of the main beam stack, and off-momentum injection with momentum cooling to the stack momentum. A disadvantage of both approaches is the need for a kicker with a moving shield to protect the stack from the kicker field. The rf stacking technique results in significant phase space dilution because the approaching buckets carrying the new beam displace the stack. The longitudinal cooling technique can eventually result in a very high longitudinal phase space density, but may be far too slow for frequent additions or, indeed, may not work at all. Stochastic cooling will fail if the beam current is too high, and electron cooling could work only over a very narrow momentum range or if the electron beam velocity could be swept over the range. Where electron cooling could be used in principle, most likely it would not be suitable in practice because impracticably high electron beam current would be necessary for the desired stacking rate.

The process described in this talk is a variant of rf stacking that has something in common with a typical multi-batch injection that adds batches into a ring at the central momentum in azimuthal slots which are free of circulating beam, like separate beads on a chain. However, the final result is to add the new emittance in uniform layers just above and below the stack. Because it causes very little dilution, it can be combined with a rather modest cooling system at the stack momentum to achieve an equilibrium condition where the emittance per batch is cooled away between batches. Then the stack momentum spread does not grow and the stacking can proceed more or less indefinitely.

The key to this technique is the use of a broadband rf system to establish longitudinal barriers to the motion of stack and the injected batches. This idea was developed for the Fermilab Antiproton Source and at that time given the name “barrier bucket”[1]. In the present context the idea of a

“bucket” is not especially apt; the effect produced will be called rf barriers. A digital waveform generator can provide a more or less arbitrary rf waveform and change it quickly and smoothly with amplitude and phase continuity. This flexibility is used to realize the somewhat elaborate sequence of steps described below. The principal point is the concept; however, following the step-by-step illustration of the scheme, the final section addresses some of the issues of practicability.

In the simplest stacking using moving azimuthal barriers, new batches are scooped up to the end of the stack and then joined to it by removal of the intervening barrier. However, there is necessarily some emittance dilution in the process, because the matched condition while the barrier intervenes has batch and stack ends bounded by parabolic trajectories. When the barrier is removed, the space outside the parabolic segments will be filled with beam from *both* the batch and the stack. The optimum situation just before the removal of the barrier is shown in Fig. 1. This illustration, and all subsequent specific examples, will use parameters of the Fermilab Recycler. The relevant parameters of the Recycler[2] are given in Table 1. The figure shows a longitudinal phase space plot for a 300 eVs stack and a 10 eVs new batch; the coordinates are harmonic 1 rf phase in degrees and energy relative to the central energy of the ring in MeV. The trajectories bounding the batch and stack have been brought to tangency by adjusting the barrier heights and phases. The dilution which would result from removing the barrier is given by the unoccupied phase space between the parabolic end trajectories and the stack energy boundary. Fig. 2 shows how the batch fits into the stack. The barrier confining the batch is simply removed and that defining the stack is jumped in phase so it just encompasses the new batch. Fig. 2 is the phase space plot just 0.2 s after the barrier change. The area of the augmented stack is greater by nearly twice the batch emittance. The Fig. 1 configuration is “optimum” within a constraint of 2 kV maximum rf voltage. If higher voltage were available, the batch would be narrowed until the momentum spread of batch and stack were equal, thereby reducing further the empty phase space between the distributions. Having reached the condition shown in Fig. 1, it is clear that what is needed is an adiabatic process to remove the barrier without disturbing the stack. For the Recycler, a few percent dilution of the stack can saturate the available cooling under certain circumstances.[3]

Table 1: Relevant parameters of the Fermilab Recycler

|                |                                                  |          |         |
|----------------|--------------------------------------------------|----------|---------|
| $R_{eq}$       | mean radius of central orbit                     | 528.30   | m       |
| $E$            | total energy on central orbit                    | 8938.27  | MeV     |
| $\gamma_T$     | lattice transition gamma                         | 21.838   |         |
| $\tau_c$       | beam circulation period                          | 11.139   | $\mu s$ |
| $\beta$        | relativistic velocity, $v/c$                     | 0.99448  |         |
| $\eta$         | phase slip factor, $\gamma_T^{-2} - \gamma^{-2}$ | -0.89223 |         |
| $V_{rf,max}$   | maximum rf voltage amplitude                     | 2000     | V       |
| $T_{b,typ}$    | typical barrier pulse width                      | 0.866    | $\mu s$ |
| $H_{B,typ}$    | typical barrier height                           | 17.56    | MeV     |
| $S_{B,typ}$    | typical phase space area for stack               | 362      | eVs     |
| $\tau_{s,typ}$ | typical oscillation period on stack boundary     | 1.424    | s       |

## 2 Adiabatic Stacking, Step-by-Step

In the process to be described here the stack is completely isolated from the manipulation which puts the batch in thin layers immediately above and below it except for the initial step of opening the injection gap. The situation shown in Fig. 1 is nearly the final stage of the fully adiabatic stacking, but the whole injection-stacking sequence will be outlined. It will be evident that the several steps constitute a highly specific rf sequence possible only with a broadband linear system. The rf voltage is limited to the 2 kV peak given in Table 1. The steps are as follows:

**Fig. 3** The initial state is a stack with a short ( $\sim 20^\circ$ ) ion-clearing gap established by the rf.

**Fig. 4** The ion-clearing gap is opened slowly (over  $\sim 7$  s) to  $\sim 60^\circ$ . The barrier is adjusted during or after the opening so that the stack fully penetrates the barrier region but does not escape it.

