# RESULTS OF A TOWER GRAVITY EXPERIMENT

Donald H. Eckhardt, Christopher Jekeli, Andrew R. Lazarewicz,
Anestis J. Romaides, and Roger W. Sands
Air Force Geophysics Laboratory
Hanscom AFB, MA 01731
USA

ABSTRACT

We have performed an experimental test of Newton's inverse-square law of gravitation. The test compared gravity values measured on a 600 m tower with upward continued gravity estimates calculated from ground measurements. A significant departure from the inverse-square law was detected, asymptotically approaching -500 ± 35 μGal (1 μGal = $10^{-8}$ ms$^{-2}$) at the top of the tower; this indicates that at the base of the tower there is a non-Newtonian attractive force that falls off rapidly with elevation. The results of the experiment are marginally consistent with a one term Yukawa type attractive force, but they are fully consistent with two Yukawa type forces, attractive and repulsive, in which case they are also compatible with Airy and Cavendish experiments. The experiment provides evidence that supports the hypothesis of a spin-0 graviscalar and of a spin-1 graviphoton. The masses (~ 1 neV) and coupling constants (not well defined, but at least 3% that of the graviton, and perhaps much more) of both particles are approximately the same, but because $m_1 > m_0$, the attractive scalar field is the dominant source of the measured effect.

**Preliminary results of this experiment were presented at the Rencontres de Moriond. The final results are now complete, and they have been submitted[1] to Physical Review Letters (PRL) for publication. This account includes some of the PRL material, including the final results.**

*"We were raising the lower pendulum up the South Shaft for the purpose of interchanging the two pendulums, when (from causes of which we are yet ignorant) the straw in which the pendulum-box was packed took fire, the lashings burnt away, and the pendulum with some other apparatus fell to the bottom. This terminated our operations of 1826"* George Biddell Airy[2]

Evidence for non-Newtonian gravitation from gravimetric measurements (Airy experiments) in mines and boreholes is suggestive, but hardly compelling. These estimates[3] of the gravitational constant, $\mathbf{G}$, are all about one per cent higher than the more precise laboratory estimates[4], but the discrepancies are just barely significant when compared with the experimental uncertainties and allowing for the possibility of unmodeled variations in regional free-air gradient anomalies and random or systematic errors in density estimates.

If there is a non-Newtonian component of gravity[5], its potential is generally assumed to be Yukawan. The gravitational potential of a point mass, $\mathbf{m}$, then has the form $-\mathbf{G}\mathbf{m}\mathbf{r}^{-1}[1+\alpha\exp(-r/\lambda)]$. For a laboratory experiment, $r/\lambda \ll 1$, and the Cavendish constant is $\mathbf{G}(1+\alpha)$. For a flat earth (radius $\gg \lambda$), the gravity perturbation caused by the Yukawan term is $\delta g = 2\pi G|\delta\sigma|\alpha\lambda\exp(-|\zeta|/\lambda)$, where $\zeta$ is the distance from the earth's surface and $\delta\sigma$ is the density difference across the surface. Generally, $|\zeta|/\lambda \gg 1$ for the Airy experiments; not being sensitive to the $\delta g$ term, they measure $\mathbf{G}$. To resolve $\alpha\lambda$ and $\lambda$ from measurements of $\delta g$, the Airy experiments require good measurements near a surface that is well mapped topographically and gravimetrically. Because $\delta g$ is symmetrical about $\zeta = 0$, $\alpha\lambda$ and $\lambda$ might also be resolved from measurements of $\delta g$ above a well mapped surface. Furthermore, $\sigma$ can be estimated (absolutely) more accurately for air than for rock, so the Newtonian gravity "noise" that impairs the precise determination of $\delta g$ (and, in turn, $\alpha\lambda$ and $\lambda$) is negligible above the earth's surface. This was a major motivation for our tower gravity experiment.

Of the (approximately) 40 TV transmission towers that rise 600 m above local ground level in the United States, we chose the WTVD tower in

Garner, NC. The tower, built in 1978 by Kline Iron and Steel of Columbia, SC, is mechanically remarkably stable; we could make reliable and repeatable gravity measurements at all tower levels when the wind speed was 3 $ms^{-1}$ or less. The tower is in a relatively flat area of the North Carolina coastal plain, 220 km from the ocean and 350 km from the mountains. The regional geology and gravity field have been well mapped, and they are rather featureless. And, most importantly, we had the hospitality and cooperation of the WTVD management and staff[6].

