*Particle Accelerators*, 1994, Vol. 44, pp. 201–214
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# SIBERIAN SNAKES WITH SMALL NUMBER OF MAGNETS FOR HIGH ENERGY ACCELERATORS

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*(Received 12 August 1993; in final form 6 December 1993)*

Considering the Siberian snake method, which is used to accelerate polarized beams, several snake configurations are investigated in assumption that for high energy machines the particle orbit excursion does not dominate in snake design requirements. Snake schemes with a small number of magnets can be effective. Using dipoles with an arbitrary field orientation various snake designs were considered containing four, five magnets and three plus two correction magnets.

KEY WORD: Polarization, Siberian snake

## 1. INTRODUCTION

The problem of accelerating polarized beam is due to spin depolarizing resonances, a number of which are crossed during beam acceleration. Among the methods used to conserve beam polarization, Siberian snakes allow a reorganization of the spin motion so that no passage through depolarizing resonances occur. Therefore, the beam can be accelerated to very high energies without polarization loss.[1] Ideally, the Siberian snake is a spin rotator which has no effect on the beam but rotates the spin around a horizontal axis by 180 degrees. For machines with energy of 20 GeV and higher a snake is composed of a specially arranged sequence of dipoles which restores the particle orbit. Transverse fields have the advantage that for a fixed angle of spin rotation, the field remains constant with energy, while longitudinal fields increase linearly. But this fact causes also a negative effect: the particle orbit excursion inside a snake is energy dependent. The angle of spin rotation is a factor $G\gamma$ larger than the particle bend in a dipole magnet, where G is the anomalous magnetic moment of particle ($G=1.79285$ for protons) and $\gamma$ is the Lorentz energy factor. The problem is difficult to resolve at low energies of the order of ten GeV, so that special steps are required to reduce the maximum orbit excursion inside a snake. It usually causes an increase in the number of snake magnets. On the other hand, at energies of a hundred GeV the orbit excursion

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inside a snake becomes small, so that for an accelerator with such an injection energy snakes can be made with a small number of magnets.

First let us estimate what is the minimum number of magnets that can be used in a snake design. The requirement of orbit restoration gives us four constraints on the snake magnet parameters for any particular snake scheme ($\triangle x = \triangle x' = \triangle z = \triangle z' = 0$). In addition, the requirements of 180 degree spin rotation and a horizontal snake axis provide three additional conditions. On the other hand, each snake dipole is characterized by two parameters: the angle of spin rotation or field integral and the orientation of the field. Therefore, the minimum number of dipoles required for an optically transparent snake is four. But to find all the parameters of the snake magnets, one should resolve a system of seven equations arising from the snake constraints. In general, solving a system of seven nonlinear equations is a non-trivial problem even using numerical methods. Here we shall consider a family of snakes where, by means of a chosen snake field configuration symmetry, many of the initial seven conditions can be satisfied automatically. Through the article we use the accelerator frame $(\hat{x}, \hat{y}, \hat{z})$ where $\hat{y}$ is the direction of the reference particle orbit and $\hat{x}, \hat{z}$ are the radial and vertical directions respectively.

## 2. SYMMETRY EFFECTS IN SNAKE CONFIGURATIONS

Studying snake schemes we will use the spinor formalism for the spin transfer matrices.[2] In this approach, the spin transfer matrix for a dipole magnet with an arbitrary field orientation $\vec{n}_b = (\cos\alpha_b, 0, \sin\alpha_b)$ is:

$$e^{-\frac{i}{2}\psi(\vec{n}_b\cdot\vec{\sigma})} = \hat{I}\cdot\cos\psi/2 - i\,(\vec{n}_b\cdot\vec{\sigma})\sin\psi/2 \qquad (1)$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ – Pauli matrices, $\hat{I}$ is the unit matrix and $\psi$ is the angle of spin rotation in the dipole which is determined by the field integral ( for protons $\psi[\text{rad}] = 0.573 \cdot \int Bdl[\text{Tesla·m}]$ ). The snake spin transfer matrix in this notation will be:

$$\hat{S}_{nake} = \exp\{\frac{i}{2}(\vec{n_s}\cdot\vec{\sigma})\pi\} = i\begin{pmatrix} 0 & e^{-i\varphi_s} \\ e^{i\varphi_s} & 0 \end{pmatrix} \qquad (2)$$

here $\vec{n}_s = (\cos\varphi_s, \sin\varphi_s, 0)$ is the snake axis. To find the sequence of magnets, which serves as a snake, the product of the magnets spin transfer matrices should satisfy two conditions. First, to obtain 180 degree spin rotation trace (sum of the diagonal elements) of the resulting matrix must be zero:

$$\operatorname{tr}\hat{S} = 0. \qquad (3)$$

The second constraint is dictated by the assumption that the snake axis must be horizontal; therefore, the difference between the diagonal elements of the spin transfer matrix also must be zero. Combining both conditions together we find that the spin transfer matrix of the snake magnet sequence must have zero diagonal elements:

$$S_{11} = S_{22} = 0. \tag{4}$$

We now discuss the condition of particle orbit restoration inside the snake. For relativistic particle (snakes become necessary in accelerator at relativistic energies) the integrated effect of the snake magnetic field on particle velocity is proportional to the integrated snake field. Hence, the necessary condition for the particle velocity restoration is the requirement of zero integrated snake field:

$$\Delta\vec{v} = \frac{e}{m\gamma c} \cdot \int_0^{L_s} \vec{B} dy = 0. \tag{5}$$

The orbit position after the snake is determined by the equation:

$$\Delta\vec{r} = \frac{e}{m\gamma c^2} \cdot \int_0^{L_s} dl \int_0^l dy \left(B_z \vec{e}_x - B_x \vec{e}_z\right). \tag{6}$$

For complete orbit restoration the orbit displacement must be zero along with the particle velocity restored. Assuming symmetry in the snake field configuration with respect to the snake center we consider two cases – the projected field is symmetric or antisymmetric:

$$\begin{aligned} B_z(y) &= B_z\left(L_s - y\right), \text{ or} \\ B_z(y) &= -B_z\left(L_s - y\right). \end{aligned} \tag{7}$$

The expression for the orbit displacement in the horizontal plane (consideration of the vertical plane is the same) can be written:

$$\begin{aligned} \Delta x = & \frac{e}{m\gamma c^2} \left\{ \int_0^{L_s/2} dl \int_0^l dy \left(B_z(y) - B_z(L_s - y)\right) + \right. \\ & \left. + \frac{L_s}{2} \int_0^{L_s/2} dy \left(B_z(y) + B_z(L_s - y)\right) \right\}. \end{aligned} \tag{8}$$

When the projected field is symmetric, the first term in the sum vanishes and the second term also goes to zero if the integrated field is zero. Therefore, the condition of orbit position restoration coincides with the requirement of particle velocity restoration when the projected field is symmetric. In the case of antisymmetric projected field the second term in the sum (8) vanishes and the requirement of zero orbit displacement becomes:

$$\Delta x = \frac{2e}{m\gamma c^2} \cdot \int_0^{L_s/2} dl \int_0^l dy B_z(y) = 0. \tag{9}$$

It is to be noted that this is the expression for the horizontal orbit offset at the snake center. Therefore, summarizing the previous arguments we conclude that the particle obit in a plane will be corrected:

If the projected field is symmetric and the integrated field is zero.

OR

If the projected field is antisymmetric, the integrated field is zero and the transverse orbit offset is zero at the snake center.

Choosing proper symmetry for the projected snake field we can also resolve the condition of horizontal snake axis. If the snake field configuration is chosen to be symmetric in the horizontal plane and antisymmetric in the vertical plane, then the snake axis is in the horizontal plane automatically. This fact can be obtained dividing the snake in the symmetry point and considering the snake spin transfer matrix as the product of two parts. Then it can be written in the following form:

$$\hat{S} = \left(A \cdot \hat{I} - i\vec{\sigma} \cdot \vec{a}\right)\left(A \cdot \hat{I} - i\vec{\sigma} \cdot \vec{b}\right) = \\ = \hat{I} \cdot \left(A^2 - \left(\vec{a} \cdot \vec{b}\right)\right) - i\vec{\sigma} \cdot \left\{A\left(\vec{a} + \vec{b}\right) + \left[\vec{a} \times \vec{b}\right]\right\}, \tag{10}$$

where because of chosen symmetry $\vec{b} = (a_x, a_y, -a_z)$. In this notation the snake axis is determined by the vector:

$$\vec{n} = -\left\{A\left(\vec{a} + \vec{b}\right) + \left[\vec{a} \times \vec{b}\right]\right\}. \tag{11}$$

This vector lies in the horizontal plane since both sum and vector product of $\vec{a}$ and $\vec{b}$ are in the horizontal plane. Therefore, the snake axis is horizontal. We can also define the condition of 180 degree snake spin rotation from the snake spin transfer matrix written in the form (10):

$$A^2 - \left(\vec{a} \cdot \vec{b}\right) = A^2 - a_x^2 - a_y^2 + a_z^2 = 0 \tag{12}$$

In this approach, one half of the snake gives the complete description of the whole snake including the orbit excursion, the spin rotation angle and the axis of the spin rotation. Further we will examine several possible practical realizations of the snake scheme with a symmetric horizontal field, while vertical – antisymmetric. To visualize different snake configurations we use vector diagrams where each vector corresponds to one snake magnet; the direction of the vector shows the magnet's field orientation or the axis of spin rotation and the length of the vector corresponds to the angle of spin rotation or the field integral of the magnet.