**Fig. 5** A batch from another ring is synchronously transferred to a matching isolated bucket in the cleared gap.

**Fig. 6** The sinusoidal portion of the waveform is shifted slowly until the stable fixed point reaches the barrier phase to squeeze all of the batch out of the gap. The shift is then continued at an accelerated rate to remove the sinusoidal segment entirely.

**Fig. 7** The original ion clearing gap is restored.

In the figures the rf waveform is shown with the dotted lines. Note that as the sinusoid is shifted left, it is truncated. The complete waveform continues to have integrated area zero because the square barriers are expanded to compensate. This addition to the barriers makes it certain that the newly stacked beam remains contained but does not influence the stack trajectories existing in the barrier region at the time. The voltage of the sinusoidal segment is raised smoothly from its initial matching value to hasten the departure of the batch particles near the synchronous energy.

The consequence of these steps is to spread the batch emittance along the top and bottom of the stack with a very small empty layer between. The empty layer represents the dilution in the process. The thickness of the layer is given by the difference of the energy spread of the stack and the barrier height. There will be dilution of the batch if the spill begins before the two distributions reach a common azimuthal boundary; there will be dilution of the stack if the barrier is reduced too far and the stack begins to spill into the batch azimuth. The process is not critically dependent on the barrier pulse shape. So long as the distributions can be brought to near contact at  $\Delta E = 0$ , the trajectories bounding the ends are not critical. If there are micro-buckets arising from ringing or other defects of the rf waveform, there could be locally turbulent or chaotic motion leading to some dilution. The paramount point is to keep the stack contained; leakage from the stack into the batch region is a source of potentially serious dilution. Depending on the stability and controllability of the rf, the margin for containment would need to be adjusted to prevent such leakage; thus the dilution of the batch will be at least what arises from the margin allowed to protect the stack.

### 3 Modeling

The figures used to illustrate the steps in stacking are results from numerical modeling undertaken both to check the qualitative scheme and to evaluate dilution to be expected in practice. The process is generally designed to isolate the stack from the various gymnastics for introducing the new batch. Certainly the spreading of the ion clearing gap to accommodate the injection does substantially reconfigure the stack. If this could not be done without diluting the stack, it would be possible always to keep open the necessary azimuth at the cost of some beam cooling capability. A preliminary step in the modeling has been to quantify the dilution with a practicable rf system.

The adiabaticity, *i. e.*, the parameter characterizing the preservation of phase space area under a variation of parameters governing the motion, is usually written

$$\alpha = \frac{\tau_s}{S} \dot{S} \quad , \quad (1)$$

where  $\tau_s$  is the phase oscillation period and  $S$  is the area of a closed phase space contour; the contour is generally taken as the boundary of stable oscillation, the so-called bucket, and  $\tau_s$  is the period of small amplitude synchrotron oscillation. The expression applying to motion between rf barriers is the same, but  $\tau_s$  is taken for the particle on the boundary, where the period is at or near its minimum. To evaluate  $\alpha$  for the moving barrier case, one needs the relevant expressions for bucket height, bucket area, and oscillation period:[4]

$$H_B = \sqrt{\frac{eVT_b}{\tau_c} \frac{2\beta^2 E}{|\eta|}} \quad (2)$$

$$S_B = 2T_g H_B + \frac{4\tau_c |\eta| H_B^3}{3\beta^2 E e V} \quad (3)$$

$$\tau_s = \frac{2T_g \beta^2 E}{|\eta| H_B} + \frac{4H_B \tau_c}{e V} \quad , \quad (4)$$

where the step-function barrier pulses have time duration  $T_b$ , constant voltage  $V$ , separation  $T_g$ . The other symbols in these expressions, except for the elementary charge  $e$ , are identified and given values in Table 1. These formulas apply to squarewave pulses, but almost any simple form is suitable dynamically. The  $VT_b$  combination generalizes to  $\int_0^{T_b} V dt$ . One advantage of the squarewave form is the highest barrier for a given peak voltage capability.