At the base and at six different levels of the tower (93.90, 192.14, 283.56, 379.51, 473.21, and 562.24 m above ground level), we measured **g** with a LaCoste-Romberg (L&R) Model G gravimeter[7]; elevations were simultaneously measured with an electronic distance meter. Altogether there were 30 tower gravity observations, tied to seven base station observations through five adjustment loops. Our estimated uncertainties for the tower measurements range from 23 (lowest level) to 27 μGal (top). At 77 sites in the inner zone (within 5 km of the tower), we measured **g** with the same gravimeter, tying the measurements to the tower base station; altogether there were 257 inner zone gravity observations, with at least two per site. DMA (the U. S. Defense Mapping Agency) surveyed the inner zone site coordinates using its Inertial Positioning System and third order differential leveling. The uncertainty of each ground survey gravity measurements is about **20** μGal. From its gravity library, DMA provided gravity measurements and coordinates at numerous sites in the region; we used data from 1784 of these DMA catalogued sites for the outer zone (between 5 and 220 km of the tower). The gravity uncertainties at these outer zone sites are estimated to be about 1 mGal. The approximate relative weights of the inner and outer zones in the upward continuation estimation of **g** at the top of the tower are 95% and 5% respectively.

Geodesists and physicists usually do not speak the same technical language; indeed, their potentials have opposite signs. Many of the words, concepts and mathematical tools for analyzing the tower gravity experiment come from geodesy. A translation for physicists is required, so here it is[8]: Let **W** be the gravity potential of the earth which is the sum of a gravitational potential **V** and a centrifugal potential $\mathbf{\Phi}$. In cylindrical coordinates ($\rho$ = distance from rotation axis), the centrifugal force is $-\nabla\Phi = \omega^2\boldsymbol{\rho}$ where $\boldsymbol{\omega}$ is the earth angular rate of rotation. Taking the divergence, $-\nabla^2\Phi = \rho^{-1}\partial(\omega^2\rho^2)/\partial\rho = 2\omega^2$, so in any coordinate system the centrifugal potential satisfies $\nabla^2\Phi = -2\omega^2$. The gravitational potential satisfies Poisson's equation, $\nabla^2 V = 4\pi G\sigma$. Thus $W = V + \Phi$ satisfies

$$\nabla^2 W = 4\pi G\sigma - 2\omega^2. \tag{1}$$

In curvilinear coordinates

$$\nabla^2 W = \frac{1}{h_1 h_2 h_3} \sum_{i=1}^{3} \frac{\partial}{\partial \xi_i} \left[ \frac{h_1 h_2 h_3}{h_i^2} \frac{\partial W}{\partial \xi_i} \right]. \tag{2}$$

Let $\partial W/\partial \xi_2 = \partial W/\partial \xi_3 = 0$; then $W(\xi_1) = \mathit{constant}$ defines an equipotential surface. This surface has the curvature $J(\xi_2, \xi_3) = \frac{1}{2}[1/h_2 + 1/h_3]$. Let $h_1 = 1$, $d\xi_1 = dH$, and $\partial W/\partial H = g$ (positive downwards). Then, from (1) and (2), using $\partial h_2/\partial H = \partial h_3/\partial H = 1$,

$$\partial g/\partial H = 4\pi G\sigma - 2gJ - 2\omega^2. \tag{3}$$

This is Bruns' equation. The second derivative of $g$ is approximately

$$\partial^2 g/\partial H^2 \approx 6gJ^2. \tag{4}$$

In geodetic practice, $W$ is reduced to a computationally manageable level by subtracting a reference field, $U$, that encompasses the central force, centrifugal and ellipsoidal components; this relegates various subsequent approximations to second order. Here we let $U$ also include the attraction of the atmosphere, which then guarantees the harmonicity of the resulting disturbing potential, $T = W - U$, above the earth's surface. A defined equipotential surface (rotating ellipsoid of revolution) completes the definition of $U$ and of its gradient, the normal gravity, $\gamma$, at any point $Q$ on the ellipsoid or at height $H$ above the ellipsoid. A truncated Taylor series, using $\gamma = g$ in (3) and (4), generally suffices. The gravity anomaly, $\Delta g$, at a point $P$ on the normal through $Q$ is defined as the difference between gravity at $P$ and normal gravity at $Q$, where $W_P = U_Q$. The height difference between $P$ and $Q$ is unknown, but the difference between the height of $Q$ above the ellipsoid and the height of $P$ above the reference equipotential surface of $W$, the geoid, is negligible in non-mountainous terrain. The spherical approximation of $\Delta g$ is given by $\Delta g_r = -\partial T/\partial r - 2T/r$, where r is the radial coordinate; and its flat earth approximation is $\Delta g_\zeta = -\partial T/\partial \zeta$, where $\zeta$ is the local vertical coordinate. Therefore, $r\Delta g_r$ is harmonic in a spherical coordinate system and $\Delta g_\zeta$ is harmonic in Cartesian and cylindrical coordinate systems. The spherical and flat earth approximations are entirely adequate for upward continuations of gravity anomalies for this experiment.

We used two methods to estimate $\Delta g$ at the gravity observation levels on the tower[9]. Method I, based on the Poisson integral and least-squares collocation, upward continues the $r\Delta g_r$'s. Method II, based on the Fourier-Bessel series, upward continues the $\Delta g_\zeta$'s. The $\Delta g$ differences between the two techniques, which are compatible with the uncertainty estimates (~20 to 50 μGal), are assuringly small: 30 μGal at the lowest level, and no more than 10 μGal at the other levels. (Errors in the $\Delta g$ estimates are highly correlated between different levels and between the two methods because they use a common set of inner zone gravity samples.)