## 3. THREE MAGNET SPIN ROTATOR

An example of a three magnet spin rotator was proposed by Derbenev and Kondratenko.[3] It consists of three identical magnets where each rotates the particle spin by 180 degrees. The second magnet has a horizontal field and the others are orientated at ±120 degrees from it. It is easy to check that the scheme provides 180 degree spin rotation around the radial direction. The magnet configuration has projected field symmetric in the horizontal plane and antisymmetric in vertical plane. However, the horizontal orbit offset is not zero at the snake center and two additional magnets are necessary to make the system optically transparent. Generalizing this model we put as the variable parameters the angles of spin rotation in the snake magnets and the magnets orientation. The vector diagram for the snake is shown in Fig. 1. Having zero integrated field the snake is optically transparent if the correction dipoles are chosen to make zero orbit offset at the snake center. To avoid uncertainty in the definition of the correction dipoles we fix the gap between the snake magnets at 0.4 m and the field strength at 1.7 Tesla. Further we use these values for all snake schemes.

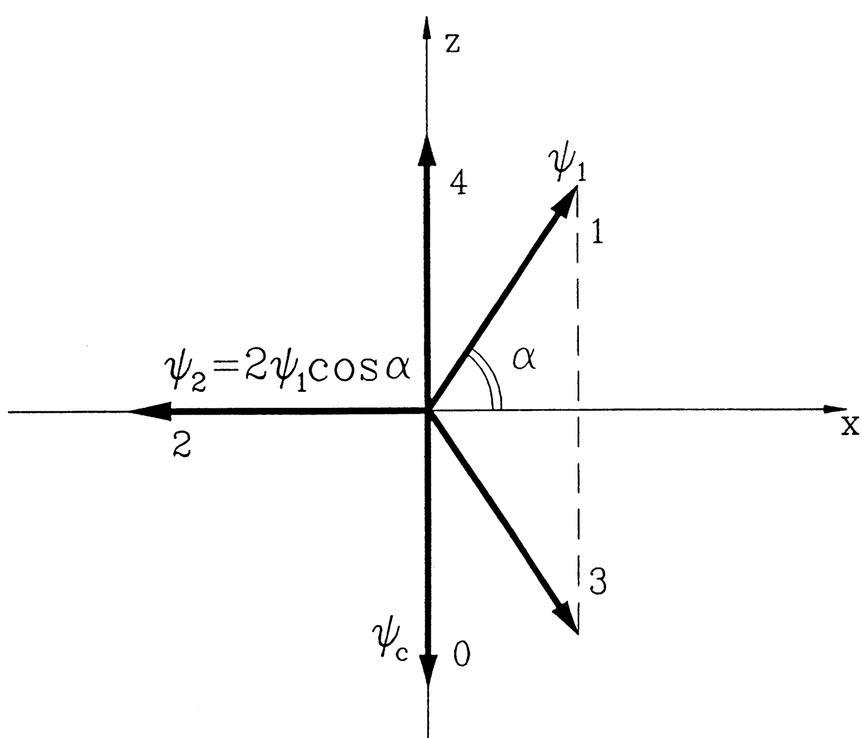


FIGURE 1: Vector diagram for the three magnet snake with two orbit correction dipoles.

TABLE 1: Examples of the three magnet snake with orbit correction ($l_{gap}$=0.4 m, B = 1.7 T).

| $\alpha$ (deg) | $\psi_2/2$ (deg) | $\psi_1$ (deg) | $\psi_c$ (deg) | $\varphi_s$ (deg) | $\int B\,dl$ (T·m) |
|---|---|---|---|---|---|
| 69.33 | 75.6 | 214.2 | 106 | 115.2 | 24.12 |
| 66.61 | 79.2 | 199.5 | 98.85 | 106.8 | 23.01 |
| 64.23 | 82.8 | 190.5 | 94.16 | 100.9 | 22.39 |
| 62.05 | 86.4 | 184.3 | 90.68 | 96.12 | 22.03 |
| 60 | 90 | 180 | 87.94 | 92.06 | 21.81 |
| 58.07 | 93.6 | 177 | 85.75 | 88.47 | 21.72 |
| 56.26 | 97.2 | 175 | 83.95 | 85.23 | 21.7 |
| 54.55 | 100.8 | 173.8 | 82.48 | 82.26 | 21.76 |
| 52.94 | 104.4 | 173.2 | 81.27 | 79.5 | 21.87 |
| 51.44 | 108 | 173.3 | 80.29 | 76.9 | 22.03 |