Irrespective of  $\alpha$ , it seems that the minimum time for a significant change in area should be  $> \tau_s/2$ , simply so that the effects of the barrier encounter have time to propagate between barriers. There is a hoary rule-of-thumb that  $\alpha < 0.25$  will result in low dilution when changing the area of sinusoidal buckets; there is no rule for dilution as a function of  $\alpha$ . The change in area of a contour containing 95% of the beam during the process of opening a 60° gap was calculated numerically for  $\alpha$  in the range from 0.5 to 0.08. The corresponding time intervals are from 0.25 to 1.5 s, and  $\tau_s$  is about 1.5 s. The results in Table 2 show that dilution increases rapidly for  $\alpha > 0.25$ , but the numerical procedure (number of macroparticles) did not provide useful results below  $\alpha = 0.1$ . The adiabaticity parameter is helpful in predicting whether major dilution will occur; it may not be so useful to predict the actual amount of dilution, at least for  $\alpha \ll 1$ . In the examples shown, the opening time and closing time for the gap was taken as 7.5 s even though the results in Table 2 suggest that less time would be sufficient. This more conservative value follows from the need to keep the barrier height just at the stack energy width. When the barrier is moving there is a periodic swing in the energy spread as the reflected charge oscillates around the periphery of the stack. The amplitude of the swing is proportional to the rate of barrier motion, and the barrier must be high enough to contain the oscillation as well as the basic stack width. Therefore, the faster the barrier moves, the bigger the gap between stack and new batch after the debunching.

Table 2: Growth of the 95% containment contour in opening a 60° gap

| Opening time [s] | $\alpha$ | % growth   |
|------------------|----------|------------|
| 0.25             | 0.50     | 15.8       |
| 0.50             | 0.25     | 2.9        |
| 0.75             | 0.17     | 1.5        |
| 1.00             | 0.12     | 0.4        |
| 1.25             | 0.10     | $\sim 0.4$ |
| 1.50             | 0.08     | $< 0.4$    |

## 4 Realization

This note has focused on the concept of on-momentum longitudinal stacking with broadband rf, an un-tried technique. These concluding remarks cite relevant experience and mention some further modeling which supports the practicability of the scheme.

The rf system must cover the frequency range from the fundamental circulation frequency, about 90 kHz, to some ill-defined upper limit which allows adequate fidelity to the details of the required waveforms. If the narrowest barrier pulse is  $T_{b,\min} \sim 1\mu\text{s}$ , then this upper limit is a few tens of megahertz, a level of performance that has been available in the Fermilab Antiproton Source since the early 1980's.[1] The system in the Accumulator has been used for the approximately inverse process of capturing beam in a sinusoidal bucket spanning one quarter of the azimuth and accelerating the captured bunch out of the stack.[5] The 2 kV broadband system in the new Recycler ring is currently being commissioned.[6] The actual waveforms produced can be used in the modeling if the system shows some special characteristic of concern. Only ideal square, sinusoidal, and sawtooth forms have been used so far. The sort of non-ideality that could interfere with the scenario described is an oscillatory over-shoot on the trailing edge of the positive-going barrier producing a small stable bucket between the stack and the batch to be merged. A little ringing on the top leading edge is to be expected but does not appear to threaten trouble.

A worry for any storage ring rf system is the effect of noise in phase or amplitude. Either variety of noise will vary the barrier height, as can be seen from eq. 2. One effect of noise, therefore, is to allow some of the stack to escape, when the barrier height is too close to the beam width. Random noise leads to a diffusive emittance growth of the stack, but for any plausible level the effect is small compared to the rate of almost any cooling system. Numerical simulations of white noise on amplitude and phase at even the 1 % level reveal practically no observable problem. Possibly this insensitivity is surprising, but note that the rf voltage is low. The steps in a particle's random walk in energy are very small. Correlated errors on the time scale of the synchrotron oscillations, on the other hand, could be very serious. However, it is a little hard to anticipate what to model; the measured properties of the rf system and the observed behavior of the beam will provide strong guidance for what modeling will be useful.

The sources of beam for the Recycler are pre-cooled batches of a few  $\times 10^{11}$  antiprotons from the Accumulator and antiprotons recovered from the Tevatron collider, as much as 400 eVs containing a few  $\times 10^{12}$  antiprotons. The accumulation cycle starts with the recovered antiprotons, which have a substantially lower phase space density. The Recycler will be commissioned with stochastic cooling; electron cooling is planned for an upgrade.[7] The dynamics of electron and stochastic cooling are not the same, and it is possible that the choice of stacking scheme could depend on that difference. For electron cooling, the rate for each particle depends only on its own velocity with respect to particles on the central orbit. Therefore, it is clear that the emittance to conserve

is the area of a contour containing a certain large fraction of the beam, like 95 % for example. For stochastic cooling, the distribution of particles within such a contour surely matters, and it may be something like the rms emittance that is the best parameter to watch for the over-all effectiveness of the stacking. Simulations with cooling have been made based on design specifications. There is a preliminary result that the Recycler will be able to stack  $2 \cdot 10^{11}$  particles every fifteen minutes onto a 300 eVs stack of  $2.5 \cdot 10^{12}$  from the Tevatron using both the original stochastic cooling[2] and a 0.5 A electron cooling system[3] simultaneously.