If Newton's inverse-square law is valid, **g** observed on the tower should agree, except for allowable error, with $\Delta g + \gamma$ modeled from surface data using either of the upward continuation methods for $\Delta g$, and using $\gamma$ for **g** in Bruns' equation (3) and its derivative (4). The 172 mGal difference between $\Delta g + \gamma$ at the top level of the tower and at its base derives 99% from the difference in $\gamma$, and 1% from the difference in $\Delta g$. The differences, **g** (observed) − **g** (modeled), and their uncertainties (in μGal) are as follows:

| Level (m) | Method I | Method II |
|---|---|---|
| 93.90 | -147±29 | -117±55 |
| 192.14 | -267±34 | -272±49 |
| 283.56 | -378±35 | -384±45 |
| 379.51 | -468±33 | -467±39 |
| 473.21 | -508±33 | -501±37 |
| 562.24 | -511±34 | -501±35 |

Unless these differences are artifacts of unsuspected errors, the data indicate that at the base of the tower there is a non-Newtonian attractive gravitational force that falls off rapidly with elevation. If it is not gravitational, the effect is one that increases with elevation and gradually levels off to a maximum in the upper reaches of the tower. We tested for possible error sources that could cause such an effect, including effects of magnetic fields, radio frequency interference, and tower motions, and found none.

A least-squares fit of $\delta g(\zeta)-\delta g(0)=2\pi G|\delta\sigma|\,\alpha\lambda\,[\exp(-|\zeta|/\lambda)-1]$ to the Method II data gives $\alpha = 0.0204$ and $\lambda = 311$ m. With this model, the Airy **G** should be about two per cent lower than the Cavendish **G**, not one per cent higher. A $\delta g$ due to a one term Yukawa potential cannot account for these results, but if $\delta g$ is due to a two term (scalar and vector) Yukawa potential[10], it can. Let the subsripts 0 and 1 denote the scalar and vector fields respectively. Then the gravitational potential has the form $-Gmr^{-1}[1+\alpha_0\exp(-r/\lambda_0)-\alpha_1\exp(-r/\lambda_1)]$, and its corresponding gravity perturbation is $\delta g = 2\pi G|\delta\sigma|\,[\alpha_0\lambda_0\exp(-|\zeta|/\lambda_0)-\alpha_1\lambda_1\exp(-|\zeta|/\lambda_1)]$. The problem now is that we have four $\alpha, \lambda$ parameters instead of two. Reducing the number of parameters to two by setting $\alpha_1 = \alpha_0 - 0.007$ (from Airy and Cavendish experiments) and $\alpha_0\lambda_0 - \alpha_1\lambda_1 = 5.1$ m (implied by $\delta g(\zeta)-\delta g(0) \rightarrow -500$ μGal as $\zeta \rightarrow \infty$), the lower limit for $\alpha_0$ is 0.03, but it has no upper limit. The $\lambda$'s fall between 20 and 180 m when $\alpha_0$ is small, but they are close to 100 m (mass $\approx$ 1 neV) for $\alpha_0 \approx \alpha_1 \geq 1$. Further gravimetric experiments will test the existence of the scalar-vector model and determine $\alpha_1 - \alpha_0$ more precisely, but other types of experiments[11] will be required to estimate $\alpha_0$.

## REFERENCES


1. D. H. Eckhardt, C. Jekeli, A. R. Lazarewicz, A. J. Romaides, and R. W. Sands, submitted to Phys. Rev. Lett. (1988).
2. G. B. Airy, Philos. Trans. R. Soc. London **146**, 297 (1856).
3. F. D. Stacey, G. J. Tuck, G. I. Moore, S. C. Holding, B. D. Goodwin, and R. Zhou, Rev. Mod. Phys. **59**, 157 (1987); A. T. Hsui, Science **237**, 881 (1987).
4. G. G. Luther and W. R. Fowler, Phys. Rev. Lett. **48**, 121 (1982).
5. Y. Fujii, Nature **234**, 5 (1971).
6. We especially thank Lew Bowers, the WTVD transmitter supervisor.
7. L. LaCoste and J. Fett of Lacoste-Romberg provided valuable support concerning the intricacies of the gravimeter.
8. The authoritative text on this subject is *Physical Geodesy*, by W. Heiskanen and H. Moritz (W. H. Freeman & Co., San Fransisco, 1967).

9. A. J. Romaides, C. Jekeli, A. R. Lazarewicz, D. H. Eckhardt, and R. W. Sands, submitted to J. Geophys. Res. (1988).
10. J. Scherk, in *Unification of the Fundamental Particle Interactions*, S. Ferrara, J. Ellis, P. Nieuwenhuizen, eds., Plenum, New York (1981).
11. T. Goldman and M. Nieto, Phys. Lett. **112B**, 437 (1982).