Besides the angle of spin rotation in the correction dipole $\psi_c$, which we determine using the orbit restoration constraint, the snake has two more variable parameters for the first magnet: the orientation angle $\alpha$ and the angle of the spin rotation $\psi_1$. The angle of the spin rotation in the second snake magnet is $\psi_2 = 2\psi_1 \cdot \cos\alpha$, since the horizontal projected field must be symmetric. One parameter we determine using the constraint of 180 degree snake spin rotation (12), where one should put $A$ and $\vec{a}$ in the form:

$$A = \cos\frac{\psi_2}{4}\cos\frac{\psi_1}{2} + \cos\alpha\sin\frac{\psi_2}{4}\sin\frac{\psi_1}{2}$$

$$\vec{a} = \begin{pmatrix} \cos\alpha\cos\frac{\psi_2}{4}\sin\frac{\psi_1}{2} - \sin\frac{\psi_2}{4}\cos\frac{\psi_1}{2} \\ -\sin\alpha\sin\frac{\psi_2}{4}\sin\frac{\psi_1}{2} \\ \sin\alpha\cos\frac{\psi_2}{4}\sin\frac{\psi_1}{2} \end{pmatrix} \tag{13}$$

The second parameter can be used for varying the snake axis. It is to be noted, that the orbit correction dipoles shift the snake axis orientation in the horizontal plane towards the longitudinal direction. The value of the shift is equal to the angle of spin rotation in one of the correction dipoles. This effect limits the range where the snake axis orientation could be varied. Numerical examples for the three magnet snake with orbit correction are given in Table 1 and Fig. 2 shows variation of the snake magnet parameters via the first magnet orientation angle.

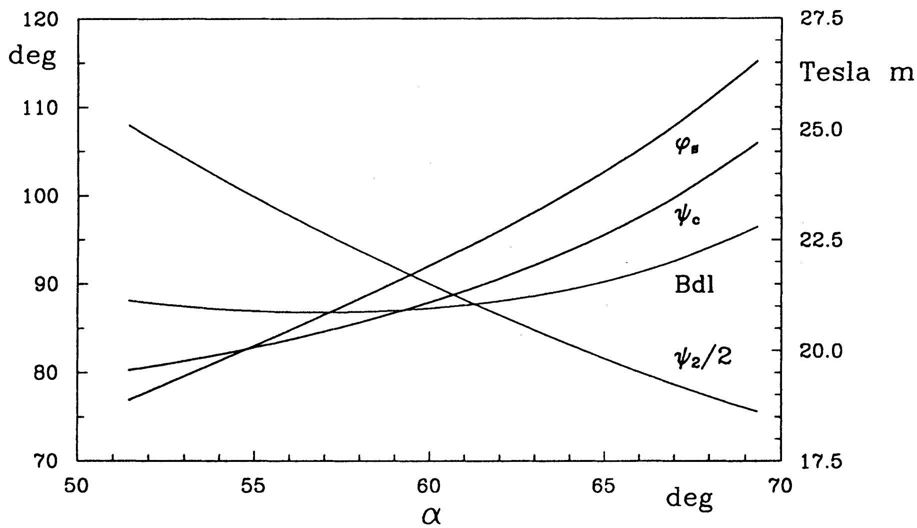


FIGURE 2: Parameters of the three magnet snake with orbit correction versus the first magnet orientation angle $\alpha$. Here $\psi_2$,$\psi_c$ are the angles of spin rotation in the second and correction snake magnets; $\varphi_s$ is the snake axis orientation angle from the radial direction and *Bdl* is the total snake integrated field including correction dipoles.

## 4. FOUR MAGNET SNAKE

We now consider the snake with only four magnets, which is the minimum number of magnets for an optically transparent full snake. The vector diagram for the four magnet snake is shown in Fig. 3. Essentially the same reasoning as for the three magnet scheme is used to determine the snake magnets parameters. As the variables we use: two angles of the magnets orientation $\alpha$ and $\beta$ and the projection of spin rotation vector of any magnet on the horizontal axis: $\psi_x = \psi_1 \cdot \cos\alpha = \psi_2 \cdot \cos\beta$. Then the expressions for $A$ and $\vec{a}$, which is used in the equation (12), can be written in the following form:

$$A = \cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2} + \cos(\beta-\alpha)\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}$$

$$\vec{a} = \begin{pmatrix} \cos\beta\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2} - \cos\alpha\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} \\ -\sin(\beta-\alpha)\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \\ \sin\alpha\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} - \sin\beta\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \end{pmatrix}. \qquad (14)$$