## References

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- [7] S. Nagaitsev, A. Burov, A. C. Crawford, T. Kroc, J. MacLachlan, C. W. Schmidt, A. Shemyakin, and A. Warner, “Status of the Fermilab Electron Cooling Project”, Proc. 1999 PAC, New York (1999)

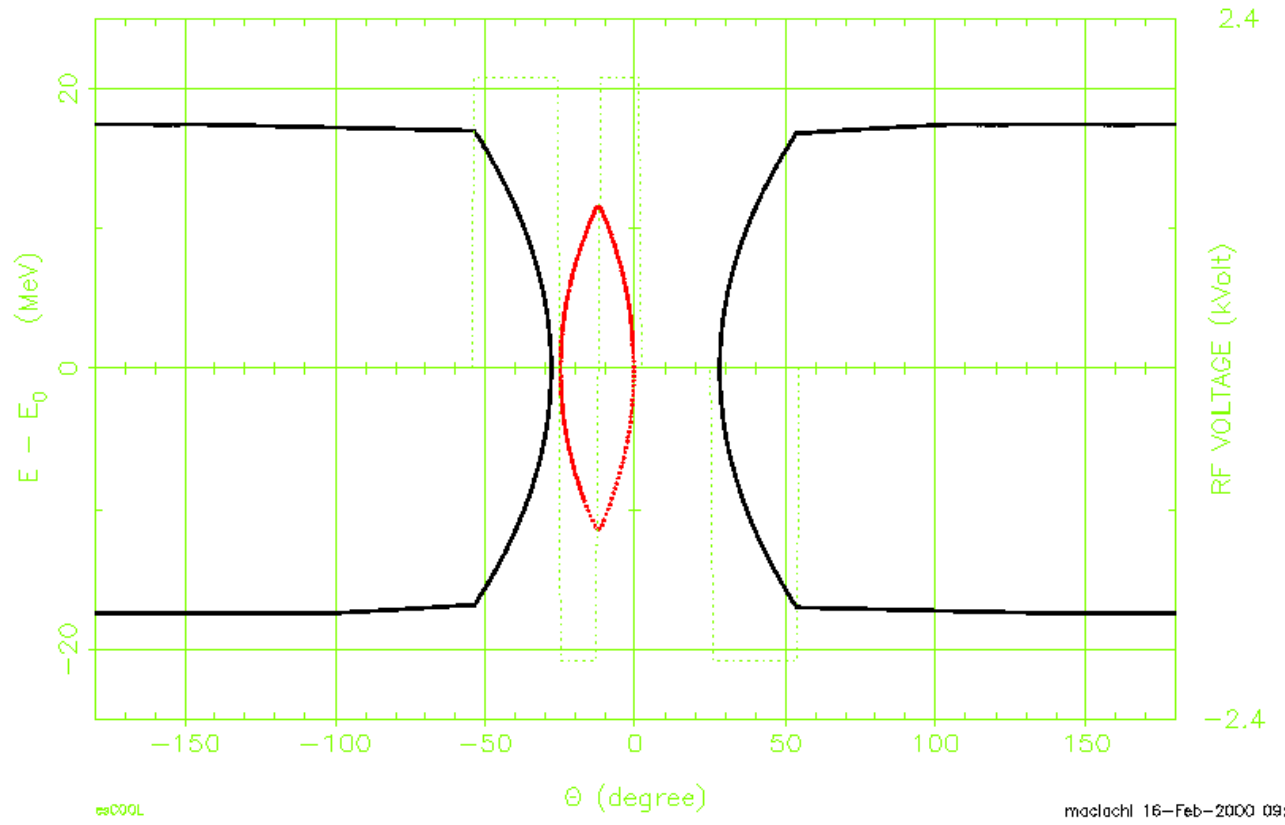


Figure 1: Longitudinal phase space plot for Recycler (parameters in Table 1). Stack of 300 eVs with a 10 eVs batch brought to closest approach with rf barriers of 2kV.

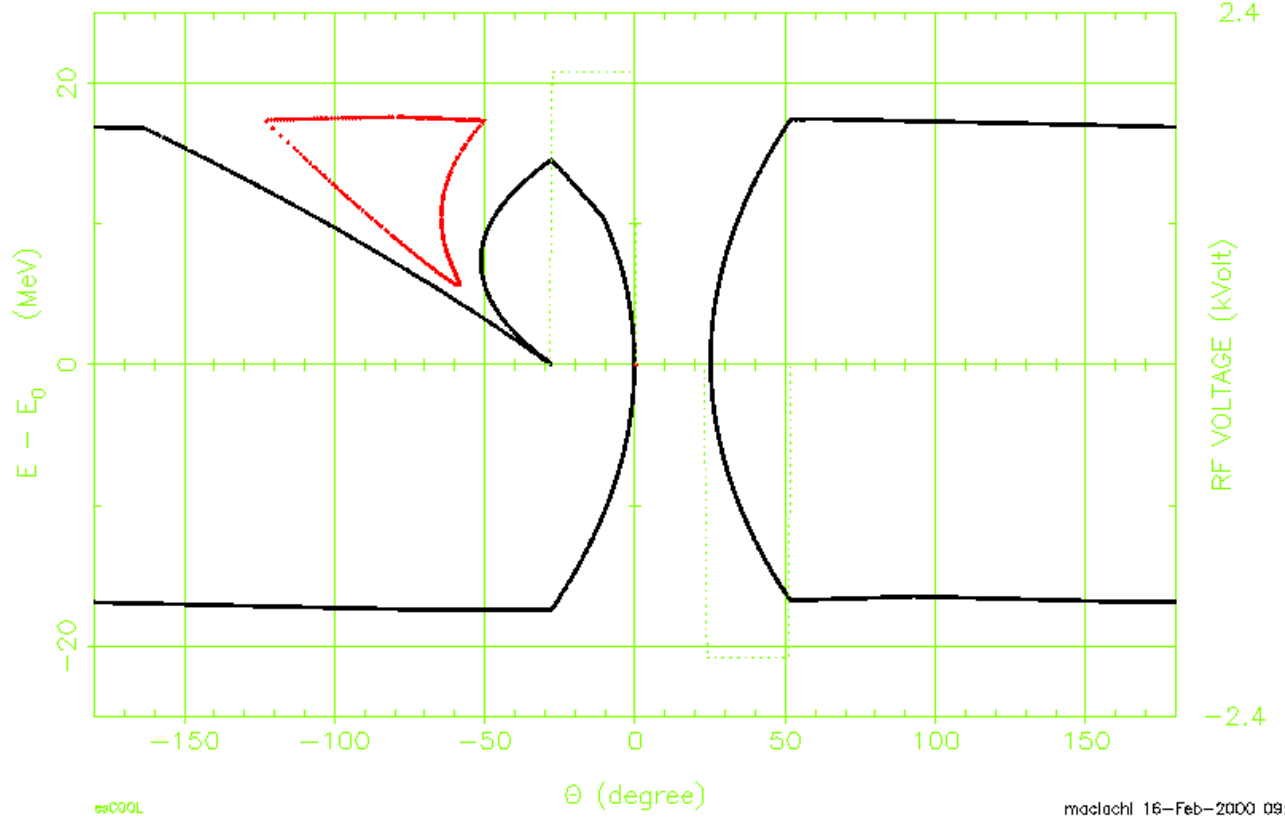


Figure 2: Distribution 0.2 s after injecting a 10 eVs batch by sudden removal of barrier separating it from the stack

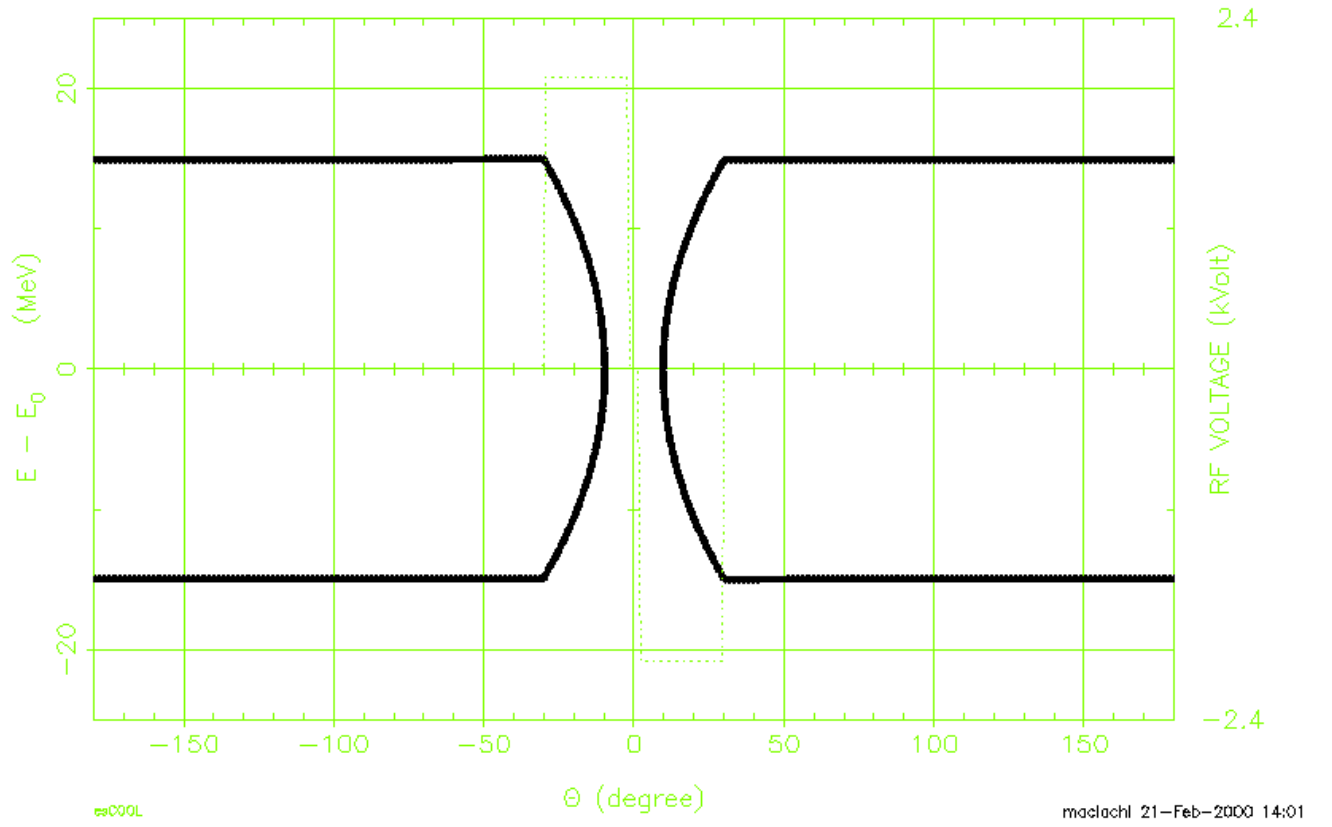


Figure 3: Recycler stack of 300 eVs in usual state with ion-clearing gap of  $0.6 \mu\text{s}$