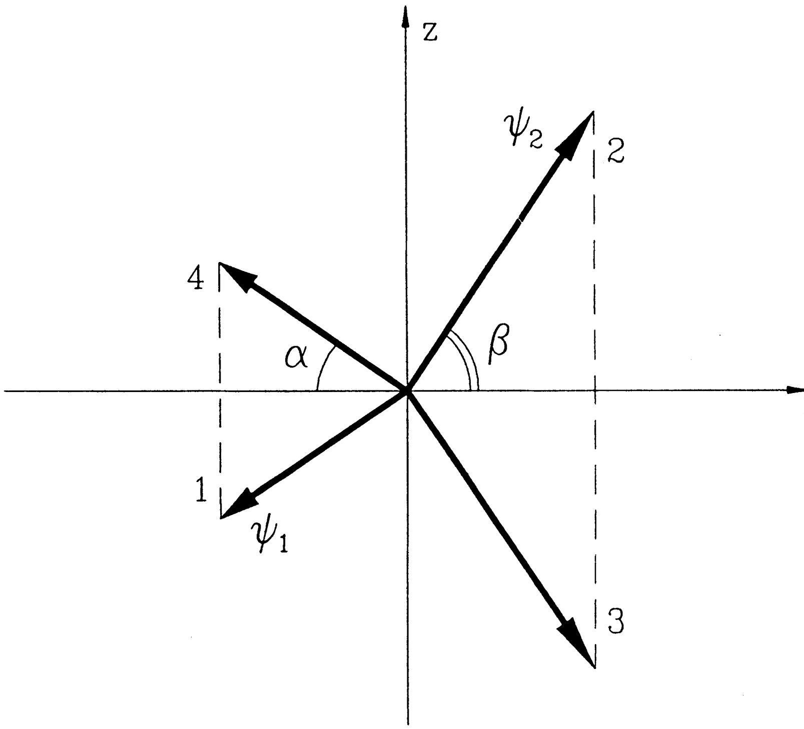


FIGURE 3: Vector diagram for the four magnet snake.

As we did for the three magnet snake we resolve the constraints of orbit restoration and 180 degree spin rotation for two snake parameters. However, it appears that the two conditions are in conflict with each other. The solutions with both constraints satisfied exist but they have total snake field integral more then 26 Tesla·m, which is significantly larger then for the three magnet snake. Large integrated field along with large orbit excursion limits practical realization of the four magnet snake, although it proves the theoretical possibility of optically transparent four magnet snake. The numerical examples are given in Table 2.

The fact that the field integral is large for this snake configuration can be understood as follows. The orbit restoration condition requires that the direction of the magnetic field in the first and the second magnets must be almost opposite and therefore these magnets have relatively small net effect on spin. In order to reduce the snake field integral we consider a modified snake scheme which includes two correction magnets and the angle $\alpha$ is assumed to be zero. Then the snake diagram becomes as shown in Fig. 4.

It is to be mentioned, that now the snake has a straightforward analogy with the previously described three magnet scheme. But the results obtained for the four

TABLE 2: Examples of the four magnet snake ($l_{gap}$=0.4 m, B = 1.7 T ).

| $\alpha$ (deg) | $\beta$ (deg) | $\psi_1$ (deg) | $\psi_2$ (deg) | $\varphi_s$ (deg) | $\int B\,dl$ (T· m) |
|---|---|---|---|---|---|
| 40 | 65.09 | 161.12 | 293.04 | 101.2 | 27.68 |
| 35 | 61.25 | 159.77 | 272.12 | 108.1 | 26.32 |
| 32.5 | 59.12 | 162.37 | 266.78 | 112.3 | 26.15 |
| 30 | 56.8 | 167.44 | 264.85 | 117.4 | 26.35 |
| 25 | 51.53 | 191.72 | 279.29 | 133.6 | 28.71 |

magnet snake with two correction dipoles shows advantages over the three magnet snake in a smaller field integral and a smaller value of the horizontal orbit excursion. On the other hand, the range where the snake axis can be varied becomes smaller too, which is again due to the correction magnets. In both snake schemes three and four magnet cores provide the required 180 degree spin rotation, but since two more magnet are necessary for the particle orbit restoration we could also call them as five and six magnet snakes respectively.