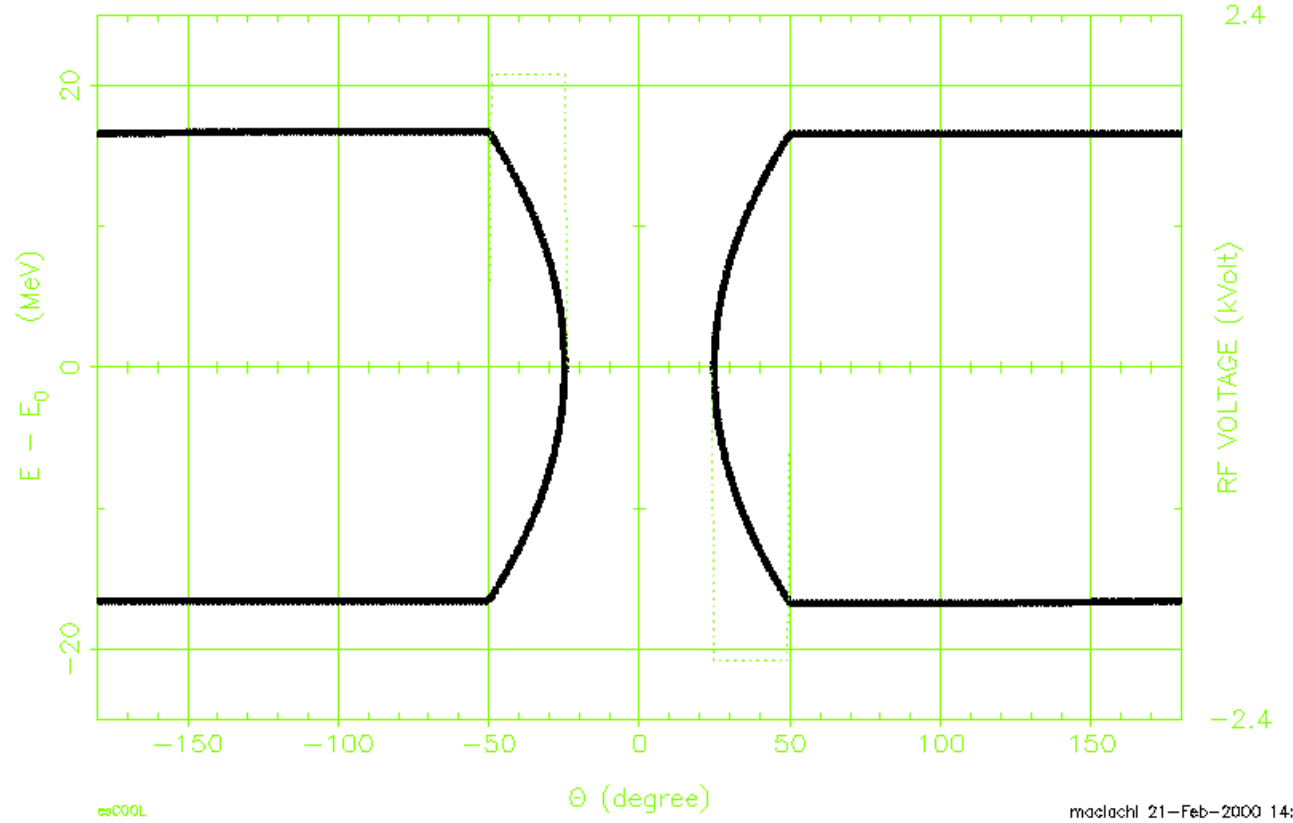


Figure 4: The ion-clearing gap is opened slowly (over  $\sim 7$  s) to  $\sim 60^\circ$  or  $1.8 \mu\text{s}$ . The barrier width is adjusted during or after the opening so that the stack fully penetrates the barrier region but does not escape it.

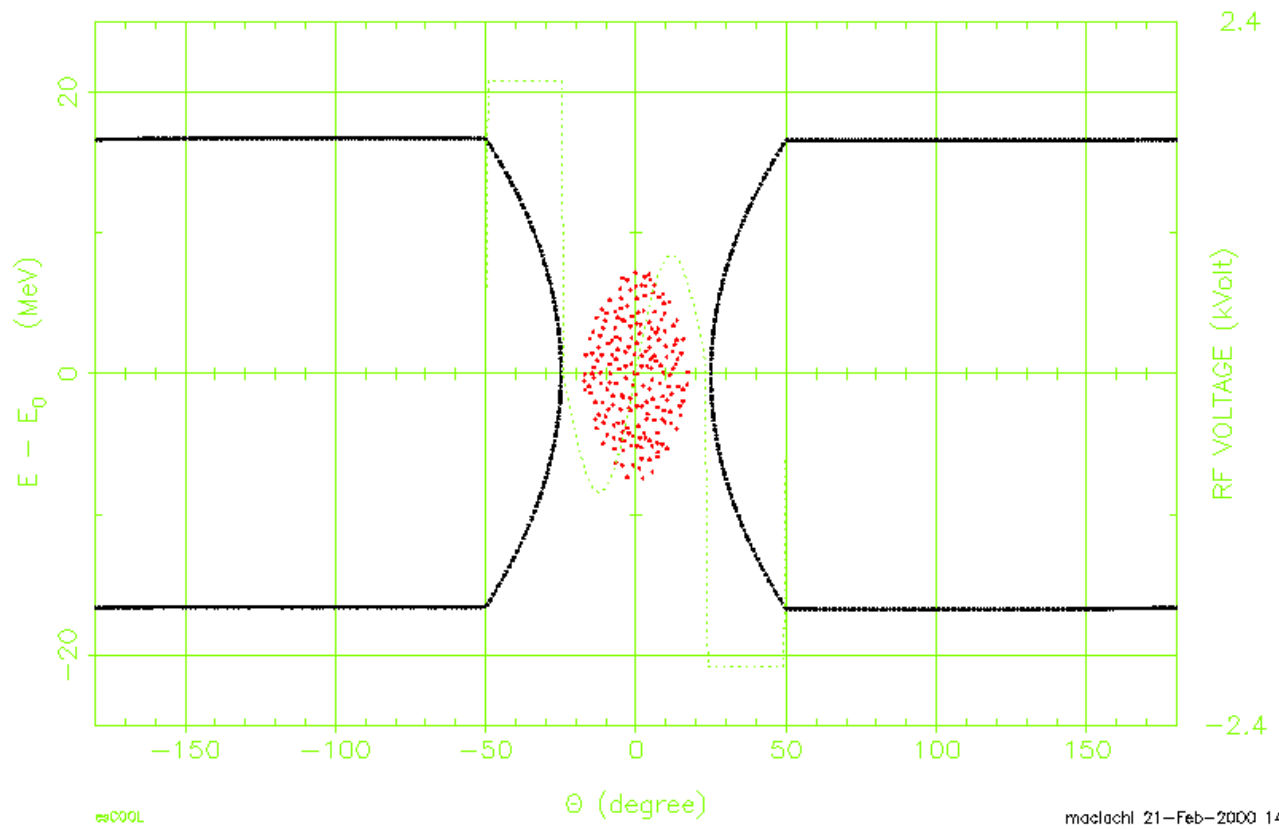


Figure 5: A 10 eVs batch from another ring is synchronously transferred to a matching isolated bucket in the cleared gap.