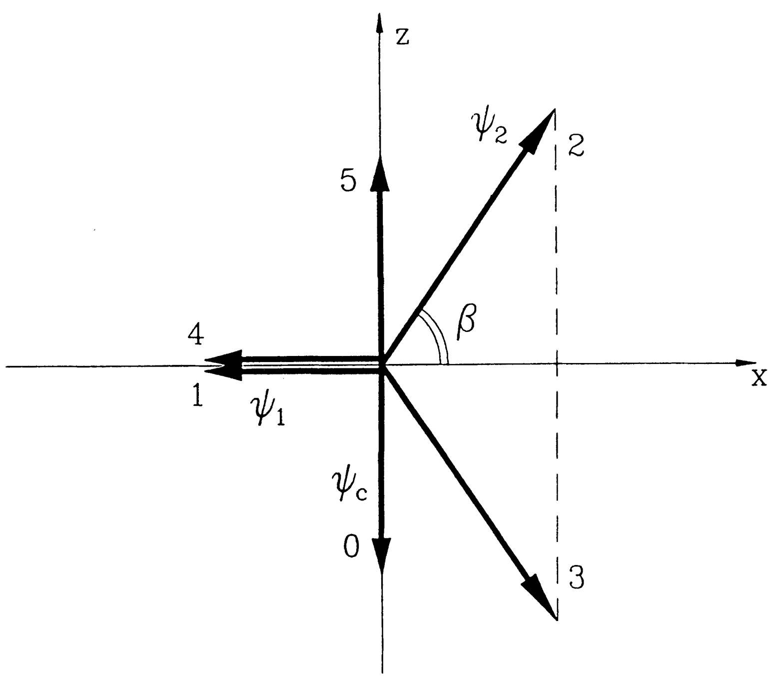


FIGURE 4: Vector diagram for the four magnet snake with two orbit correction dipoles.

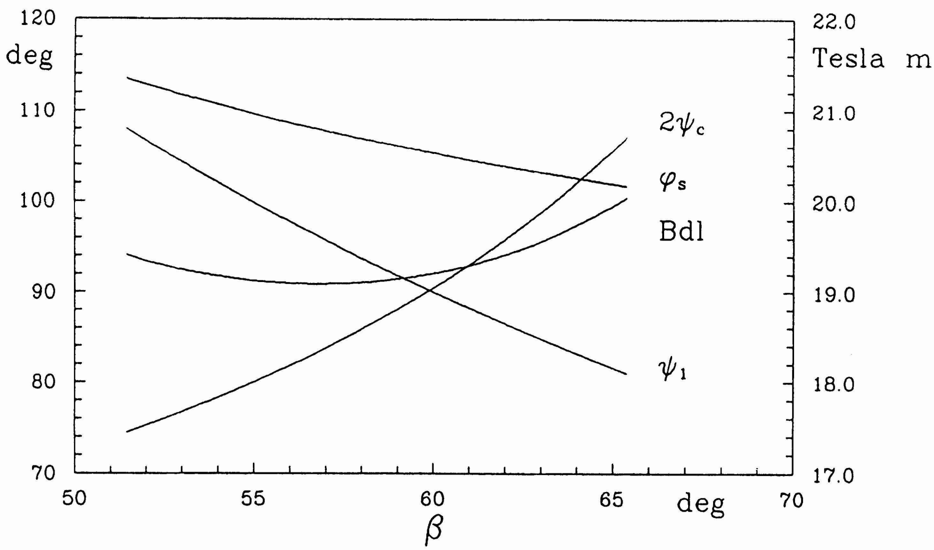


FIGURE 5: Parameters of the four magnet snake with orbit correction versus second magnet orientation angle *beta*. Here $\psi_1, \psi_c$ are the angles of spin rotation in the first and correction snake magnets; $\varphi_s$ is the snake axis orientation from the radial direction and *Bdl* is the total snake integrated field.

Numerically obtained results for the modified four magnet snake are summarized in Fig. 5 , where the snake parameters are plotted against the angle of orientation of the second/third magnet $\beta$ and Table 3 shows the examples for this snake. Now we proceed further and consider the five magnet snake, where no correction magnets are required.

## 5. FIVE MAGNET SNAKE

The scheme of this snake, shown in Fig. 6, has a structure very similar to all the previously described cases. Moreover, one can find correspondence with the Steffen-Lee snake,[4] which is given by a sequence of magnets (-H, -V, mH, 2V, -mH, -V, H) and H denotes here horizontally bending magnet with vertical field. The important distinction of the five magnet snake is that the range where the snake axis orientation can be varied is not limited by orbit correction magnets. The variable parameters for this snake are orientation of the first magnet $\alpha$ and the spin rotation angles in the first and the second magnets $\psi_1, \psi_2$. The angle of the spin rotation in the third magnet is: $\psi_3 = 2\psi = 2\psi_1 \cdot \cos\alpha$.