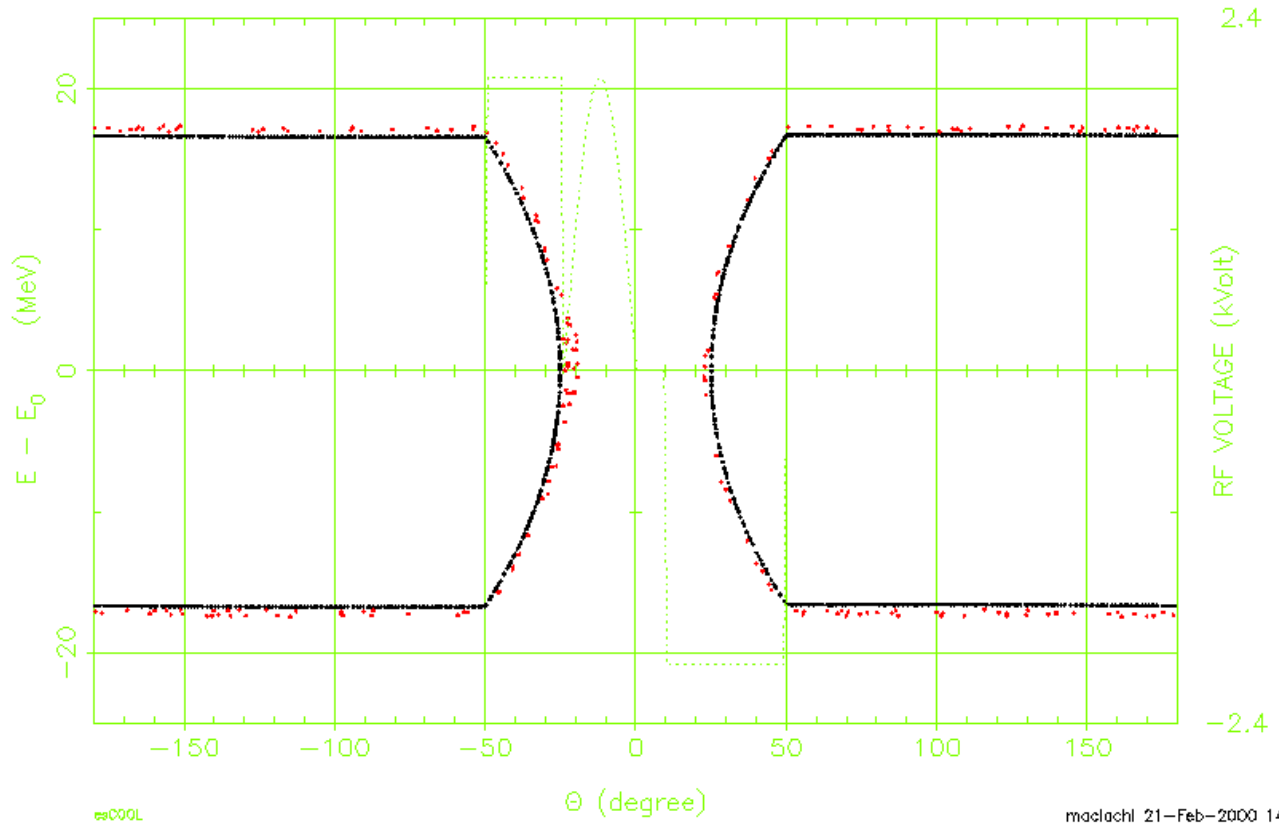


Figure 6: The sinusoidal portion of the waveform is shifted until the stable fixed point reaches the phase of the positive-going barrier, thereby squeezing all of the batch out of the gap. The shift continues after this at an accelerated rate until the sinusoidal segment is entirely removed.

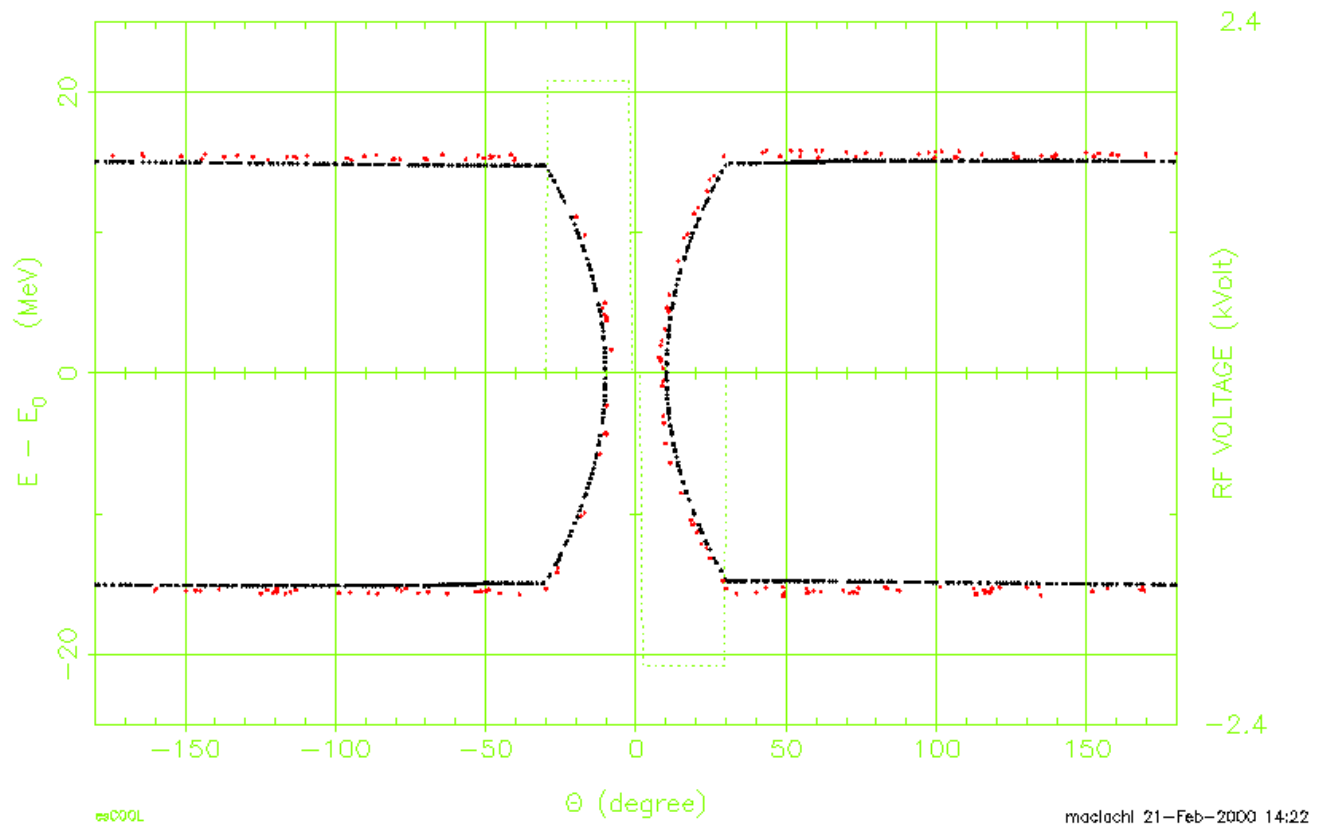


Figure 7: The ion clearing gap re-established in the augmented stack