TABLE 3: Examples of the four magnet snake with two orbit correction magnets ($l_{gap}$=0.4 m, B = 1.7 T).

| $\beta$ (deg) | $\psi_1$ (deg) | $\psi_2$ (deg) | $\psi_c$ (deg) | $\varphi_s$ (deg) | $\int B\,dl$ (T·m) |
|---|---|---|---|---|---|
| 65.39 | 81 | 194.5 | 53.54 | 101.7 | 20.05 |
| 63.12 | 84.6 | 187.1 | 49.61 | 103.1 | 19.58 |
| 61.01 | 88.2 | 182 | 46.55 | 104.5 | 19.3 |
| 60 | 90 | 180 | 45.26 | 105.3 | 19.21 |
| 58.07 | 93.6 | 177 | 43.03 | 106.8 | 19.11 |
| 56.26 | 97.2 | 175 | 41.18 | 108.4 | 19.1 |
| 55.39 | 99 | 174.3 | 40.38 | 109.2 | 19.12 |
| 54.55 | 100.8 | 173.8 | 39.64 | 110.1 | 19.15 |
| 52.94 | 104.4 | 173.2 | 38.34 | 111.8 | 19.26 |
| 51.44 | 108 | 173.3 | 37.25 | 113.5 | 19.41 |

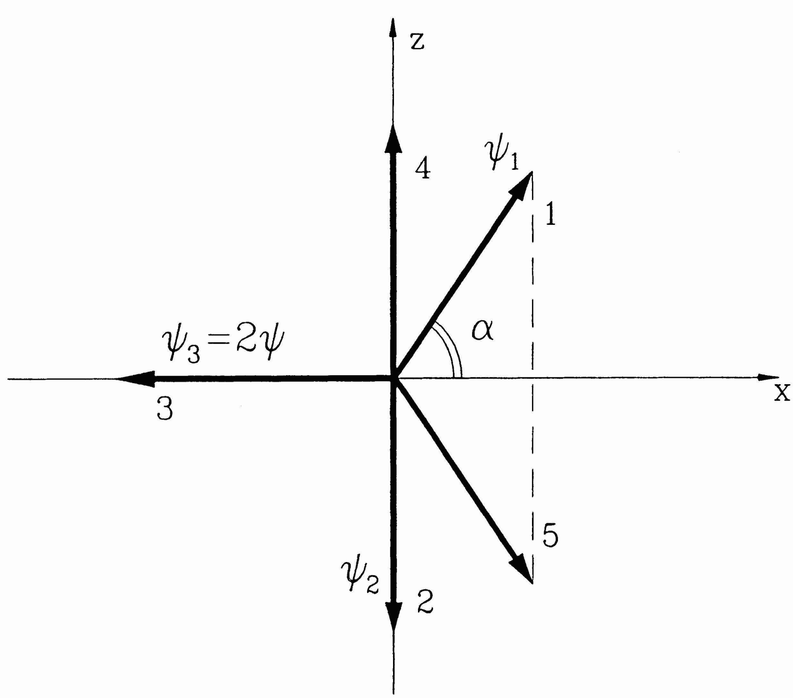


FIGURE 6: Vector diagram for the five magnet snake.

As well as for the previous snake schemes the two constraints to determine the snake magnets parameters are 180 degree spin rotation and zero orbit offset in the snake center. The given sequence of five snake magnets determines the expressions for $A$ and $\vec{a}$ in the equation of 180 degree spin rotation (12):

$$A = \cos\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2} + \sin\alpha\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) + \cos\alpha\sin\frac{\psi}{2}\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2},$$

$$\vec{a} = \begin{pmatrix} \cos\alpha\cos\frac{\psi}{2}\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} - \sin\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2} + \sin\alpha\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) \\ \sin\frac{\psi}{2}\left(\sin\alpha\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} - \cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) - \cos\alpha\cos\frac{\psi}{2}\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \\ \cos\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2} - \sin\alpha\cos\frac{\psi_2}{2}\sin\frac{\psi_1}{2}\right) - \cos\alpha\sin\frac{\psi}{2}\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \end{pmatrix}. \tag{15}$$

Following the procedure of excluding two of the three snake variable parameters one can finally obtain how the snake axis can be varied using the last parameter. Table 4 gives numerical examples of the snake configuration and Fig. 7 shows the variation of the snake magnet parameters via the snake axis orientation angle. It is important for the snake scheme that the snake axis can be oriented far from the longitudinal direction. This allow one to match the snakes axes when several snakes are required for an accelerator. For instance, if an accelerator needs two snakes then their axes should be orthogonal and one can use the five magnet configuration with 45 degree axis as the first snake and as the second one the same snake with a reversed magnet order, so that the snake axis will be –45 degrees. The five magnet snake configuration also can be used with the schemes discussed earlier since they cover different regions of the snake axis orientation.

TABLE 4: Five magnet snake examples ($l_{\rm gap}$=0.4 m, B = 1.7 T ).

| $\alpha$ (deg) | $\psi_1$ (deg) | $\psi_2$ (deg) | $\psi$ (deg) | $\varphi_s$ (deg) | $\int B\,dl$ (T·m) |
|---|---|---|---|---|---|
| 49.82 | 101.7 | 148.3 | 65.61 | 75.28 | 19.24 |
| 47.97 | 102.6 | 144.3 | 68.69 | 71.23 | 19.23 |
| 45.79 | 104.4 | 140.3 | 72.8 | 66.02 | 19.35 |
| 43.13 | 108 | 136.9 | 78.82 | 58.48 | 19.73 |
| 41.38 | 111.6 | 135.7 | 83.74 | 52.23 | 20.17 |
| 40.12 | 115.2 | 135.8 | 88.1 | 46.49 | 20.66 |
| 39.17 | 118.8 | 136.7 | 92.1 | 41 | 21.18 |
| 38.46 | 122.4 | 138.2 | 95.84 | 35.62 | 21.72 |
| 37.94 | 126 | 140.2 | 99.38 | 30.27 | 22.28 |

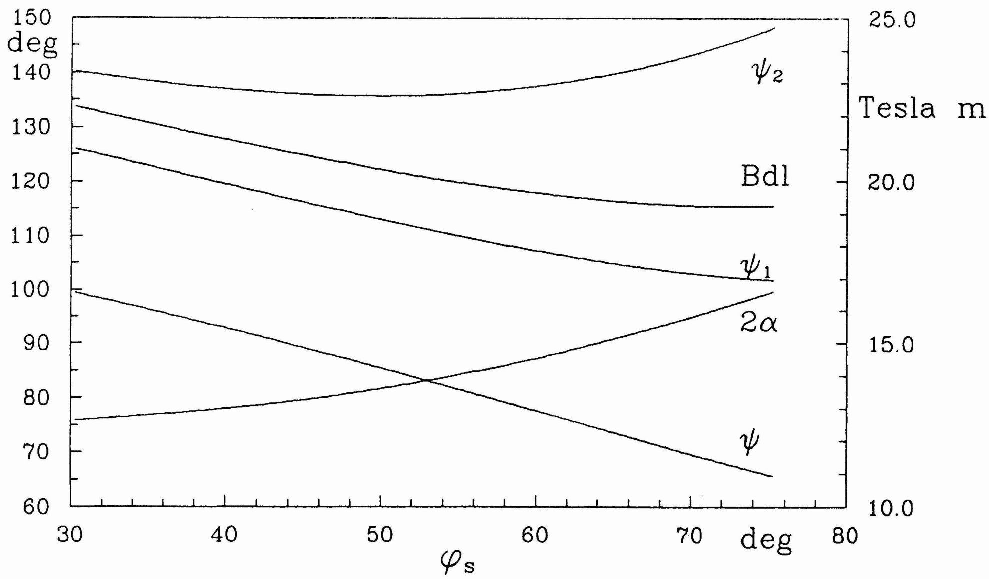


FIGURE 7: Five magnet snake parameters versus the resulting snake axis angle $\varphi_s$. Here $\psi_1$,$\psi_2$,$2\psi$ are the angles of spin rotation in the first, second and third snake magnets respectively; $\alpha$ is the field orientation in the first magnet and *Bdl* is the snake integrated field.

## 6. CONCLUSION

Let us discuss the obtained results. Even though the initial goal was to find an appropriate snake scheme with small number of magnets which can serve at high energy accelerators, the snake configurations presented here can be used over a wide energy range. For the particle energy of $\gamma = 160.9$ typical values of orbit excursion varies from 1 to 2 cm, so at the energy of $\gamma = 16.1$ it will be from 10 to 20 cm, which is appropriate for practical use. On the other hand, the main disadvantage of the three and the four magnet schemes with orbit correction is that the snake axis orientation lies within a small range around the longitudinal direction. Only in the five magnet scheme the snake axis can be as far as 60° from longitudinal direction. The five magnet snake may be a perfect choice for such accelerators as RHIC, Tevatron and SSC. The application of this snake becomes difficult at energies lower than 15 GeV, where the orbit excursion becomes significant.

All the snakes except the four magnet configuration have essentially the same field integral about 20 Tesla·m, so that if using dipoles with field strength 1.7 Tesla the snakes can fit in a 15 m straight section. For example the five magnet snake with a 46.5° snake axis has integrated field 20.7 Tesla·m and length 13.76 m. The four magnet snake, even though it has a large field integral, is still a good illustration of the possibility to make an optically transparent snake with as low as four magnets.

## ACKNOWLEDGEMENT

This research was supported by a grant from the U.S. Department of Energy.

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